Mathematical explanation and indispensability*

Susan Vineberg

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ABSTRACT: This paper discusses Baker's Enhanced Indispensability Argument (EIA) for mathematical realism on the basis of the indispensable role mathematics plays in scientific explanations of physical facts, along with various responses to it. I argue that there is an analogue of causal explanation for mathematics which, of several basic types of explanation, holds the most promise for use in the EIA. I consider a plausible case where mathematics plays an explanatory role in this sense, but argue that such use still does not support realism about mathematical objects.

Keywords: indispensability, explanation, realism, structuralism.

RESUMEN: Este artículo analiza la versión 'mejorada' del argumento de la indispensabilidad (EIA) de Baker y varias de las respuestas que ha recibido. Este argumento pretende establecer el realismo matemático en virtud del papel indispensable que las matemáticas juegan en las explicaciones científicas de los hechos físicos. Argumento que hay un análogo de explicación causal para las matemáticas que, de los varios sentidos básicos de explicación, parece el más adecuado para el EIA. Analizo un caso plausible en el que las matemáticas juegan un papel explicativo en este sentido, pero concluyo que este uso no es suficiente para establecer el realismo acerca de los objetos matemáticos.

Palabras clave: indispensabilidad, explicación, realismo, estructuralismo.

Introduction

The so-called 'Quine/Putnam indispensability argument' has been discussed over the past half century as the main impediment to mathematical antirealism. Although they are often lumped together, the arguments given by Quine and Putnam are significantly different.[1] Quine's argument depends upon his controversial claim of confirmational holism, and concludes that given the use of mathematics in our best (and well-confirmed) scientific theories, we must accept the existence of mathematical objects. Putnam's argument, by contrast, emphasizes the use of mathematics in formulating physical theories that he thinks (in accordance with scientific realism) we have reason to regard as true. Putnam's claim,

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[1] A number of crucial differences are noted in (Vineberg 1998). Colyvan (Colyvan 2015) presents a version of the argument he identifies as 'The Quine-Putnam Indispensability argument', but also notes that Quine and Putnam gave different arguments. (Quine 1961a, 1961b, 1976, 1969) (Putnam 1979a, 1979b)

that the indispensable use of mathematics in formulating statements of physical theory that are taken as true requires accepting the mathematics so used, has generally been read as carrying a commitment to the existence of mathematical objects as in Quine, and so, at least in broad outline, the arguments of Quine and Putnam have been taken to have much the same core. Nonetheless, Putnam makes clear that he regards his indispensability argument as one for the truth of mathematical claims, but not for traditional Platonism about mathematical objects (Putnam 2012).

More recently, Baker has presented and defended what he calls an 'enhanced' indispensability argument (EIA) for the existence of mathematical objects that stems from the use of mathematics in explaining physical facts (Baker 2009). I begin with a discussion of this version of the indispensability argument and its presuppositions, along with ways of responding to it and their relative merits. This sets up the main focus of the paper, which concerns how mathematics figures in explaining physical facts and whether mathematical claims should thereby be accepted. Different ways in which explanation could be understood in Baker's argument are considered, with a focus on their potential to support inference to the existence of mathematical objects. I explore the idea that a kind of explanation that is analogous to causal explanation might serve this function, and consider in broad outline how explanation along these lines functions both within mathematics and in explaining physical facts by focusing on the famous problem of the Seven Bridges of Königsberg. This leads to a suggestion about how mathematics can explain physical facts without embracing the realism about mathematical objects that the indispensability argument is typically invoked to support.

Baker's indispensability argument and strategies of response

Baker's enhanced indispensability argument (EIA) is similar to those of Quine and Putnam, but intended to complement them by focusing on the apparent explanatory role of mathematics in science:[2]

1. We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
2. Mathematical objects play an indispensable explanatory role in science.
3. Hence, we ought rationally to believe in the existence of mathematical objects. (Baker 2009)

This argument is valid, so any quarrel must rest with the premises. One way of resisting the conclusion denies that explanation is tied to belief as required by (1). For Baker (1) derives from the Principle of Inference to the Best Explanation (IBE), which he holds as a basic tenet of scientific realism. However, many forms of scientific realism reject the idea that we must accept a literal reading of all claims figuring in scientific explanations and also reject that we must accept the existence of every object mentioned in those explanations. Accordingly, at least some qualification of (1) is an option.

Several programs developed in response to Quine and Putnam's versions of the indispensability argument, if successful, allow resisting Baker's EIA by opposing (2). Most

[2] This apparent explanatory role is discussed by Field (Field 1989).

prominently, Field tries to show that physical theories can be replaced by nominalized versions that quantify only over physical objects and relations to establish that mathematics is dispensable in formulating physical theory (Field 1980). However, Baker thinks that Field's nominalization program cannot be successfully carried out, and that there are indispensable uses of mathematics in explaining physical facts. These include his explanation in support of premise (2) of why the cicadas of the genus Magicica in a particular region have prime (17-year) life cycles:

E1. Having a life-cycle period that minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.
E2. Prime periods minimize intersection.
E3. Hence, organisms with periodic life cycles are likely to evolve periods that are prime.
E4. Cicadas in ecosystem-type E are limited by biological constraints to periods from 14 to 18 years.
E5. Hence, cicadas in ecosystem-type E are likely to evolve 17-year periods.

Baker regards E1-E5 as a mathematical explanation of the fact

E*. Cicadas in ecosystem-type E have a life cycle of 17 years. (Baker 2005)

Other alleged mathematical explanations of physical facts include explanations of the division of honeycombs into hexagonal cells and the impossibility of a continuous walk over the Seven Bridges of Königsberg.

Such examples raise many questions, including the precise role that mathematical objects play in providing explanations of physical facts. One response to Baker's cicada example is that the mathematics is dispensable in that reference to mathematical objects can be eliminated in the explanation of E*, or at least avoided in a closely related explanation of it. Saatsi provides such an alternative explanation by replacing E2-3 with the claim that for periods in the range 14-18 years the intersection minimizing period is 17, where this is to be understood as a claim about durations rather than number (Saatsi 2011).

However, Baker replies that his mathematical explanation is more general and unified, and so superior to such alternatives that attempt to dispense with mathematics (Baker 2017). IBE then sanctions accepting his mathematical explanation, and with it mathematical objects.

Baker's reasoning raises questions concerning IBE, as to the appropriate criteria for the best explanation, whether by those criteria his explanation is the best, and whether this supports realism about mathematical objects. Advocates of IBE cite various virtues, often including generality, simplicity, and unity, but this leaves open the precise form of these theoretical features that is associated with explanatory power. Predictive power and accommodation of known data are sometimes regarded as important features of scientific explanation, whereas minimality may be considered a metaphysical virtue. In particular, for Field, nominalized versions of physical theory are superior to those formulated mathematically in that they avoid the misleading appearance that the physical facts depend upon non-physical facts (Field 1989).

IBE is notoriously vague and just how such dimensions of goodness are to be weighted remains unclear. Beyond the problem of resolving these issues with IBE, Field's program faces the disadvantage that the mathematics used in describing or explaining physical facts must be replaced with domain specific predicates, and is thus a piecemeal and open-ended project. Even if mathematics is dispensable to the (best) explanation of the cicada life-cycle, it may not be dispensable to the best explanations of other facts, since it is unclear that minimality, or some other virtue(s) of explanations that dispense with mathematics, will dominate over other merits in every case.

Other antirealist responses to the EIA avoid having to show that mathematics is dispensable in physical explanations by arguing that mathematical claims don't carry with them a commitment to abstract mathematical objects. One approach tries to show that mathematical truths can be understood as modal claims of possibility rather than about existing abstract objects.[3] Baker does not address such projects in conjunction with the EIA, but his response to Field-style attempts to reformulate the alleged mathematical explanations of physical facts to remove mathematical terms suggests the position that casting the mathematical claims that figure in those explanations in modal terms involves giving more complex and thus inferior explanations. Although antirealists who invoke modality in understanding mathematical truth have grounds to dispute a rebuttal along these lines, a more direct response to Baker's argument that avoids the details of such programs is desirable.

Wakil and Justus do respond more directly against Baker's EIA by arguing that he provides the wrong sort of explanation of E* (Wakil & Justus 2017). They claim that his mathematical explanation of why such species evolved to have 17-year life-cycles errs in supposing that this trait occurred because it was selected for. Instead they suggest a different kind of explanation that does not appeal to mathematics, which, unlike Saatsi's alternative, does not merely try to dispense with the reference to mathematical objects in Baker's explanation. However, their response again follows a piecemeal approach; and, they admit that there appear to be other cases in which mathematics plays an indispensable explanatory role, making this an inadequate reply to the EIA generally.

In contrast to the responses to the EIA noted above, I focus here on the concept of explanation in accounting for physical facts and the conditions under which an explanation, or a portion thereof, provides grounds for inference. I argue that even given the sort of explanation most favorable to such inference, typical explanations of physical facts involving mathematics do not commit us to mathematical objects. As such the position will be consistent with antirealist views that understand mathematical truths as modal claims of possibility, in line with the suggestion of Putnam (2012), and the accounts proposed by Chihara (1990) and Hellman (1989).

Explanation and inference

As discussed, Baker's EIA turns on the idea that mathematical objects play a role in the best explanation of some physical facts and that this compels at least scientific realists to accept such objects in accordance with IBE. The claim that one ought to infer to the best explanation of a given fact presupposes both that there are grounds for taking an explanation of it as best and that having this status as the best explanation warrants belief or acceptance. Despite the prevalence of appeals to IBE, precisely what such grounds amount to, and the extent to which they license inference, remains unclear and controversial. Adding to the dif-

[3] Putnam suggests such a view in various places as noted in (Putnam 2012). For detailed positions of this sort see (Chihara 1990) and (Hellman 1989).

ficulty is that what constitutes an explanation is itself unclear. Indeed, it is plausible that there are distinct notions of explanation, each with their own measures of assessment, whose domains overlap at least to some extent,[4] which looks to complicate claims about what is to be inferred on explanatory grounds.

Connected with the problem of characterizing explanations and comparing them is that of determining exactly what it is that is doing the explaining and might be held to be accepted on this basis. Clearly it is not sufficient for the claim that mathematics explains a physical fact that mathematics is used or appealed to in some way in the explanation. Many scientific explanations invoke laws that are stated using mathematics, but this does not make such explanations mathematical. Mathematics may also be used in deriving consequences of laws in an explanation, but again this does not make the explanation mathematical. Rather, a mathematical explanation of a physical fact must somehow turn upon a mathematical fact; the mathematics cannot be simply a mechanism for conveying the explanation of the physical fact.[5] An account of explanation appropriate for the EIA should facilitate showing how a given explanation is genuinely mathematical.

Despite the fact that various different accounts of explanation have been proposed and a number of distinct virtues suggested as contributing to explanatory power, which make appeals to IBE in the EIA appear difficult to assess, many particular ideas about explanation may be thought of as falling under one of several basic conceptions of explanation. Of these, I suggest that only one is a basic tenet of scientific realism and might be held to function in a non-question-begging way in the EIA.

Where explanation is construed as providing understanding, an explanation may be taken as making plain and/or reducing the phenomena to be explained to common or familiar terms. This sort of explanation is important in conveying scientific ideas to non-experts, but is not in line with the fundamental understanding sought by science, which provides deep and novel explanations in what were previously unfamiliar terms. It is further doubtful that this sense of explanation supports IBE as required by the EIA. Quine did list familiarity of principle as a theoretical virtue that supports theory acceptance (Quine 1976); however, scientific practice suggests that making use of familiar principles is better taken as a guideline for extending what are already well-confirmed scientific theories to account for phenomena in their domain than as a reason for favoring theories that have yet to be established.

A broad view of scientific explanation, beginning with Hempel, who took explanations to be arguments, is that it involves assimilation of a fact to a general pattern or regularity. Baker's emphasis on generality and the fact that in defending his 'mathematical explanation' of the cicada's 17-year life cycle he presents the explanation as an argument in much the style of Hempel suggests a conception of explanation along these lines. However, Hempel's particular account of scientific explanation, whereupon scientific explanations involve deductions from laws, cannot be satisfactorily modified to cover mathematical explanation. Instead, Kitcher's unification theory (Kitcher 1989), which, with Hempel, links explanations with generality and argument,[6] appears more promising as having the potential to apply to mathemati-

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[4] A case for this is made in (Salmon 1989).

[5] Lange discusses these points in (Lange 2013), adding that what he calls distinctively mathematical explanations of physical facts involve a deeper sort of necessity than mere physical necessity.

[6] See Salmon (1989).

cal explanations of physical facts while conforming with Baker's assertions about explanation. Kitcher's theory takes explanation to consist in unifying a body of facts through their derivation from a small set of restricted patterns of argument, with greater unification achieved by using fewer and more restricted (stringent) argument patterns. Baker's claim that his explanation of the prime life cycle of the 17-year cicada is more general than Saatsi's fits with this idea in that Baker can make use of a more restricted pattern of argument than Saatsi in deriving the periods of other species of cicada with different prime life-cycles.

Although Kitcher's unification theory of explanation fits well with Baker's apparent views about scientific explanation, the unification theory faces various problems,[7] and is unsuitable in arguing for mathematical realism via the EIA. There is little agreement, even among scientific realists, that the greater generality, unification or simplicity of a proposed explanation makes it more likely to be true. Many scientific realists reject the broad acceptability of IBE, particularly where generality, unification, and/or simplicity figure as pivotal in determining the best explanation, even where the inference concerns the existence and properties of physical objects. To suppose that scientific realism is committed, over and above using the principle in making inferences about physical objects, to a broader principle covering mathematical objects referenced in an explanation of physical fact that counts as best on grounds of generality, unification, and /or simplicity is question begging.[8]

Some realists who reject IBE in general, nonetheless accept (and/or require) a restricted version of the principle. Notably Cartwright has argued that scientists do accept inference to the most probable cause (Cartwright 1983). Whether or not it is correct that scientists do so reason, it is central to many versions of scientific realism that, at least under certain conditions, those objects that figure as the causes in the best causal explanations of observed phenomena are to be accepted. Such acceptance appears grounded in the idea that there are some underlying properties of physical objects that bring about the observed facts, and that the success of scientific methodology in predicting and controlling empirical phenomena is to be accounted for in terms of tracking its causes.

Taking scientific realism as adhering to a restricted principle of IBE in which we infer to the most probable cause, while less controversial in requiring only that we accept the best causal explanation of a given empirical fact, initially seems unhelpful for the EIA, since mathematical objects, on virtually every account, are causally inert. This would be right if scientific realism employs only a very narrow version of IBE in which we accept the existence of the physical objects that figure in the best causal explanation of the phenomena. Instead scientific realism might embrace a still restricted but somewhat broader principle according to which we are to accept the conditions upon which the phenomena depend, where such dependence need not be strictly causal. Some scientific realists, along with antirealists, will reject the broader principle as contentious, but a principle of at least this strength appears required for the EIA. The issue here is whether the EIA succeeds given the stronger principle. Towards this possibility, I suggest that there is a partial analogue of causal explanation that applies to mathematics. Although this analogue helps to illuminate

[7] See Woodward (2014).

[8] The fact that it is question begging to take what would be a unifying explanation if true to be a reason to accept a given explanation as true does not count against the unification theory of explanation per se as an account of what an explanation from previously accepted facts may consist in.

how mathematics works in the explanation of certain physical facts, ultimately it does not support acceptance of mathematical objects as needed for the indispensability argument.

Analogues of causal explanation in mathematics

Many scientific explanations are considered causal, although there has been some disagreement as to the form (or forms) that such explanation can take. Salmon wrote of causal explanation in terms of locating the underlying mechanism that brings about the fact to be explained. More recently, Woodward has emphasized the idea that "(causal) explanation is a matter of exhibiting systematic patterns of counterfactual dependence" (Woodward 2003, 191), with such patterns describing how things would be different under alternative conditions. A key feature of this view is that a causal explanation of a fact is associated with the conditions that make a difference to whether the fact occurs, where such difference makers can be exhibited through interventions. In considering causal explanation proper, mechanisms and interventions are understood as physical processes, and as such do not belong to mathematics and do not figure in mathematical explanation, but both mechanisms and interventions have something of an analogue in mathematics. Moreover, both conceptions of causal explanation belong to a more general idea that to explain is to give the reason for the fact, locate its source, or find the conditions upon which it depends. It is particularly significant here that the identification of difference makers appears tied to certain mathematical explanations, which suggests that these explanations are analogous, in some ways, to causal explanations. I examine this idea and then indicate how this permits us to see how mathematics can be taken in some cases as explaining physical fact without a commitment to mathematical realism.

Mathematical explanation and difference makers

To show how there can be a kind of mathematical explanation of a physical fact that parallels causal explanation, it will be useful to focus attention on the solution to the problem of the Seven Bridges of Königsberg. The problem concerns whether it is possible to walk in a continuous path over each of the seven bridges of old Königsberg, crossing over each bridge only once.

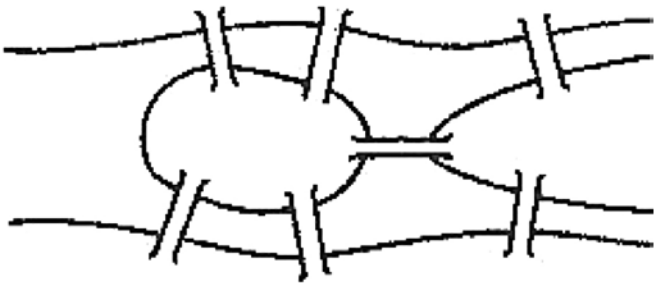

Fig. 1

Schematic diagram of the Seven Bridges of Königsberg

Euler established that this is impossible by associating a graph with the configuration of bridges in which each landmass is replaced by a point (also node or vertex) and the bridges connecting them by lines (or edges). The resulting Seven Bridges graph corresponds to the physical arrangement of landmasses and bridges, and gives rise to a mathematical problem about whether there is a continuous path (or Eulerian path) traversing the graph that covers each edge only once.

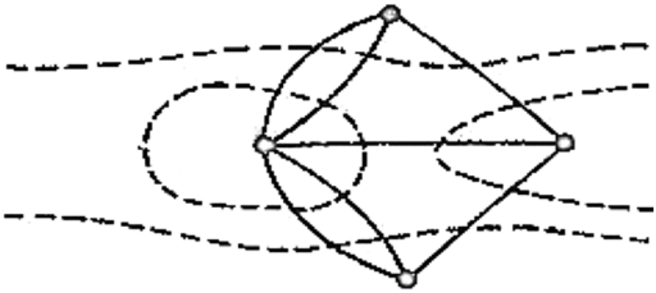

Fig. 2

Diagram of Bridges of Königsberg and the Seven Bridges graph with points on each landmass and edges representing bridges

Euler argued that there can be no such path, for in the Seven Bridges graph each point touches an odd number of edges. Any continuous path covering each line segment of the graph only once contains one starting point and one end point (possibly the same as the starting point). A starting or end point may touch an odd number of edges. But each non-starting point must touch an even number of edges so that once a path touches a point via an edge, there is a new edge, not previously traversed by the path, from which to exit. Since the Seven Bridges graph contains more than two points touching an odd number of edges, there is no continuous path through the Seven Bridges graph traversing each edge only once.

In constructing the Seven Bridges graph, which represents the structure of the physical arrangement of bridges, Euler passes from a physical problem to one of pure mathematics concerning the associated graph. His solution to the mathematical problem proceeds by the argument above concerning possible paths over the edges of the graph and passing through the vertices. Importantly, Euler's argument provides an internal mathematical explanation for why there is no path through the Seven Bridges graph and plausibly also gives us a mathematical explanation of the physical fact that it is impossible for someone to take a walk that crosses over each bridge only once. Here the features of the internal mathematical argument account for how it explains the mathematical fact, but also suggest how the argument can provide a mathematical explanation of the physical impossibility of a continuous walk across the Seven Bridges of Königsberg. I first discuss the idea that Euler provided an internal mathematical explanation, which bears a similarity to causal explanations, and then take up the implications for mathematical explanations of physical facts and the EIA.

How Euler's proof explains the impossibility of a continuous walk may be seen by first contrasting the proof with one that does not explain this fact. An inelegant proof by brute force can be given by considering each possible starting point and showing that each partial path beginning with the point cannot be completed by following it until arriving at a point from which the path cannot be continued without re-crossing an edge. This argument establishes the result, but does not explain why there is no Eulerian path through the Seven Bridges graph, leaving us to account for the difference between the two proofs. One suggestion is that Euler's proof is somehow more general; however, both proofs cover only the Seven Bridges graph. However, Euler's argument is immediately generalizable to other graphs and in so doing establishes that any graph with more than two points touching an odd number of edges contains no continuous path. This suggests focusing on the generalizability of a proof, rather than on whether it has been so generalized. However, generalizability per se is insufficient to distinguish the two proofs, for the brute force proof can also be generalized.

Instead of considering generalizability there is another facet of Euler's proof that does distinguish it from the brute force proof and is plausibly its key explanatory feature. Namely, the proof exhibits, and argues from, those features of the graph that make a difference to the existence of an Eulerian path. In particular, the completion of such a path depends upon having no more than two points touching an odd number of line segments. A general (or generalizable proof) need not involve locating such difference-makers (although many do), and so the conception of explanation that involves the identification of difference-makers does not reduce to that of generality within mathematics. The fact that Euler's proof stems from locating those factors that make a difference to the possibility of a continuous path further distinguishes it from the brute force proof. This suggests that, as with causal explanations on Woodward's account, Euler's graph theoretic proof explains through identifying the relevant features of counterfactual dependence.

The idea that there are mathematical proofs that proceed through identifying difference-makers, and thus that there is something of an analogue of causal explanation in mathematics, is related to the view of Steiner in an early paper on mathematical explanation. Steiner's idea is that explanatory proofs make reference to what he terms a characterizing property of an object or structure mentioned in the theorem, where substituting a different object from the same domain results in the collapse of the theorem (Steiner 1978). It is clear that on his view explanatory proofs, of which he gives various examples many of which are not the most general, involve identifying difference makers linked with conditions of counterfactual dependence. Steiner took explanatory proofs in general to have this feature, although the essential claim here is just that Euler's proof is an explanatory proof with these features and that there is a broad class of other explanatory proofs within mathematics that do also, regardless of whether all explanatory proofs have this similarity with causal explanations.

The link observed here between explanatory proofs and the identification of certain conditions of counterfactual dependence leaves much unsettled about just how both internal mathematical explanations and mathematical explanations of physical facts compare with causal explanations proper. Note first that Steiner's account is incomplete in that he leaves the notion of a characterizing property unanalyzed. Still individual graphs can clearly be characterized by the arrangement of their nodes and edges. The existence of a continuous path depends upon having an arrangement where no more than two nodes touch an odd number of edges, so that Euler's argument meets Steiner's condition for an explanatory proof. Still, seeing that this and other cases fit Steiner's account leaves open the extent to which it is, or is not, analogous to particular conceptions of causal explanation proper.

Explanation of a fact on Woodward's view involves both an explanatory generalization providing counterfactual information and a specification of the variables involved in the generalization as it applies to the explanation. Central to his idea of causal explanations is that the explanatory generalization remain invariant under interventions to such variables appearing in the explanans. Importantly, deeper generalizations are associated with broader explanatory generalizations. In a recent paper, Jansson and Saatsi (Jansson & Saatsi 2017) sketch how the general features of Woodward's account can be applied in explaining why there is no continuous walk over the Seven Bridges of Königsberg and to related configurations of islands and bridges. They classify various limited configurations of islands and bridges, involving limits on the number of islands and bridges, into whether they include islands connected by an odd number of bridges or not (note that they understand the general problem to be one of tourability, which involves returning to the initial starting point to the conclude the walk, rather than merely traversing each edge in a continuous path). In accordance with Woodward's framework, they provide a generalization for various limited configurations of bridges that relates whether the configuration is odd or even to its tourability. Even configurations correspond to a variable that yields an output corresponding to tourability and odd configurations correspond to a variable that yields a value corresponding to no tourability. In their treatment invariant generalizations that cover more arrangements of islands and bridges are considered more general and provide deeper explanations, with the broadest possible case invoking the mathematical objects of graph theory.

In addition to trying to show how an explanation of the impossibility of a walk over the Seven Bridges of Königsberg may be said to be akin to causal explanation by applying Woodward's account, Jansson and Saatsi here continue with Saatsi's earlier project of attempting to show that mathematics is dispensable, for the explanation of the tourability/nontourability of any particular configuration of islands and bridges (or collection of such configurations) can be given without reference to the objects of graph theory. Thus their recent development along Woodwardian lines retains the claim that mathematical objects are dispensable in the explanation of physical fact and that it is physical structure, rather than mathematics, that does the explaining.

Jansson and Saatsi's development bolsters the idea that at least certain mathematical explanations work by exhibiting counterfactual dependencies, and in this way at least are analogous to causal explanations. However, their account involves a discontinuity between what they take as the non-mathematical explanation of the nontourability of the Seven Bridges of Königsberg provided by considering classes of physical island and bridge structures and the mathematical explanation from graph theory. Furthermore, there is some question about whether their understanding of interventions requires giving up the idea that each of these explanations is essentially causal. This is particularly acute in the mathematical case where it is unclear how interventions are to be understood. As Pincock points out (Pincock 2015), Woodward requires that causal explanations answering the questions of counterfactual dependence must do so in terms of interventions, where the intervention would be made to the object that figures in the explanandum. That is, such explanations require invariance under interventions to the object of the explanandum. Jansson and Saatsi instead allow interventions to apply to objects of the same type. This also fits with Steiner's account upon which we are to consider the resulting theorem upon substitutions of objects of the same type. One possibility for accommodating Jansson and Saatsi's treatment is to broaden Woodward's concept of intervention, and with it casual explanations. This might be done by counting substitutions of objects of another type as interventions, but another possibility with some promise for understanding interventions in mathematics is to take them as involving transformations applied to mathematical objects. One could then take interventions to be made on the very object mentioned in the explanandum, with the idea that maintaining invariance would require certain limitations on the allowable transformations. An alternative is to maintain the restriction on causal explanations and simply allow that mathematical explanations may involve a different (non-causal) type of counterfactual dependence. The issues about how to understand causal explanation and its connection with interventions, as well as the extent to which certain mathematical explanations may be regarded as causal, are complex. But what matters for the purpose of addressing the EIA is the idea that mathematical explanations such as that of the impossibility of a continuous walk over the Seven Bridges of Königsberg appeal to a kind of mathematical dependence, regardless of whether this is to be understood as causal in Woodward's sense.

The role of mathematics in explaining physical facts

Given the suggestion that there is something of an analogue of causal explanation within mathematics that involves identifying difference makers, I turn to how such mathematical difference makers play a role in explaining physical facts and whether this provides reason to accept mathematical objects, focusing as before on the problem of the Seven Bridges of Königsberg. For Baker the appeal to the graph in Euler's solution is part of the best explanation of the physical fact and thus requires accepting the mathematics so involved, in parallel with his reasoning in his cicada example. But recall that the mere use of mathematics in the explanation of a physical fact, including use in representing the physical scenario involved in that explanation, does not make the explanation mathematical. Establishing that some mathematical fact explains a physical one requires a reason to take the latter to depend upon the mathematical fact. Saatsi appears to hold that the physical fact does not depend upon a mathematical fact, because the impossibility of a continuous walk does not depend upon the existence of mathematical objects. Although in his paper with Jansson it is suggested that the deepest explanation in one sense is provided by invoking graph theory, and thus mathematical objects, the particular fact is nevertheless explained in Woodwardian style without invoking graph theoretic objects by means of more limited generalizations holding of physical structures alone. The view seems to be that the physical impossibility does not depend upon a purely mathematical fact, but that mathematics merely plays a representational role by providing a general explanatory framework where this does not require the acceptance of mathematical objects. The rejoinder for Baker is that the best explanation requires mathematical objects, and so we should accept them. I suggest that a third view avoids this impasse.

There is a case to be made that the impossibility of a continuous walk over the Seven Bridges depends upon the mathematical impossibility established by Euler's graph theoretic proof and that his explanatory proof, which locates the relevant mathematical difference-makers, indicates how the physical fact may be said to depend upon the mathematical one, while still not requiring the acceptance of mathematical objects. Observe first that what matters to the impossibility of a path over the Seven Bridges graph is the structural relationship of the points and edges. It is not any property of the points or edges that are the difference-makers to the impossibility of a continuous path, nor does such an impossibility appear to depend upon the existence of the particular graph itself. The difference-makers do not consist in the features of the individual nodes and edges, but in their relational structure. The role of these objects is to provide a framework to characterize that structure. Accordingly, the fact that there is no possible path through the graph, does not appear to depend upon the existence of the graph, but only upon the relationships between the points and edges were the relations of the graph to be realized. Insofar as the graph itself plays an explanatory role which accounts for the physical facts, the thought is that it is the structural dependencies revealed by the graph that explain those facts rather than the existence of mathematical objects.

The question that remains is whether the structural dependences depicted by the graph as used in accounting for the physical fact should be taken as mathematical. Euler's use of the Seven Bridges graph is an example of what Pincock calls an abstract explanation, in that it involves a classification of systems that appeals to a more abstract entity that is linked to the phenomena to be explained (Pincock 2015, pg. 867). The entity here is a mathematical one, and so for Pincock yields a mathematical explanation. By contrast, Jansson and Saatsi do not take the use of mathematics as providing a special sort of explanation; rather, they view many ordinary cases of causal explanation in science as more or less abstract, but not thereby mathematical. Considering arrangements of bridges involves abstraction on their view, but mathematics only enters when the description involves explicit mathematical objects, such as graphs with their nodes and edges.

It is clear if we consider the arrangement of landmasses and bridges found in the original bridges of Königsberg along with the graph that we can apply Euler's argument using the latter to the physical arrangement directly. Indeed, there is a sense in which the same argument is applied to the arrangement of landmasses and bridges as to the arrangement of nodes and edges in the connected graph. Whereas this might be taken as evidence that the explanation of the impossibility of a continuous walk is physical rather than mathematical,[9] I suggest instead that the structural relationships that are appealed to in the physical arrangement might be seen as mathematical. This corresponds to a structuralist view on which mathematics is concerned with spatial relations, shape, quantity, order, etc., and that doing mathematics involves reasoning about such concepts in accordance with certain methods.

It is common to conceive of mathematics as consisting of theories of numbers, sets, various geometric objects, and so on. Instead for the structuralist, mathematics is concerned with the relations that hold between objects in structures; it is the relational features between the objects referenced in the structure that they seek to describe. In any particular instance of the structure, the objects that appear are place-holders in the structure, rather than the subject matter of interest. This structuralist interpretation of mathematics allows for the view that mathematics plays a genuinely explanatory, rather than a merely representational, role in accounting for such facts as the impossibility of a continuous walk over the Seven Bridges of Königsberg.

[9] The argument applied to the graph references odd and even, but these terms can be avoided.

In general, on a structuralist understanding of mathematics, those explanations of physical facts that turn upon locating structural differences may be seen as being mathematical, for on this conception to be a mathematical explanation is not itself to concern individual mathematical objects. Euler can be said to explain the impossibility of a continuous walk over the Seven Bridges of Königsberg by bringing out those structural features of the associated graph that make a difference to the possibility of a continuous path, where the structural features that preclude a path through the graph are the structural features of the physical arrangement of the bridges that make a continuous walk impossible. Euler's argument shows the common structure to be configured so that a walk is impossible, thus explaining why one cannot traverse the Seven Bridges of old Königsberg in a continuous path crossing each bridge only once. The graph functions here to make the mathematical structure apparent. Showing that mathematical objects are dispensable in formulating scientific theories and explanations by giving an alternative that does not make reference to such objects isn't to have shown that *mathematics* is dispensable. In particular, it isn't to have shown that such explanations are non-mathematical. For on this view mathematics isn't fundamentally about such objects, but about structures.[10]

An attractive feature of this position is that it fits well with the intuition that Field's nominalized theories, which if successful remove reference to individual abstract mathematical objects, do not seem to have removed the mathematics, for Field obtains correlates of many standard mathematical theorems employed in science, and intuitively they seem to be mathematical too. That is, it isn't reference to abstract mathematical objects that makes a subject mathematical, but it is the description of the relations that hold between (hypothesized) mathematical objects, which are captured by theorems, along with the methods of demonstration, that constitutes mathematics. Where those relations hold of physical objects and predicates that can be described using correlates of the ordinary theorems stated in terms of mathematical objects, the subject matter on this view remains mathematical.[11] In particular, this has the advantage that there is no discontinuity between the (abstracted) physical arrangements of islands and bridges considered by Jansson and Saatsi, and mathematical graphs. Although the graphs involve a greater degree of abstraction, the reasoning concerning the essential (mathematical) concepts is the same.

A structuralist view of mathematics has considerable promise to account for the relationship between pure and applied mathematics. Here, it makes good sense of the connection between the argument of Euler, which references the Seven Bridges graph, in arguing that there is no continuous path through the graph and the argument that references only the physical arrangement of landmasses and bridges. The former provides an explanatory proof of the fact of pure mathematics that there is no continuous path through the graph. Given that the proof exhibits the structural features that make such a path through the graph impossible, and the physical arrangement shares the structural features of the graph, the proof may be said to yield a mathematical explanation of the corresponding fact about the physical bridges, on a structuralist understanding of mathematics.

[10] One notable feature here is that it captures the similarity between the argument within mathematics concerning the Seven Bridges graph and the argument concerning the physical arrangement.

[11] This differs from Field's view according to which mathematics is about mathematical objects.

There are many versions of mathematical structuralism, some of which assume a realist ontology of structures. Others adopt an antirealist version according to which mathematics concerns possible structures. The latter sort of position permits taking reference to possible mathematical structures to yield mathematical explanations of physical facts, without having to accept mathematical objects.[12] What is essential here is not so much whether pure mathematics concerns actual or possible structures, but that it is possible to take a fictionalist stance towards the mathematical objects that traditional mathematical realists accept, while accepting the relations between such hypothesized objects as objective mathematical truths. This in turn provides a way of allowing for mathematical explanations of physical facts without accepting mathematical objects. On this view mathematics may be seen as indeed playing an indispensable role in explaining physical facts, but as I have argued, even insofar as mathematics is considered to be supported through its role in the explanation of physical facts, this does not require accepting mathematical objects. As such, it may be said that there is a version of the indispensability argument highlighting the explanatory role of mathematics, but whose conclusion only requires the truth of mathematical statements, rather than the existence of mathematical objects, and which fits closely with Putnam's understanding of the indispensability argument (Putnam 2012).

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[12] It is here where the main difference with Pincock lies. To the extent that he may be said to take mathematical explanations to be concerned with abstract structures, he appears to link these essentially with objects, whereas I am suggesting the view that particular mathematical objects are not the fundamental subject matter, but rather provide a means of characterizing structural relations.

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Susan Vineberg is Associate Professor of Philosophy at Wayne State University. She received her Ph.D. in Logic and the Methodology of Science at UC Berkeley, where she wrote a dissertation on rational belief change. She has authored numerous articles on Dutch Book arguments and other topics concerning probability and decision. Her research papers also include work on realism and explanation in science and mathematics.

Address: Susan Vineberg, Department of Philosophy, Wayne State University (5057 Woodward Ave. Detroit, MI 48202. USA). Email: susan.vineberg@wayne.edu