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Theoretical Models of the Brans-Dicke Parameter for Time Independent Deceleration Parameters

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Abstract: **The dependence of the dimensionless Brans-Dicke (BD) coupling parameter upon time and the scalar field has been determined, for an isotropic and homogeneous space of zero curvature, by solving BD field equations and the wave equation for the scalar field. For this purpose, very simple expressions of empirical scale factors, that generate constant deceleration parameters, have been used here in two theoretical models. The characteristics of time dependence of the BD parameter, obtained from these two models, are in qualitative agreement with each other. The mathematical expressions representing the rate of change of the BD parameter with time, based on these two models, are found to have identical forms. Combining the expressions of the BD parameter obtained from these two models, a method for the determination of the present value of the equation-of-state (EoS) parameter of the cosmic fluid has been discussed. Its value is found to be consistent with the ranges of values obtained from other studies based on recent observations.**

Keywords: **Brans-Dicke Theory, Scalar Field, Empirical Scale Factor, Equation-of-State (EoS) Parameter, Constant Deceleration Parameter, Cosmology.**

1. INTRODUCTION

The information regarding several interesting phenomena in the field of cosmology, particularly the accelerating expansion of the universe, have emerged from astrophysical observations [1, 2]. The recent observations reveal that the accelerated expansion of the universe is caused by an exotic form of energy with negative pressure, referred to as dark energy (DE). This DE is one of the most important subjects of scientific investigation in physics and astronomy. To explain the late-time acceleration of the universe, lots of models have been proposed. The cosmological constant (Λ) is the simplest representative of DE [3]. But the models involving Λ are known to have cosmological constant (CC) problem (the fine-tuning problem) and the problem of cosmic coincidence [4]. To explain the accelerated expansion by solving such problems, many alternative models have been proposed either by modifying the right hand side of the Einstein field equations by incorporating specific forms of energy-momentum tensor or changing the left hand side of Einstein field equations. The first category includes the DE models of quintessence [5], k-esssence [6], the family of chaplygin gas [7], holographic [8], new agegraphic [9] dark energy etc. The second category models are based on modified gravity, such as f(R) gravity, scalar-tensor theories [10] etc. The Brans-Dicke (BD) theory of gravity is a path breaking study of scalar-tensor theory by Brans and Dicke, incorporating Mach's principle into gravity [11]. In BD theory, the space-time dynamics is represented by the metric tensor and the dynamics of gravity is represented by a scalar field. The gravitational constant in BD theory is replaced by the reciprocal of a time dependent scalar field (φ). There is a dimensionless parameter (ω) that couples this scalar field (φ) with gravity. BD theory has successfully explained the experimental findings from the solar system [12]. Brans-Dicke theory has been found to be very effective regarding the recent study of cosmic acceleration [13]. From general theory of relativity (GR), this can be obtained by letting $\omega \to \infty$ and $\varphi =$

constant[14]. The early and the late time behaviour of the universe have been very effectively solved by this theory. According to a study by Banerjee and Pavon, BD theory can potentially solve the quintessence problem [13]. The generalized BD theory is an extension of the original BD theory where the BD parameter ω is regarded as a function of the scalar field φ [15]. The generalized BD theory gives rise to a decelerating radiation model where the big-bang nucleosynthesis scenario is not adversely affected [13]. A self-interacting potential have also been introduced in its modified form. Bertolami and Martins have used this theory to explain accelerated cosmic expansion for a spatially flat model [16]. According to these theories, ω should have a low negative value, of the order of unity, to account for the accelerated expansion. This is in contradiction with the solar system experimental bound of $\omega \geq 500$. Bertolami and Martins have successfully explained the accelerated expansion with a potential φ^2 and large |ω| [16].

In the present study, the time dependence of the BD parameter (ω) has been determined from the field equations of the Brans-Dicke theory of gravity and the wave equation for the scalar field (φ). Here we have formulated two theoretical models of the BD parameter, based on two empirical expressions of the scale factor. One of them has a power-law type of dependence upon time and the other one has an exponential dependence upon time. Each of these very simple scale factors produces a time independent deceleration parameter. Although the deceleration parameter is known to be a function of time, these scale factors have been chosen for the simplicity of mathematical calculations, without much loss of generality.The scalar field, in each model, has been assumed to have a power law type of dependence upon the corresponding scale factor. The values of the constants involved in all these empirical relations have been evaluated from the field equations. The properties of time variation of the BD parameter, obtained from these two models, are found to be in qualitative agreement with each other. They are consistent with the findings of other resent studies in this regard [17, 18]. Apart from time dependence, the dependence of the BD parameter upon the scale factor and the scalar field has also been determined in the present study. Although the entire formulation is based upon a time independent equation-of-state parameter, an attempt has been made, at the end of this analysis, to determine the present value of the EoS parameter. Combining the results obtained from the two models, a mathematical exercise has been carried out for an approximate estimation of its present value. The findings of this theoretical method are in agreement with the range of possible values of this parameter, obtained from recent astrophysical observations [19, 20]. This result seems to provide justification for the choice of scale factors that generate time independent deceleration parameters.

2. SOLUTION OF FIELD EQUATIONS

In the generalized Brans-Dicke theory, the action is expressed as [11],

$$S = \int d^4x\sqrt{-g}\left(\frac{\varphi R}{16\pi G} + \frac{\omega(\varphi)}{\varphi} g^{\mu\nu}\partial_\mu\varphi\, \partial_\nu\varphi + L_m\right) \tag{01}$$

Here, g is the determinant of the tensor metric $g^{\mu\nu}$, R is the Ricci scalar, L_m is the Lagrangian of matter, φ is the Brans-Dicke scalar field and ω is a dimensionless Brans-Dicke parameter. In generalized BD theory, this coupling parameter ω is considered to be a function of φ.

The line element for a non-flat universe, in the framework of the Friedmann-Robertson-Walker (FRW) cosmology, can be expressed in polar coordinates as,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\, d\theta^2 + r^2 sin^2\theta\, d\xi^2\right] \tag{02}$$

Here $a(t)$ is the scale factor of the universe, t represents the cosmic time, r is the radial component, k is the curvature parameter and θ and ξ are the two polar coordinates.

The gravitational field equations of Brans-Dicke theory, for a universe filled with a perfect fluid and described by FRW space-time with scale factor $a(t)$ and spatial curvature k, are given by,

$$3\frac{\dot{a}^2+k}{a^2} + 3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} - \frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2} = \frac{\rho}{\varphi} \tag{03}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2+k}{a^2} + \frac{\omega(\varphi)}{2}\frac{\dot{\varphi}^2}{\varphi^2} + 2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} + \frac{\ddot{\varphi}}{\varphi} = -\frac{\gamma\rho}{\varphi} \tag{04}$$

The wave equation for the scalar field (φ), in generalized Brans-Dicke theory of gravity, where ω is a time dependent parameter, is expressed as,

$$\frac{\ddot{\varphi}}{\varphi} + 3\frac{\dot{a}\dot{\varphi}}{a\varphi} = \frac{\rho}{\varphi}\frac{(1-3\gamma)}{2\omega+3} - \frac{\dot{\varphi}}{\varphi}\frac{\dot{\omega}}{2\omega+3} \qquad (05)$$

Combining equations (3) and (4), and taking $k = 0$ (for flat space), one obtains,

$$2\frac{\ddot{a}}{a} + 4\frac{\dot{a}^2}{a^2} + 5\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi} + \frac{\ddot{\varphi}}{\varphi} = \frac{\rho(1-\gamma)}{\varphi} \qquad (06)$$

The energy conservation equation for the cosmic fluid is written as,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(1+\gamma)\rho = 0 \qquad (07)$$

In the above expressions, $\gamma(\equiv P/\rho)$ is the *equation of state* (EoS) parameter for the cosmic fluid, with specific values for different cosmological periods. The values of γ are, 1 (massless scalar field dominated era), $1/3$ (radiation dominated era), 0 (matter dominated era), -1 (vacuum energy dominated era).

Considering this EoS parameter γ to be independent of time, the solution of equation (7) is obtained as,

$$\rho = \rho_0 a^{-3(1+\gamma)} \qquad (08)$$

Combining equation (3) with (8) one gets,

$$\omega = \frac{3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{\varphi}}{a\varphi} - \frac{\rho_0 a^{-3(1+\gamma)}}{\varphi}}{\frac{1}{2}\frac{\dot{\varphi}^2}{\varphi^2}} \qquad (09)$$

Combining equations (5) and (8) one gets,

$$\omega = \frac{1}{2}\left[\frac{\rho_0 a^{-3(1+\gamma)}(1-3\gamma) - \dot{\omega}\dot{\varphi}}{\ddot{\varphi} + 3\dot{a}\dot{\varphi}/a} - 3\right] \qquad (10)$$

To determine the time dependence of ω from the above equations we have used two sets of empirical expressions for the scale factor (a) and the scalar field (φ) in the following two models. In these two models, the scale factors are such that one obtains constant deceleration parameters from them. The scalar field in each model has been chosen to have a power-law dependence on the corresponding scale factor.

The values of different cosmological parameters used in this article given below:

$$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18} sec^{-1},\ q_0 = -0.55,\ \ \rho_0 = 2.83 \times 10^{-27} Kg\ m^{-3}$$

$$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kg\ s^2 m^{-3}\ ,\ t_0 =\ 1.4 \times 10^{10}\ Years = 4.42 \times 10^{17}\ s$$

MODEL – 1

The empirical expressions for the scale factor and the scalar field chosen for model-1 are given by,

$$a = a_0\left(\frac{t}{t_0}\right)^{\alpha} \qquad (11)$$

$$\varphi = \varphi_0\left(\frac{a}{a_0}\right)^{n} = \varphi_0\left(\frac{t}{t_0}\right)^{n\alpha} \qquad (12)$$

The scale factor (in eqn. 11) produces a constant deceleration parameter, which is given by,

$q_0 = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{\alpha} - 1$. Thus α can be written as,

$$\alpha = \frac{1}{1+q_0} \qquad (13)$$

Using equation (11) in (8) we get,

$$\rho = \rho_0 {a_0}^{-3(1+\gamma)}(t/t_0)^{-3\alpha(1+\gamma)} \qquad (14)$$

Using equations (11), (12) and (14) in equation (6) and writing all parameter values for the present epoch ($t = t_0$), one obtains,

$$\left(\frac{\alpha^2}{{t_0}^2}\right)n^2+\left(\frac{5\alpha^2}{{t_0}^2}-\frac{\alpha}{{t_0}^2}\right)n+\left\{\frac{6\alpha^2}{{t_0}^2}-\frac{2\alpha}{{t_0}^2}-\frac{\rho_0}{\varphi_0}(1-\gamma_0)\right\}=0 \tag{15}$$

Equation (15) is quadratic in n. Two solutions for n are given by,

$$n_{\pm}=-\frac{5}{2}+\frac{1}{2\alpha}\pm\frac{1}{2}\sqrt{\left(1-\frac{1}{\alpha}\right)^2+\frac{4\rho_0{t_0}^2}{\alpha^2\varphi_0}(1-\gamma_0)} \tag{16}$$

For the present matter dominated era one may take $\gamma_0=0$.

One thereby obtains its two values as $n_{\pm}=-1.987,-2.563$.

Using equations (11) and (12) in equation (9), one gets,

$$\omega=\frac{2}{n^2\alpha^2}\left[3\alpha^2(1+n)-\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0}\left(\frac{t}{t_0}\right)^{2-3\alpha(1+\gamma)-n\alpha}\right] \tag{17}$$

The time derivative of the BD parameter from equation (17) is given by,

$$\dot{\omega}=\frac{\rho_0 t_0 {a_0}^{-3(1+\gamma)}\{2\alpha(3+3\gamma+n)-4\}}{\varphi_0 n^2\alpha^2}\left(\frac{t}{t_0}\right)^{1-3\alpha(1+\gamma)-n\alpha} \tag{18}$$

Using equations (11) and (12) in equation (10), one gets,

$$\omega=-\frac{3}{2}+\frac{(0.5-1.5\gamma)\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0 n\alpha(3\alpha+n\alpha-1)}\left(\frac{t}{t_0}\right)^{2-3\alpha(1+\gamma)-n\alpha}-\frac{0.5}{3\alpha+n\alpha-1}\dot{\omega}t \tag{19}$$

Substituting for $\dot{\omega}$ in equation (19) from equation (18) one obtains,

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}(4-6\alpha-6\alpha\gamma-3n\alpha\gamma-n\alpha)}{\varphi_0 n^2\alpha^2(3\alpha+n\alpha-1)}\left(\frac{t}{t_0}\right)^{2-3\alpha(1+\gamma)-n\alpha} \tag{20}$$

Combining equation (11) with (20), one gets the following expression (eqn. 21) for ω as a function of the scale factor (a).

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}(4-6\alpha-6\alpha\gamma-3n\alpha\gamma-n\alpha)}{\varphi_0 n^2\alpha^2(3\alpha+n\alpha-1)}\left(\frac{a}{a_0}\right)^{\frac{2-3\alpha(1+\gamma)-n\alpha}{\alpha}} \tag{21}$$

Combining equation (12) with (20), one gets the following expression (eqn. 22) for ω as a function of the scalar field (φ).

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}(4-6\alpha-6\alpha\gamma-3n\alpha\gamma-n\alpha)}{\varphi_0 n^2\alpha^2(3\alpha+n\alpha-1)}\left(\frac{\varphi}{\varphi_0}\right)^{\frac{2-3\alpha(1+\gamma)-n\alpha}{n\alpha}} \tag{22}$$

MODEL – 2

The empirical expressions for the scale factor and the scalar field assumed for model-2 are given by,

$$a=a_0 Exp\left[\beta\left(\frac{t}{t_0}-1\right)\right] \tag{23}$$

$$\varphi=\varphi_0\left(\frac{a}{a_0}\right)^m=\varphi_0 Exp\left[m\beta\left(\frac{t}{t_0}-1\right)\right] \tag{24}$$

The scale factor (in eqn. 23) produces a constant deceleration parameter, which is given by,

$q_0=-\frac{\ddot{a}a}{\dot{a}^2}=-1$. Here, the Hubble parameter is $H=\frac{\dot{a}}{a}=\frac{\beta}{t_0}$. Since it is independent of time, it must be equal to the present value of Hubble parameter (H_0). Thus, the value of β can be expressed as,

$$\beta=H_0 t_0 \tag{25}$$

Using equation (23) in (8) we get,

$$\rho=\rho_0{a_0}^{-3(1+\gamma)}Exp\left[-3\beta(1+\gamma)\left(\frac{t}{t_0}-1\right)\right] \tag{26}$$

Using equations (23), (24) and (26) in equation (6) and writing all parameter values for the present epoch ($t=t_0$), one obtains,

$$\left(\frac{\beta^2}{{t_0}^2}\right)m^2+\left(\frac{5\beta^2}{{t_0}^2}\right)m+\left\{\frac{6\beta^2}{{t_0}^2}-\frac{\rho_0}{\varphi_0}(1-\gamma_0)\right\}=0 \tag{27}$$

Equation (27) is quadratic in m. Its two solutions are given by,

$$m_{\pm}=-\frac{5}{2}\pm\frac{1}{2}\sqrt{1+\frac{4\rho_0{t_0}^2}{\beta^2\varphi_0}(1-\gamma_0)} \tag{28}$$

For the present matter dominated era one may take $\gamma_0=0$.

One thereby obtains its two values as $m_{\pm}=-1.966,-3.034$.

Using equations (23) and (24) in equation (9), one gets,

$$\omega=\frac{2}{m^2\beta^2}\left[3\beta^2(1+m)-\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0}Exp\left\{(-3\beta-3\beta\gamma-m\beta)\left(\frac{t}{t_0}-1\right)\right\}\right] \tag{29}$$

The time derivative of the BD parameter from equation (29) is given by,

$$\dot{\omega}=\frac{2\rho_0t_0{a_0}^{-3(1+\gamma)}(3+3\gamma+m)}{\beta\varphi_0m^2}Exp\left\{(-3\beta-3\beta\gamma-m\beta)\left(\frac{t}{t_0}-1\right)\right\} \tag{30}$$

Using equations (23) and (24) in equation (10), one gets,

$$\omega=-\frac{3}{2}+\frac{(1-3\gamma)\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}\mathrm{Exp}\left\{(-3\beta-3\beta\gamma-m\beta)\left(\frac{t}{t_0}-1\right)\right\}}{2\beta^2\varphi_0m(3+m)}-\frac{t_0\beta m}{2\beta^2m(3+m)}\dot{\omega} \tag{31}$$

Substituting for $\dot{\omega}$ in equation (31), from equation (30), one obtains,

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{(-6-6\gamma-3m\gamma-m)\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0m^2\beta^2(3+m)}Exp\left\{(-3-3\gamma-m)\beta\left(\frac{t}{t_0}-1\right)\right\} \tag{32}$$

Combining equation (23) with (32), one gets the following expression (eqn. 33) for ω as a function of the scale factor (a).

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{(-6-6\gamma-3m\gamma-m)\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0m^2\beta^2(3+m)}\left(\frac{a}{a_0}\right)^{-3-3\gamma-m} \tag{33}$$

Combining equation (24) with (32), one gets the following expression (eqn. 34) for ω as a function of the scalar field (φ).

$$\omega=-\frac{3}{2}+\frac{1}{2}\frac{(-6-6\gamma-3m\gamma-m)\rho_0{t_0}^2{a_0}^{-3(1+\gamma)}}{\varphi_0m^2\beta^2(3+m)}\left(\frac{\varphi}{\varphi_0}\right)^{\frac{-3\beta-3\beta\gamma-m\beta}{m}} \tag{34}$$

3. RESULTS AND DISCUSSIONS

The values of the BD parameter at the present epoch ($t=t_0$), from model-1, for two values of n (i.e. $n_{\pm}$, as in eqn. 16) are given by,

$$\omega_{0\pm}=-\frac{3}{2}+\frac{1}{2}\frac{\rho_0{t_0}^2{a_0}^{-3(1+\gamma_0)}(4-6\alpha-6\alpha\gamma_0-3n_{\pm}\alpha\gamma_0-n_{\pm}\alpha)}{\varphi_0{n_{\pm}}^2\alpha^2(3\alpha+n_{\pm}\alpha-1)} \tag{35}$$

For the present matter dominated era we have taken $\gamma_0=0$.

Using the values of the parameter n, i.e., $n_+=-1.987$ and $n_-=-2.563$,

one gets $\omega_{0\pm}=-1.503,-1.430$.

The values of the BD parameter at the present epoch ($t=t_0$), from model-2, for two values of m (i.e. $m_{\pm}$, as in eqn. 28) are given by,

$$\omega_{0\pm}=-\frac{3}{2}+\frac{1}{2}\frac{(-6-6\gamma-3m_{\pm}\gamma_0-m_{\pm})\rho_0{t_0}^2{a_0}^{-3(1+\gamma_0)}}{\varphi_0{m_{\pm}}^2\beta^2(3+m_{\pm})} \tag{36}$$

For the present matter dominated era we have taken $\gamma_0=0$.

Using the values of the parameter m, i.e., $m_+=-1.966$ and $m_-=-3.034$, one gets $\omega_{0\pm} = -1.518, -1.333$.

According to a study by Banerjee and Pavon, the value of ω must be greater than $-3/2$ [13]. This requirement is satisfied if we take $n = n_-$ in model-1 and $m = m_-$ in model-2. Therefore, to determine the time dependence of ω, one should use these values of n and m in the equations (20) and (32), for model-1 and model-2 respectively.

Using all parameters values and taking $n = n_-$, the expression of the BD parameter ω (eqn. 20), based on model-1, takes the following form.

$$\omega = -1.5 + (0.071 - 0.073\gamma)\left(\frac{t}{t_0}\right)^{1.029-6.667\gamma} \tag{37}$$

From equation (37), the time derivative of ω is given by,

$$\frac{d\omega}{dt} = \frac{(0.073-0.548\gamma+0.487\gamma^2)\left(\frac{t}{t_0}\right)^{1.029-6.667g}}{t} \tag{38}$$

From equation (38), the value of $\frac{d\omega}{dt}$ at the present epoch $(t = t_0)$ is obtained as,

$$\left(\frac{d\omega}{dt}\right)_{t=t_0} = \frac{0.073-0.548\gamma_0+0.487\gamma_0{}^2}{t_0} \tag{39}$$

Using all parameters values and taking $m = m_-$, the expression for the BD parameter ω (eqn. 32), based on model-2, takes the following form.

$$\omega = -1.5 + (0.168 - 0.175\gamma)Exp\left[(0.035 - 3.045\gamma)\left(\frac{t}{t_0} - 1\right)\right] \tag{40}$$

From equation (40), the time derivative of ω is given by,

$$\frac{d\omega}{dt} = \frac{(0.006-0.518\gamma+0.533\gamma^2)\,Exp\left[(0.035-3.045\gamma)\left(\frac{t}{t_0}-1\right)\right]}{t_0} \tag{41}$$

From equation (41), the value of $\frac{d\omega}{dt}$ at the present epoch $(t = t_0)$ is obtained as,

$$\left(\frac{d\omega}{dt}\right)_{t=t_0} = \frac{0.006-0.518\gamma_0+0.533\gamma_0{}^2}{t_0} \tag{42}$$

The expressions of (39) and (42), based on model-1 and model-2 respectively, are identical in form. Here, t_0 denotes the age of the universe, with a very large value of 1.4×10^{10} years or 4.42×10^{17} seconds. Due to this extremely large value, $\left|\left(\frac{d\omega}{dt}\right)_{t=t_0}\right|$ would be very small. The sign of $\left(\frac{d\omega}{dt}\right)_{t=t_0}$ is determined by the value of γ of the cosmic fluid at the present epoch. This value has been taken to be zero for the present era of matter dominated universe (pressureless dust) by Sahoo and many researchers [17]. Setting the right hand sides of equations (39) and (42) equal to each other, one gets,

$$0.046\gamma_0{}^2 + 0.030\gamma_0 - 0.067 = 0 \tag{43}$$

Equation (43) is quadratic in γ_0, leading to the following solutions.

$$(\gamma_0)_\pm = \frac{-0.030\pm\sqrt{(0.030)^2-4\times0.046\times(-0.067)}}{2\times0.046} = 0.924, -1.576 \tag{44}$$

The negative value of γ_0, in equation (44), is consistent with recent observations [19, 20].

The results of equation (44) have their origins in equations (37) and (40). We took $\gamma_0 = 0$ to obtain the values of n and m, used for evaluating the constants in the equations (37) and (40).

We should therefore discuss a more generalized method to estimate the value of the EoS (equation of state) parameter γ_0.

According to this new method, one may determine $\left(\frac{d\omega}{dt}\right)_{t=t_0}$ for model-1 and model-2, from equations (20) and (32) respectively. Since they represent the time dependence of the same entity (ω) at the present epoch, their values are likely to be equal to each other. Setting them equal to each other and taking $m = n$, one gets,

$$A\gamma_0{}^2 + B\gamma_0 + C = 0 \tag{45}$$

In equation (45),

$$A = 18\alpha^2\beta n^2 - 54\alpha^3\beta n^2 + 54\alpha^2\beta^2n^2 + 9\alpha^2\beta n^3 - 45\alpha^3\beta n^3 + 45\alpha^2\beta^2n^3 - 9\alpha^3\beta n^4 + 9\alpha^2\beta^2n^4$$

$$B = 36\alpha^2\beta n^2 - 108\alpha^3\beta n^2 - 72\alpha\beta^2n^2 + 108\alpha^2\beta^2n^2 + 18\alpha^2\beta n^3 - 90\alpha^3\beta n^3 - 42\alpha\beta^2n^3 + 90\alpha^2\beta^2n^3 + 3\alpha^2\beta n^4 - 27\alpha^3\beta n^4 - 6\alpha\beta^2n^4 + 27\alpha^2\beta^2n^4 - 3\alpha^3\beta n^5 + 3\alpha^2\beta^2n^5$$

$$C = 18\alpha^2\beta n^2 - 54\alpha^3\beta n^2 + 24\beta^2n^2 - 72\alpha\beta^2n^2 + 54\alpha^2\beta^2n^2 + 9\alpha^2\beta n^3 - 45\alpha^3\beta n^3 + 8\beta^2n^3 - 42\alpha\beta^2n^3 + 45\alpha^2\beta^2n^3 + \alpha^2\beta n^4 - 12\alpha^3\beta n^4 - 6\alpha\beta^2n^4 + 12\alpha^2\beta^2n^4 - \alpha^3\beta n^5 + \alpha^2\beta^2n^5$$

Equation (45) is quadratic in γ_0.

Here, the values of α and β can be obtained from equations (13) and (25) respectively.

The reason for taking $n = m$ is due to the closeness of their values, obtained from equations (16) and (28). Since the expressions of φ in equations (12) and (24) are not the exact solutions of the field equations, the values of n and m can also be other than those given by equations (16) and (28) respectively. The only restriction imposed upon their choice of values is the requirement that $-1.5 < \omega_0 < 0$, as obtained by Banerjee and Pavon [13].We have only negative values n and m in equations (16) and (28). Negative values of n and m indicate that the scalar field (φ) decreases with time, as evident from their defining equations (eqns. 12, 24). Consequently, the gravitational constant $G \equiv 1/\varphi$ increases with time. There are other recent studies showing a decrease of φ (an increase of G) with time [17, 18].

The values of $\gamma_0$obtained from equation (45) are given by,

$$\gamma_{0_\pm} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{46}$$

For $-2 < n < 0$, one finds from equation (46) that $\gamma_{0_}$vary approximately in the range of $-1.9 \leq \gamma_{0\_} - 0.6$, which is consistent with the recent experimental findings regarding the EoS parameter [19, 20]. The value of γ_{0_+} is positive for this range of n values. These positive values are not at all consistent with the range of values for γ_0 obtained from scientific studies based on recent astrophysical observations [19, 20]

4. CONCLUSIONS

In the present study we have used scale factors that generate time independent deceleration parameters. As per scientific studies based on recent observations, the deceleration parameter has changed its sign with time, from positive to negative, indicating a change of phase of the cosmic acceleration from deceleration to acceleration [18]. The scale factor for our model-1 is identical in form to that used by Sahoo et al. [17], although our method of deriving the time dependent expression for ω is different. These scale factors can be used for the simplicity of mathematical analysis in a theoretical study. They are required to satisfy a condition that the constant deceleration parameter obtained from them is negative and $q_0 \geq -1$. The time dependence of ω, for different eras of the expanding universe (characterized by different values of the EoS parameter γ), is found to be qualitatively the same for these two models. The choice of the scalar field in both models is such that it has the same functional dependence on the corresponding scale factor. According to generalized Brans-Dicke theory, the BD parameter (ω) is a function of the scalar field (φ). In both models, the choices of empirical relations regarding the scale factors and the scalar fields have allowed us to formulate expressions of ω as functions of φ. Using the results of these two models together, the range of values for γ_0 (EoS parameter at the present epoch) has been estimated. One could also have taken $n \neq m$ for a more generalized form, leading to a larger mathematical expression which might be untenable. An improvement over this method of determination of the EoS parameter can be made by using scale factors that are obtained as solutions of the field equations for different logical choices of the scalar field.

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