Realism about the Wave Function

Eddy Keming Chen*

Draft for *Philosophy Compass*. October 13, 2018

Abstract

A century after the discovery of quantum mechanics, the meaning of quantum mechanics still remains elusive. This is largely due to the puzzling nature of the wave function, the central object in quantum mechanics. If we are realists about quantum mechanics, how should we understand the wave function? What does it represent? What is its physical meaning? Answering these questions would improve our understanding of what it means to be a realist about quantum mechanics. In this survey article, I review and compare several realist interpretations of the wave function. They fall into three categories: ontological interpretations, nomological interpretations, and the *sui generis* interpretation. For simplicity, I will focus on non-relativistic quantum mechanics.

Keywords: quantum mechanics, wave function, quantum state of the universe, scientific realism, measurement problem, configuration space realism, Hilbert space realism, multi-field, spacetime state realism, laws of nature

Contents

*Department of Philosophy, 106 Somerset Street, Rutgers University, New Brunswick, NJ 08901, USA. Website: www.eddykemingchen.net. Email: eddyckm@gmail.com

1 Introduction

Quantum mechanics is one of the most successful physical theories to date. Not only has it been confirmed through a wide range of observations and experiments, but it also has led to technological advances of a breathtaking scale. From electronics and optics to computing, the applications of quantum mechanics are ubiquitous in our lives.

As much as it has given rise to technological innovations, the meaning of quantum mechanics remains elusive. Many curious features of quantum mechanics, such as entanglement, non-locality, and randomness, are taken to be *prima facie* challenges for a clear understanding of quantum mechanics. These puzzles are related to the *wave function*, the central object in quantum mechanics. Understanding the meaning of quantum mechanics seems to require a good understanding of the meaning of the wave function.

What does the wave function represent? That is the main concern of this survey article. The answer to that question is complicated by the fact that the wave function does not look like anything familiar. It is a function defined on a vastly high-dimensional space, with values in complex numbers, and unique only up to an "overall phase." Nevertheless, we have devised many ways of using wave functions in making predictions and explaining phenomena. We use wave functions to calculate the probabilities of microscopic and macroscopic behaviors of physical systems. These led to the successful explanations of the double-slit experiment, the Stern-Gerlach experiment, and the stability of the hydrogen atom. The wave function is indispensable for making these predictions. However, the predictions are probabilistic. (More on this later.)

Roughly speaking, there are three main views about the wave function:

Instrumentalism: The wave function is merely an instrument for making empirically adequate predictions.

Epistemicism: The wave function merely represents the observer's uncertainty of the physical situation.[1]

Realism: The wave function represents something objective and mind-independent.

[1]The recently published theorem of Pusey et al. (2012) shows that a certain class of epistemic interpretations of the wave function are incompatible with the empirical facts.

In this article, I focus on the realist interpretations of the wave function. They seem to be the most interesting and promising ways of understanding quantum mechanics.

Let me make four remarks. First, the meaning of the wave function is related to solutions to the quantum measurement problem. Hence, we will start in §2 with an introduction to this topic, along with some mathematical preliminaries. Second, I left the definition of *realism* open-ended. This is because we will consider proposals for specific versions of realism about the wave function. The proposals are grouped into three categories: ontological interpretations (§3), nomological interpretations (§4), and the *sui generis* interpretation (§5). Third, because of the prevalence of quantum entanglement, "the wave function" should be understood to refer to the wave function of the universe, or the universal wave function. The wave functions of the subsystems are thought to be derivative of the universal one. Fourth, for simplicity, I will focus on non-relativistic versions of quantum mechanics.[2]

The issues taken up here are continuous with the general question about how to interpret physical theories. They offer concrete case studies for scientific realism, and they might be useful for philosophers of science, metaphysicians, and anyone with an interest in understanding quantum mechanics.

2 Background

In this section, we will review some basic facts about the wave function and its connection to the probabilistic predictions. We will then consider the quantum measurement problem and three realist theories that solve it. The upshot is that the wave function occupies a central place in their descriptions of physical reality.

2.1 The Wave Function

It will be useful to have a brief review of classical mechanics. To describe a classical mechanical system of N particles, we can specify the position $\boldsymbol{q}$ and momentum $\boldsymbol{p}$ of each particle in physical space (represented by $\mathbb{R}^3$). We can represent the classical state of an N-particle system in terms of $6N$ numbers, $3N$ for positions and $3N$ for momenta. The classical state can also be represented as a point in an abstract state space called the *phase space* $\mathbb{R}^{6N}$. Once we specify the forces (or interactions) among the particles, they evolve deterministically, by the Hamiltonian equations of motion:

$$\frac{\partial \boldsymbol{q}_i}{\partial t} = \frac{\partial H}{\partial \boldsymbol{p}_i}, \frac{\partial \boldsymbol{p}_i}{\partial t} = -\frac{\partial H}{\partial \boldsymbol{q}_i}, \tag{1}$$

where H stands for the Hamiltonian function on the phase space, which is a short-hand notation that encodes classical interactions such as Newtonian gravitational potential and Coulomb electric potential. The Hamiltonian equations are differential equations, and the changes in the particles are obtained from taking suitable derivatives of H. In this sense, H is the generator of motion. For every point in the phase space, H

[2]For complications that arise in the relativistic theories, see Myrvold (2015).

generates a curve starting from that point. In other words, for every initial condition of the N particle system, H determines the future trajectories of the particles.

Now let us introduce the quantum mechanical way of describing a system of N "particles."[3] Instead of describing it in terms of the positions and momenta of N particles, we use a wave function for the system. The wave function represents the *quantum state* of the system. In the position representation, the wave function, denoted by $\psi(q)$, is a particular kind of function from configuration space $\mathbb{R}^{3N}$ to complex numbers $\mathbb{C}$. Let us elaborate on this definition:

- Domain: the domain of the wave function ψ is $\mathbb{R}^{3N}$, or N copies of physical space $\mathbb{R}^3$. N is the total number of particles in the system. When N is large, $\mathbb{R}^{3N}$ is vastly high-dimensional. Each point in $\mathbb{R}^{3N}$ is an N-tuple $(\boldsymbol{q_1}, ..., \boldsymbol{q_N})$. Each $\boldsymbol{q_i}$ corresponds to particle i's position in physical space $\mathbb{R}^3$. Hence, the N-tuple lists the positions of N particles. We use a point in $\mathbb{R}^{3N}$ to represent a particular configuration (arrangement) of N particles in $\mathbb{R}^3$. Hence, $\mathbb{R}^{3N}$ is called the configuration space.[4] The wave function $\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N})$ is a function whose domain is the configuration space, which is vastly high-dimensional when the system has many particles.

- Range: the range of the wave function ψ, in the simplest case, is the field of complex numbers $\mathbb{C}$. A complex number has the form $a + bi$, where $i = \sqrt{-1}$; in polar form, it is $Re^{i\theta}$, where R is the amplitude and θ is the phase.[5]

- Restrictions: the wave function is a particular kind of function from $\mathbb{R}^{3N}$ to $\mathbb{C}$. It has to be a "nice" function that we can take certain operations of integration and differentiation.[6]

- Abstract state space: each wave function describes a quantum state of the system. The space of all possible quantum states is called the *state space* of quantum mechanics. The state space will include all possible wave functions for the system, that is, all the "nice" functions from configuration space $\mathbb{R}^{3N}$ to complex numbers $\mathbb{C}$. The Hilbert space is the abstract mathematical space that we use to describe such a state space. The Hilbert space is a high-dimensional vector space, in which each wave function is represented as a vector.

In classical mechanics, the state of a system is represented by the positions and momenta of all the N particles (a point in phase space) that changes deterministically

[3] In some ways of thinking about quantum mechanics, particles are not fundamental. Hence the quotation marks.

[4] This is the ordered configuration space, in which a permutation of the particle labels creates a different configuration. If the particles are indistinguishable, then it is more natural to use the *unordered configuration space*, ${}^N\mathbb{R}^3$. This has implications for the nature of the wave function. See Chen (2017) and the references therein.

[5] If we include spinorial degrees of freedom, the range is $\mathbb{C}^k$. We set spins aside in this paper.

[6] It has to be "square-integrable." That is, if we take the square of the amplitude of the wave function value at every point, and integrate over the entire configuration space, we will get a finite value. This is to ensure that we can normalize the squared value of the wave function to 1 so that it has meaningful connections to probabilities. To ensure that we can take suitable derivatives on the wave function, we often also require the wave functions to be sufficiently *smooth*.

according to (1). If the wave function represents the quantum state of a system at a time, how does it change over time? It obeys another differential equation called the *Schrödinger equation*:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi, \tag{2}$$

where i is the complex number $\sqrt{-1}$, $\hbar$ is the Planck constant divided by 2π, and $\hat{H}$ is the Hamiltonian operator that encodes the energy and fundamental interactions in nature. It is also deterministic: given any vector in the Hilbert space, the Schrödinger equation (2) produces a determinate curve in the Hilbert space. Another feature of (2) is that it is linear: if ψ_1 and ψ_2 are solutions to the equation, then their linear combinations are also solutions to the equation. A surprising consequence of linearity is that, in the Schrödinger's cat thought experiment, the cat can be in a superposition of the alive state and the dead state.

$$\psi_{cat} = \frac{1}{\sqrt{2}}\psi_{alive} + \frac{1}{\sqrt{2}}\psi_{dead} \tag{3}$$

A cat in this quantum state is not alive, and it is not dead. The linear Schrödinger equation (2) ensures that the wave function of the system will not change into ψ_{alive} (the cat is alive) or ψ_{dead} (the cat is dead). Thus, the Schrödinger equation does not determine a unique experimental outcome. To resolve this, textbook quantum mechanics supplements the Schrödinger equation with additional collapse postulates. Whenever we open the box and "observe" the cat, the system will suddenly change (collapse) into one of the two states: ψ_{alive} or ψ_{dead}. An important role of the wave function is determining the probabilities of experimental outcomes, which are taken to be the results of wave function collapses. For example, the probability of finding the system in any set of configurations is given by the Born rule:

$$P(q \in A) = \int_A |\psi(q)|^2 dq, \tag{4}$$

where A is a set of points in configuration space, $|\psi(q)|^2$ is the squared amplitude of the wave function, and dq is the Lebesgue measure on $\mathbb{R}^{3N}$. In the cat example, the probability of finding the cat to be alive is equal to $\frac{1}{2}$, since $\int |\frac{1}{\sqrt{2}}\psi_{alive}|^2 + 0 = \frac{1}{2}$. The Born rule has the consequence that wave functions that differ only by an *overall phase* (multiplied by a complex number $e^{i\theta}$, where $\theta \in [0, 2\pi]$) will give rise to the same observable phenomena ($|\psi|^2 = |e^{i\theta}\psi|^2$). That is called the *overall phase symmetry*, which motivates the common view that two wave functions that differ by an overall phase represent the same quantum state.

2.2 Quantum Measurement Problem

Notwithstanding the empirical success of quantum mechanics, the collapse postulates seem out of place for a fundamental theory of the world. If the wave function (of the system and the measurement device) obeys the Schrödinger equation, how can it also obey the collapse postulates that contradict the linearity of the Schrödinger equation? But if the wave function does not collapse, how can we obtain unique experimental outcomes? In short, we have the *quantum measurement problem*:

(P1) The wave function is the complete description of the physical system.

(P2) The wave function always obeys the Schrödinger equation.

(P3) Every experiment has a unique outcome.

Each of these three propositions is, on its own, plausible. However, together they lead to a contradiction. To see the contradiction, let us apply them to Schrödinger's cat thought experiment. If P1 is true, the system is completely described by (3). If P2 is true, the wave function never collapses into one of the definite states. If P3 is true, the cat is nonetheless in one of the definite states—either alive or dead.[7]

Since P1—P3 are inconsistent, at least one of them is false. Rejecting P1 or P2 would require us to develop alternative theories of quantum mechanics, since we would need to find additional variables omitted by the wave function, or we would need to modify the Schrödinger equation. Rejecting P3 would lead to major revisions of our assumptions about the world. There are three main "interpretations " of quantum mechanics that carry out such strategies. They all contain significant revisions of quantum mechanics, so we should call them realist theories of quantum mechanics instead of interpretations.

First, the de Broglie-Bohm theory, or Bohmian mechanics (BM), rejects P1. According to BM, the wave function is not the complete description of the physical system. There are actual particles with precise positions in physical space. The wave function still obeys the Schrödinger equation. But the wave function also determines the velocity of the particles according to the *guidance equation*.[8] In the cat example, the cat is made out of particles in physical space. There is always a determinate configuration of particles, so the cat is either alive or dead. The probabilities of quantum mechanics become epistemic uncertainties over initial particle configurations.[9]

Second, the Ghirardi-Rimini-Weber theories of spontaneous collapse (GRW) reject P2. According to GRW theories, the wave function ψ_t does not always obey the Schrödinger equation. It undergoes spontaneous collapses with a fixed rate per particle per unit time. In the cat experiment, given the vast number of particles in the system, it will quickly collapse into a determinate state in which the cat is either

[7]For a more thorough discussion about the quantum measurement problem, see Myrvold (2017) and Bell (1990).

[8]The particles move according to the guidance equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_i\psi}{\psi}, \tag{5}$$

where Q_i and m_i are the position and mass of particle i, Im means taking the imaginary part, and ∇_i means taking the gradient with respect to the i-th particle. The particles are initially distributed according to the Born rule, and their distribution will always agree with the Born rule because of the mathematical properties of the Schrödinger equation and the guidance equation.

[9]For a survey of BM, see Goldstein (2017); for the original paper, see Bohm (1952); for a modern version, see Dürr et al. (1992).

alive or dead. Collapses are represented by Gaussian functions with a fixed width in physical space. Due to entanglement, collapses on a single particle has the effect that the universal wave function will collapse into a definite state. On the macroscopic scale, the collapse will give rise to (approximately) Born rule probabilities. Each version of GRW postulates specific values for the collapse rate and the Gaussian width. Moreover, there can be additional variables representing ontology in physical space. GRWm adds a mass-density ontology that specifies the amount of mass in physical space by a real-valued function $m(x,t)$, where (x,t) is a space-time point.[10] In contrast, GRWf adds a flash ontology that postulates the existence of space-time events at the center of the Gaussian function. It can be represented as a function $F(x,t)$ with $x \in \mathbb{R}^3$ and $F(x,t) = 1$ if (x,t) is the center of some GRW collapse and 0 otherwise.[11]

Third, many-worlds interpretations of Everettian quantum mechanics (EQM) reject P3.[12] According to these interpretations, there is no need to ensure that there is a unique outcome in the cat experiment. There simply are two branches of the wave function, one in which the cat is alive and the other in which the cat is dead. Both branches co-exist. Because of a property called *decoherence*, the branches do not interfere much with each other. The branches of the wave function are emergent worlds, the wave function is the complete description of the "multi-verse," and it always obeys the Schrödinger equation. Similarly to GRWm, we can devise a version of EQM with a mass-density ontology. This is called Sm and was first proposed by Allori et al. (2010). A challenge for any version of EQM is how to make sense of probability in a world in which every possible outcome of every quantum experiment happens with certainty.[13]

The upshot is that the wave function figures prominently in all three realist quantum theories. In BM, although the wave function is not the complete description of the system, it is still part of the description. Moreover, the wave function guides particle motion. In GRW, the wave function collapses and gives rise to unique outcomes of experiments. In (many-worlds interpretations of) EQM, the wave function never collapses but gives rise to emergent parallel worlds. For quantum theories with additional ontology, such as BM, GRWm, GRWf, and Sm, the wave function is also tied to the dynamics of the additional ontology. But their relationship

[10] The mass-density function is defined from the wave function:

$$m(x,t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d^3x_1 ... d^3x_N \delta^3(x_i - x) |\Psi_t(x_1, ..., x_N)|^2 \tag{6}$$

[11] For a survey of GRW, see Ghirardi (2018); for the original paper, see Ghirardi et al. (1986); Bell (2004), Ch 22, contains a clear presentation of the theory.

[12] The many-worlds interpretations are popular among Everettians. However, Conroy (2012) has provided textual evidence that Everett himself might endorse a single-world interpretation of quantum mechanics.

[13] For a survey of EQM, see Vaidman (2018); for the original paper, see Everett III (1957); for an updated book-length development of the theory, see Wallace (2012). There has been significant progress in addressing the probability challenge with the tools of typicality, decision theory, and self-locating probabilities. For some recent examples, see Barrett (2017), Wallace (2012), Sebens and Carroll (2016), and the references therein.

is different in these theories. Bohmian particles have independent dynamics: even if the wave function were not to change, Bohmian particles would still move in a non-trivial fashion. That is not the case in GRWm, GRWf, and Sm. Had there been no change to the wave function, the additional ontology would not change either. It is in this sense that the dynamics of mass-densities and flashes are not independent of the dynamics of the wave function.

3 Ontological Interpretations

In this section, I review four *ontological* interpretations of the wave function. However, the label "ontological" could be misleading. These four interpretations share the feature that the wave function is interpreted as part of the fundamental *material ontology*, on a par with particles, fields, space-time events or properties, which are the kind of microscopic things that make up macroscopic objects such as tables and chairs. In §4 and §5, we will review nomological interpretations and the *sui generis* interpretation of the wave function, which are compatible with the position that the wave function is part of the ontology but just not in the same ontological category as particles or fields.

3.1 A Field on a High-Dimensional Space

According to the first ontological interpretation, the fundamental space is a high-dimensional space, and the wave function is a field in that space. This was introduced by Albert (1996). Albert calls this view *wave function realism*.[14] However, as we shall see in the later sections, that label is no longer appropriate given the abundance of other approaches that are also realist about the wave function.

It is counterintuitive how the fundamental space can be high-dimensional. It might help to compare this idea with something familiar—classical physics. In classical field theories such as Maxwellian electrodynamics, electromagnetic fields are fields on the four-dimensional physical space-time. A field on physical space-time can be interpreted as an assignment of monadic properties (field strength and direction) to each point in space-time. Such an assignment is determined by Maxwell's equations and certain boundary conditions.

In a similar way, the wave function can be interpreted as a physical field. However, the wave function cannot be interpreted as a field on physical space, as its domain is the high-dimensional configuration space, represented by $\mathbb{R}^{3N}$. If we take configuration space to be the fundamental space, then the wave function can be interpreted as a field that assigns properties to each point in configuration space. The properties assigned by the field, represented as complex numbers, change according to the Schrödinger equation. On this view, the high-dimensional configuration space

[14]It has been developed and defended by Loewer (1996), Ney (2012, 2013), and North (2013), although Ney and North are primarily concerned with the first part of the thesis, i.e. the fundamental space is a high-dimensional space.

is ontologically prior to physical space(time), and the latter somehow comes out of the fundamental structure. This is despite the fact that we call the high-dimensional space "configuration space," which seems to imply the reverse order of ontological dependence.[15]

The high-dimensional field interpretation prioritizes the structure of the wave function and its dynamics. The fundamental physical events are those that happen on the high-dimensional space. A key challenge to this view is to explain our apparent experiences in a three-dimensional space. This is not just a question about recovering the manifest image, but it is also about whether such an interpretation of quantum mechanics can be "empirically coherent," in the sense that if our evidence for quantum mechanics comes from instrument readings in the three-dimensional space, the theory should not undermine such evidence. It should explain how the appearances of three-dimensional objects come out of the high-dimensional fundamental space.[16]

Albert (1996) suggests that the explanation lies in the dynamics—in the structure of the Hamiltonian operator. Although all the $3N$ dimensions are metaphysically on a par:

$$\{q_1, q_2, q_3, q_4, q_5, q_6, ..., q_{3N-2}, q_{3N-1}, q_{3N}\} \tag{7}$$

the Hamiltonian operator has a term that encodes fundamental interactions and it takes on a particular form:

$$\sum_{0\leq i<j\leq N}\sum V_{ij}[(q_{3i-2} - q_{3j-2})^2 + (q_{3i-1} - q_{3j-1})^2 + (q_{3i} - q_{3j})^2] \tag{8}$$

The Hamiltonian operator groups the coordinates in the $3N$-dimensional configuration space into triplets, such that there might be emergent objects that have the same functional profile as what we take to be ordinary objects in the 3-dimensional space. This provides reasons to believe that there might be an emergent 3-dimensional

[15]There are three versions of this view:

- Bohmian version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a point particle located in that space and a field that assigns properties to points of that space. The field always evolves by the Schrödinger equation. The point particle moves along in the field according to the guidance equation, much like corks move along in flowing river. Here we see a dis-analogy with the classical field. In classical physics, the field and the particles satisfy the action-reaction principle; the fields and the particles can influence each other. In Bohmian mechanics, the wave function interpreted as a field can influence the particle but not vice versa.
- GRW version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a field that assigns properties to points of that space. The field evolves by the Schrödinger equation most of the time but sometimes collapses by the GRW collapse mechanism.
- Everettian version: the fundamental space is represented by $\mathbb{R}^{3N}$. The fundamental ontology consists in a field that assigns properties to points of that space. The field always evolves by the Schrödinger equation.

The high-dimensional field interpretation of the wave function is incompatible with GRWm, GRWf, or Sm.

[16]See Barrett (1999) and Barrett (1996).

physical space. However, Albert's proposed explanation has been challenged by Monton (2002), Lewis (2004), and Chen (2017). See Emery (2017) for an objection based on conservativeness principles.

Maudlin (2013) has criticized Albert's proposal on the ground that it reifies too much structure. The common view (§2.1) holds that two wave functions that differ only by a complex multiplication constant represent the same physical state. But if we interpret the wave function as a field that assigns monadic properties to points in configuration space, then we would distinguish two wave functions related by a constant, for the numbers assigned to the points are different. This problem can be avoided if we adopt an *intrinsic* (or gauge-free) characterization of the wave function, in terms of comparative relations that are invariant under the change by a constant (Chen (2018a)).

3.2 A Multi-field on Physical Space

The high-dimensional field interpretation of the wave function faces difficulties, primarily because it privileges configuration space over physical space. There are many good reasons to take physical space to be ontologically more basic. First, it underlies many important symmetries in physics. Second, it is much easier for a theory to be empirically coherent if it does not undermine the relative fundamentality of physical space(time).

These difficulties are avoided in the second ontological interpretation, according to which the fundamental space is the ordinary physical space(time). On this view, the wave function is not a field in the traditional sense, but a *multi-field* on physical space. (See Forrest (1988), Belot (2012), Chen (2018a, 2017), Hubert and Romano (2018).) A multi-field is similar to a field. However, unlike fields, multi-fields assign properties not to individual points but to *regions* of points in space. Such regions can be connected or disconnected. The wave function is a function from N copies of $\mathbb{R}^3$ to complex numbers. Instead of thinking of it as a field that assigns properties to every point in $\mathbb{R}^{3N}$, we can think of it as a "multi-field" that assigns properties to every region of $\mathbb{R}^3$ that is composed of N points. The multi-field interpretation is a more faithful representation for "indistinguishable particles," for which particle labels do not matter. This is because spatial regions understood as N-element subsets of $\mathbb{R}^3$ (or mereological fusions of N points in $\mathbb{R}^3$) are *unordered*. Thus, the multi-field interpretation has the additional advantage of automatically enforcing what is called "permutation invariance": mere permutations of a configuration of N particles do not change the physical state.[17] Similar to the previous interpretation, here we can avoid postulating too much structure by using an intrinsic account of the multi-fields.

[17] Chen (2017) suggests that the symmetrization postulate is better explained by the low-dimensional interpretation than the high-dimensional interpretation.

3.3 Properties of Physical Systems

The third ontological interpretation, proposed by Wallace and Timpson (2010), affirms the (relative) fundamentality of the physical space(time). On this view, the universe is divided into subsystems that occupy some spatial-temporal regions. Larger systems can be made out of unions of smaller systems. And the universe is the union of all systems. Although not every system has a wave function (because of entanglement), we can still associate to each system a determinate property represented by what is called a *density matrix*. A density matrix will encode all the dynamical variables of the system. Here we should not think of the density matrix as a field or multi-field. Rather, it is thought to be an abstract operator in the Hilbert space.[18] This view was introduced as an alternative to the high-dimensional field interpretation. It is also an alternative to the low-dimensional multi-field interpretation. However, it is still a version of realism about the wave function, since the universal wave function (or the universal density matrix) is to be found in the ontology—the property of the entire universe.

Wallace and Timpson call this approach *spacetime state realism*. They argue that this approach avoids privileging the position representation of the wave function, and that it has significant advantages in reconciling with relativistic invariance. See Swanson (2018) for some discussions about the relativistic extensions.

A question about spacetime state realism is whether the fundamental ontology contains redundant information. If we help ourselves to a decomposition of the universe into subsystems, and if we have the quantum state of the universe, then we can obtain density matrices of the subsystems by a purely mathematical procedure of tracing out the environmental degrees of freedom. Since they can be derived from the quantum state of the universe, the properties of the subsystem need not be placed in the fundamental ontology. If we get rid of the subsystem properties and only keep the universal property (the universal density matrix), then this approach would be in the same spirit as the low-dimensional multi-field approach in §3.2.[19]

[18] For example, if the universe consists in two systems A and B, and if their joint quantum state is this:

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle_A |\beta\rangle_B - |\beta\rangle_A |\alpha\rangle_B) \tag{9}$$

then the density matrix associated with system A (called the *reduced density matrix*) will be:

$$\rho_A = \frac{1}{2}(|\alpha\rangle_A \langle\alpha|_A + |\beta\rangle_A \langle\beta|_A) \tag{10}$$

On this view, neither $|\Psi_{AB}\rangle$ nor ρ_A are understood as functions or fields on some spaces. Rather, they are understood as structures in the abstract Hilbert spaces: $|\Psi_{AB}\rangle$ is a vector in the total Hilbert space $\mathscr{H}_A \otimes \mathscr{H}_B$, and ρ_A is an operator that maps vectors to vectors in the system A's Hilbert space $\mathscr{H}_A$.

[19] See Monton (2006) and Monton (2013) for another view that interprets the wave function as properties of physical systems.

3.4 A Vector in the Hilbert Space

The final ontological interpretation of the wave function takes the abstract Hilbert space more seriously. Recall that the wave function is represented as a vector in Hilbert space, and the Schrödinger equation can be represented as an equation for vector rotation in that space. Carroll and Singh (2018) suggest that the Hilbert space is the fundamental space, and the wave function is just a vector in that space. Every goings-on in the world corresponds to some particular direction the vector is pointing.

Since the Everettian interpretation of QM is the most natural place for this view, Carroll and Singh (2018) call this approach *Mad-Dog Everettianism*. In their words, the label is "to emphasize that it is as far as we can imagine taking the program of stripping down quantum mechanics to its most pure, minimal elements."

It is already difficult to recover ordinary objects from configuration space. It is even more difficult to recover them from the Hilbert space. For one thing, there is no space-time structure in the Hilbert space. The state of the world corresponds to a vector, which is just like every other vector. How can anything familiar, such as space, time, and ordinary objects, come out of a vector in a high-dimensional Hilbert space? Like Albert (1996), Carroll and Singh propose that the answer lies in the structure of the Hamiltonian operator. The Hamiltonian provides a privileged way to decompose the total Hilbert space into smaller spaces, which may explain the emergent structure.[20] This proposal is more speculative than the high-dimensional field interpretation. However, it is in part motivated by the non-fundamentality of space-time in several theories of quantum gravity. As such, it could be a fruitful project to explore.

4 Nomological Interpretations

According to the previous ontological interpretations, the wave function is part of the fundamental material ontology of the world, such as particles and fields in classical mechanics. In contrast, nomological interpretations hold that the wave function is nomological, i.e. on a par with laws of nature. In this section, I survey two kinds of nomological interpretations of the wave function: the strong nomological interpretations and the weak nomological interpretations.

These interpretations are most compelling from a Bohmian point of view. However, they might be adaptable for some versions of GRW theories and Everettian theories with additional ontologies.

[20]Their analysis is restricted to *locally finite-dimensional Hilbert spaces*. See Cotler et al. (2017) and Bao et al. (2017) for more details.

4.1 Strong Nomological Interpretations

The guiding idea of nomological interpretations is that the wave function is on a par with laws of nature (Goldstein and Zanghì (2013)). To appreciate the strong nomological interpretations, it would be helpful to review the status of the Hamiltonian function in classical mechanics. As mentioned in §2.1, the Hamiltonian equations (1) govern the motion of classical particles in physical space, represented by a curve in phase space. The Hamiltonian function is the generator of such motion. It is a convenient short hand for the kinetic energy term and the pair-wise interactions of the particles. We can, if we like, write out H explicitly as a function (of position and momentum) on the right hand sides of the equations. For the Hamiltonian equations to be simple *laws* of nature, H has to be a simple function. In this sense, we give H a nomological interpretation. Although it is a function on phase space, we do not treat it as part of the material ontology.[21]

Let us now consider Bohmian mechanics. In this theory, the guidance equation governs the motion of the Bohmian particles in physical space, represented by a curve in configuration space. The wave function is the generator of such motion. If the wave function turns out to be a simple function, then we can write out ψ explicitly as a function (of configuration variables) on the right hand side of the equation. In that case, we can give it an analogous nomological interpretation. Although it is a function on configuration space, we do not need to treat it as part of the ontology but only part of the law system. I call this a *strong nomological interpretation*, for it affords the same status to the wave function as it does to the classical Hamiltonian function.

The strong nomological interpretation requires the universal wave function to be simple. Although generic wave functions of quantum systems are very complex, there are reasons to be optimistic. Goldstein and Zanghì (2013) have offered one. The universal wave function can be quite distinct from the wave functions of the subsystems. If we were to extend quantum mechanics to quantum gravity, then it is possible that the wave function of the universe will be stationary. This is seen in the Wheeler-DeWitt equation of canonical quantum gravity:

$$\hat{H}\Psi = 0 \tag{11}$$

If we understand (11) as telling us about the time evolution of the wave function, then it tells us that the wave function does not change over time, i.e. it is stationary. Since the Schrödinger equation governs how the wave function changes over time, it is to be treated not as a fundamental equation but only as an effective equation—describing the behavior of subsystems.

It is plausible to think that a stationary wave function contain many symmetries, because usually only symmetrical wave functions are stationary. Such symmetries might ensure that the wave function is simple. For example, a translationally invariant function on physical space can only be a constant function, which is relatively simple. Therefore, if the wave function of the universe satisfies (11), and if

[21] To borrow a term from Barry Loewer and Tim Maudlin (p.c.), on this view, the wave function is part of the *nomology* of the theory.

we understand it as telling us about time evolution, it is plausible that the universal wave function is simple. On the Bohmian theory, then, the universal wave function can be treated on a par with laws of nature.[22]

This nomological interpretation faces some challenges. First, it is controversial whether the Wheeler-DeWitt equation governs the universal wave function. For example, there are research programs in quantum gravity that do not depend on it. Second, since the wave function is stationary, it requires some revisions about how we think about the arrow of time.[23] Chen (2018b) develops a new framework of quantum mechanics that avoids these problems. However, in that approach, the fundamental quantum state has to be represented by a density matrix instead of a wave function.

These interpretations of the wave function are most compelling in the Bohmian framework. However, In Everettian and GRW theories with additional ontologies, we might also give a nomological interpretation of the wave function.

4.2 Weak Nomological Interpretations

The literature on the nomological interpretation of the wave function is growing. However, much of that is directed at a weaker thesis, which I will call the *weak nomological interpretation*. On this view, the wave function does not need to be like the classical Hamiltonian to fit into the law system. It recommends a weaker criterion for being nomological. This idea is most plausible in some extended Humean framework. In the original Humean framework, laws of nature are the axioms of the best system that summarizes the mosaic. In Loewer (2001), the Humean framework has been extended to allow for deterministic "chances." In Hall (2015), it has been further extended to allow intrinsic properties such as mass and charge to be non-fundamental and to be merely part of the best system.

According to the weak nomological interpretation (Humean version), what is fundamental is just the distribution of matter in the four-dimensional spacetime, and the wave function is just a dynamical variable that assists in a simple and informative summary of the mosaic. (See Miller (2014), Esfeld (2014), Bhogal and Perry (2015), Callender (2015), and Esfeld and Deckert (2017).) Although the wave function is part of the best system, it does not have to be simple *simpliciter*. It just needs to be the simplest one among all competitors. Even though the exact specification of the wave function is complicated, the best system involving the wave function might still be the simplest overall. Albert (p.c.), Maudlin (p.c.), and Dewar (2017) have raised the worry that the complete specification of particle trajectories, which will form another system, seems to postulate less information than the wave function. This is

[22]Relatedly, Allori (2017) has proposed a new argument for nomological interpretations based on symmetry principles.

[23]The problem is that in standard Boltzmannian quantum statistical mechanics, the arrow of time is associated with the increase of entropy of the quantum system, which is a property of the wave function. If the universal wave function is stationary, then there is no increase of Boltzmann entropy. Perhaps the Bohmian approach can help by providing an alternative definition of entropy in terms of particle configurations. But that has not been done.

because the particle trajectories form a single curve in configuration space, while the wave function assigns values to every point in configuration space. Moreover, they have raised the worry that the wave function does not supervene on the particle trajectories, since *prima facie* the particle trajectories do not determine the exact values of the wave function. However, it is true that physicists who have access only to position facts nonetheless postulate wave functions to make explanations and predictions, and they often agree on the exact wave function of the system. So the best system comparisons and the issue of supervenience may be more complicated than what the debate has assumed.

At any rate, the weak nomological interpretation demands less of the wave function of the universe. It does not have to be a simple function or determined in a simple way. It can be highly complex, as long as it is the simplest among all the choices. The weak nomological interpretation is less realist than the previous approaches, but it could still be realist if the extended Humean model can be understood as a realist view about laws and properties.

5 The *Sui Generis* Interpretation

It is possible to be not persuaded by any of the above strategies. The high-dimensional field interpretation and the Hilbert space interpretation require sophisticated stories about the emergence of the apparent three-dimensional objects and ordinary space-time. The low-dimensional multi-field interpretation and the subsystem property interpretation may seem to be trying too hard to squeeze the wave function into familiar ontological categories.

Perhaps the lesson of quantum mechanics is that the wave function does not fit into any familiar categories of things; it is a new kind of entity. Perhaps it is neither ontological nor nomological. In that case, the wave function has its own category of existence that is distinct from anything we have considered. In other words, the wave function is ontologically *sui generis*. Maudlin (2013) suggests that we should be open to that possibility.

6 Conclusion

In this article, we have surveyed three kinds of realist interpretations of the wave function: ontological interpretations, nomological interpretations, and the *sui generis* interpretation. (See Table 1 for a summary.) A century after the discovery of quantum mechanics, although there is no consensus on what it means, we have made significant progress in constructing several realist interpretations. Almost every interpretation requires further developments, and it is too early to say which one is the best or the most fruitful. It is also too early to think that those are exhaustive of all the options available to the realist. In all likelihood, there will be other ways to think about the wave function from the realist perspective that we have never

Interpretation	BM	GRW0	GRWm	GRWf	S0	Sm	HD	LD
High-Dimensional Field	✓	✓	✗	✗	✓	✗	✓	✗
Low-Dimensional Multi-field	✓	✓	✓	✓	✓	✓	✗	✓
Properties of Systems	✓	✓	✓	✓	✓	✓	✗	✓
Vector in Hilbert Space	✗	?	✗	✗	✓	✗	✓	✗
Strongly Nomological	✓	✗	?	?	✗	✓	✓	✗
Weakly Nomological	✓	✗	✓	✓	✗	✓	✓	✓
Sui Generis	✓	✓	✓	✓	✓	✓	✓	✓

Table 1: The first column lists all the realist interpretations reviewed in this article. In the first row, we have the main solutions to the quantum measurement problem: BM (Bohmian mechanics with a particle ontology), GRW0 (GRW theory without additional ontologies), GRWm (GRW theory with a mass-density ontology), GRWf (GRW theory with a flash ontology), S0 (Everettian theory without additional ontologies), and Sm (Everettian theory with a mass-density ontology). "HD" stands for the view that the fundamental physical space is high-dimensional (10^{80} dimensions in configuration space fundamentalism or possibly infinity in Hilbert space fundamentalism), and "LD" stands for the view that the fundamental space is low-dimensional (3 dimensions of ordinary physical space). We mark their compatibility with a check (compatible), a cross (incompatible), or a question mark (unknown compatibility).

considered.[24]

Acknowledgement

I am grateful for helpful discussions with David Albert, Valia Allori, Sean Carroll, Christina Conroy, Alexander Ehmann, Sheldon Goldstein, Veronica Gomez, Mario Hubert, Barry Loewer, Ricardo Martinez, Bradley Monton, Bradley Rettler, Thad Roberts, Davide Romano, and Ashmeet Singh.

References

Albert, D. Z. (1994). *Quantum Mechanics and Experience*. Harvard University Press.

Albert, D. Z. (1996). Elementary quantum metaphysics. In Cushing, J. T., Fine, A., and Goldstein, S., editors, *Bohmian Mechanics and Quantum Theory: An Appraisal*, pages 277–84. Kluwer Academic Publishers, Dordrecht.

Allori, V. (2013). Primitive ontology and the structure of fundamental physical theories. *The Wave Function: Essays on the Metaphysics of Quantum Mechanics*, pages 58–75.

Allori, V. (2017). A new argument for the nomological interpretation of the wave function: The galilean group and the classical limit of nonrelativistic quantum mechanics. *International Studies in the Philosophy of Science*, 31(2):177–188.

[24]For book-length treatments of the subject, see Ney and Albert (2013), Lewis (2016), Gao (2017), and Ney (2018). For clear introductions to the foundations of quantum mechanics, see Bell (2004), Albert (1994), Ghirardi (2005), Bricmont (2016), and Norsen (2017). For an alternative perspective on realism about the wave function, see Halvorson (2018).

Allori, V., Goldstein, S., Tumulka, R., and Zanghì, N. (2008). On the common structure of bohmian mechanics and the Ghirardi–Rimini–Weber theory: Dedicated to Giancarlo Ghirardi on the occasion of his 70th birthday. *The British Journal for the Philosophy of Science*, 59(3):353–389.

Allori, V., Goldstein, S., Tumulka, R., and Zanghì, N. (2010). Many worlds and schrödinger's first quantum theory. *British Journal for the Philosophy of Science*, 62(1):1–27.

Bao, N., Carroll, S. M., and Singh, A. (2017). The hilbert space of quantum gravity is locally finite-dimensional. *International Journal of Modern Physics D*, 26(12):1743013.

Barrett, J. A. (1996). Empirical adequacy and the availability of reliable records in quantum mechanics. *Philosophy of Science*, 63(1):49–64.

Barrett, J. A. (1999). *The quantum mechanics of minds and worlds*. OUP Oxford.

Barrett, J. A. (2017). Typical worlds. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 58:31–40.

Bell, J. (1990). Against 'measurement'. *Physics world*, 3(8):33.

Bell, J. S. (2004). *Speakable and Unspeakable in Quantum Physics: Collected Papers on Quantum Philosophy*. Cambridge University Press.

Belot, G. (2012). Quantum states for primitive ontologists. *European Journal for Philosophy of Science*, 2(1):67–83.

Bhogal, H. and Perry, Z. (2015). What the humean should say about entanglement. *Noûs*.

Bohm, D. (1952). A suggested interpretation in terms of "hidden variables": Part i and part ii. *Phys. Rev*, 85(166179):180193.

Bricmont, J. (2016). *Making sense of quantum mechanics*, volume 37. Springer.

Callender, C. (2015). One world, one beable. *Synthese*, 192(10):3153–3177.

Carroll, S. M. and Singh, A. (2018). Mad-dog Everettianism: Quantum mechanics at its most minimal. *arXiv preprint arXiv:1801.08132*.

Chen, E. K. (2017). Our fundamental physical space: An essay on the metaphysics of the wave function. *Journal of Philosophy*, 114: 7.

Chen, E. K. (2018a). The intrinsic structure of quantum mechanics. *Philsci-archive preprint:15140*.

Chen, E. K. (2018b). Quantum mechanics in a time-asymmetric universe: On the nature of the initial quantum state. *The British Journal for the Philosophy of Science*, forthcoming.

Conroy, C. (2012). The relative facts interpretation and Everett's note added in proof. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 43(2):112–120.

Cotler, J. S., Penington, G. R., and Ranard, D. H. (2017). Locality from the spectrum. *arXiv preprint arXiv:1702.06142*.

Dewar, N. (2017). La Bohume. *Philsci-archive preprint:13235*.

Dürr, D., Goldstein, S., and Zanghì, N. (1992). Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics*, 67(5-6):843–907.

Emery, N. (2017). Against radical quantum ontologies. *Philosophy and Phenomenological Research*, 95(3):564–591.

Esfeld, M. (2014). Quantum humeanism, or: physicalism without properties. *The Philosophical Quarterly*, 64(256):453–470.

Esfeld, M. and Deckert, D.-A. (2017). *A minimalist ontology of the natural world*. Routledge.

Everett III, H. (1957). "Relative state" formulation of quantum mechanics. *Reviews of modern physics*, 29(3):454.

Forrest, P. (1988). *Quantum metaphysics*. Blackwell Publisher.

Gao, S. (2017). *The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics*. Cambridge University Press.

Ghirardi, G. (2005). *Sneaking a look at God's cards: unraveling the mysteries of quantum mechanics*. Princeton University Press.

Ghirardi, G. (2018). Collapse theories. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition.

Ghirardi, G. C., Rimini, A., and Weber, T. (1986). Unified dynamics for microscopic and macroscopic systems. *Physical Review D*, 34(2):470.

Goldstein, S. (2017). Bohmian mechanics. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2017 edition.

Goldstein, S. and Zanghì, N. (2013). Reality and the role of the wave function in quantum theory. *The wave function: Essays on the metaphysics of quantum mechanics*, pages 91–109.

Hall, N. (2015). Humean reductionism about laws of nature. *A companion to David Lewis*, 57:262.

Halvorson, H. (2018). To be a realist about quantum theory. *Philsci-archive preprint:14310*.

Hubert, M. and Romano, D. (2018). The wave-function as a multi-field. *European Journal for Philosophy of Science*, 8(3):521–537.

Lewis, P. J. (2004). Life in configuration space. *The British journal for the philosophy of science*, 55(4):713–729.

Lewis, P. J. (2016). *Quantum Ontology: A Guide to the Metaphysics of Quantum Mechanics*. Oxford University Press.

Loewer, B. (1996). Humean supervenience. *Philosophical Topics*, 24:101–27.

Loewer, B. (2001). Determinism and chance. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 32(4):609–620.

Matarese, V. (2018). A challenge for super-humeanism: the problem of immanent comparisons. *Synthese*, pages 1–20.

Maudlin, T. (2013). The nature of the quantum state. *The wave function: Essays on the metaphysics of quantum mechanics*, pages 126–53.

Maudlin, T. W. (2007). Completeness, supervenience and ontology. *Journal of Physics A: Mathematical and Theoretical*, 40(12):3151.

Miller, E. (2014). Quantum entanglement, bohmian mechanics, and humean supervenience. *Australasian Journal of Philosophy*, 92(3):567–583.

Monton, B. (2002). Wave function ontology. *Synthese*, 130(2):265–277.

Monton, B. (2006). Quantum mechanics and 3n-dimensional space. *Philosophy of science*, 73(5):778–789.

Monton, B. (2013). Against 3n-dimensional space. *The wave function: essays on the metaphysics of quantum mechanics*, 154.

Myrvold, W. (2017). Philosophical issues in quantum theory. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2017 edition.

Myrvold, W. C. (2015). What is a wavefunction? *Synthese*, 192(10):3247–3274.

Ney, A. (2012). The status of our ordinary three dimensions in a quantum universe. *Noûs*, 46(3):525–560.

Ney, A. (2013). Ontological reduction and the wave function ontology. *The wave function: Essays on the metaphysics of quantum mechanics*, 168.

Ney, A. (2018). *Finding the World in the Wave Function*. Unpublished Manuscript.

Ney, A. and Albert, D. Z. (2013). *The wave function: Essays on the metaphysics of quantum mechanics*. Oxford University Press.

Norsen, T. (2017). *Foundations of Quantum Mechanics: An Exploration of the Physical Meaning of Quantum Theory*. Springer.

North, J. (2013). The structure of a quantum world. In Albert, D. Z. and Ney, A., editors, *The Wavefunction: Essays in the Metaphysics of Quantum Mechanics*. Oxford University Press, Oxford. Forthcoming.

Pusey, M. F., Barrett, J., and Rudolph, T. (2012). On the reality of the quantum state. *Nature Physics*, 8(6):475–478.

Sebens, C. T. and Carroll, S. M. (2016). Self-locating uncertainty and the origin of probability in everettian quantum mechanics. *The British Journal for the Philosophy of Science*, 69(1):25–74.

Sider, T. (forthcoming). *The tools of metaphysics and the metaphysics of science*. Oxford University Press, Oxford.

Swanson, N. (2018). How to be a relativistic spacetime state realist. *Philsci-archive preprint:14412*.

Vaidman, L. (2018). Many-worlds interpretation of quantum mechanics. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2018 edition.

Wallace, D. (2012). *The emergent multiverse: Quantum theory according to the Everett interpretation*. Oxford University Press.

Wallace, D. and Timpson, C. G. (2010). Quantum mechanics on spacetime I: Spacetime state realism. *The British Journal for the Philosophy of Science*, 61(4):697–727.