ALOYSIUS LILIUS AUTHOR OF THE GREGORIAN REFORM OF THE CALENDAR

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Abstract

During the 16th century the disagreement between the dates of the Julian calendar, that had been in use since 46 BC and the vernal equinox, necessitated a correction to the computational rules used to regulate the flow of time. This was a very difficult task as it was necessary to resynchronize the civil time with celestial indicators, maintaining a lien adamant: the date of the vernal equinox, conventionally fixed perennially on March 21. In fact, during the Nicaea Council (325) the celebration of Easter was fixed on the first Sunday following the XIV Moon (Full Moon) belonging to the first month after the vernal equinox.

Aloysius Lilius, a physician, astronomer and mathematician, using imprecise astronomical data contained in tables from three centuries before, was able to elaborate a calendar that has stood the test of time. By the use of two equations he was able to synchronize the solar and lunar cycles and to develop a useful tool, named the *epact cycle*, to determine without uncertainty the Easter date. Furthermore, the Lilian method offers the possibility to correct the calendar according to the variation of the tropic year during time.

Unfortunately, only a few details of his personal life are known. Indeed, he has left only a few faint traces in public or private archives, so that today his name is almost unknown.

In this paper are reported the few details known of his life and a reconstruction of his plan for the calendar reform.

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1. Biography of Aloysius Lilius

Lilius was born during 1510 at Psycrön, today known as Cirò, a rich feud that made up a part of Citeriore Calabria. There is little record of his existence, indeed doubt has existed in the past to his origins in Calabria. In order to overcome such doubts that Lilius was indeed born in the town of Cirò, it is sufficient to read what the German Jesuit priest Christopher Clavius, a mathematician and a member of the Commission created by Gregory XIII to study the reformation of the calendar, wrote in 1603:

"If only he was still alive, Aloysius Lilius of Cirò, a man more than worthy of immortality, who was the principle author of a correction, so true that it splendours amongst the many other of his great discoveries".

Only guesses can be made about Lilius's early years, as the only record that exists comes from the 1530s. From a letter dated the 28th of January 1532, addressed to Lilius from the humanist Giano Teseo Casopero, it is clear that Lilius is no longer in Calabria but in Naples, where he is undertaking medical studies. This is confirmed by Juan Salon who in 1576 assigns to Lilius in his *Romani calendarii nova emendatione, ac Paschalis solennitatis reductione* the qualification of medical doctor as well as mathematician: "Aloysius Lilius Medicus execellentissimus & Mathematicus haud vulgaris Alfonsum Regem in anni quantitate *imitates, cyclum magnum* (...)".¹

In Naples Lilius was in service to the Carafa family, not having enough resources personally to continue his studies. After becoming aware of this fact Casopero, who probably having a good idea of Lilius's promise as scientist, sent him a letter in which he writes politely but firmly, reprimanding Lilius at the same time recommending that he dedicate his time solely to scientific study.²

This letter, dated Psycrò, V Kalendas februarii MDXXXII, represents one of the only two documents that provides direct proof to Aloysius Lilius's existence.

In Naples, Lilius was able to study in a very stimulating atmosphere. In that period (1532-1540), Naples was the home to numerous scholars of Calabrian origin, bound together by their common love for the classics.

Most of these came from the celebrated Parrasio School of Cosenza and attended the splendid Villa Leucopetra. This Villa, home to an actual Academia, collected the best minds of southern Italy, whom during this period had a notable role in the political, civil and religious life in Naples and Rome.

Due to the lack of any documentation, we are only able to hypothesise about Aloysius's life in Naples. No trace remains of either Aloysius, not even in the register of the University of Naples, as during the first half of the 16th century, there was no obligation to enrol or attend regularly. It was only after 1562, that the Spanish government ordered the university to register matriculation.³

After obtaining his degree in medicine, Aloysius Lilius moved to Rome, where using the scientific experience obtained in Naples, conceived and advanced the reformation of the calendar. The road to the Vatican state was probably smoothed by the knowledge obtained in Villa Leucopetra, or perhaps Guglielmo Sirleto who lived in Naples around the same time as Lilius played an important role.⁴

Guglielmo Sirleto, was born in 1514 in Stilo in Calabria.⁵ At the age of eighteen, he moved to Naples where he lived until 1539 creating a solid reputation as scholar. At this time, he moved to Rome where he met powerful Roman families, like the Carafa and Farnese. A

meeting with Cardinal Marcello Cervini (1501-1555), descendant of a Tuscan noble family, who later became Pope Marcellus II, was for him very important. The Cardinal took him into his service, trusting him with the schooling of the children of his family and in 1554 appointed him as custodian of the Vatican Library.

S. Carlo Borromeo, Marcellus II and Paul IV proposed Sirleto three times during the Papal elections, without success, as the times did not call for an academic Pope, but for a strong leader, pragmatic and above all else able to defend the Church from the threats posed by the Protestants and the Turks.⁶

Sirleto influenced greatly work undertaken for the reformation of the calendar undertaken by the Commission created by Gregory XIII. He was a great patron for his fellow countrymen and therefore it is reasonable to suppose that it was he who convinced Lilius to transfer to Rome, where he was introduced into the high ecclesiastical circles. We know almost nothing of Lilius's life in Rome due to the lack of any true documentation, on the contrary, on the basis of incontrovertible documentation, we know that he lived in Perugia during 1552.

A personal letter dated the 25th of September 1552 and addressed to Guglielmo Sirleto from the Cardinal Marcello Cervini, attests that around this period "*messer Aloysius Gigli*", was a teacher of medicine at the University of Perugia.⁷ In order to guarantee to Lilius an increase in salary, which was granted to the best teachers of the department, Cardinal Marcello Cervini requests Guglielmo Sirleto to intervene personally with Cardinal Girolamo Dandini a powerful exponent of the Church.

The two letters examined in this article, firstly, that signed by Giano Teseo Casopero and secondly, that from Marcello Cervini, are the only documents that provide sure evidence to the actual existence of the scientist from Cirò. In the absence of these two letters, it could be easily concluded that Lilius never existed, if not only in the imagination of his brother

Antonio.

The last years of Aloysius Lilius also remain a mystery. We only know that he died, exact date uncertain, before the implementation of the reform, leaving to his brother Antonio the task of making known his work. In the absence of actual data, most academics agree that he died in Rome in 1576, but we can say that, with good probability, his death came in 1574. In fact, it was in 1574 that Alessandro Piccolomini was informed of the hypothesis of reform not from Aloysius but from Antonio. This circumstance leads us to believe that Aloysius was already dead, but we do not know where he died because his tomb has never been found. ⁸

2. Gregory XIII appoints the Papal Commission for the reform

Pope Gregory XIII immediately after his appointment made great efforts to put into practise the various outcomes of the Council of Trent. The objective being to maintain in all the Christian nations harmony in the celebration of Easter and of all the festivities derived from such, and was in great haste to reform the old Julian calendar, with the primary objective, the restoration of the coincidence between the date of Easter and the dictates of Council of Nicea.⁹ To this end, he nominated a Commission of experts, with the task of investigating and approving a reformation project.

The date of the beginning of the Commissions work is not known exactly, as is not known either the members who took part from the beginning. Christopher Clavius, Professor of the Roman College, states in his *Explicatio*,¹⁰ that the Commissions' work lasted 10 years. If we take 1585 as the last year of the Commissions activities, because both Sirleto and Gregory XIII, died in this year, we can say that the Commission started its work in 1575. If however, we consider 1582 as the last year of work, because in that year the decree *Inter gravissimas*¹¹ was signed, then the Commission was inaugurated in 1572.

In the final report that was presented to the Pope, dated the 14th of September 1580, in addition to Cardinal Guglielmo Sirleto who was president, there are listed the names of the other eight members:

- *Vincenzo di Lauro*, from Tropea in Calabria, astronomer and medical doctor, Bishop of Mondovì, theologian expert.
- Christopher Clavius, German Jesuit, mathematician and professor of the College of Rome;
- *Pedro Chacòn*, Spanish theologian, expert in Patristics and history of the Church, that helped the Commission regarding movable feasts and the Martyrology.
- *Ignatius Nehemet*, Patriarch of Antiochia in Syria, expert in ecclesiastic dates and rituals of the eastern and western churches.
- Antonio Lilius, Doctor of medicine and art, Aloysius Lilius's brother.
- *Leonardo Abel*, of Malta, interpreter of eastern languages and witness to the presence and signature of Ignazio Nehemet;
- Serafino Olivari, French from Lione (born of a Italian mother), legal Counsel and auditor of the Rota.
- Ignazio Danti, Domenican Frier from Perugia, Bishop of Alatri, cartographer, mathematician and astronomer.

Aloysius Lilius does not appear amongst the representatives of the Commission, as he was no longer alive. All, with the exception of Antonio Lilius who had to be a person of great standing in the fields of astronomy and mathematics, came from church backgrounds.

During the numerous sessions that were held in Rome, other researchers gave their contributions to the debate on the reform. Amongst these figure: Tommaso Giglio, Bishop of Sora and treasurer to the Pope, was president of the Commission, but was shown to be incompetent and was replaced by Sirleto; Juan Salon, Spaniard, Franciscan observer, president of the congregation of the movable feasts; Giovanni Battista Gabio, greek professor at the Sapienza; Giuseppe Moleto of Messina to whom it was entrusted the task of reelaborating the Calendar tables.

The Commision examined various projects for the reform presented by: Pietro Pitati of Verona, Basilio Lupi and Antonio Lupi from Florence, Giustino Ristori, Giovanni Tolosani from Colle Val D'Elsa, Filippo Fantoni, Giovanni Padovani e Juan Salon. These proposals were all rejected and attention concentrated on an ingenious project for the reform of the calendar that had been elaborated by Aloysius Lilius. This project, presented by his brother Antonio, allowed for maintaining the vernal equinox on a fixed date, the 21st of March and also allowed for the precise determination of the date of Easter.

In 1575 Tommaso Giglio, the Commissions' president, requested various experts to examine Lilius's proposal. Amongst these was, apart from Juan Salon, who expressed a negative opinion, Giovanni Carlo Ottavio Lauro, a legal astrologer, who remained for several months in possession of Lilius's work, with the excuse of correcting the various errors. In reality, he used Lilius's work to produce his own proposal for the reform that was also rejected. On the 12th of november 1576, Tommaso Giglio was appointed arch bishop of Piacenza and the presidency of the Commission passed to Guglielmo Sirleto. The Commission finally accepted Lilius's work, and on the 5th of January 1578, it was printed as a *Compendium*, and sent by the Pope to the scientific community and to various Christian princes, inviting them to give a clear opinion. An important role was played by Vincenzo di Lauro, a famous doctor of medicine and mathematician, of Tropea in Calabria, who according to some, coordinated the Commissions work before Sirleto and until 1580, the date given for his transfer from Rome to Turin, where he took the role of Apostolic.



FIG. 1. This painting shows Pope Gregory XIII on the throne chairing the Commission for the reform of the calendar, while a member of the Commission (Antonio Lilius or Christopher Clavius) indicates his proposal showing some signs of the zodiac. Courtesy of State Archives of Siena (Biccherna, A.S. Siena, n.72).

3. The role of Antonio Lilius

As protagonist, Antonio Lilius both promoted and explained in detail to the Commission the work of his brother Aloysius. However, he did not limit his role to this and his contribution given to the reformation of the calendar was of fundamental importance. This can be deduced from the significant words of the Bishop of Siena Alessandro Piccolomini that, during his stay in Rome in 1574, discussed at length the reform with Antonio Lilius, with whom had become friends:

"(...) quite often I had the chance to speak with the distinguished doctor Antonio Lilius, brother of Aloysius Lilius; a man also expert in this type of study; in fact it was he the partner in the composition of the book in which is contained the new form of the calendar. Certainly, from his book this compendium has been made, given to us by your serene highness (...)".¹²

One month after having decreed the reform, the Pope in order to reward him for the work undertaken, with a decree of the 3rd April 1582 gave to Antonio and his heirs the exclusive rights to publish the calendar for a period of 10 years.

In "*Lunario Novo*" printed in 1582 by Vincenzo Accolti, one of the first examples of the calendar printed in Rome after the reform, the hand written signature of Antonio Lilius can be seen and the Papal authorization "*et permissu Ant(oni) Lilij*".¹³ The Pope successively revoked the decree on the 20th of September 1582 due to delays in printing, as Antonio was not able to meet the increasing demand for copies. Printing of the new calendar then became freely open to anyone.

Significant testimony to the role that Antonio played in the calendar reform can be seen in his sculptured image in bas-relief, on the monument dedicated to Gregory XIII situated in Saint Peter's Cathedral where a kneeling Antonio can be seen offering the book of the new calendar to Pope Gregory.



Fig. 2. Monument dedicated to Gregory XIII situated in Saint Peter's Cathedral in Rome where Antonio Lilius kneeling offers the book of the new calendar to Pope Gregory.

4. The proposal of Aloysius Lilius

Not only are Aloysius Lilius's biographical details obscure, uncertainty exists even regarding the work he undertook for the reformation of the calendar, since the autographic manuscript, that contained his calculations, was never printed, in fact it has been lost without a trace. What remains is a short booklet, the *Compendium*, which is a brief summary of his proposals. Initially, the Reform Commission thought of accompanying the reformation project with a book containing the details of the reform. The task was initially assigned to Antonio Lilius but, in order to save time, it was decided to draft a summary of Lilius's manuscript and this task was entrusted to the spaniard Pietro Chacòn.¹⁴

Since the Commission had collectively decided in favor of Lilius's project, the Compendium

was considered a work written by more than one person, thus ignoring its original author who is not mentioned at all in the text. The work was commonly known as the *Lilius Compendium* also by Lilius's contempories, but no part of the manuscript was written by him.

The work broadly entitled "*Compendium novae rationis restituendi kalendarium*", described the essential points of Lilius's manuscript; Chacòn does not however describe the manner in which Lilius had defined his reform, nor does he clarify the improvements made by the Commission to the original reform.

The compendium was printed in Rome in 1577 in the typographic workshop managed by the heirs to Antonio Blasio "*Impressores camerales*", under the care of Guglielmo Sirleto, Cardinal of S. Lorenzo in Panisperna. Copies of the *Compendium* were sent to various Christian princes and to important Universities and Academia with the invitation, written in the first few pages of the *Compendium*, to examine, add corrections or approve. Chacòn in the *Compendium* explains:

"After the Council of Trent, the reform of the calendar did not seem absolute and perfect in all its elements, if you do not add also corrections for the year and the ecclesiastic calendar. While Pope Gregory XIII with all willingness and reflection thought to resolve this problem, a book written by Aloysius Lilius was presented to him that appeared to propose a simple and easy method and a project to finish the reform. However, as this reform of the calendar is very difficult to do due to enormous astronomical problems and as up to now has not been resolved by all great mathematicians, the wise pope is convinced that it is necessary to have the opinion on this argument from all competent men in this science, until this work, that concerns all of the community, is brought to fusion with the favorable council and opinion of everyone. For this reason, he thought to send the book to all Christian princes until they, after having consulted expert mathematicians, were able to examine and approve the proposal or if they had found some error, they could correct and refine it. At the same time if someone has discovered in some place a better method, if it is communicated to the Pope, he will take it under serious consideration".¹⁵

Various experts in astronomy and mathematics examined the *Compendium* and sent their comments to their respective Universities and monarchs, who in turn forwarded these to Pope Gregory, along with their declarations. It was the secretary of State Tolomeo Gallio, Cardinal of Como, who received these comments that were then given to Cardinal Sirleto.¹⁶

The *Compendium* was long believed to have been lost. Merit for its rediscovery in 1975 belongs to the historian of Science and Technology at the Polytechnic Institute of New York, Thomas B. Settle. In 1981, Settle informed Gordon Moyer of its existence, who in turn, announced the startling discovery.¹⁷

The *Compendium*, which was found in the National Central Library of Florence, was inserted in a volume that contained various documents regarding the reformation and was catalogued as a work of unknown authors, even if in the first lines of text, there is a clear reference to the author: "*A book written by Aloysius Lilius*".

After Settle's discovery, other copies were then found in Florence, Rome and Siena. The original manuscript written by Aloysius Lilius, has however never been found and is probably lost.

A well-known expert in the field of the history of the calendar reform, Ferdinand Kalterbrunner, notes that "*it cannot be found in the Vatican Archives as probably it passed between the hands of Clavius and Antonio Lilius, as they had the task to prepare a complete explanation of the new calendar. Antonio, after much work, finished his book in 1585 but it was ever published*".¹⁸

Recently, Thomas Settle consigned a copy of the Manuscript of the *Compendium* written in Italian to the author of this article. It is contained in a dossier of 1670, belonging to Carlo di Tommaso Strozzi (1587-1670), that collects various manuscripts and is found in the National Central Library of Florence.¹⁹ The manuscript is well organized and with characteristics that give it authenticity dated close to the middle of the 1500s.



FIG. 3. The first page of the *Compendium* where Lilius presents his plan for the calendar. Courtesy of National Central Library of Florence.

5. The ten days lost forever

In 46 BC Julius Caesar, with his reform, had not been able to resolve definitely the problem associated with the confusion between the astronomical year and the civil year. At that time the duration of the tropic year was not well defined and the suggestion of the astronomer Sosigene, inventor of the Julian calendar, to introduce a leap year every 4 years, lead to a calendar year of 365.25 days.

During Sosigene's time, the tropic year was comprised of 365.24231 days and the difference with respect to the year of the Julian calendar was around 0.00769 days, given the difference between 365.25 and 365.24231, which corresponds to 11 minutes and 4.416 seconds.

The duration of the tropic year changes in time, about 0.5 seconds every 100 years; in 1500 it was 365d 5h 48m 48s, with a difference with respect to the Julian calendar of 11 minutes and 12 seconds. This difference in 25 periods of 4 years, corresponds to the bringing forward of the real position of the sun during the equinox, equal to 18h and 40m. After about 12.5 centuries from the Council of Nicaea this error brought the astronomical vernal equinox to occur around 10 days before (exactly 9g 17h 20m) the vernal equinox calculated from the Julian calendar. To be precise half way through the 1500s, the calendar showed the day of the vernal equinox to be the 21st of March, as decided by the Council of Nicaea, but astronomers had calculated the 11th of March, approximately 10 days earlier.

Additionally, the dates of the New Moon, not defined and put in relation with the *Gold Numbers*²⁰ shown by the calendar, were behind by more than 4 days. This difference had an immediate effect on the church feasts, in particular with respect to the celebration of Easter, the date of which is related directly to the vernal equinox. Clavius writes in the *Romani calendarij a Gregorio XIII P.M. restituti explicatio* that as a result of the incorrect vernal equinox and the determination of the XIV Moon (the moon of reference which determines the

date of Easter), Easter had been celebrated at times 5, or 8 or even 35 days late with respect to that which was decreed by the father of the Council of Nicaea.

The fact that Easter was not celebrated on the day of the year that astronomically coincided with the resurrection of Jesus Christ gave grave concerns to the Church, and in the absence of a common solution, warned of the threat of controversies and possible divisions as had occurred in the past. In order to avoid such difficulties and stop the accumulation of errors with time, it was necessary firstly to match the date of the spring equinox to the 21st of March; secondly it was necessary to precisely regulate the *New Moons* on which depended upon the *XIV Pascal Moon*; finally it was necessary to renew the *solar cycle* so that the *litterae dominicales*²¹ could be determined exactly and as a consequence the exact date of Easter.

Several Popes, not a few councils and many great scholars learned in the disciplines of mathematics and astronomy had all attempted in vain to agree the two periods of the lunar year and the solar year. This was a task essential in order to be able to predict perpetually the date of Easter.

Ptolemy, already in the second century, highlighted the errors contained in the Julian calendar. Roger Bacon in 1267 in his *De reformatione calendarii*²² had observed to Pope Clement IV an error of 9 days in the vernal equinox. Bede the Venerable in the 700s had already discovered the errors contained in the Roman calendar and the same was indeed also determined by Campano of Novara and the English monk John of Holywood. In 1344 Clement VI and 10 years later his successor Innocent VI gave the task of reforming the calendar to the best astronomers of the time. The councils of Constance and Basel, in the first half of the 15th century, instituted various Commissions for the reform. The problem was

tackled by many great astronomers and mathematicians and in 1436 Cardinal Nicolaus Cusanus, wrote of this argument in his *De reparatione calendarii*.²³

In 1476 Pope Sixtus IV, in order to bring about the reform, summoned to Rome Johannes from Koenigsberg known as the Regiomontanus, a famous astronomer and humanist, who immediately after his arrival, died in unusual circumstances. Finally, during the Lateran Council, under Leo X, many aimed to resolve the reform. Amongst these, emerged a principle figure in the German astronomer Paul of Middelburg, who was teaching in Padua in 1479 and afterwards between 1480 and 1494, was at the service of the duke of Urbin before becoming the Bishop of Fossombrone. His greatest work *Paulina, sive de recta Paschae celebratione et de die passionis domini nostri Jesu Christi*, written in 1513 was the basis used by the commission instituted by Leo X.²⁴

Other well-known astronomers had worked on the project, including Giovanni Solferino, Alberto Pighi and Luca Gaurico. In a famous letter written by Galileo Galilei to the Grand Duchess of Tuscany Cristina of Lorraine,²⁵ he states that Copernicus had been summoned to Rome from far way Germany in order to participate in the reform. It is known that Copernicus' role in this question was relatively limited. In fact he did not participate directly in the reform, but as noted in the list of correspondence written by Paul of Middelburg, attached to the conclusive report that he sent to Leo X, Copernicus was called to give his opinion. The content of the letter is not known but probably summarizes his knowledge and opinions regarding the effective duration of the tropic year. In the dedications to Paul III in *De revolutionibus*, Copernicus writes:

"Not long ago, under Leo X, when at the Lateran Council, there was talk of reforming the ecclesiastic calendar, this was not done because the measurement of the year and the months were not taken with precision. From that time I have retaken to study these aspects driven by the Bishop of Fossombrone Paul who was chairman of this question".

Copernicus did not believe it was possible to have a perfect calendar, as the solar year was too variable.²⁶ He attributed the variability of the solar year to the irregular movement of the equinox. It was for this reason in his *De revolutionibus* he based the most stable sidereal year corresponding to the time necessary for the earth to return to the same point of the orbit from which it had started taking as the reference the fixed position of a star in the sky. However, the problem of the calendar was to create a year that agreed with the seasons and not with the position of the earth in space.

Many proposals were put forward to resolve the problem of the calendar. Some proposed to celebrate Easter always on the same date every year (at the end of March), while others proposed a modification of the canon stabilized by the Nicaea Council and others, like that made by Paul of Middelburg, proposed the movement of the vernal equinox from the 10th to the 11th of March and to slowly recover the days lost from the Julian calender losing a day every 134 years. Veronese astronomer Girolamo Fracastoro wrote *Homocentrica* in 1538 underlining the importance of the reformation of the calendar.²⁷

Pietro Pitati was more precise in his *Compendium super annua solaris atque lunaris anni quanti tate* where he suggests to take into account the length of the tropic year *Alfonsine*,²⁸ but did not arrive at any practical solution. The Domenican Giovanni Maria Tolosane was also interested in the question of the calendar and in 1537 published a *Opuscolum de emendationibus temporum* in which proposed to fix the vernal equinox at either the 10th or 11th of March and to eliminate one day every 104 or 108 years.²⁹ In the end no proposal for the reform was accepted apart from that of Aloysius Lilius. Gregory XIII defined the reason adopted by the Church, in favor of Lilius's proposals in the bull *Inter gravissimas*.



Fig. 4. The first page of the decree Inter gravissimas with which Pope Gregory XIII on the 24th of February 1582 ordered the adoption of the new calendar. Courtesy of Central National Library of Florence.

The *bull* is an actual synthesis of the astronomical problem represented by the determination of the tropic year, which had been for centuries the true obstacle to overcome in order to obtain a correct reform of the calendar.

In the course of the centuries, every astronomer performed his own determination of the length of the tropic year.

In table 1 are reported the length of the tropical years attributed to different astronomers, their

transformation into a decimal number and the difference with the tropical year of our era.

				Calculated error
Year	Source	Measurement	Fractional	from the actual
			value	value of the tropic
				year
2000 2100	Measurement	265 a 5h 19m 15a	265 24219750	No orror
2000 - 2199	with atomic	505g 5ff 48fff 458	505.24218750	NO EITOI
1799	Delambre	365g 5h 48m 52s	365.24226850	+7s
1627	Kepler	365g 5h 48m 45s	365.24218750	No error
1582	Gregorian	365g 5h 49m 12s	365.24250000	+27s
	Calendar			
1574-75	Ignazio Danti	365g 5h 48m	365.24166667	-45s
1551	Prutenic	365g 5h 55m 58s	365.24719907	+7m 13s
1543	Copernicus	365g 5h 49m 29s	365.24269676	+44s
1252	Alfonsine	365g 5h 49m 16s	365.24254630	+31s
Ca 1440	Tables	365g 5h /19m 15s	365 24253472	+30s
Ca 1440	Chug Deg	505g 511 4911 155	505.2-255-72	1 503
Ca 1100	Omar	365g 5h 49m 12s	365.24250000	+27s
882	Al-Battani	365g 5h 48m 24s	365.24194444	-21s
499	Aryabhata	365g 8h 36m 30s	365.35868056	+2h 47m 45s
139	Ptolomy	365g 5h 55m 10s	365.246644351	+6m 25s
45 B.C.	Caesar	365g 6h	365.25000000	+11m 15s
185-127	Hipparchus	365g 5h 55m 12s	365.2466(66)	+6m 27s

TABLE 1. Length of the tropical years attributed to different astronomers.

Many astronomers had produced not only written works and correspondence that give a significant testimony to the study undertaken to find, through the determination of the tropic year, a solution to the disagreement between astronomical and civil time, but also highly ingenious and artistic works, like that of the Dominican Ignazio Danti, mathematician, astronomer and cartographer, who in order to observe the errors accumulated by the Julian

calendar, constructed in the Church of S. Petronio in Bologna a sun dial (since destroyed) with which he observed the solstice of 1576.³⁰ Danti also drew a horizontal sundial in the Vatican Tower of the Winds, in order to calculate the mismatch of the dates of the vernal equinox.³¹

The problem of the difference between the solar year and calendar year had been long accepted, but no-one before Lilius had been able to elaborate an accurate method for measuring effectively the length of the civil year, a problem which even today has not been completely resolved, even with the use of atomic clocks, given the complex variability in the earth's movements.

6. The Lilian Calendar

Aloysius Lilius was well aware of the astronomical debate regarding the true length of the tropical year, that had been debated through centuries. He believed that the development of a calendar based on sidereal year, as Copernicus and initially Clavius wanted, was far too complex to be translated into a readily usable instrument able to correctly account for the passage of time.

The elaboration of the Lilian calendar was based on the mean length of the tropical year,³² obtained using astronomical tables known as the *Alfonsine Tables*; as reported by Clavius in *Romani calendarij a Gregorio XIII P. M. restituti explicatio*, this was "365d 5h, 49m and 16s" that corresponds to 365.2425463 days.

The decimal number 365.2425463, approximated to 365.24254 can be resolved into:

365 + (0.25 - 0.01 + 0.0025 + 0.00004); the same numerical value can be written according to the fractional decimal formulation as:

 $365 + \frac{1}{4} - \frac{1}{100} + \frac{1}{400} + \frac{4}{100,000}$.

A civil year is necessarily composed of 365 days as an entire number. When a calendar formulation is based on the entire number (365) excluding the fractional components (+1/4 - 1/100 + 1/400 + 4/100.000), the following errors will be obtained:

minus 1 day every 4 years;

plus 1 day every 100 years;

minus 1 day every 400 years;

minus 4 days every 100,000 years (that means minus 1 day every 25,000 years).

It is likely that Lilius, "*Medicus execellentissimus & Mathematicus*", was able to obtain an accurate estimation of the mean length of the calendar year through a careful evaluation of the decimal number 365.2425463. In order to remove those shift mistakes, Lilius established the leap year schedule using the following corrections following the list of possible errors presented above:

1) To correct the shift "*minus 1 day every 4 years*" Lilius suggested maintaining a leap year for every year identified by a number divisible by 4, as it had under the old Julian calendar. The added day was the 29th of February;

2) To correct the shift "*plus 1 day every 100 years*" he suggested the cancellation of the leap day for the centenarian years, even if they resulted divisible by 4;

3) To correct the shift "*minus 1 day every 400 years*" he suggested the introduction of a leap day for the years whose number was divisible by 400 (that means for the numbers constituted by the first two figures divisible by 4);

4) No correction was added for the last case (minus 1 day every 25,000 years) even if the addition of 4 days every 100,000 years should be required.

According to the above plan, 1980 and 1984 resulted as leap years (*bis-sextile years*); years 1800 and 1900 were not leap years; the same for the future year 2200. Additionally, the past and future centenarian years 1600, 2000, 2400, 2800 etc., are scheduled as leap years.

Summarizing in a single sentence the general intercalation rules adopted by Lilius results as follows: if the identifying number is not divisible by 4, it will be a common year composed of 365 days; on the contrary a year divisible by 4 will be a leap year. Certain exceptions to these simple rules referred to centenarian years: among these, only those characterized by the number, after removing the two zeros, divisible by 4, were scheduled as leap years.

Therefore, to avoid the progressive increase of shift mistakes, the following changes to the Julian calendar were adopted: 3 days every 400 years were removed while the Julian rule of a leap day (1 day every 4 years) was maintained. Furthermore, the centenarian years, which were all leap years in the Julian calendar, were converted to common years (with the exception as previously described).

Actually we currently know that the tropic year of the present age is composed of 365.24218750 days, that means 365d 5h 48m 45s. This value can be approximated to 365.2422 and can be resolved as:

365 + (0.25 - 0.01 + 0.0025 - 0.0003).

This equation could be presented at the same time by the following decimal fractional form: 365 + 1/4 - 1/100 + 1/400 - 3/10,000.

The rigorous use of this fractional equation, starting from the numerical value of 365 days for a civil year, leads to the following errors in a hypothetical calendar formulation:

minus 1 day every 4 years;

plus 1 day every 100 years;

minus 1 day every 400 years;

plus 1 day every 3333 years.

The first, second and third errors can be corrected as previously according to the *Alfonsine year* approximated to 365.24254; in order to avoid the fourth error *plus 1 day every 3333 years*, a further 3 leap years every 10,000 should be removed.

To obtain an even higher level of accuracy in the calculation both the decrease of the tropic year extension and the increase of the day extension need to be taken into account; nevertheless, this correction represents only a theoretical modification: in fact, the time extension range involved is over 20,000 years.

To be correct, 400 years of Lilius's Calendar includes 146,097 days; this value, divided by 400, leads to 365.2425d (year extension) that means 365d 5h 49m 12s:

 $(400 \times 365 + 100 - 3) \div 400 = 365 + 1/4 - 3/400 = 365.2425$ days.

The new civil calendar year introduced by Lilius is based on old astronomical data (recorded in approximate tables) and on a basic approximate value, despite this, the corresponding result is extremely precise, that means only 26,7648 seconds higher than that of the tropic year extension related to the current age (Age 2000-2199, extension: 365d 5h 48m 45s = 365.24218750).

Historians and scientists are at a loss as to the method used by Lilius to calculate the mean civil calendar year value of 365.2425 (365d 5h 49m 12s). In fact Pietro Chacon in the *Compendium* does not give any explanation about the mathematical procedure adopted by Lilius.

The only reliable values available to Lilius were: the Alfonsine value of 365.24254630d (365d 5h 49m 16s); Copernicus's contribution of 365.24269676d (365d 5h 49m 29s); and Reinhold's value (included in the *Prutenic Tables*) of 365.24719907d (365d 5h 55m 58s);

none of the above values gave a precise intercalation of 97 days every 400 years (datum achievable only when the medium year extension is that used by Lilius, i.e. 365.2425000) It is known that Lilius chose the length of the *Alfonsine year* as a reference (365g 5h 49m 16s); Copernicus's and Reinhold's year values were respectively 13 seconds and 7 minutes longer than the Alfonsine value.

The difference between the *Alfonsine* and *Julian years* is 11m and 12s; this gives a shift increase of 3.11 days over 400 years.

Many researchers have suggested that Lilius approximated the increase of 3.11 days to the value of 3; the value of 3 corresponds to the 3 days subtracted every 400 years. The *Alfonsine year* length is shorter than that of the Julian by 1/134 parts of a day. The extension of 1 day corresponds to 86,400 seconds, which divided by 134 gives 644 seconds (or 10 minutes and 44 seconds). Summarizing this calculation the Julian calendar was going to lose 3 days every 400 years".³³

The Commission of the Calendar in its report underlined the removal of 1 day every 134 years that equals 3 days every 400 years, but nothing else is reported about the calculations used by Lilius. M. Swerdlow suggested that Lilius could have used every mean tropic year value available at the time for his calculations (*i.e.* the Alfonsine mean tropical year of 365d 5h 49m 15.58s, the mean tropical year value introduced by Copernicus of 365d 5h 49m 16.28s or the mean value included inside the Prutenic Tables of 365d 5h 49m 15.45s); in fact the difference between them is about 1 second while the shift respect to Lilius's calculation is counted at 4 seconds. ³⁴ In his papers Swerdlow stated that all of these values are equal in the sexagesimal fractional equation form up to the second position:

365; 14.33.

The value conversion of (14 + 33/60)' (that is [(14+33/60)/60]d) from sexagesimal to decimal form leads to 365 plus 97/400, that corresponds to 365.2425.

Four centuries after Lilius's corrections, Swerdlow thought to have found a resolute explanation concerning the debate regarding the tropical year used by Lilius. In reality this remains an open question; in fact Swerdlow's theory is not supported by any documents and it was in contrast with respect to the Compendium text in which Chacon stated: *"For he himself proposes the length of the Alfonsine years, because it is an average of the various measurements and therefore less subject to error"*. This means that Lilius based his intercalary plan on the length of the Alfonsine year.

It is likely that Lilius's calculation steps can be summarized as follows: starting from the Alfonsine-based decimal value, he converted it into the fractional equation; then he imported the corresponding corrections to the Julian calendar.

It can be suggested that Lilius's starting point was the value of 365.2425 (decimal number on the basis of Alfonsine based mean tropical year value).

Only by the discovery of the original Lilius's manuscript (containing all of his mathematical calculations) can this hypothesis be proved. It is likely that the original calculations would appear simple by current astronomical and mathematical knowledge. The reader should keep in mind the scarce availability of mathematical instruments at the time. Indeed the first mathematical resolution of fractional decimal numbers in Europe occurred in 1582; the author was the mathematician Stevin who according to the basis introduced by the French Viète, in 1579, proposed the substitution of the sexagesimal fraction with a decimal fraction. The introduction of the point to decimal number representations has been attributed to either Giovanni Magini in his work *De planis triangulis* (1555-1617) or to Christopher Clavius in a specific *table of sines* in 1593³⁵. The cited works were subsequent to Lilius's reform, but if

that proposed by Swerdlow is true (the conversion of (14 + 33/60)) from sexagesimal to decimal form) then we must admit that Lilius had already used long before decimal fractions. Concerning the equinox shift of spring caused by the Julian calendar, Lilius suggested dropping 10 days from the calendar schedule; in this way equalling the series of days lost and fixing precisely the equinox date (21^{st} of March). Two different approaches could be adopted to introduce this kind of correction: the complete introduction from the beginning of the revised calendar or a gradual insertion over an extended period (from 1584 to 1620); both of these possibilities are mentioned in the *Compendium*, reported inside the table named *Tabula Aequationis Epactarum (a part of the Tabula Epactarum expansa*), in which the leap years were labelled as "B"; furthermore the correction to the "Epact" was labelled with a specific alphabetic letter.

The adjustment period for leap years of the Julian Calendar is reported for the centenarian years up to 4300 (the case of 10 days gradual removal) and up to 5000 (case of 10 days instantaneous removal).

Clavius suggested the jump from the 4th to the 15th of October during 1582. Responding to Michael Maestlin, Clavius explained that there was nothing mysterious in the choice of month: October contained the fewest religious observances and was therefore the least difficult to change for the Church.³⁶

Lilus's corrections were not limited to matching the civil and astronomical year of the time; his calculations gave us a very accurate instrument to correct any variation of the tropic year extension. The Compendium text explains well this concept:

"But if the Alfonsine calculations seems to anyone too unreliable to be trusted, and he thinks rather that reliance should be given to modern calculations, he will understand that the arrangement and ordering of this ingenious Cycle and the table of the Epactes devised by Lilius is such that it can easily be adapted to the calculations of Copernicus or anyone else, if only a little table of corrections made from them be substituted for the one that we report in the margin".

The same concept was reported inside the decree *Inter gravissimas* document; this underlines that Lilius was perfectly aware of the tropic year variation; in fact he established the possibility to modify again the "intercalation rule" in the case that another more accurate year extension would be proposed.

Lilus's calculations lead Clavius to send the *Compendium* text to the scientific community in order to obtain a critical opinion about the intercalation rules using either the tropical year of Copernicus, or the mean tropical year Alfonsine.

At the time most Church members and scientists had not understood the revolutionary concept of the universe reported by Copernicus in *De revolutionibus; Lilius*'s opinion about it is not known. However, we know that Lilius, using all astronomical data collected by Ptolemy up to Copernicus's time, elaborated a calendar correction matching at the same time the complex and variable movements of the Earth, independent from both geocentric and heliocentric models of solar system.



FIG. 5. Tabula Epactarum expansa (contained in the Compendium), which determines the *epact* of a given year.Courtesy of Central National Library of Florence.

7. Easter reckoning: the Epact Cycle

Together with the civil calendar correction, Lilius also had to solve the difficult task of dating the New Moons. This issue represented the most interesting part of the reform; as scholars of the time needed to find the right correction in order to maintain the date of Easter as established by the Council of Nicaea: Christian Easter has to be celebrated on the first Sunday after the Full Moon following the spring equinox.

Before the calendar reform the lunar phases were determined with the help of the *Metonic cycle*. At the end of the V Century B.C., the Greek astronomer Metone discovered that 235 lunar months equals exactly 19 solar years. Therefore, after a cycle composed of 19 years (named "*Metonic cycle*" or "*lunar cycle*") the lunar phases correspond to the same days of the solar year; so it is possible to maintain a "solar – lunar calendar".

Metone divided the scale of time in 19 year steps and named each period with a number ranging from 1 to 19 ("Golden Number") corresponding to a specific lunar period.

The exact dating of Christ's resurrection was a debated issue between the eastern and western churches. During the IV Century, they had celebrated Easter on different days seven times. The calculation of the date of Easter was related to the lunar year, shorter with respect to the solar one by about 11d and 6h.

On the western side, Saint Ippolito (3rd Century) first tried to schedule Easter; the African Augustalis (second half of the 4th Century) and other scholars followed him. However, in the east the Easter celebration day was established according to the calculations made by Anatolius of Laodicea (second half of the 2nd Century), followed by Theophilus of Alexandria and Cyril of Alexandria (end of the 4th Century). Only a few details about this calculation are available but it's likely that the *Metonic cycle* had been used.

In the year 525, as required by Bonifacius, the notary belonging to the court of Pope John I the monk Dionysius Exiguous calculated the Easter day, by labelling each year with one "*Golden Number*" then he formulated a table containing all the new moon days with the corresponding "*Golden Number*", the "*Epact number*"³⁷ and finally the "*Easter Term*".³⁸ That table allowed for the calculation of the date of the Easter just by knowing the Easter Term.

Everything seemed to have been resolved; but the slower flowing of the lunar cycle with respect to solar cycle of 19 years was rapidly discovered.

Lilius calculated that the difference between solar and lunar cycle was about 1h 27m 28s in 19 years; that means a lunar shift from the lunar – solar calendar of 1 day every 312.8 years.

In the period of 1570 that value had resulted in a shift of 4 days. Clavius found a different shift value equal to 1h 27m 31s 9d, that means 1 day anticipation of the lunar phases every 312.5 years. Therefore, in the middle of 1500 the lunar months resulted 3 days in advance with respect to those previously established by the Church. The Commission eventually accepted this Clavius's correction that is the only modification to Lilius's calculations.

Furthermore, Lilius decided to revise the *Metonic cycle* and he elaborated a new method to avoid the lunar month shift of 1 day every 312.5 years (this value was fixed by the Commission) and obtained from the initial value of 312.8. He introduced a "solar equation", starting from the removal of 3 leap years (this means minus 3 days in the calendar schedule) every 400 years; he shortened the "*Julian Epact*" of 3 days every 400 years; furthermore he formulated a "lunar equation" capable of increasing the *epact* value of 8 days every 25,000 years. The formulation, with modern mathematical notation, can be expressed as follows:

$$E = G - S + L;$$

S = 3C/4; L = (8C + 5)/25

where:

G represents the *Epact* deriving from the Julian calendar;

S is the solar equation used to remove the 3 days every 400 years (3C / 4);

L is the lunar equation, related to the difference between the 19 years extension and 235 lunar months;

C is the number of the century considered;

Value 25 represents the secular part of 2500 years, while the *epact cycle* increase of 8 units; the value number 5 is the *Julian epact* correction up to 2100.

In the *Metonic cycle*, every *Golden Number* was related to an *epact* value characterized by only 19 possible numbers, while the lunar months were considered closed alternatively every 29 and 30 days.

Lilius attributed 30 possible values to his new *epact*, to define the day of the new moon for every month of the year. By knowing that, he was able to define the day of the *Pascal Full Moon* and therefore the Easter day celebration. Based on this method of correction, the Easter celebration is always located in the period ranging from the 22^{nd} of March to the 25^{th} of April. After 300,000 years the Lilius's *epact tables* repeats itself. The period of *epacts* is 5,700,000 years. This value is the result of the following equation: 19 (*Metonic cycle*) x 400 (Lilus's *solar cycle* equivalent) x 25 (cycle related to the *epact*) x 30 (*epact* values).³⁹

The calculation of Easter day represents the most complex part of the entire work. For instance the great mathematician Carl Friedrich Gauss, who investigated this question during the first part of his career, was not able to develop an exact mathematical algorithm for determining the date of Easter. The mathematician Ciccolini revised and corrected the Gauss algorithm in 1817.⁴⁰

In summary, Lilius's contribution lead to the removal of 10 days from the Julian calendar;

furthermore only the centenarian years divisible by 4 remained as leap years. Finally, the *Metonic cycle* was substituted by the *epact cycle*.

The Gregorian calendar came into use in Italy, Spain, Portugal and Poland on the 15th of October 1582 and was adopted by the other catholic states over the following 5 years. The calendar was not accepted by protestant countries until the XVIII century. The Swiss Cantons, protestant Germany, Denmark, Norway and the Netherlands adopted the calendar during the 1700s, followed by England and Ireland in 1752 and by Sweden in 1753.

Last to adopt the calendar were the orthodox countries that accepted the new calendar after the end of the first world war, however only for civil purposes while the Julian calendar was still used for religious purposes. Bulgaria joined with the other states in 1917, Russia in 1918, Serbia and Romania in 1919, Yugoslavia in 1923, Turkey in 1927 and lastly Greece in 1928. Outside of Europe, the Japanese adopted the calendar in 1873 and the Chinese in 1911. To this day both Hebrews and Muslims have refused to adopt the Gregorian calendar.

Conclusions

Five hundred years have passed since the birth of Aloysius Lilius, an extraordinary mathematician and astronomer; unfortunately, only few details of his personal life are known. His name and cleverness have remained unknown to most of the world because his calendar reform is associated with the name of Pope Gregory XIII who promulgated it in 1582. Yet the genius of Lilius is unquestionable; notably, as he lived at a time when astronomical and mathematical knowledge was only in gestation. Indeed, Lilius was already dead when Galileo introduced the Copernican system and scientific methodology. Lilius's work preceded the Thyco Brahe model of the non-perfect and non-immutable Universe; he also preceded the

rules of planetary motion of Kepler and the gravitational laws of Newton.

Nevertheless, he was able to solve important problems such as both the correction of the civil calendar and the exact reckoning of the date of Easter. During the Nicaea Council (325) the celebration of Easter was fixed on the first Sunday following the XIV Moon (Full Moon) belonging to the first month after the vernal equinox. A long time before the birth of Modern Science, Lilius was able to elaborate an almost perfect calendar. By the use of two equations, he was able to synchronize the solar and lunar cycles and to develop a useful tool, named the *epact cycle*, to determine the Easter date. Furthermore, the Lilian calculations offer the possibility to correct the calendar according to the variation of the tropic year during time.

We don't know exactly how Lilius realized such a great invention. Notably, at the time the fractional decimal numbers were not in use, they appeared from 1582; starting from 1593 a symbol known as "point" was introduced by Clavius to indicate decimal numbers.

For this reason, the discovery of Lilius's original manuscript would be of great importance in particular to help better understand his plan for the calendar reform.

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