Universality Reduced

Alexander Franklin[*†]

October 2018

Forthcoming in *Philosophy of Science: Proceedings of the PSA 2018*

Abstract

The universality of critical phenomena is best explained by appeal to the Renormalisation Group (RG). Batterman and Morrison, among others, have claimed that this explanation is irreducible. I argue that the RG account is reducible, but that the higher-level explanation ought not to be eliminated. I demonstrate that the key assumption on which the explanation relies – the scale invariance of critical systems – can be explained in lower-level terms; however, we should not replace the RG explanation with a bottom-up account, rather we should acknowledge that the explanation appeals to dependencies which may be traced down to lower levels.

1 Introduction

While universality is best explained with reference to the Renormalisation Group (RG), that explanation is nonetheless reducible. The argument in defence of this claim is of philosophical interest for two reasons: first, the RG explanation of universality has been touted by Batterman (2000, 2017) and

[*]alexander.a.franklin@kcl.ac.uk

[†]I am grateful to Eleanor Knox, and to the audience of the IMPS 2018 conference in Salzburg for helpful comments. This work was supported by the London Arts and Humanities Partnership.

Morrison (2012, 2014) as a significant impediment to reduction. Second, universality is a paradigm instance of multiple realisability (MR) in the philosophy of physics; as such it is regarded as irreducible by those who accept the multiple realisability argument against reduction. My account charts a middle course: I deny claims that RG explanations are irreducible, and I deny that universality is *best* explained from the bottom up.

The view of reduction advocated here is non-eliminativist; the best explanations are often higher-level explanations: such explanations are more parsimonious, more robust, and have broader applicability than lower-level explanations. In general, such higher-level explanations ought not to be replaced by lower-level explanations, rather the parts of theories on which such explanations rely may be understood in lower-level terms; reducible explanations satisfy the following two conditions: (a) each higher-level explanatory dependency is explained by or derived from a lower-level dependency, and (b) the abstractions involved in constructing the higher-level explanations are justified from the bottom up.[1]

In §2 I outline the RG explanation of universality. Although my reductive claims may generalise, I focus exclusively on the field-theoretic approach to the RG.[2] I claim that this explanation follows a general formula for explaining multiply realised phenomena. §3 considers the arguments of Batterman and Morrison, and analyses their force against any putative reduction.

In §4 I note that the RG explanation is a higher-level explanation. As it is less contentious that the common features of each universality class are reducible, I simply assume that that's the case in this paper. The nub of the debate rests on the RG: I show that the RG arguments rely on the assumption of scale invariance and the abstractions engendered by that assumption. I argue that the applicability of this assumption may be explained from the bottom up. Thus, I claim, that my reduction satisfies (a) and (b) above.

[1]While I expect the claims in this paper to be compatible with many different accounts of explanation, they are most straightforwardly cashed out on an interventionist approach – see Woodward (2003).

[2]See Franklin (2018) and Mainwood (2006) for arguments that only this approach provides an adequate explanation of universality.

2 The RG Explanation of Universality

'Universality' refers to the phenomenon whereby diverse systems exhibit similar scaling behaviour on the approach to a continuous phase transition. Continuous phase transitions occur at the critical temperature, a point beyond which systems no longer undergo first-order phase transitions.[3] The approach to this phase transition can be very well described by power laws of the form $a_i(t) \propto t^\alpha$ where t is proportional to the temperature deviation from the critical temperature and α is the critical exponent – a fixed number which leads to a characteristic curve on temperature-density plots.[4]

Different physical systems can be categorised into universality classes: members of the same class have identical critical behaviour – the same set of critical exponents $\{\alpha, \beta, ...\}$ for several power laws – while their behaviour away from the critical point and microscopic organisation may be radically different. For example, fluids and magnets are in the same universality class despite otherwise having totally different chemical and physical properties.

Each physical system which exhibits critical phenomena may be described at the critical point by the same mathematical object – the Landau-Ginzburg-Wilson (LGW) Hamiltonian. That Hamiltonian will include the features – the symmetry and dimensionality – which sort these systems into their universality classes. The RG argument demonstrates that the LGW Hamiltonian applies to a wide range of systems at the critical point by showing that any additional operators which may be appended to that Hamiltonian will fall away on approach to criticality, where only the central LGW operators will remain. The following steps are essential to the explanation thus on offer:[5]

1. Define the effective Hamiltonian for your system of interest:
   (i) Specify the order parameter with symmetry and dimensionality.
   (ii) Specify the central operators of the LGW Hamiltonian.

[3]Note that not all continuous phase transitions are associated with first-order phase transitions in this way.

[4]E.g. the specific heat (in zero magnetic field) c scales as $c \sim (t^{-\alpha})/\alpha$ as $t \to 0$ where $t = \frac{T-T_c}{T_c}$.

[5] To see a full account of the physics of universality and details of the RG see Binney et al. (1992) and Fisher (1998); the philosophical aspects of such an explanation are discussed in detail in Batterman (2016) and Franklin (2018).

(iii) Specify operators in addition to the terms in the LGW Hamiltonian.

2. Apply the RG transformations to that Hamiltonian.

3. Examine the flow towards fixed points in the critical region and note that some operators are irrelevant to the critical behaviour.

4. Thus divide the set of operators into subsets: 'relevant', 'irrelevant' and 'marginally relevant'.

5. Repeat for other systems of interest.

In order to explain universality we must identify commonalities between the different systems in the same universality class – 1(i) and 1(ii) above – and show that such commonalities are sufficient for the common behaviour – 2-4 above. Although 1(iii) can't, in general, be done explicitly, the explanation only depends on the RG demonstration that all distinguishing features are irrelevant – it's not necessary to say exactly which those distinguishing features are. As discussed below, the infinities which are central to some of the anti-reductionist arguments feature in steps 3 and 4.

Overall the explanation takes the following form: consider a universality class composed of four different physical systems A-D. Each of A-D is described in step 1 by an effective Hamiltonian; effective Hamiltonians are ascribed to systems on the basis of various theoretical and empirical data. The RG explanation of universality, by virtue of steps 2-4, tells us that all the details which distinguish A-D, i.e. their irrelevant operators, are, in fact, irrelevant to the critical phenomena. Thus we have an explanation for how otherwise different systems exhibit the same phenomena at the critical point. This explanation relies, of course, on the RG transformations which allow for the categorisation of certain operators as irrelevant.

Importantly, this explanation takes the form of a general explanation of multiply realised phenomena: such phenomena are explained if commonalities are identified among the realisers and these are shown to be sufficient for the multiply realised phenomena to occur. Note that such explanations may be higher level and nothing written so far establishes their reducibility.

3 Anti-reductionist Arguments

Batterman (2000, 2017) and Morrison (2012, 2014) offer two arguments in defence of the view that the explanation just outlined is irreducible. The more general argument is that universality, *qua* instance of multiple realisability, is irreducible because multiple realisability requires abstracted explanations of a particular form.

However, one goal of this paper is to demonstrate that just such abstracted explanations may be reducible. Insofar as my reduction of the RG explanation goes through, we are thus faced with a dilemma: either some instances of MR are, in principle, reducible, or universality is not a case of MR. While I would opt for the former horn, nothing in the rest of the paper hangs on that choice.

The second anti-reductionist argument is much more specific to the case at hand and involves various demonstrations that the RG explanation requires infinities which are inexplicable from the bottom up. As noted by Palacios (2017), two different limits are invoked in the case of continuous phase transitions – the thermodynamic limit and the limit of scale invariance. There is an extensive literature on the thermodynamic limit as it appears in first order phase transitions; as I see no salient differences between appeal to this limit in the two contexts, I do not discuss this further here – see e.g. Butterfield and Bouatta (2012) for a reductionist account of that limit.[6]

The second limit is discussed by Butterfield and Bouatta (2012), Callender and Menon (2013), Palacios (2017), and Saatsi and Reutlinger (2018), among others, and these papers undermine claims that continuous phase transitions are irreducible. However, they pay insufficient attention to the specific role played by the RG (and by the limit of scale invariance) in establishing the irrelevance of certain details, and it is this role which is crucial to the anti-reductionist arguments.[7]

For Batterman, the RG is required because it allows us to answer the following question:

[6]The reductionist claims made here are conditional on a successful resolution of such issues.

[7]For example, Saatsi and Reutlinger (2018, p. 473) do not consider a counterfactual of the form 'if a physical system S did not exhibit effective scale invariance at criticality, then S would not exhibit the critical phenomena of any universality class' in their list of counterfactuals which the RG account is supposed to underwrite.

> **MR**: How can systems that are heterogeneous at some (typically) micro-scale exhibit the same pattern of behavior at the macro-scale? ...
>
> if one thinks (**MR**) is a legitimate scientific question, one needs to consider different explanatory strategies. The renormalization group and the theory of homogenization are just such strategies. They are inherently multi-scale. They are not bottom-up derivational explanations.
>
> [Batterman (2017, pp. 4, 14-15)]

As further elaborated below, the RG seems to Batterman to preclude "bottom-up derivational explanation" because it requires the following infinitary assumption:

> This [fixed point] is a point in the parameter space which, under τ [the RG transformation], is its own trajectory. That is, it represents a state of a system which is invariant under the renormalization group transformation. Of necessity, such a fixed point has an *infinite correlation length* and so lies on the critical surface S_∞. The singularity/divergence of the correlation length ξ is *necessary*.
>
> [Batterman (2011, p. 1045), original emphasis]

I accept that the RG formalism makes use of infinite limits. The salient question, to borrow Norton's (2012) distinction, is whether such infinities are approximations which allow one to use the more tractable infinitary mathematics to approximate features of the finite systems, or, alternatively, idealisations which describe a distinct infinite system. Claiming that the infinities are idealisations would preclude reduction because the macroscopic system with infinite properties has features which may not be reductively explained.

As Batterman demonstrates, the RG argument rests on the assumption of the infinite correlation length which generates absolute scale invariance. In §4 I claim that the physical systems under consideration are not absolutely scale invariant: in fact, one may abstract from the details of the underlying system insofar as such systems are effectively scale invariant; thus the infinitary assumption is best viewed as an approximation.

While Morrison (2014, p. 1155) likewise focusses on explanations of MR phenomena, she claims that RG explanations are irreducible for a different, but related, reason: the "RG functions not only as a calculational tool but as the source of physical information as well". Morrison (2012) makes a similar argument in relation to symmetry breaking in the physics of superconductors. She argues that, in both cases, top-down constraints play an essential role in the physical descriptions which thus rules out reduction. In the present context, Morrison's views may be understood as taking the RG invocation of scale symmetry to be a necessary physical assumption which cannot be understood from the bottom up. Below I argue that the effective scale invariance on which the RG rests is, in fact, reductively explicable. As such, no top-down organising principles are required and Morrison's claims are deflated.

4 Reducing the RG Explanation

Arguments for the reducibility of the explanation of universality have primarily been targeted at Batterman's claims that infinities are essential to the models used to describe continuous phase transitions. I do not have space to consider these arguments in any detail. Suffice it to say that, in my view, none succeeds in reducing the principal feature of the renormalisation group – the assumption of scale invariance. Thus I focus on that aspect of the RG, and claim that it, too, is reducible.

Furthermore, with the notable exception of Saatsi and Reutlinger (2018), not much attention has been paid to the explanation of universality *per se*. This, of course, makes a difference for MR-based objections to reduction, which raise doubts that a reductionist account could explain why the same phenomenon is exhibited in multiple different systems.

As far as the physics is currently developed, the RG plays an ineliminable role in the explanation of universality: it is the only mathematical framework available to predict the precise extent of observed universality of critical phenomena. If its application were truly mysterious, if we had no idea why it worked, then, infinity or no infinity, this would provide exactly the right kind of failure of explanation on which the anti-reductionist could hang their arguments.

I argue in the following that the applicability of the RG to systems undergoing continuous phase transitions is not mysterious. The RG exploits effective scale invariance to set up equations which tell us how certain properties vary with respect to the variation of other properties. It is a piece of mathematics whose applicability is deeply physical – where the assumptions invoked in applying the RG do not hold, the RG's predictions go wrong.

In order fully to reduce the RG explanation, one also must consider the common features shared by each member of the same universality class, and argue that these, too, are reducible to aspects of the microphysical description. Such arguments have been given by the reductionists mentioned above. The innovation of this paper lies in reducing the RG framework, and the assumptions on which it relies; thus, given space constraints, I do not consider the reduction of the symmetry, dimensionality and representation by common Hamiltonians.

4.1 Reducing the Renormalisation Group

The RG argument rests on the assumption of scale invariance, and this is crucial to the demonstration that a class of operators are irrelevant at criticality. I claim that we can provide a bottom-up explanation of this scale invariance and that, as such, the RG arguments provide a mathematical apparatus for relating scale invariance to the irrelevance of certain details. One can see, heuristically, how scale invariance relates to universality: if the system at criticality is effectively scale invariant then many of that systems' features – those which are scale dependent – will turn out to be irrelevant at criticality, and all that will remain are those shared features such as the symmetry and dimensionality.

To argue that the RG explanation is reducible, I first give a more general characterisation of an RG flow. The calculation of each system's dynamics involves integration over a range of scales and energies. The highest energy (smallest scale) cutoff (denoted Λ) corresponds to the impossibility of fluctuations on a scale smaller than the distance between the particles in the physical system. The RG transformation involves decreasing the cutoff thereby increasing the minimum scale of fluctuations considered. Iterating this transformation generates a flow through parameter space designed to maintain the Hamiltonian form and qualitative properties of the system in question.

The RG transformation $\mathcal{R}$ transforms a set of (coupling) parameters $\{K\}$ to another set $\{K'\}$ such that $\mathcal{R}\{K\} = \{K'\}$. $\{K^*\}$ is the set of parameters which corresponds to a fixed point, defined such that $\mathcal{R}\{K^*\} = \{K^*\}$. This fixed point corresponds to the critical point defined physically. At the fixed point, the RG transformation (which changes the scale of fluctuations) makes no difference. Thus the fixed point encodes the property of scale invariance.

Given the Hamiltonian of one of our models, one can define an RG transformation which generates a flow that allows one to: (i) classify certain of the coupling parameters of the system in question as (ir)relevant to its behaviour near the fixed point, (ii) extract the critical exponents from the scaling behaviour near the fixed point.

The RG may be understood as a mathematical framework for exploring how certain properties vary with changing energy, length-scale, or, by proxy, temperature, on approach to the scale invariant critical point. Philosophical discussions of the RG are occasionally prone to mysterianism, but the RG should be considered to be no different from, for example, the calculus. As Wilson (1975, p. 674) notes: "the renormalization group ... is the tool that one uses to study the statistical continuum limit [the point of scale invariance] in the same way that the derivative is the basic procedure for studying the ordinary continuum limit".

The Hamiltonian which represents the system at the critical point, from which the critical exponents are extracted, is scale invariant at the fixed point – all the scale dependent contributions have gone to zero. Such Hamiltonians are known as 'renormalisable'. As such, the explanation provided below for the effective scale invariance of physical systems at criticality underlies the fact that such systems are well-described by renormalisable Hamiltonians at fixed points.

My argument has two steps: I demonstrate that scale invariance is implicit in the power law behaviour which is intrinsic to universality; then I provide a bottom-up explanation of the effective scale invariance for liquid-gas systems, a story somewhat motivated by the observation of critical opalescence. Thus, I show how scale invariance features in the mathematics – the Hamiltonian's renormalisability and the power laws, and how it features in the observed physics – the critical opalescence is a direct consequence of the bottom-up story.

The universality of critical phenomena lies in the sharing of power laws, and hence critical exponents, between members of the same universality class. In what sense are such power laws scale-free? As Binney et al. (1992, p. 20) explain, a phenomenon obeying a power law is independent of scale because one could multiply its characteristic scale length by some factor and the ratio of values will remain constant. For example, consider the power law $f_1 = (r/r_0)^\eta$, and its measurement in the range $(0.5r_0, 2r_0)$. The ratio of largest to smallest value will be identical for measurements centred on $r_0, 10r_0, 100r_0$ – it will always be $4^{|\eta|}$, thus one may superimpose all the power laws by a simple change of scale. By contrast, for $f_2 = \exp(r/r_0)$ the ratio of values will change on scale changes.

Such systems are therefore described as scale-free; the RG is used to predict that at the point of scale invariance the heterogeneous features will be irrelevant. So, in order to work out when this framework is applicable, and why it works, we ought to look at each individual system, (for our purposes let's reserve inquiry to liquid-gas and ferromagnetic-paramagnetic systems) and identify the underlying processes which lead to effective scale invariance at the critical point. The following two caveats apply to this proposal for reduction:

First, it might be objected that universality may only be explained if the same processes are identified across all the systems exhibiting the universal behaviour; if that were so, the strategy employed here would be inadequate. However, universality may be explained by demonstrating that two conditions are fulfilled: that all the systems share common features, and that their heterogeneous details are irrelevant. While it's essential that the common features are shared by all the systems, the mechanism by which the heterogeneities are irrelevant may differ, so long as all the heterogeneities in fact end up as irrelevant.

Second, although the power laws and renormalisable Hamiltonians at the fixed point are absolutely scale invariant, the physical systems will, at best, be effectively scale invariant – that is, scale invariant within a certain range of length-scales. That should be acceptable because we know that scale invariance is never exactly true of a system: any real system will be finite and thus violate the assumption at some scale. Moreover, this will not generate empirical problems because the power laws are observed for systems approaching criticality – they are predictions about $T \to T_c$, not $T = T_c$. Thus one should only assume that critical exponents asymptotically approach those predicted at the fixed point. While infinite assumptions are required in order to impose the full scale invariance for RG analysis, I claim that we can explain effective scale invariance for finite systems, and that absolute scale invariance is an approximation invoked to make the mathematics tractable.

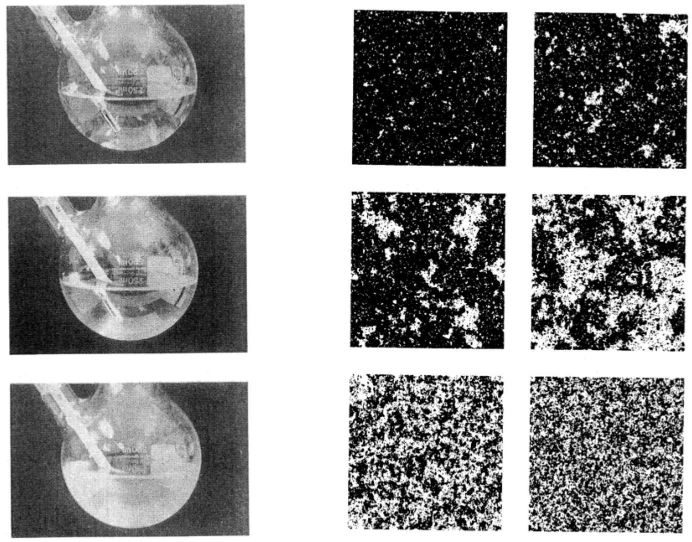
Scale invariance, as it manifests in systems at criticality, is known as 'self-similarity': as scales change the system resembles itself. How do we account for such self-similarity? The critical point, at which a continuous phase transition occurs, corresponds (for liquid-gas systems) to the highest temperature and pressure at which liquid and gas phases can be distinguished.

As is well known, there is a plateau in pressure-volume diagrams, which corresponds to the latent heat (or enthalpy) of vapourisation. This, roughly, is the extra energy needed to break the intermolecular bonds which distinguish liquids from gases and vapours. At the critical point this plateau, and the latent heat of vapourisation vanishes. Now it's difficult precisely to work out the binding energies of the intermolecular bonds. The values for this will be material dependent, and surface tension dependent, and will change at different pressures. But the heuristic argument tells us that the reason the plateau vanishes is because the system has enough temperature, and thus the molecules have sufficient energy to equal the binding energy. The point at which binding energy is exactly matched by kinetic energy will be the critical point.

The isothermal compressibility (κ) is defined as $\kappa_T = \frac{-1}{V}\left(\frac{\partial V}{\partial p}\right)_T$. This corresponds to how much the volume will change (∂V) with a given pressure change (∂p) at fixed temperature (T). As supercritical fluids have far higher compressibility than liquids, and both are present at the critical point, the compressibility diverges. Given, in addition, that the latent heat is zero at criticality, there's nothing to prevent a given bubble expanding arbitrarily. Thus we ought to expect the system to have bubbles of all sizes: this is what is meant by the claim that the system is dominated by fluctuations and has no characteristic scale.[8]

Negligible energy cost for transitions and infinite compressibility leads to self-similarity, and, in certain fluids, the bubbles at all scales lead to a high refraction of visible light. Thus otherwise transparent fluid may become opaque and milky-white. This is known as 'critical opalescence' – see figure 1(a) – and is a visible correlate of a system at criticality.

[8]Note that, for first order phase transitions, the compressibility also diverges; this doesn't lead to scale invariance because latent heat is finite.



(a) (b)

Figure 1: From Binney et al. (1992, pp. 10,19). (a) Critical opalescence is visible when arbitrarily large bubbles form in liquid at criticality. (b) Increasing loss of characteristic scale as $T \to T_c$ in simulations of the Ising model.

Such self-similarity is conceptually crucial to the applicability of the renormalisation group: in order to extract critical exponents from RG equations one identifies a renormalisable Hamiltonian which is scale invariant at the fixed point. Without fluctuations across all scales, systems would fail to be well modelled by such Hamiltonians. The physical argument for diverging fluctuation size justifies the use of a scale invariant mathematical model to represent such systems. Thus, for critical phenomena, the applicability of the RG depends on scale invariance, where this assumption is explicable from the bottom up.

Demonstrating these claims quantitatively is difficult, but the heuristic argument is convincing. Kathmann (2006) reviews theories of the nucleation of gas bubbles in water which generate accurate predictions concerning the rate of bubble growth and the threshold for stability over a range of temperatures; although these models do not reach the critical point, progress is being made.[9]

[9]Constructing exact models is especially difficult because of the fluctuations at a wide range of length scales – precisely the reason that the RG is employed.

Of course, further work could be done to develop these arguments and make them more precise. But there seems to be, in the above, a sound qualitative argument and no in-principle barriers to full derivation. This 'in-principle' ought not to be problematic: we know the relevant physical principles, even if quantitative models are still unavailable.

Moreover, as discussed below, and depicted in figure 1(b), the Ising model allows us quantitatively to predict analogues of the results for liquid-gas systems. While well short of a full explanation, the following discussion illustrates how self-similarity may be reduced for magnetic systems. By treating the Ising model as a stand-in for such systems, a similar kind of reasoning to that given above will go through.

Below the critical point, energy fluctuations will lead to random isolated spin flips. Such flips will be energetically costly and tend to be reversed. The higher the energy, the more likely these are to occur, and if sufficiently many occur then a patch will form, and other spins will have some tendency to align themselves with this patch. However, below the critical point, such patches beyond a certain size will be too costly and spins will overall remain aligned (there is some small probability of net magnetisation flipping, but this is increasingly unlikely further below the critical point).

At the critical point, the energy of the atoms in the lattice is greater than the energetic cost of violating spin alignment, and patches can become arbitrarily large. This results from the latent heat's vanishing and the divergence of the magnetic susceptibility (χ) on approach to the critical point. $\chi_T = \left(\frac{\partial m}{\partial B}\right)_T$ where m is the magnetisation and B represents an external magnetic field. Universality is manifested by the fact that the susceptibility and the compressibility both diverge according to identical power laws with the same critical exponent γ: $\chi_T,\ \kappa_T \sim (T - T_c)^{-\gamma}$. Thus, we have self-similarity and effective scale invariance with bubbles or patches arbitrarily large up to the size of the system.

My aim is to establish the reducibility of the RG relevance and irrelevance arguments. I have demonstrated that the RG is a mathematical procedure that extracts information based on the empirically and theoretically justified assumption of effective scale invariance; this has been shown to be a property shared by different systems at criticality. The key ingredients for effective scale invariance are features of the interactions of neighbouring sub-systems, and the particulate constitution of the materials. While that suggests that these materials are not so different after all, it's worth emphasising that the systems which exhibit universal behaviour are nonetheless dissimilar away from the critical point – it's clear that magnets and liquids have many distinct chemical and physical properties.

The assumption of scale invariance plays a crucial role for the RG – it licences the discarding of scale dependent details; it is precisely this discarding of details which ensures that all systems are commonly described at the critical point. Moreover, discarding such details is what gives the higher-level explanation its stability and parsimony. It is thus incumbent on the reductionist to explain how the higher-level RG account is successful despite its leaving out such details. So, the reductionist should identify physical processes at the lower level which ensure the irrelevance of the discarded details.

As argued above, the physical processes in question are exactly those which lead to effective scale invariance. The fluctuations at all scales make it such that the scale-dependent properties which distinguish systems away from criticality are irrelevant at criticality, when the system is effectively scale invariant. We have identified, at the molecular level, the physical mechanisms which prevent variations in the discarded details from leading to changes in the higher-level description of the system. As such, we are assured that the explanatory value of the higher-level explanation is a consequence of features of the lower-level system.

One upshot of this reductionist account is that we may specify the conditions under which the higher-level description remains a good one. The discarded details are irrelevant while the large scale fluctuations – the bubbles or patches – dominate the physics. As we move to systems which are less scale invariant, as the bubbles die down, the critical point becomes a less accurate description and each system in the class will start to exhibit distinct behaviour. This is reflected in the fact that the macroscale RG description only derives the shared behaviour at the fixed point of scale invariance and predicts distinct behaviour away from the fixed point.

I end this section with the following intuitive physical gloss on the RG explanation: "[b]ecause the fluctuations extend over regions containing very many particles, the details of the particle interactions are irrelevant, and a great deal of similarity is found in the critical behavior of diverse systems" (A. L. Sengers, Hocken, and J. V. Sengers (1977, p.42)). Since we can explain the wide-ranging fluctuations from the bottom-up, the RG explanation of universality is reducible.

5 Conclusion

The field-theoretic RG framework, together with the common features of physical systems in the same universality class, explains how those systems all display the same critical phenomena when undergoing continuous phase transitions. That explanation is a higher-level explanation.

That higher-level RG explanation is nonetheless reducible. That is, we may explain in terms of the microstructure of each system how it is that each aspect of the higher-level explanation is explanatory. We may, in particular, show why the RG categorisation of operators as relevant and irrelevant works. That division depends on the assumption of scale invariance, and the assumption of scale invariance is justifiable when systems are effectively scale invariant at criticality.

The anti-reductionist claim that universality is MR, and MR is essentially irreducible has been undermined by demonstrating that we may arrive at a bottom-up understanding of the common features and of what makes such features sufficient for the common behaviour.

The further argument that the use of the infinite limit imposes an irreducible divide between the higher-level and lower-level models has similarly been countered: while we move to the infinite limit in order to make the mathematics simpler, the effective scale invariance can be shown to follow from details of the particle interactions at criticality – that's what identifies the critical point and allows us to make the corresponding abstractions from scale dependent details. Provided with this bottom-up explanation, there is no further reason to claim that the infinite limit is an idealisation rather than an approximation: for we have explained from the bottom up how the system is approximately self-similar.

One upshot of this discussion is that the RG is not to be regarded as mysterious, or, somehow, as the source of physical information. It is applicable only insofar as the systems to which it is applied have the relevant properties, and their having such properties may be reductively explained.

References

Batterman, Robert W. (2000). "Multiple Realizability and Universality". In: *The British Journal for the Philosophy of Science* 51.1, pp. 115–145.

Batterman, Robert W. (2011). "Emergence, singularities, and symmetry breaking". In: *Foundations of Physics* 41, pp. 1031–1050. DOI: 10.1007/s10701-010-9493-4.

— (2016). "Philosophical Implications of Kadanoff's work on the Renormalization Group". In: *Journal of Statistical Physics (Forthcoming)*.

— (2017). "Autonomy of Theories: An Explanatory Problem". In: *Noûs*. DOI: 10.1111/nous.12191.

Binney, James J. et al. (1992). *The Theory of Critical Phenomena: an Introduction to the Renormalization Group*. Clarendon Press, Oxford.

Butterfield, Jeremy and Nazim Bouatta (2012). "Emergence and Reduction Combined in Phase Transitions". In: *AIP Conference Proceedings* 1446, pp. 383–403. DOI: 10.1063/1.4728007.

Callender, Craig and Tarun Menon (2013). "Turn and Face the Strange ... Ch-ch-changes Philosophical Questions Raised by Phase Transitions". In: *The Oxford Handbook of Philosophy of Physics*. Ed. by Robert W. Batterman. Oxford University Press, pp. 189–223.

Fisher, Michael E. (1998). "Renormalization group theory: Its basis and formulation in statistical physics". In: *Reviews of Modern Physics* 70.2, p. 653.

Franklin, Alexander (2018). "On the Renormalization Group Explanation of Universality". In: *Philosophy of Science* 85.2. DOI: 10.1086/696812.

Kathmann, Shawn M. (2006). "Understanding the chemical physics of nucleation". In: *Theoretical Chemistry Accounts* 116.1, pp. 169–182. DOI: 10.1007/s00214-005-0018-8.

Mainwood, Paul (2006). "Is More Different? Emergent Properties in Physics". PhD thesis. University of Oxford.

Morrison, Margaret (2012). "Emergent Physics and Micro-Ontology". In: *Philosophy of Science* 79.1, pp. 141–166. DOI: 10.1086/663240.

— (2014). "Complex Systems and Renormalization Group Explanations". In: *Philosophy of Science* 81.5, pp. 1144–1156. DOI: 10.1086/677904.

Norton, John D. (2012). "Approximation and Idealization: Why the Difference Matters". In: *Philosophy of Science* 79.2, pp. 207–232.

Palacios, Patricia (2017). *Phase Transitions: A Challenge for Reductionism?* URL: philsci-archive.pitt.edu/13522/.

Saatsi, Juha and Alexander Reutlinger (2018). "Taking Reductionism to the Limit: How to Rebut the Antireductionist Argument from Infinite Limits". In: *Philosophy of Science* 85.3, pp. 455–482. DOI: 10.1086/697735.

Sengers, Anneke Levelt, Robert Hocken, and Jan V. Sengers (1977). "Critical-point universality and fluids". In: *Physics Today* 30.12, pp. 42–51.

Sober, Elliott (1999). "The multiple realizability argument against reductionism". In: *Philosophy of Science*, pp. 542–564.

Wilson, Kenneth G. (1975). "The renormalization group: Critical phenomena and the Kondo problem". In: *Reviews of Modern Physics* 47.4, pp. 773–840.

Woodward, James (2003). *Making Things Happen: A Theory of Causal Explanation*. Oxford Studies in Philosophy of Science. Oxford University Press.