Tool Migration: A Framework for Analyzing Cross-disciplinary Use of Mathematical Constructs

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Abstract. Mathematical formalisms that are constructed for inquiry in one disciplinary context are sometimes applied to another, a phenomenon that I call 'tool migration.' Philosophers of science have addressed the advantages of using migrated tools. In this paper, I argue that tool migration can be epistemically risky. I then develop an analytic framework for better understanding the risks that are implicit in tool migration. My approach shows that viewing mathematical constructs as tools while also acknowledging their representational features allows for a balanced understanding of knowledge production that are aided by the research tools migrated across disciplinary boundaries.

Keywords: Cross-disciplinarity, tool migration, epistemic risks

1. Introduction

Mathematical formalisms that are constructed for scientific inquiry in one disciplinary (or sub-disciplinary) context are applied to another. Philosophers of science have started paying attention to this cross-disciplinary aspect of scientific practice. For instance, the discussion of 'model transfer' concerns a relatively small set of mathematical models that are applied in multiple disciplinary contexts. Humphreys (2004) proposes that models that are transferred to study phenomena of a different domain owe their versatility to the computational tractability they afford. In contrast, Knuttila and Loettger (2014, 2016) suggest that in addition to tractability, versatile models also offer conceptual frameworks for theorization, which they label 'model templates.' However, these analyses do not deal with the risks inherent in this aspect of scientific practice. Consider the use and development of game theory in evolutionary biology as an example. In importing game theory, which was originally conceived to describe strategic interaction between rational agents typically studied by social scientists, evolutionary biologists may need to modify the theory in order to generate knowledge about presumably non-rational agents, at least in many cases. One can then assume that any changes to the theory--between its established applications in social sciences and its novel uses in evolutionary biology--require special attention so as to avoid misinterpreting an analysis.

Despite the advantages, there might be risks associated with using mathematical constructs across disciplines. In this paper, I ask: might there be patterns of transfer that may undermine the effectiveness of the imported mathematical formulation? What would these
patterns, if any, look like? This paper is an attempt to explore the conditions in which importing mathematical constructs may be epistemically risky. To begin, I develop a framework to systematically characterize the landscape of mathematical importations. The goal of such a framework is two-fold. Proximately, the framework captures characteristics of migration that the current terminology, such as 'model transfer' or 'importing/exporting,' fails to discern. Ultimately, with this additional discernibility, I suggest that one may start to explore and identify patterns of importation that may be subject to epistemic risks, such as misinterpretation of an outcome produced by using a imported mathematical construct.

In Section 2, I argue that one can view mathematical constructs in science in terms of 'research tools' and that transporting such tools across disciplines, which I call 'tool migration,' can in some cases be a disservice to science. Next, I classify tool migration based on two kinds of contextual details that bear significance to the effectiveness of the migrated research tool in a foreign context. In Section 3, I apply this approach to the use and development of game theory in evolutionary biology. Finally, in Section 4, I discuss in what ways this tool migration framework, which is essentially a typology of four types of tool migration, may help to characterize epistemically risky patterns of tool migration.

2. Theoretical Background
Although the notion of epistemic risks associated with migration of mathematical constructs has not been explicitly addressed, the idea of viewing mathematical constructs as research tools follows from the discussion on the ontology of scientific models. Ever since the shift of attention to scientific practice (e.g., Hacking 1983), there has been a growing literature in which models in science are viewed as entities detachable from theory and data (e.g., Morrison 1999; Morgan and Morrison 1999). One recent predecessor to my tool migration account is a pragmatic approach to scientific models put forth by Boon and Knuuttila (2008). In their paper, which uses examples from engineering, they argue that scientific models are better understood as 'epistemic tools' instead of as representations of some target systems in the world. Boon and Knuuttila's argument draws heavily on the epistemological roles of scientific models in relation to the scientists who use them. According to them, scientific models allow their users “to understand, predict, or optimize the behavior of devices or the properties of diverse materials” (2008, 687). Thus, for an ontological account of scientific models to be productive and realistic, as they argue, it should be sensitive to the relation between the models and the modelers, i.e., the tools and their users. An adequate evaluation of Boon and Knuuttila's argument will take us far afield, but my work will show that both the representational and the pragmatic aspects are indispensable to a better understanding of the epistemic risks in tool migration.

2.1 Viewing mathematical constructs as research tools
In general terms, any mathematical construct that is to be *used or operated* in an algorithmic manner, and the outcome of whose operation is to be *interpreted* in order to answer a research question, is an example of what I am calling a research tool. Let me first unpack the operational aspect of a research tool.

Let's assume that the proper use of any mathematical constructs employed in scientific research is expected to produce consistent results. To achieve this consistency, then, a well-defined procedure needs to accompany such a construct so that anyone who follows the procedure expects, and is expected, to obtain the same outcome given the same input. For instance, when performing a game-theoretic analysis, one goes through a sequence of steps, such as: (i) identify the players and the acts available to them, (ii) identify the payouts in every set of acts, (iii) find the ‘Nash equilibria,’ which refers to a set of acts, one for each player, in which no player could improve his or her payoff by unilaterally changing act. A similar algorithmic procedure can be seen when applying, say, Newton's law of gravitation:

\[ F_{\text{grav}} = G \frac{m_1 m_2}{r^2}. \]  

(1.1)

For example, the sequence of steps to obtain the magnitude of the gravitational force, \( F_{\text{grav}} \), between any two objects includes: (i) identify the mass of each object, (ii) identify the distance between them, (iii) complete the equation in which \( m_1 \) and \( m_2 \) refer to the masses of the two objects, \( r \) the distance in between, and \( G \) the gravitational constant. In these two examples, when the first two steps produce consistent input, the third step is expected to generate the same output.

Moreover, concerning the interpretational aspect of a research tool, the output of a series of symbol assignments and manipulations can be understood *only through the lens of some interpretation*. The Nash-equilibrium of a game is a meaningful 'solution' in virtue of the usual understanding of the game-theoretic formulation of a problem. Similarly, the meaning of the value obtained through completing the equation in (1.1) is derived from the usual interpretation of the quantities appearing in the equation and the theoretical context in which those quantities are defined.

Finally, assume that something can be viewed as a tool if it serves as a means to an end. In this case, then, mathematical constructs like game theory or mathematical formulas can be seen as research tools. In the case of applying a mathematical construct, the goal of performing a sequence of prescribed steps goes beyond merely completing the calculation and obtaining a result. Instead, the output is to be interpreted so that one may solve a problem, answer a research question, or gain knowledge about a subject-matter. Thus, a mathematical construct that prescribes algorithmic symbol manipulation can be seen as a research tool, assisting its users to meet an end. Manipulating symbols is a means to the end that was specified during the mathematical formulation of the research problem.
2.2 Epistemic risks of tool migration

Another predecessor to my account is Morgan's discussion of the re-situating of knowledge (2014). According to her, knowledge production is necessarily 'situated,' and consequently, applying a piece of knowledge outside its initial context requires effort - different contextual situations require different 're-situating' strategies. The term 're-situation' thus captures what scientists do in practice to transport locally generated knowledge across contexts. As she argues, to make an instance of scientific knowledge accessible outside its production site, one needs to establish inferential links between the production site and the destination site. However, she suggests, whether a re-situation of knowledge contributes to scientific progress depends on whether the transport secures some sort of inferential safety.

Building from Morgan's notion of the re-situation of knowledge, I argue that cross-disciplinary use of research tools is epistemically risky. Given the locality of scientific knowledge production, applying scientific knowledge outside its production site may come with epistemic risks. For example, between the production site and a destination site, there may be incongruent disciplinary characteristics (e.g., implicit theoretical assumptions) that fail to be captured by the inferential strategy, such that knowledge from the former cannot be transferred to the latter. Similarly, we can assume that the construction of a research tool is also situated in nature. Namely, a research tool is conceived to be operated and to extend our knowledge concerning a subject-matter given a particular disciplinary context. It follows that cross-disciplinary use of research tools is as epistemically risky as re-situating knowledge. That is, the epistemic reliability (i.e., general ability or tendency to produce knowledge) of some research tool in one disciplinary context does not necessarily carry over to another.

The concept of 'tool migration' captures both the 'situated-ness' of a research tool that was established in its native discipline and the effort it takes to 're-situate' the tool in a foreign discipline. Naturally, in the process of uprooting a research tool, significant contextual details—ranging from implicit expertise to important background assumptions—may be stripped away. Likewise, during re-situation, new features may be introduced to the tool so as to treat a different subject matter in a new disciplinary context. Together, due to the possibility of losing or gaining significant contextual details, or both, a cross-disciplinary tool migration risks undermining the effectiveness of the tool. These risks include, for example, misinterpretation of the research result or failure to produce genuine knowledge. Thus, it follows that tool migration can in some cases be a disservice to the production of knowledge.

Acknowledging these challenges, some have argued against the cross-disciplinary effort to integrate disciplinary knowledge (e.g., van der Steen 1993). Alternatively, one might try to overcome these challenges so long as the risks are better understood and managed. To understand the risks, I suggest that we first look at the patterns of tool migration. Among these patterns, we might find that some of them could be epistemically risky. Having established the
notions of research tools and risks involved with tool migration, I turn to the contextual details that are closely related to a tool's epistemic performance.

2.3 **Contextual details of a research tool: the target profile and the usage profile**
The construction of a research tool is necessarily situated within a context. In order to compare and contrast between the native (or established) context and the foreign context of a migrated tool, I single out two major types of details.

The first type concerns the assumptions about the entities that are studied by a subject-matter for which the tool is developed. For instance, game theory defines what it considers as a game, a player, or an act. For simplicity, I call *all* the assumptions that a tool makes about its target entities the tool’s 'target profile.'

The second type considers *the ways* in which one interprets the output from applying a tool in his or her research. In a game-theoretic analysis, for example, by following an algorithmic procedure, one obtains a solution of a game in the form of a Nash equilibrium. Depending on the game that one was analyzing, the solution could be understood as an explanation of economic behavior, or a prediction about it, or it could be used to optimize an strategic interaction. For simplicity, I call *all* the ways in which a tool is intended to be used, e.g., describing, predicting, optimizing, or explaining its target phenomenon, the tool's 'usage profile.'

Together, as I demonstrate in Section 4, the 'target profile' and 'usage profile' allow one to detect patterns of changes in the contextual details between the established use and the novel use of a research tool. They are able to do this because these two profiles offer a coarse resolution; looking through the lens of the target profile and usage profile, one zooms out from particular cases of tool migration so as to detect patterns of cross-disciplinary transport. Further analyses of these patterns will then shed lights on their associated epistemic risks.

2.4 **Four types of tool migration**
With the two profiles of a research tool and the two contexts in which the tool is used, i.e., a novel use and an established use, one can distinguish four types of tool migration.

First, compared to its established use, when a novel use of a tool catalyzes changes in both target and usage profiles, the tool migration is transformative, and therefore I call it a *tool-transformation*. Second, in contrast, when both target and usage profiles remain more or less intact after the migration, the tool's novel use is considerably similar to its previous applications. Thus, I call such a case *tool-application*. Between these two extreme types, there are novel uses of a research tool that alter only one of the two profiles but not both. When a tool changes its target profile but not its usage profile, I call it a *tool-transfer*, and when a tool changes its usage profile but not the target profile, I call it a *tool-adaptation*. See Table 1 for a summary.
### Table 1
A Typology of Tool Migration

<table>
<thead>
<tr>
<th>Between established and novel uses of a research tool</th>
<th>Usage profile remains</th>
<th>Usage profile deviates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Target profile remains</strong></td>
<td>'Tool-application'</td>
<td>'Tool-adaptation'</td>
</tr>
<tr>
<td><strong>Target profile deviates</strong></td>
<td>'Tool-transfer'</td>
<td>'Tool-transformation'</td>
</tr>
</tbody>
</table>

Among these four types of tool migration, tool-transfer is arguably the most familiar to the philosophers of science. Humphreys coins the term 'computational templates' to refer to a relatively small number of mathematical equations that are applied to investigate different domains of phenomena (2002, 2004). Bailer-Jones (2009) discusses such a scientific practice in terms of mathematical analogy. For one example, Newton’s law of gravitation was intentionally sought after to model electrostatic force (see Bailer-Jones 2009 for a detailed account). The important parallel between the two formulas, shown in (1.2), is that both types of forces (gravitational and the electrostatic) are proportional to the inverse of the square of the distance, \( r \), between two masses, \( m_1 \) and \( m_2 \), or two charges, \( q_1 \) and \( q_2 \). The constants that appear in both formulas scale the quantities to match empirical phenomena.

\[
F_{grav} = G \frac{m_1 m_2}{r^2} \quad \text{and} \quad F_{el} = k \frac{q_1 q_2}{r^2}
\]  

(1.2)

In contrast, the other three types of tool migration, despite prominent examples, are less explored in regard to their general features. One prominent example of tool-transformation is the development of game theory to be used in evolutionary biology.

### 3. The Migration of Game Theory From Social Sciences to Biology

In this section, I show in what sense the novel use of game theory in evolutionary biology, which is now known as 'evolutionary game theory' ('EGT') can be considered as a tool-transformation. I should mention that my account of the migration of game theory in this paper is not meant to address all the limitations of both game theory and EGT in their respective disciplinary contexts. Instead, the purpose of this account is to show that one can detect patterns of migration that have epistemic implications by focusing on the target profile and usage profile of a research tool.

#### 3.1 Game theory in social sciences

Game theory was initially formulated to mathematically model strategic interactions between intelligent, rational agents. In game theory, a game is defined as an interaction between two or
more players in which each player's payoff (e.g., profit) is affected by the decisions made by other players. Typically, such a game assumes both \textit{perfect information} and \textit{common knowledge}. \textit{Perfect information} assumes that all players know the entire structure of the game (all moves and all payouts) as well as all previous moves made by all players in the game (if it is an iterated or multi-move game). \textit{Common knowledge} is the assumption that all players know that all players have perfect information, and that all players know that all players know that all players have perfect information, and so on. That is, \textit{common knowledge} concerns what players know about what other players know. Moreover, the players also recognize that all players are cognizant that all players are rational, i.e., there is common knowledge of the game and of the \textit{unbounded rationality} of all players. As such, all players will act in the way that takes all other players' potential moves into account in order to maximize their odds of winning. In addition to these assumptions regarding the players of a game, the structure of a game, which refers to the combinations of each move and its payout, is usually summarized in a 'payoff matrix.' Typically, an analysis of a game aims to find out its 'solution,' a unique Nash equilibrium (or sometimes equilibria) of the game.

Game theory has been used in economics, as well as other social sciences, to describe, predict, optimize, or explain a variety of human interactions, such as the economic behaviors of firms, markets, and consumers (e.g., Brandenburger and Nalebuff 1995; Casson 1994) military decisions (Haywood 1954) or international politics (e.g., Snidal 1985).

\section*{3.2 Game theory in evolutionary biology}

Game theory was later used in evolutionary biology, where a game is understood as phenotypes (or heritable traits) in contest. In 1973, John Maynard Smith and George Price borrowed the formalism of a payoff matrix from game theory to mathematically model the evolution of phenotype frequencies in a population of organisms (see Grüne-Yanoff 2011). Their modeling method assumed that phenotypes are in contest with other phenotypes in a population of organisms. For instance, in a Hawk-Dove game, the contest is embodied by organisms with the phenotype of being aggressive and other organisms that are peaceful. In such a context, the payoff of a move is interpreted as the reproductive success of the phenotype (i.e., the number of copies it will leave to the next generation). Moreover, while the terminology such as 'game,' 'payoffs' and the formalism of a payoff matrix can be seen in the novel use of game theory in biology, the solution to a game in evolutionary biology is decidedly different from the Nash-equilibrium. An evolutionary game theoretic analysis typically looks for an evolutionarily stable strategy (ESS), i.e., a distribution of phenotypes in a population that is stable.

\section*{3.3 Epistemic implications of tool transformation}

It is clear that the target profile of game theory is no longer the same between its established use
in social sciences and its novel use in biology. First, none of the assumptions of perfect information, common knowledge, and unbounded rationality in what is now known classical game theory (CGT) remain in the novel use of game theory in biology. Second, the moves in EGT are heritable phenotypes exhibited by a group of organisms instead of acts available to players. Third, the payoffs in EGT are the reproductive success of the heritable traits. In this sense, the three assumptions concerning the players were stripped away from the tool - as a result of uprooting game theory from social sciences, and the heritability assumption about the moves as well as Darwinian fitness interpretation of the payoff were introduced to the tool - as a result of re-situating it to evolutionary biology.

Note that the change in the target profile forces a limitation to the usage profile of the migrated tool. For instance, nullifying the unbounded rationality assumption concerning the players, EGT can no longer be used to optimize a game, i.e., discovering the rationally optimal strategy, which is a common use of game theory in social sciences. For instance, in the prisoner's dilemma, the Nash-equilibrium is for both players to defect. This solution is often interpreted as a prescription for the game; the players are irrational not to defect. However, in a Hawk-Dove game, the ESS obviously has no such normative use. Because the 'moves' of being an aggressive type or a peaceful type are not 'chosen,' the idea of there being normatively better or worse choice of moves is therefore questionable. Moreover, the organisms are not assumed to be rational. Thus, while the players in the prisoner's dilemma could be said to be irrational for choosing to cooperate, this sense of normativity does not carry over to the evolutionary game theoretic analysis of the Hawk-Dove game. One would be mistaken to say that it is ‘irrational’ for the doves to be doves. Thus, the change in the target-profile of game theory, especially the stripping away of the unbounded rationality assumption, has resulted in how the migrated tool should or should not be used.¹

Moreover, applying EGT to study social phenomena (e.g., Axelrod 1984) or cultural evolution (e.g., Skyrms 2010) requires a careful re-defining of the terms (such as fitness) so as to avoid misinterpretation. Using EGT in social sciences, which can be considered as a ‘homecoming’ of the migrated tool, is not uncommon. However, the notion of payoffs in EGT refers to, roughly, the overall biological reproductive success of a group of organisms that exhibit a phenotype. Obviously in a social context, reproductive success of the members of some group is not, very often, the feature of interest. A careful reinterpretation of payoffs is thus needed in every analysis to prevent misleading conclusions.

¹ Of course, a more interesting prescriptive use of the ESS of a Hawk-Dove game might be, for example, to manage ecosystems for optimal predator-prey balance. Nevertheless, it should be noted that a justification for this type of prescriptive use of EGT would require further analysis because it is apparently not be derived from CGT.
To generalize, this example suggests that at least in some cases, a change in the target profile requires a corresponding change in the usage profile, or failure of producing genuine knowledge may follow. So far, I have shown that a solution of an ESS analysis may not be interpreted as an optimization to a Hawk-Dove game. Applying EGT to study social phenomena also requires careful treatment to the notion of payoff. Now if, hypothetically, some researcher were to make either of these two mistakes, his or her novel use of the tool would have been classified as tool-transfer - the novel use changes only the target profile without also changing the usage profile. It suggests that in some cases, tool-transformation may not be as risky as tool-transfer. I will come back to the issue of tool-transfer after some remarks related to the migration of game theory.

4. Contributions of the Tool Migration Analysis

The tool migration typology and its focus on tracking both similarities and differences meets the needs to sharpen discussions concerning inter- or cross-disciplinary use of research tools. Current literature seems to lack a framework to capture important, relational characteristics of the research tools that appear in multiple disciplinary contexts. For instance, 'tool-transformation' captures significant differences in details between CGT and EGT without losing sight of the contextual relationship between the two. In contrast, other terms in the literature, such as 'imports' or 'transfers,' fall short of doing so.

'Imports' signals the importation of research tools from a foreign discipline. In contrast, 'transfers' refers to the use of a scientific model, which was established to study phenomena of one domain, to study phenomena of a different domain. Neither term captures the migration of game theory to biology. As Grüne-Yanoff argues,

[B]iologists constructed the more sophisticated formal [evolutionary game theoretic] concepts themselves. One could speak of the import of formal concepts only with respect to very basic notions such as strategies or pay-off matrices, and it may be more appropriate to refer to formal inspirations rather than imports or transfers in these contexts. (2011, 392)

Moreover, I have suggested that a change in a tool's target profile without a corresponding change in the tool's usage profile may lead to misinterpretation and hence misuse of the tool. If this observation is generalizable, which is debatable, then it follows that cases of tool-transfer are epistemically riskier than cases of tool-transformation. On the other hand, if this observation applies only to some cases, it nevertheless reveals at least two epistemic implications concerning tool migration: 1) when the target profile changes, one must be careful not to draw conclusions that might be natural in the old context but may not make sense within the new context, given the new target, and 2) sometimes a change in target profile can, force a change in usage profile. Potentially failing to recognize when these changes occurred in a migration leads
to risky uses of the migrated tool.

Morgan (2011) has argued that while not all scientific knowledge travels far, those that travel with integrity (i.e., maintaining their content more or less intact during its travels) and travel fruitfully (i.e., finding new users or new functions) are considered to be traveling well. It is relatively easy to quantify the latter feature – one needs to look at just the number of a tool's novel applications. However, determining whether a tool has traveled with integrity is not straightforward. As a starting point, this proposed tool migration framework—especially its distinction between the target profile and the usage profile of a tool—provides a starting point that is crucial for assessing the integrity of a migrated research tool. With this framework, one may discover more patterns of tool migration that impact the epistemic integrity and, consequently, effectiveness of a migrated research tool in a foreign discipline.

5. Conclusion
I have argued that mathematical constructs used in science can be viewed as research tools and their cross-disciplinary novel use as tool migration. I have also argued that making novel use of established tools has its risks, but such an implication is not meant to deter cross-disciplinary sharing of tools. Indeed, certain important breakthroughs in the history of science are due to creative, unconventional, uses of research tools (e.g., the use of Fourier's mathematical treatment of heat to study electrostatics [Thomson 1842] or the use of Faraday's mechanical model of fluid motion to model the electromagnetic field [Maxwell 1861]). Versatile research tools are not rare in science. A framework of tool migration aims to offer not only a useful terminology to characterize the diverse landscape of their versatility but also a groundwork to investigate risky patterns of making novel use of established research tools. Finally, this tool migration approach shows that viewing these constructs as tools whilst acknowledging their representational features (i.e., as captured in their target profile) allows for a balanced understanding of knowledge production - especially those productions that are aided by research tools that have migrated across disciplinary boundaries.
References