Putnam and contemporary fictionalism*

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ABSTRACT: Putnam rejects having argued in the terms of the argument known in the literature as "the Quine-Putnam indispensability argument". He considers that mathematics contribution to physics does not have to be interpreted in platonist terms but in his favorite modal variety (Putnam 1975; Putnam 2012). The purpose of this paper is to consider Putnam's acknowledged argument and philosophical position against contemporary so called in the literature 'fictionalist' views about applied mathematics. The conclusion will be that the account of the applicability of mathematics that stems from Putnam's acknowledged argument can be assimilated to so-called 'fictionalist' views about applied mathematics.

Keywords: Putnam, indispensability, fictionalism, Yablo, Field, ontology.

RESUMEN: Putnam rechaza haber suscrito el argumento conocido como «argumento de la indispensabilidad de Quine y Putnam». Considera que la contribución de la matemática a la física no tiene por qué interpretarse en términos platónicos sino en términos modales (Putnam 1975; Putnam 2012). En este trabajo se contrapone el argumento que Putnam sostiene haber defendido—y la posición filosófica que lo sustenta—, a ciertas concepciones ficcionalistas contemporáneas de la matemática aplicada. De ese análisis se concluye que la concepción de la aplicabilidad de la matemática que subyace a la propuesta de Putnam coincide en muchos aspectos con las concepciones ficcionalistas analizadas.

Palabras clave: Putnam, indispensabilidad, ficcionalismo, Yablo, Field, ontología.

Introduction

Traditional interpretations of Putnam's indispensability argument endorse him with a defense of the so-called Quine-Putnam indispensability argument,[1] an argument for ontological platonism in mathematics. This interpretation has been disputed in the literature

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[1] For instance, Colyvan 2001; Field 2001, 329.

and by Putnam himself.[2] Putnam rejects that the only way of arguing for the objectivity of mathematics—the truth of mathematical statements—is by committing to ontological platonism. He contends that there is an alternative, namely, not to take mathematics at face value. The aim of the paper is to see, in the light of his acknowledged indispensability argument, where he stands in relation to certain contemporary fictionalist positions.

The worry presented above is not an unheard of one. Field (2001, 315 and ff.) analyses Putnam's position, among others, in relation to the problem of accounting for the objectivity of *pure* mathematics. As a result, he establishes a link between several conceptions of pure mathematics that acknowledge limits to its objectivity while postulating one kind of ontological anti-platonism or other. In particular, all the following—according to Field's analysis—, go together: the intuitionist who considers mathematical objects are mind-dependent;[3] she who considers them as dependent on the mathematical community;[4] the ones defending there are multiple set-theoretic universes (Balaguer 1995, 1998); those contending that there is a unique universe of sets while being convinced by Putnam's model-theoretic argument (1980);[5] and fictionalists like himself. That is so because all of them agree that—in the case of pure mathematics—there is no specific mathematical objectivity, "understood as objectivity in the choice of axioms," beyond logical objectivity, "completely objective standards of mathematical proof" (Field 2001, 316-317).

Field (2001, 329) ends his analysis considering Putnam's position (1971) in relation to applied mathematics to endorse him with a defense of the so-called Quine-Putnam indispensability argument. Field claims that, according to Putnam, the only way of accounting for the applicability of mathematics—"its utility in science and metalogic" (Field 2001, 328)—is that mathematics is true. Since he considers that Putnam's modal paraphrase of mathematics cannot be used to explain mathematical applicability to contingent disciplines such as physics (Field 1989/91, 252-69), Field concludes that Putnam is arguing for ontological platonism. Putnam (2012), of course, disagrees.

The purpose of this paper is not to assess Field or Putnam's arguments in relation to the possibility of a paraphrase of pure mathematics. Rather, the objective is to show—in the light of Putnam's preferred paraphrase of classical mathematics (Hellman's 1989 according to Putnam 2012, 182-3, ft.5)—on one hand, that many of the arguments that Putnam (2012) waves against other authors apply to his proposal as well. On the other, we aim to maintain that, in spite of the differences in the particular strategies that Field (1980), Yablo (2005; 2013) and Putnam advocate in order to account for the applicability of mathematics, there are also enough coincidences as to contend that their views are similar and compatible with a certain conception of abstract objects.

The paper starts characterizing fictionalism and Putnam's acknowledged argument for the truth of mathematics and his objections to other proposals in order to characterize the kind of view about the applicability of mathematics that stems from it. Then, we introduce the details of Hellman's reconstruction of mathematics and see that Putnam's concerns against other positions apply to the reinterpretations provided by Hellman, Field, and Yablo.

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[2] Liggins (2008); Bueno (2013); Putnam (2012).

[3] He mentions Dummett (1959).

[4] For a contemporary view of the kind see Feferman (2014, 76).

[5] See also Field (2001, 323).

1. *Putnam's acknowledged version of the indispensability argument versus Field's and Yablo's fictionalisms*

Putnam's acknowledged (non-Quinean) indispensability argument (2012, 182-3) has been reconstructed by Marcus (2017) as Putnam's success argument for mathematics:

Argument 1:

1. Mathematics succeeds as the language of science.
2. There must be a reason for the success of mathematics as the language of science.
3. No positions other than realism in mathematics provide a reason.

Therefore, realism in mathematics must be correct.

There is a second part of the argument, or a second argument, whose conclusion is that the truth of mathematics does not commit us to the existence of mathematical objects:

Argument 2:

1. We should believe that mathematics is true (the conclusion of ARGUMENT 1 above).
2. If mathematics is true in the real world, then we should believe that there are mathematical entities if and only if there are not equivalent descriptions avoiding the commitment to the existence of mathematical entities.
3. Mathematics can be paraphrased in modal terms (there are equivalent descriptions avoiding the commitment to the existence of mathematical entities).
4. We should believe that there are mathematical entities if and only if there are not equivalent descriptions avoiding the commitment to the existence of mathematical entities (MP 1,2).
5. We should not believe there are mathematical objects. (E$\leftrightarrow$, Modus Tollens 3 and 4).

Hence, Mathematics is true though there are not mathematical objects. (I& 1, 5)

In short, according to Putnam, mathematics is not to be taken at face value since there are equivalent descriptions, in particular, his preferred modal paraphrase, Hellman's. Since the modal paraphrase does not commit us to the existence of mathematical entities, the truth of mathematics is independent of the existence of mathematical objects.

Argument 1 above coincides with Balaguer's (2015) reconstruction of what he takes to be Quine's version of the indispensability argument. Balaguer refers to the latter as "the most important and widely discussed argument against fictionalism:"

> "(i) mathematical sentences form an indispensable part of our empirical theories of the physical world—i.e., our theories of physics, chemistry, and so on; (ii) we have good reasons for thinking that these empirical theories are true, i.e., that they give us accurate pictures of the world; therefore, (iii) we have good reasons to think that our mathematical sentences are true and, hence, that fictionalism is false." (2015, §2.1)

Balaguer (2015, Introduction) characterizes mathematical fictionalism as the view that (a) our mathematical sentences and theories do purport to be about abstract mathematical objects, as platonism suggests, but (b) there are no such things as abstract objects, and so (c) our mathematical theories are not true.

The sort of explanation searched for by the fictionalist pursues the idea that mathematical entities are fictions, they do not exist in the real world, and hence our mathematical theories are not true in the real world, though they can be true according to the fiction. For Putnam, "[t]he *internal success and coherence of mathematics* is evidence that it is true under *some* interpretation" (2012, 182), this is why both Putnam and the fictionalist agree in relation to the status of *pure* mathematics as we mentioned above Field pointed out.

Disagreement between them has to do with their accounts of applied mathematics; in particular, they differ about premise 3 in argument 1 above. While the fictionalist considers that the truth of applied mathematical sentences conveys platonism, Putnam (1967; Hellman 1989) embraces the possibility of a reinterpretation of mathematics that is compatible with its truth and does not commit us to platonism. Balaguer (2015) presents the main argument for fictionalism—an argument he describes as "trying to eliminate all of the alternatives to fictionalism"—and he introduces Putnam's position as arguing *against* its first premise, namely, "Mathematical sentences should be read at face value." As a result, while Putnam defends that only the truth of mathematics can explain its success, fictionalists champion other elucidations.

Balaguer introduces the two fictionalist authors we are interested in as authors that try to argue back against the indispensability argument. Field is the champion of the nominalization response, the so-called 'hard road' (Colyvan 2010), while Yablo advocates a no-nominalization strategy (the easy road).

Field (1980) rejects that mathematics is indispensable for science; his strategy to prove that mathematics is dispensable consists in providing a paraphrase of applied mathematical sentences in which appropriate nominalistic subrogates replace references to mathematical entities. In particular, he provides a reconstruction of Newtonian physics that does not use mathematical terms. He contends, as it is well known, that mathematics applies because it is consistent and conservative over natural science, where being conservative means "if you take any body of nominalistically stated assertions N, and supplement it with a mathematical theory S, you don't get any nominalistically statable conclusions that you wouldn't get from N alone." (Field 1980, 9) Hence, Field's paraphrase, if successful, would explain why mathematics is applicable in a substantial way, namely, in terms of its being conservative.

Yablo considers there is no need for a paraphrase of mathematics, since,

> [W]hether A (a sentence, a theory) has truth in it ought not to depend on whether the language happens to contain a sentence B that drops what is false in A and retains the rest. The unavailability of such a sentence could be the reason we are using A in the first place; we don't know how to put the truth it contains into words. (Yablo 2014, 48)

Contrary to Putnam, he thinks that mathematics does not have to be true in order for the nominalistic part of the theory to be true, and contrary to Field and Putnam he contends that there might be no paraphrase and we might not need it. He claims that the problem is to find an account of the semantics of the contents of a scientific theory that distinguishes between the true contents and the false contents of the theory. The philosopher explains why the mathematics is useful by explaining how physical propositions expressed by sentences that combine mathematical and physical terms can be taken to be t*rue about the nominalistic content they express*. Hence, a statement N in a scientific theory that mixes mathematical terms and non-mathematical terms is *partiall*y true because the nominalistic content of the statement N is true in the actual world. There will be alternative worlds in which the whole theory is true (worlds in which there are mathematical objects) that are equivalent to the actual world in relation to the nominalistic content of the theory. A is partly true if it includes a truth, possibly a propositional truth not expressed by any readily available sentence.[6]

In any case, Putnam and the fictionalists considered in this paper agree that (a) our mathematical sentences and theories purport to be about abstract entities; (b) there are no such things as abstract objects; (c) mathematics is not to be taken at face value; (d) any alternative interpretation must allow for enough mathematics for applications, must be compatible with scientific realism.

We will argue, we are in front of different strategies to show why mathematics is indispensable for science (explain the applicability of mathematics) that share relevant features. In particular, they commit to similar assumptions and face similar problems when answering Putnam's worries about different accounts of the applicability of mathematics. Besides, they are compatible with a certain metaphysical view about mathematical entities, also in the case of applied mathematics. Hence, the fictionalist should be happy if Putnam's preferred paraphrase succeeded, and so should be Putnam if either Field's or Yablo's account of the applicability of mathematics did.

2. *Putnam's concerns about the applicability of mathematics*

When Putnam argues against several positions about the applicability of mathematics in his 2012, he gives different reasons that tell about the issues that concern him and about his views in relation to them.

2.1. Appropriate semantics

One worry has to do with the appropriate semantics for mathematical statements. In particular, Putnam considers whether intuitionist semantics is compatible with the use of mathematics in applications. His verdict is clear (Putnam 1975, 75; 2012, 189-201): intuitionist semantics does not allow (in all cases) for something to be true beyond what is provable and, as a result, it is not compatible with scientific realism. To develop her task, the scientist needs the difference between what is the case and what she knows.[7] If mathematics is interpreted intuitionistically, the distinction is not available. For example, it is sometimes the case that the existence of a given mathematical solution for a differential equation cannot be assumed to be there, given the intuitionistic semantics for mathematical statements. Hence, being provable is not an adequate surrogate for being true in mathematical applications. To satisfy this requirement, the semantics for mathematics has to be a realist one,

[6] I am paraphrasing Yablo (2012: 1018).
[7] See Putnam (1982).

where 'realist' is meant in Dummett's sense: every statement applied in mathematics has to be true or false independently of what we know or may know.

2.2. How much mathematics is indispensable for science?

A second concern has to do with how much mathematics is indispensable for science. Putnam argues against the predicativist claiming that she has to face the same difficulty as Wittgenstein's finitism.[8] The problem is that the predicativist interpretation, even if, as Feferman (1998) has argued and Putnam (2012, 200-1) acknowledges, allows for enough mathematics for applied physics, it does not allow for quantification over the whole set of real numbers. The issue is interesting in itself, and particularly relevant both to any position that assumes the indispensability argument in Quine's version, and to positions that do not take mathematics at face value and advocate a reinterpretation of mathematical statements that mutilates classical mathematics. Hence, it is relevant, at least, to the positions he explicitly considers, namely, the intuitionist, the predicativist, the nominalist and to Field's and Yablo's since their reinterpretations of mathematics have to allow for enough mathematics for science.

Since Putnam rejects Quine's standard version of the argument while he advocates Hellman's modal-structuralist reconstruction that includes set theory (Hellman 1989), his preferred reading of mathematics does not mutilate classical mathematics. Hence, while he acknowledges that the problem is interesting in itself his position is not affected by it. On the contrary, he contends this is an advantage of his proposal.

2.3. Rejection of instrumentalism

Finally, another issue we have already mentioned and that Putnam emphasizes (2012, 197-8) is that mathematical instrumentalism cannot possibly be right: the truth of mathematics is the only explanation that does not make of the applicability of mathematics in physics a miracle. The fact that mathematics leads the scientist to successful predictions or that it is nominalistically adequate does not explain why this is so.

Putnam considers Field's position and the Bedrocker's position. In relation to Field's position, Putnam argues (2012) that Field's intended reconstrual of our best scientific theories in nominalistic terms does not work for Quantum Physics. Putnam (2012, 195) doubts that such a reconstrual is available for Quantum Theory.[9] In plain words, the diffi-

[8] Predicativists, unlike constructivists, do not reject classical logic; like constructivists, they reject transcendent mathematics. As a result, they do not take transcendent mathematics at face value. (Thanks to the referee who helped me realize this). Their position is compatible with scientific realism in the sense that they do not promote a replacement of the truth conditions of mathematical statements for proof conditions. Predicativism differs from nominalism in that it quantifies over abstract objects such as sets, but only if those sets are defined predicatively; that is to say if the definition of the set does not generalize over a totality to which the set to be defined belongs. Sets that can only be defined impredicatively do not exist, according to the Predicativist. For further information, see Linnebo on "Predicative and Impredicative Definitions" at the *Internet Encyclopedia of Philosophy,* http://www.iep.utm.edu/predicat/

[9] Even Field himself doubts it, see his 2016, preface.

culty stems from the fact that Field's project demands a nominalistic version of the mathematically formulated theory and it is difficult to see what the surrogates for the 'entities' described by Quantum Theory with the help of a strong mathematical apparatus could be. Nevertheless, Putnam (2012, 190) sounds sympathetic to Field's project.

Rosen's Bedrocker (Rosen 2001) contends that nominalistic adequacy is sufficient to account for the role of mathematics in applications. Rosen (2001) describes the Bedrocker as a person who accepts a nominalistic modern science and uses the Bedrocker's view of science to argue for the possibility of an equally rational nominalistic version of science. Bedrockers consider that only concrete things exist; discourse about abstracts is a fiction. The world W has a concrete core formed by the largest wholly concrete part of the mereological sum of all the concrete objects that exist in W. The Bedrocker contends that we are concerned with nominalistic adequacy, not with truth since, if we were concerned with truth we would never say, for instance, that

(M) 'The number of moons of the Earth is 1'[10]

is false because numbers do not exist. A sentence S is nominalistically adequate (at the actual world) if and only if the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which a sentence S is true; that is, just in case things are in all concrete respects as if S were true. Hence, (M) is nominalistically adequate if and only if the concrete core of the actual world is an exact duplicate of the concrete core of some world at which (M) is true. As we shall see below, Yablo's notion of partial truth sounds a lot like an improvement on this idea.

Putnam (2012) provides three reasons to argue that Bedrockers have not found a rationally permissible substitute for any physical theory that quantifies over numbers and sets: 1) the trick described by the Bedrocker does not work because it depends on the notions of concrete object and exact intrinsic duplicate. Putnam (2012, 195) doubts we are clear about how these notions apply in the case of modern physical entities such as electrons. He thinks that the trick Rosen describes depends on assuming that all possible things are "good old-fashion concrete objects, which we know how to describe in nominalistic language or good old-fashioned abstract entities which are not in space time and which are causally inert. But the entities of modern physics are neither of the above." 2) The predicate that is the substitute for truth, nominalistic adequacy, does not have the properties of truth. That would be so, because from the fact that a theory T is nominalistically adequate and that a theory T' is also nominalistically adequate, it does not follow that the conjunction of both theories T and T' is nominalistically adequate. 3) The account is not interested in correct explanations, and this is a rational shortcoming.

To sum up, an adequate explanation of the applicability of mathematics should satisfy the following requisites. i) It cannot explain the semantics of applied mathematical statements proof-conditionally; ii) it must allow for enough mathematics for science; iii) it must explain applicability in terms of a predicate that has the logical properties of truth; and it must not assume that there is an exact nominalistic duplicate, or that there is a 'concrete core' of the actual world that is detachable from the abstract.

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[10] This is Yablo's example (Yablo 2014)

Below, we will analyze Hellman's reconstruction and compare and contrast it to Field's and Yablo's; our aim is to show that Putnam's 'being true in a modal-structural interpretation' (understood as described in Hellman's 1989) might face difficulties that are similar to the ones encountered by 'being conservative', 'being nominalistically adequate' or 'being partially true' (respectively Field, the Bedrocker's and Yablo's counterparts of Putnam's notion).

3. *Hellman's modal-reconstruction for applied mathematics*

Putnam's preferred reconstruction of applied mathematics contends that mathematics is true because there is an equivalent description of classical mathematics that does not refer to mathematical entities; the equivalent description is a modal one that uses second-order S5 modal logic without the Barcan Formula. Mathematics is true and it is not about actual mathematical structures or numbers, it is about *possible* structures. Hellman claims that the following is the version of the indispensability argument that results from his reconstruction:

— (P1) We ought to have an ontological commitment to all the entities that are indispensable to our best scientific theories.
— (P2) Modal-structural translations of mathematical theories are indispensable to our best scientific theories.

We ought to have an ontological commitment *to the possibility of the corresponding structures*.[11]

Hellman's reinterpretation of mathematical statements has two components: hypothetical and categorical. For instance, when we say that 2+2=4, what we are really saying is that:

1) Hypothetical:
Necessarily, if a structure satisfies Second-Order Peano Axioms, then $2 + 2 = 4$. Or,
$\Box\forall X\forall f(\wedge PA^2 \supset 2 + 2 = 4)\,(\overset{s}{f})$ in which X is a class variable, and *f* is a two-place relational variable that replaces the constant s throughout the conditional.
2) Categorical:
It is logically possible that there is a structure that satisfies second-order Peano axioms: $\Diamond\exists X\exists f(\wedge PA^2)^X\,(\overset{s}{f})$ in which X, s, and *f* should be interpreted as in 1 above.

The categorical element is necessary to guarantee that the translation scheme respects classical negation: if mathematical entities were not possible, then all the hypothetical conditionals would be true. Moreover, second-order Peano Arithmetic is necessary in order to guarantee that the possibility of a standard model obtains.

[11] Hellman (1989, 96-7); Bueno (2014) makes it explicit (the italics are mine). To keep it in line with Putnam's rejection of confirmational holism (Putnam 2012) the 'only if' is eliminated in the first premise of Bueno's reconstruction.

Hellman provides modal reformulations for number theory, analysis, and set theory. He also accounts for the practice of theorem proving by applying the translation scheme to all the lines in a proof. Finally, he addresses the applicability problem as well.[12]

The form of applied mathematical statements is (Hellman 1989, 130):

$$\Box\forall X\forall f\,((\wedge Z^{+})^{X}\left(\substack{\in\\ f}\right)\ \&\ \forall x(x \text{ an urelement of } X \equiv @\vee_{i=1}^{n} R_i(x_1\ldots x_k))$$
$$\&\ \wedge_{i=1}^{n}\forall x_1\ldots x_k\,[R_i^{k}(x_1\ldots x_k) \equiv @R_i^{ki}(x_1\ldots x_k)] \supset \ldots)$$

Where, in the antecedent of the conditional,

- — The first conjunct "is an abbreviation for the results from writing out the axioms of Zermelo set theory with all quantifiers relativized to the second order variable X, replacing each occurrence of the membership symbol '∈' with the two place relation variable '*f*'." (Bueno 2014) This corresponds to the specification of the mathematical structure that is being used.
- — The second conjunct says that the urelements of the Z+ model are just those items that are actually relata of the R_i, where the R_i are a finite list of synthetic predicates.
- — The third conjunct says that each of the R_i behaves in the model just as it actually does in the actual world. This element is meant to specify the background conditions in which the counterfactual statement holds. In the case of pure mathematical sentences, there is no need for this since all relevant conditions are stated when the axioms for the structure are given as the categoricity proofs show. (Hellman 1989, 95-96)
- — The consequent would be the considered applied mathematical statement.

The statement says that if there were structures satisfying the conjunction of the axioms of Zermelo set theory Z+, including some non-mathematical objects referred to in U and that are the relata of the R_i, and each of the synthetic predicates behaves in the model as it does in material reality then, the consequent would hold in such structures (Bueno 2014).

Hellman acknowledges there is a difficulty he intends to solve with the third conjunct above (1989, 124ff.). The problem is that to specify in detail how material things are (to specify the background conditions), we use applied mathematical statements. Hence, Hellman would use his modal paraphrase, but in order to use the modal paraphrase, he needs to stipulate that material conditions keep as they actually are, so, he would be in a circle. Hellman considers two ways out. The first assumes the world is out there as it is (entities and relations among them as well) independently of our descriptions. Subscribing this view, the circle is avoided but one is committed to (some sort of) metaphysical realism. Since Putnam's account of the applicability of mathematics is not committed to metaphysical realism, only to scientific realism, this way out is not a real possibility for him. Hellman does not take this line either. The second line grants that it is necessary to fix how things stand at the material level. This is Hellman's line and presumable Putnam's. Hellman proposes to avoid the circularity by framing the relevant conditions in synthetic relational terms "which can be utilized in the antecedent of the counterfactuals to achieve the effect of 'fixing the actual material situation'." (Hellman 1989, 129)

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12 See Bueno (2014, §4) for a concise presentation of Hellman's proposal.

The conditions that those synthetic relational terms, predicates, are to satisfy are the following:

1. A finite list of predicates $R_1,\ldots, R_n$, that is ontologically adequate, that is to say: "every non-mathematical object of interest for the mathematical application in question falls within the actual extensions of some predicate on the list."[13]
2. We need to be convinced that "the R predicates sufficed to "determine uniquely" a mathematical-physical description of the situation" (Hellman 1989, 130). The problem is that there might be mathematical descriptions $d_1 ,\ldots, d_n$, incompatible among them but compatible with a given description of the situation in terms of the R predicates. Hence, given a statement like '*o* has mass *r*', its modal-structural reinterpretation could not be asserted, since it would be false that "necessarily, were there a suitable mathematical structure respecting the R predicates (i.e. in which the R predicates had their actual extensions), there would be a mass-representing function assigning *o* the value *r*. For, by hypothesis, there could be mathematical structures respecting the R predicates and different mass-representing functions associated with those structures assigning to *o* different values, each compatible with the R descriptions." (Idem, 129)

In relation to the first issue, Hellman simply claims that the problem is not so difficult to address (1989, 132), though we shall see below it is not so easy either. In relation to the second, Hellman (1989, 132) proposes "to invoke models of an overall theory T' including both the vocabulary of the applied mathematical theory T, and the proposed synthetic vocabulary S. T' links both vocabularies and it may be assumed to be an extension of T. T specifies up to isomorphism a particular type of mathematical structure. Then, we may explicate determination along the following lines. Let α be the class of mathematically standard models of T' and V the full vocabulary of T'. Then, S (the proposed synthetic vocabulary) determines V in α iff for any two models m and m' in α, any bijection ϕ between their domains, if ϕ is an S-isomorphism, it is also a V isomorphism. The last clause means that if ϕ preserves the synthetic vocabulary, it also preserves the rest of the vocabulary of the total theory T', including all relations between the non-mathematical part of the domain and the mathematical part."

Now, once we have a purported synthetic base for an applied mathematical statement, Hellman continues, the problem is how do we convince ourselves that the predicates are ontologically adequate and that any two models of the theory that include both the mathematical and the synthetic vocabulary, T' above, satisfy the determination condition. He advances two possibilities: to proceed inductively by examining a variety of particular systems to which T' is applied or to prove that the conditions on the models of the theory hold. He discards the first option because "it becomes unwieldy when T is global cosmology and you are not god". This leaves him with the second option. We need to prove determination theorems (See Hellman, 132ff.) Therefore, he does.

This should be enough to get a glimpse of what we need to know about Hellman's reconstruction in order to follow the argument. Now it is time to contrast and compare Hellman's, Field's and Yablo's positions with Putnam's desiderata and worries.

[13] Hellman initially allows for a finite list of predicates, but then generalizes to an infinite one (1989, 129ff.)

4. *The reconstructions by Hellman, Field, and Yablo in front of Putnam's concerns about the applicability of mathematics*

Our discussion of Putnam's worries should illustrate that Hellman's reconstruction, Putnam's preferred reconstruction, faces similar problems and assumes comparable commitments to those faced and assumed by Field and Yablo. That would be so because the success of all the accounts considered, even Putnam's, depends on a non-interference requirement between the mathematical and the nominalistically acceptable realms.[14]

4.1. Appropriate semantics

The first worry we mentioned above has to do with the appropriate semantics for mathematical statements. In this sense, it is clear they agree at the semantic level that the adequate reading is a truth-conditional one.[15]

4.2. How much mathematics is indispensable for science?

Putnam's second worry has to do with how much mathematics is indispensable for science. It is important to note that how much mathematics is needed for science cannot be established independently of one's understanding of the expression 'applied mathematics'. Our authors attach two senses to the expression. We will use the term 'narrow' to refer to the one Field and Yablo use which is the one that underlies Feferman's and Burgess' assertion that nominalism and even predicativism[16] allow for a reconstruction of as much mathematics as is used in contemporary science (Feferman 1998; Burgess 1984, 386). And we will use 'wide' for the sense in which Putnam and Hellman use it, namely, as including the mathematics needed in order to *understand* or *to prove* the mathematical results that are used in science.

To defend that the expression 'applied mathematics' should be understood in its wider sense, Hellman mentions the extrinsic and the intrinsic formulations of General Relativity (Hellman 1989, 47 ff.). The extrinsic formulation uses coordinate systems and is the one used by physicists, while the intrinsic one is done in purely geometric terms. The example provides an issue of theoretical importance that can only be *understood* in the light of the intrinsic representation of General Relativity, that is to say, in the light of the presentation of the theory that requires more than RA^2. RA^2 is the part of mathematics that is available to the nominalist and that is all that is required for the extrinsic presentation: "As long as one works within RA^2, one can conceive of its entities as "physical" in the sense of space-time physics in its usual formulations. [...] Thus, all the mathematics of RA^2 could arguably be represented in terms of these "nominalistically acceptable" entities."[17] Hence, if the expression "applied mathematics" is understood in Hellman's wide sense, nominalism fails to allow for enough mathematics for physics.

[14] Mary Leng argued in this sense in a workshop that took place in November 2016, at the University of Santiago de Compostela "Updating Indispensabilities: Hilary Putnam in Memoriam."

[15] See Balaguer (2014).

[16] Hellman (1989, 106).

[17] Hellman (1990, 324; see 321 for the details).

In general, the reasons Hellman provides against "modal nominalism" being sufficient for applied mathematics have to do with our needing mathematics beyond RA^2 in order to *understand* what is going on in physics or to *prove* the mathematical results that are used in physics.[18] The chances that his argument results in a commitment to platonism are faint.[19] Lately, a new version of the indispensability argument, the so-called 'enhanced indispensability argument', has prompted discussion about the ontological commitments of mathematical explanation. Even if the debate is far from ended, it seems clear that if an explanation is only epistemically or representationally relevant, then it conveys no ontological commitment (Baker 2005). It seems clear that the cases put forward by Hellman in his example above, are cases in which we need mathematics in order to achieve a certain *understanding* of what is going on in physics.

Hellman does not intend his argument to establish ontological platonism either. Rather, he means it to motivate the possibility of a wider range of mathematical structures, "without benefit of any assumption as to 'the nature' of the objects" (Hellman 1989, 117). This is consistent with Putnam's argument that tries to conclude that mathematics has to be true under an interpretation that squares with the applicability of mathematics, but aims to establish that ontological platonism is false. Hellman's argument is an argument for the modal-structural reconstruction versus nominalist positions, not an argument for platonism. This might be the reason why Putnam claims, "the most serious objections to [his] 'indispensability arguments' depend, albeit in very different ways, on considering *nominalist* alternatives to present-day theoretical physics" (Putnam 2012, 184).

From an ontological viewpoint, the nominalist can still be right, that the relevant conception of *applied mathematics* is the narrow one. In other words, it does not seem that Hellman's argument would convey a real problem for nominalist or fictionalist programs in case they succeeded to provide an account of the applicability of mathematics understood in the narrow sense mentioned above. After all, they are happy to accept that mathematics has an indispensable descriptive and structural role (Yablo 2012, 1020-21), or even that mathematics is indispensable in order to prove what follows from his synthetic version of Newtonian Physics (Field 1989, 241).

4.3. Rejection of instrumentalism

Putnam's third worry has to do with his rejection of instrumentalism. In particular, we saw the problems he identified in relation to the Bedrocker's stance.

4.3.1. The need for the notions of concrete object and exact intrinsic duplicate

The first objection to the Bedrocker is that her trick depends on the notions of concrete object and exact intrinsic duplicate. As we have seen above, Putnam (2012) doubts it is possible to establish a sharp distinction between the abstract and non-abstract component of a scientific assertion. Avoiding this difficulty might be what Putnam has in mind when

[18] Hellman uses the expression 'modal nominalism' to refer to that part of his proposed re-interpretation that is compatible with nominalism. (See Hellman 1989, 47-52)

[19] Unless heavy duty platonism is the case. See Knowles (2015) for discussion.

he contends that all of mathematics should be paraphrased in a way that the resulting paraphrase is universally applicable; since there is no clear distinction between theoretical terms of scientific theories and mathematical entities, paraphrasing the mathematics first saves us the problem.

Nevertheless, going through the details of Hellman's reconstruction of applied mathematical statements shows that his reconstruction requires assuming a separation between the non-mathematical and the possible mathematical structures, and specifying that the material world remains untouched. We have seen that the ontological adequacy of the synthetic relational predicates demands that non-mathematical objects of interest for the mathematical application in question fall within the actual extensions of some of those predicates. Hellman does not take the issue seriously because "the context of application usually involves a given domain of material objects to which a piece of mathematics is to be applied, and it suffices to cover this domain with the synthetic predicates." (Hellman 1989, 132) However, this argument sounds a lot like assuming that all possible things are "good old-fashioned concrete objects, which we know how to describe in nominalistic language or good old-fashioned abstract entities which are not in space time and which are causally inert. But the entities of modern physics are neither of the above." (Putnam 2012, 195) In other words, Hellman seems prone to Putnam's criticism to the Bedrocker.

Field also has to presuppose there is a separation between the non-mathematical and the mathematical in order for conservativeness to obtain. Hawthorne (1996, 367-8) explains that, if M is a mathematical theory and T is a non-mathematical physical theory, it is necessary to disallow that M+T is inconsistent (imagine T rules out the existence of mathematical entities by claiming that there are no non-physical entities) or that from M+T it follows that mathematical entities have physical properties. Field solves the problem by introducing a predicate, M, for being a mathematical entity "so that where T says that all things are located in space time, T* says that all non-mathematical things are located in space-time." However, Field is well aware that finding subrogates for the mathematical expressions is not an easy task. As mentioned above, he thinks it might not be done in the case of Quantum Physics.

Hellman's reconstruction also requires being sure that the synthetic predicates determine uniquely a mathematical description of the situation. Hellman explains in detail (1989, 135-144) that although his characterization of how it is possible that the synthetic vocabulary determines the full vocabulary of the reconstructed theory coincides in some aspects with Field's, it is nevertheless different in a relevant sense. While Field needs to find acceptable substitute theories in which mathematical physics can actually be carried out, his reinterpretation is less demanding. That would be so, because in order for it to satisfy the determination condition, it is enough to be confident that a synthetic vocabulary is descriptively adequate in a limited sense; it is not necessary that a good theory in this vocabulary is forthcoming over which the applied mathematics is conservative.

Field (1989, 235) has objected to Hellman's reconstrual that the task of splitting up mixed statements into purely mathematical and purely nonmathematical components is a highly non-trivial one and "*it is precisely the same as the task of showing that mathematics is indispensable in the empirical sciences*".[20] Even Putnam's objection to Field (2012, 191-2) in

[20] This brings us back to the second conjunct in Hellman's account of scientific statements: $\forall x$(x an urelement of $X \equiv @v_{1=1}{}^{n}R_i(x_1 \ldots x_k)$)

relation to theories of Quantum Gravity sounds like a problem for Hellman's determination condition: "That means that space-time does not have a determinate metric at all, and hence, one cannot speak of determinate 'straight lines' (geodesics) at all".

Yablo assumes that mathematical content and nominalistic content are orthogonal. His proposal offers a third way to express detachability, one that neither commits to metaphysical realism nor compromises with reformulating applied mathematical statements. He contends that applied mathematical statements are partially true. Yablo's proposal can be seen as an improvement on the Bedrocker's and an alternative to Field's or Putnam's (Hellman's reconstruction). One relevant difference between Yablo's proposal and those by Putnam and Field is that no paraphrase is needed. This is something the Bedrocker also avoids; but while the Bedrocker 'trick' relies on the notion of nominalist adequacy, Yablo's proposal relies on the notion of partial truth. Where the Bedrocker says that a sentence S is nominalistically adequate if and only if the concrete core of the actual world is an exact intrinsic duplicate of the concrete core of some world at which the sentence is true, Yablo says "something is partly true to the extent it has (nontrivial) parts that are wholly true" (2014, 11). Applied mathematical statements are partially true because only their nominalistic (material) content is true. He defines partial truth in terms of the notion of partial extricability. Consider Yablo's (2013, 1018ff.) example:

Let T be a scientific hypothesis about the number of new stars in the nth millennium after redshift 2, in particular, T says:

> T: The number of stars in millennium n is equal to k/2n, for some suitable k.
>
> M, the mathematical component, is 'Numbers exist with all the expected properties'.

How extricable M is from T? M is partially extricable if in some numberless worlds, T adds just truth to M®T and in other numberless worlds, it adds just falsity. In our case, T adds just truth to M®T in a numberless world, if each millennium sees in twice as many new stars as the next. In that case, M®T will be true, while M®¬T will be false. On the other hand, if the considered world is such that it is not true that each millennium sees in twice as many stars as the next, then M®¬T will be true.

Of course, the problem remains how we convince ourselves that mathematics is in fact partially extricable.

4.3.2. The properties of the truth predicate

Putnam's second objection to the Bedrocker is that the predicate that is the substitute for truth, nominalistic adequacy, does not have the properties of truth. Field contends, as it is well known, that mathematics applies because it is conservative, and being conservative has the logical properties of truth as Hawthorne proved (Hawthorne 1996). The problem for Field is that for the moment, there are not alternative synthetic versions for theories such as Quantum Theory, hence no conservatiness results are available in those cases.

Nevertheless, we should remember that Putnam, Field, the Bedrocker, and Yablo understand the term 'applied mathematics' in two different senses, wide and narrow. While Putnam understands it in the wide sense, the others understand it in the narrow sense. In order to argue that mathematics is non-conservative for scientific theories for which a conservativeness result is available, we have to understand the term 'applied mathematics' in the wider sense in which Putnam and Hellman do. Yet, they should agree that conservativeness is appropriate if applied mathematics is understood in the narrow sense.

Beyond that, Hellman puts forward a possibility according to which, even understanding 'applied mathematics' in the narrow sense, his approach would be more adequate. That would be in case parts of mathematics that go beyond RA^2 turned out to have physically significant corollaries, for instance, in case large-cardinal axioms were needed in scientific applications as certain investigations by Harvey Friedman suggest (Hellman 1989, n. 120). In this hypothetical case, there would be parts of mathematics that are not available to the nominalist that would be indispensable for physics. Nevertheless, for the moment, large cardinals have not been used in scientific applications.

4.3.3. No interest in correct explanations

Finally, Putnam's third objection to the Bedrocker is that the account is not interested in correct explanations, and this is a rational shortcoming. This objection corroborates that this worry of Putnam's is epistemological. However, it is doubtful that allowing for a descriptive, structural or representational explanatory role for mathematics, something our fictionalists are ready to accept, amounts to not being interested in correct explanations.

Conclusion

Finally, a couple of consequences of our analysis. In the light of Putnam's concerns about how to account for the applicability of mathematics, it is far from clear that Putnam's preferred reconstruction provides a more successful explanation than the considered fictionalist positions. Field, Yablo, and Putnam coincide, *pace* what Putnam has claimed, in adopting a non-interference proviso and their reconstructions face analogous difficulties. If this is the case as we have contended above, Putnam's argument and strategy can be seen as coincident with those of Field's and Yablo's. Any of the nominalistic strategies advocated could serve the objectives of the others. The remaining worry is then that it could be that the non-interference proviso obtains or it could be that it does not. The three approaches fight to show that it does not. The issue is an empirical one.

At the ontological level, a certain underlying conception of abstract objects stems from these views. In relation to pure mathematics, abstract objects are claimed to exist inside the fiction of mathematics. What abstract objects are depends on what the fiction says; hence, they are not independent of us.

Likewise, in relation to applied mathematics, a clear contention of the views is that abstract objects are sharply different from concrete objects while they do not exist in the real world. From this viewpoint, Putnam's opponent is the heavy duty platonist (Field 1989, 186-193; Knowles 2015). Field characterizes heavy duty platonists as those platonists who believe that "there are *relations of physical magnitude* that relate physical things and numbers: for instance, the mass in kilograms' relation that might hold between a physical object and the real number 15,3" (1989, 186) and that those relations are a brute fact. Moderate platonists hold, on the contrary, that those relations are something conventional "derivative from more basic relations that hold among physical things alone." (Idem) Hence, it seems that the kind of objections coming from that debate with the heavy duty platonist, *pace* Putnam, are the more serious ones.[21]

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[21] See Plebani's paper in this monograph for a more detailed reflection on the view of abstract objects compatible with Putnam's argument.

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