ADAMS THESIS AND THE LOCAL INTERPRETATION OF CONDITIONALS

ABSTRACT. Adams' Thesis states that the probability of a conditional is the probability of the consequent conditional on the antecedent. S. Kaufmann introduced a rival method, the so-called "local interpretation", for calculating the probability of a conditional that, according to a purported majority, squares better with intuition in some circumstances. He also gives an example purporting to show that this new method sometimes corresponds to rational action. We challenge the intuitions and expose a mathematical error in the example. We also offer a model for the local interpretivist semantics. This model puts *theoretical* local interpretivists on ground as solid as that of Thesis abiders for whom conditionals have truth conditions.

1. LOCAL PROBABILITIES FOR CONDITIONALS

Stefan Kaufmann (2004) introduces a method for predicting "strength of belief" (of a purported majority of speakers) in some conditionals. It (the method) is intended as a complement to (and sometime rival of) the so-called *Adams Thesis* (Adams 1996, p. 3), according to which the "probability" of a conditional $A \rightarrow C$ should be the conditional probability $P(C|A)$. Kaufmann presents a scenario in which one is about to draw a ball from one of two bags. The likelihood of drawing from each bag, as well as the contents of each bag, are given in the following table.

$P(\text{Bag } X) = \frac{1}{4}$	$P(\text{Bag } Y) = \frac{3}{4}$
10 red balls,	10 red balls,
9 of them with a black spot	1 of them with a black spot
2 white balls	50 white balls

Kaufmann asks whether strength of belief in

(1) If I pick a red ball, it will have a black spot

ought to be 'high', 'fifty-fifty', or 'low'. Kaufmann writes:

> The judgment of nine out of ten informants to whom I posed this question in an informal survey, as well as my own intuition, is that the answer should be 'low'.

Accordingly, Kaufmann postulates a new, "local" method of calculating a probability for (1). This method may be motivated as follows. Consider an "expert" who knows which bag is in play. (Kaufmann doesn't resort to this device, but it's useful.) By Expert Reflection, our credence in (1) ought to be the expectation of the expert's credence in (1). If it's Bag X, the expert's credence in (1) will be $\frac{9}{10}$ (Kaufmann assumes that speakers do defer to conditional probability, conditional on the bag being fixed), and if it's Bag Y the expert's credence in (1) will be $\frac{1}{10}$. So our credence in (1) (says Kaufmann) might plausibly be

$$\text{(2) } P_l(R \to B) = P(R \to B|X)P(X) + P(R \to B|Y)P(Y)$$
$$= P(B|RX)P(X) + P(B|RY)P(Y) = (\tfrac{9}{10})(\tfrac{1}{4}) + (\tfrac{1}{10})(\tfrac{3}{4}) = \tfrac{3}{10}.$$

(See Section 2 for more on the above display.) Here of course $B =$*Black Spot*, $R =$*Red*, etc. Kaufmann contrasts (2) with what he calls the "global" interpretation (i.e. the interpretation consistent with The Thesis):

$$\text{(3) } P_g(R \to B) = P(B|RX)P(X|R) + P(B|RY)P(Y|R) = P(B|R) = \tfrac{6}{10}.$$

He calls the move from $P(X)$ in (2) to $P(X|R)$ in (3) an "*abductive* inference to the best explanation for the (hypothetical) observation that the ball is red," observing that "This step is evidently not performed by those who give the conditional (1) a 'low' rating."

Kaufmann's work has been influential. Michael Zhao (2014), in particular, attempts to characterize the conditions under which a "local" reading exists.[1] As to the descriptive aspects of Kaufmann's work, we are at least intrigued. Something like the local interpretation may be what some speakers use, at least some of the time. (Though our own "informal survey" suggests an approximate wash in the Bags-and-Balls case; see Section 3.) A further question is whether the local interpretation can be employed as a basis for rational decision in some circumstances where local and global interpretations come apart. Kaufmann attempts to establish that it can, but this attempt founders on a mathematical error, as we observe in Section 4.

There is already one critical reply (Douven 2008) to Kaufmann's paper in the literature. Is there really need for another? We believe that there is, for at least four reasons. First, Douven's main argument is that the local interpretation is inconsistent. To show this, he changes the partition (here $\{X, Y\}$) used in the calculation (2) to come up with a value different than $\frac{3}{10}$. We don't consider this to be damaging to Kaufmann's position; indeed (3) already accomplishes this much, being a version of (2) for the trivial partition $\{X \vee Y\}$, and it yields a value different from $\frac{3}{10}$. What Kaufmann would say here, we think, is that the local interpretation is consistent, but dependent on the choice of partition–so that one ought, technically, to speak of the "local interpretation relative to Ω", where Ω is a partition.

[1] In his paper's final section, "Causation and the origin of local conditionals" Zhao writes: "What is the purpose of having such a way of evaluating conditionals, given that, evaluated this way, their probability does not reflect the credence we should have in the consequent upon observing the antecedent? Call *causal conditionals* any conditional whose assertibility depends on the existence of a causal relation between the antecedent and consequent: for example, 'If the Fed lowers interest rates, then stock prices will go up.' Here...there are two natural readings: one on which we pretend to observe the antecedent, and one on which we pretend to intervene causally to make the antecedent so. ...the first is just the global reading, and the second, the local reading." The problem we have with the second reading is that to "pretend to intervene causally to make the antecedent so" is to pretend also that the probabilities are different than they actually are–indeed, it is to pretend that the global probability of the conditional is the same as its local probability is actually. In the balls and bags scenario, for example, one could pretend that the bag is just translucent enough that one can "make the antecedent so" by choosing a red ball intentionally, or that one can "make the antecedent so" by sampling balls and throwing them back until one has got a red one. Conditional on the chooser's determination to get a red ball, however, $P(B|R)$ is then no longer $\frac{6}{10}$, but $\frac{3}{10}$. So this is hardly any way to generate sympathy for the idea that the global reading isn't always salient.

Second, Douven does little to undermine the claim that the local interpretation is predictive of speaker attitude. Indeed, we think this is a rather hard task, owing to the fact that Kaufmann was definitely onto something: it does appear that many speakers do report something like local probabilities, at least some of the time. However, it isn't hard to show that local interpretation commits speakers to combinations of attitudes that, intuitively, cohere very poorly. This strategy, which we pursue in Section 3, is we think very promising, and is, so far as we know, unique to the literature.

Third, Douven does not satisfactorily rebut Kaufmann's claims of rational efficacy for the local interpretation. Kaufmann's rhetoric places a great deal of importance on this claim–he essentially concedes that if there aren't scenarios where speakers are justified in basing decisions on locality, there is no reason to think that the local interpretation isn't a mere fallacy.[2] But the scenario that he claims does this work does not, owing to a mathematical error. Douven (in his footnote 4) misdiagnosed the example as a case in which true credence and rational action come apart.

Finally, Kaufmann doesn't indicate clearly what he takes conditional sentences to mean; indeed, he doesn't even indicate whether or not he takes them to have truth conditions. Taking cues from his exposition, we pinpoint his commitments and provide, based on these, a semantic model for the local interpretation. The model, which is based on Van Fraassen's (1976) model in support of Adams' Thesis, brings into relief the (very close) relationship between the two theories. All the more so, in fact, in that Van Fraassen's model is only technically appropriate for agents who are certain that their credences are ideal. In the more typical case of an agent with a distribution over the ideal credences that is not concentrated at a point, the correct model to employ (as even Thesis supporters must acknowledge) is formally equivalent to a local interpretativist one.

2. Triviality, Total Probability and the Resilient Equation

Our first task is to pin down Kaufmann's position precisely, and offer a model for its semantics.

In the first line of (2) there appears to be an appeal to the law of total probability

$$P(A) = \sum_{i=1}^{n} P(B_i)P(A|B_i) \text{ for any partition } \{B_1, \ldots, B_n\} \tag{TP}$$

over the partition $\{X, Y\}$, with A the conditional $(R \to B)$. In the second line, meanwhile, the identity $P(R \to B|X) = P(B|RX)$ is used; Fitelson (2015) refers to this identity as the "Resilient Equation". The conjunction of these practices, taken as generally valid rules, has been shown to imply "triviality" by David Lewis (1976). Lewis employs (TP) for $A \to C$ over the partition $\{C, \neg C\}$. This appears to commit Thesis defenders who accept the Resilient Equation to the absurd conclusion $P(A \to C) = P(C)$, which shows in its turn that no Thesis-friendly probability space

[2] We make no such claim, though we do think that most speakers who appear on the surface to be local interpretivists are, to the contrary, just probabilistically naive speakers who are attempting to report a conditional probability and getting it wrong. We take issue, that is, with Kaufmann's claim that speakers employ this very theoretical device "widely and systematically".

admits of a three cell measurable partition $\{E_1, E_2, E_3\}$ with positive measure cells: otherwise, one could set $A = E_1 \cup E_2$ and $C = E_1 \cup E_3$.

Thesis defenders therefore face a choice. Either they must disavow that (TP) applies to conditionals, or they must disavow the Resilient Equation. Kaufmann (who grants that "The Thesis generally accords well with pre-theoretical intuitions", and generally admits that the global reading is always available) disavows the Resilient Equation. For though he uses it in (2), he notes that it is 'not warranted by an "official" rule of the probability calculus', suggesting that its legitimacy there is exceptional.[3] Even so, as a sometime employer of this equation, he feels obligated to at least address the threat of "triviality", writing "I do not claim that conditionals have local interpretations with respect to just any variable."

Lewis (1976) discusses two types of Thesis proponents–those for whom conditionals are propositions, so that their "probabilities" are true probabilities, and those for whom conditionals are not propositions, so that their "probabilities" are "assertabilities", "felicities" or other "ersatz probabilities". The former are obligated to deny the Resilient Equation; the latter are obligated to deny the Law of Total Probability (as applied to conditionals). Those who believe conditionals to be propositions believe them to be highly context-sensitive; those who believe they aren't employ a highly non-classical notion of (ersatz) conditional probability. Stalnaker and Van Fraassen are of the first type, Adams is of the second. Kaufmann's position is close enough to that of a Thesis defender that he must, to some approximation, fall into one camp or the other. Since he unflinchingly employs the Law of Total Probability, but flags his one use of the Resilient Equation as exceptional, we judge him to fall into the first: he views conditionals as genuine (context sensitive) propositions.

This is enough information to evolve Van Fraassen's model for Thesis abidement into a semantics for the local interpretation. We relegate this task to an appendix, as it is orthogonal to our primary purpose, but it's an important exercise in that it shows the local interpretation to be coherent and not to lapse into triviality. Another reason it's important is that Van Fraassen's model is technically appropriate only in the rare case that the agent knows herself to maintain ideal credences. In cases where the agent's distribution over the ideal credence function fails to be concentrated at a point, Van Fraassen's model must be modified so as to become formally equivalent to the local interpretation model. Kaufmann's position is, therefore, not an idle curiosity.

3. On the intuitions underwriting the local interpretation

Despite its virtues, the local interpretation is, we think, widely misunderstood. Kaufmann, in particular, believes that the local interpretation is applied by speakers "widely and systematically". We doubt this. Speakers who answered "low" to Kaufmann's original question aren't, for the most part, local interpretivists; they are global interpretivists who are bad at computing conditional probabilities.

[3]Douven (2008) appears to agree, stating that the first line of (2), "being a mere application of the law of total probability, is unassailable."

Assume for example that after the draw mentioned in (1) you'll continue to draw balls from the (same) bag, without replacement. We take the status of (1) to be the same in this revised scenario; in particular it still has local probability $\frac{3}{10}$ with respect to $\{X, Y\}$. Consider:

(4) If I pick a red ball first, the first red ball picked will have a black spot

and

(5) If I pick a white ball first, the first red ball picked will have a black spot

Notice that (4) is essentially equivalent to (1). In particular it has the same local probability, $\frac{3}{10}$, and (owing to the equivalence), ought to have similar "salience". But the local probability of (5) is also equal to $\frac{3}{10}$. Local interpretivists should, therefore, report equal strengths of belief in (4) and (5). Our intuition was that most speakers would not. In an informal survey, only of 5 of 16 calculus students did. (6 reported higher strength of belief in (4), 2 reported higher strength of belief in (5), and 3 answered "Does not exist".)

A more extreme example: imagine that someone present (Rose) knows which bag is in play. Presently you will ask her which. She's pretty reliable; you think there is a probability of .999 that she will report the correct bag. Consider

(6) If Rose says "Bag Y is in play" then Bag Y is in play

(7) If Rose says "Bag X is in play" then Bag Y is in play

The local interpretivist has strength of belief $\frac{3}{4}$ in each of (6), (7). But it's beyond far-fetched that many speakers would pretheoretically assign (6) and (7) equal strengths of belief. The best explanation for this is the obvious one: local interpretation is *not* something that speakers do "widely and systematically". (Perhaps they do it widely, but they don't do it systematically.) Most don't report strengths of belief consistent with the local interpretation when correlations are obvious, which suggests that they do it in oblique cases only because they fail to judge that (or how) the relevant antecedents correlate with the corresponding background variables.

Another reason to think that local interpretation isn't intuitively attractive is that it violates the following intuitive desideratum.[4]

Weights: Suppose $\{A_1, \ldots, A_n\}$ is a partition of event space. For any event C, $\min\{P(A_i \to C) : i = 1, \ldots, n\} \leq P(C) \leq \max\{P(A_i \to C) : i = 1, \ldots, n\}$.

Note that if *Weights* is satisfied, $P(C)$ is always a weighted average of the values $P(A_i \to C)$.[5] To motivate the independent plausibility of *Weights*, consider the (pre-theoretical) awkwardness of the following combination of attitudes:

(i) strength of belief in "if we watch the game then we'll order pizza" is $\frac{1}{2}$;
(ii) strength of belief in "if we don't watch the game then we'll order pizza" is $\frac{1}{2}$;
(iii) strength of belief in "we'll order pizza" is $\frac{1}{3}$.

[4]The law of total probability agrees with *Weights* under The Thesis. More generally, if C is a conditional $D \to E$, where D and E are in the event space, then under The Thesis $P(C) = P(E|D)$ is, by (TPC), a weighted average of $\{P(A_i \to C) : i = 1, \ldots, n\} = \{P(E|DA_i) : i = 1, \ldots, n\}$.

[5]This is trivial; every element of a closed bounded interval is a weighted average of its endpoints.

Such combinations are realized under local interpretation. Consider the following:

$P(\text{Bag } X) = \frac{1}{2}$	$P(\text{Bag } Y) = \frac{1}{2}$
10 red balls,	20 red balls,
9 of them with a black spot	2 of them with a black spot
20 white balls	10 white balls
2 of them with a black slot	9 of them with a black spot

Suppose you are about to start picking a balls uniformly at random from whichever of the two bags is before you. Local interpretivists will subscribe to:

(I) strength of belief in "if I pick a red ball first, the first ball picked will have a black spot" is $\frac{1}{2}$;

(II) strength of belief in "if I pick a white ball first, the first ball picked will have a black spot" is $\frac{1}{2}$;

(III) strength of belief in "the first ball picked will have a black spot" is $\frac{11}{30}$.

Philosophers having a theoretical story to tell about local interpretation will explain such examples away, of course, but the majority of speakers (who aren't theorists) will reject such combinations of attitudes out of hand.[6] Therefore, we hold that similarity between majority speaker response and the local interpretation is accidental.

4. On a book of bets said to support the local interpretation

In Section 7 of his paper, Kaufmann writes:

> Assuming that the present proposal is descriptively correct, it raises a deeper question: Is it an account of a fallacy–one that is committed widely and systematically, but fallacious nonetheless–or is the departure from (The Thesis), at least in some circumstances, the "correct" interpretation of a conditional? (...) Are there situations in which it would be detrimental to base one's actions upon (The Thesis) and advantageous to follow the local interpretation? A negative answer would not imply that the local interpretation is not what speakers use, but only that it is not what they *should* use.

Kaufmann claims that the answer is "not negative". In support of this claim, he sets up the following scenario. At time 0, B pays a bookie $P(C|A)$. X's return on the wager is: $P(C|A)$ if $\neg A$, 1 if AC, 0 if $\neg CA$. B regards this wager as fair.

Between time 0 and time 1, both B and the bookie will learn whether A is true, but if in fact A is true the bookie will find out, in addition, "in what way" it is true, i.e. which of $AX_1, \ldots, AX_n$ is true, "where the X_i are the values of some variable $\mathbf{X}$ that we take to be causally relevant.... The conditional probability of C is not evenly

[6] In a previous draft, we examined cases such as $Q =$ "If the first two picks are red with black spots, the third pick will be white." The local probability of Q is not defined (since the conjunction of Y with "the first two picks are red with black spots" is null), yet speakers overwhelmingly report non-vacuous strengths of belief (most often reverting to the global probability $\frac{2}{10}$, in our informal survey) for Q. Such "strength of belief pluralism" is disconcerting, particularly in that few speakers are likely to take themselves to employ different methods in their evaluations of Q and (1).

distributed over all AX_i, and $\mathbf{X}$ does not causally depend on A." B will not have this additional information, but does know that the bookie will have it.

At time 1, B perceives her expected net payoff to be zero. (If $\neg A$ she knows that payoff to be exactly zero, and if A she knows it will be $1-P(C|A)$ if C and $-P(C|A)$ otherwise, the former with probability $P(C|A)$.) The bookie's expected payoff, however (from her own perspective), can be positive or negative. If negative, she would like to make a new bet to exactly cancel the first.

Although this bet looked fair to B prior to the offer of it, the mere fact that the bookie wants to make it is evidence, for B, that she should not. Kaufmann claims (equation 30 in his paper) that the bookie's expected payoff at time 0 is now curiously influenced by B's refusal to do business with the bookie at time 1. Indeed, he claims that it is now "the weighted sum of these posterior payoffs for each X_i: $\sum_{X_i \in \mathbf{X}}(P(C|A) - P(C|AX_i))P(X_i)$."

That isn't right. Expected payoff for the bookie on AX_i is indeed $P(C|A)-P(C|AX_i)$, but on $\neg AX_i$ it is zero. The correct time zero expectation is therefore

$$\begin{aligned}
&\sum_{X_i \in \mathbf{X}} (P(C|A) - P(C|AX_i))P(AX_i) \\
&= \sum_{X_i \in \mathbf{X}} \Big(\frac{P(CA)}{P(A)}\Big)P(AX_i) - \sum_{X_i \in \mathbf{X}} \Big(\frac{P(CAX_i)}{P(AX_i)}\Big)P(AX_i) = P(CA) - P(CA) = 0.
\end{aligned}$$

The fair price for the wager, therefore, is $P_g((A \to C) = P(C|A)$, not $P_l(A \to C)$ as Kaufmann (based on the fact that $\sum_{X_i \in \mathbf{X}}(P_l(A \to C) - P(C|AX_i))P(X_i) = 0)$ claims. It is implicit that Kaufmann thinks the fair price is the probability we should assign the conditional $A \to C$, so this is actually an argument *for* The Thesis.

5. Appendix: Semantics for The Thesis and the Local Interpretation

Thesis proponents face pitfalls. For an example, assume that $\{A, B, C, D\}$ partitions event space into equal measure events. A Thesis literalist might be tempted to write

$$P(\neg D \to A) = P(A|\neg D) = \frac{1}{3} \tag{5.1}$$

$$\neq \frac{1}{2} = P(A \vee D)P(A|A) + P(B \vee C)P(A|B \vee C) \tag{5.2}$$

$$= P(A \vee D)P(\neg D \to A|A \vee D) + P(B \vee C)P(\neg D \to A|B \vee C). \tag{5.3}$$

Passage from (5.2) to (5.3) goes, e.g., by the Resilient Equation which, recall, says that $P(X \to Y|Z) = P(Y|XZ)$ in general. A Thesis defender ought (so it might seem) to endorse the Resilient Equation, because if she were to learn Z, her new probability function would be $Q(\cdot) = P(\cdot|Z)$, and the resilient equation looks to follow in a line or two from $Q(X \to Y) = Q(Y|X)$. However, if we accept the Resilient Equation then (5.1)-(5.3) appears to violate the Law of Total Probability.

Thesis proponents have two options for explaining this away that are worth considering. In the first, indicative conditionals aren't propositions, and have neither truth conditions nor true probabilities. On this view, it would be better to write, say,

$$\begin{aligned} &P^*(\neg D \to A) = P(A|\neg D) = \frac{1}{3} \\ \neq &\frac{1}{2} = P(A \vee D)P^*(\neg D \to A|A \vee D) + P(B \vee C)P^*(\neg D \to A|B \vee C). \end{aligned}$$

Here $P^*(Z \to W|K) = P(W|ZK)$ (by definition) whenever Z and K are classical events (i.e. sentences containing no occurence of "$\to$"). P^* might be said to denote "assertability", "felicity", "strength of belief" or some other such notion, and would not be assumed to obey the probability axioms. It would have a few probability-like features (whatever is inherited from its definition), but lack others. Changing to P^* the instances of P evaluated at conditionals in (5.1)-(5.3) would thus eliminate the apparent violation of the Law of Total Probability there. Lewis (1976) writes:

> Adams himself seems to favor this hypothesis about the semantics of conditionals. ...I have no conclusive objection to the hypothesis that indicative conditionals are non-truth-valued sentences, governed by a special rule of assertability that does not involve their non-existent probabilities of truth. I have an inconclusive objection, however: ...(a) need to explain away all seeming examples of compound sentences with conditional constituents.

Such compound sentences appear to be well-formed and sometimes useful. ("That car is old, and if you honk its horn, a fuse will blow out.") Under the current proposal, the rules for assigning them ersatz (conditional) "probability" would include

$$\begin{aligned} &P^*((A \to C) \wedge B) = P(B|A)P^*(A \to C|B) \\ =&P(B|A)P(C|AB) = P(BC|A) = P^*(A \to BC). \end{aligned}$$

The proponent of this method will, in particular, deny the more familiar-looking identity $P^*((A \to C) \wedge B) = P(B)P^*(A \to C|B)$. On this account, the following "Total Law of Probability for Conditionals" replaces the usual Law:

$$P^*(A \to C) = \sum_{i=1}^{n} P(B_i|A)P^*(A \to C|B_i) \text{ for any partition } \{B_1, \ldots, B_n\}. \quad (TPC)$$

Note the corresponding identity $P(C|A) = \sum_{i=1}^{n} P(B_i|A)P(C|AB_i)$. So this method of explaining away (5.1)-(5.3) is to hold onto the Resilient Equation and simply assert that the law corresponding to the Law of Total Probability for conditionals is (TPC).[7]

The second method is to relinquish the Resilient Equation by giving conditionals truth conditions (and hence true probabilities) in accord with The Thesis. We'll adapt

[7] This account of ersatz conditional probabilities appears also to sidestep triviality arguments such as that of Lewis (1976) and Fitelson (2015).

an approach from Van Fraassen (1976).[8] Let $\mathbf{N}$ denote the set of natural numbers (with zero). Let $(\mathcal{A}, P)$ be the agent's personal probability space (here $\mathcal{A}$ is a finite Boolean algebra of "classical events" and P a probability function having domain $\mathcal{A}$). Denote by Ω the set of ordinal numbers $\{\sum_{i=0}^{k} n_i\omega^i : k \in \mathbf{N}, n_1, \ldots, n_k \in \mathbf{N}\}$. For each $\beta \in \Omega$, independently sample x_β, an atomic event in $\mathcal{A}$, in accord with the probability function P. (One says that one is conducting "Bernoulli trials".) Let Q be product measure on the set of functions taking Ω to the set of atomic events in $\mathcal{A}$.

Given the complete sampling $\{x_\beta : \beta \in \Omega\}$, an event $A \in \mathcal{A}_0 = \mathcal{A}$ is true at β if and only if $x_\beta \subset A$. A *degree 1 conditional*, i.e. a conditional having the form $A \to C$, where $A, C \in \mathcal{A}_0$, is true at β if and only if $x_{\beta+k} \subset C$, where k is the least natural number such that $x_{\beta+k} \subset A$. Let $\mathcal{A}_1$ be the Boolean algebra generated by $\mathcal{A}_0$ and the degree 1 conditionals. A *degree 2 conditional* is a conditional not having lesser degree and having the form $A \to C$, where each of $A, C \in \mathcal{A}_1$. Such a conditional is true at β if and only if $x_{\beta+k\omega} \subset C$, where k is the least natural number such that $x_{\beta+k\omega} \subset A$. Let $\mathcal{A}_2$ be the Boolean algebra generated by $\mathcal{A}_1$ and the degree 2 conditionals.

Continue in this fashion. Having constructed the finite Boolean algebra $\mathcal{A}_m$, define a *degree* $m+1$ *conditional* to be a conditional not having lesser degree and having the form $A \to C$, where each of $A, C \in \mathcal{A}_m$. Such a conditional is true at β if and only if $x_{\beta+k\omega^m} \subset C$, where k is the least natural number such that $x_{\beta+k\omega^m} \subset A$. Let $\mathcal{A}_{m+1}$ be the Boolean algebra generated by $\mathcal{A}_m$ and the degree 1 conditionals. Finally, for any sentence A, let $P(A) = Q($"A is true at 0"$)$.

The proof of Thesis confirmation is trivial. For sentences A and B, let m be the least natural number such that $\{A, B\} \subset \mathcal{A}_m$. (So that $A \to B$ is a degree $m+1$ conditional.) Let $x = P(A)$ and let $y = P(A \wedge B)$. The event "$A \to B$ is true at 0" is the disjoint union of the events "$A \wedge B$ is true at 0", "A is false at zero and $A \wedge B$ is true at ω^m", "A is false at zero and ω^m and $A \wedge B$ is true at $2\omega^m$", ... Thus $P(A \to B) = Q($"$A \to B$ true at 0"$) = y + (1-x)y + (1-x)^2y + \cdots = \frac{y}{x} = P(B|A)$.

This proof highlights the role of the independence of the trials: the truth of A at 0 shouldn't correlate with the truth of A at any other $\beta = k \in \mathbf{N}$ (for classical events A); the truth of $A \to B$ at 0 shouldn't correlate with the truth of $A \to B$ at any other ordinal $\beta = k\omega$ (for classical events A, B), etc. This assumption is only realistic, however, when the agent's distribution over the "ideal credence function" (given her epistemic position) over $\mathcal{A}_0$ is concentrated at a point.

Since the probability of a conditional is a true probability on this approach, the Law of Total Probability will be valid under its auspices. The Resilient Equation, however, fails. For example, the passage from $P(\neg D \to A|A \wedge D)$ in (5.2) to $P(A|A) = 1$ in (5.3) is denied. In fact, $\neg D \to A$ will come out false whenever $x_0 = D$ and

[8]Our model was developed independently of (if decades later than) Van Fraassen's, which may serve to indicate that it isn't as arbitrary as it might appear (to some). Van Fraassen's method, incidentally, only succeeds in verifying the Thesis (which he attributes to Stalnaker) for relatively simple conditionals, namely of the forms $A \to B$, $(A \to B) \to C$ and $A \to (B \to C)$. Though our approach treats more general indicative conditionals, there is a small cost–we are forced to use different "closeness" relations for conditionals of different complexities. This approach therefore does not arise from Stalnaker's semantics precisely. (This was one of Van Fraassen's concerns.)

$x_1 \in \{B, C\}$. It's important to note that this doesn't imply that the agent's Thesis-abiding habits aren't "resilient" under conditionalization. Should the agent learn that $A \vee D$ obtains, she will come to have full credence in the proposition she will then express using the sentence "$\neg D \to A$"; this just won't be the same proposition that she intended by that sentence before. Lewis wrote of such context sensitivity:

> But presumably our indicative conditional has a fixed interpretation, the same for speakers with different beliefs, and for one speaker before and after a change in his beliefs. Else how are disagreements about a conditional possible, or changes of mind? Our question, therefore, is whether the indicative conditional might have one fixed interpretation that makes it a probability conditional for the entire class of all those probability functions that represent possible systems of beliefs.

To understand Lewis's objection, let's go back to the agent who originally holds that $\neg D \to A$ is true at 0 when, in a sequence of possible $(A \vee B \vee C \vee D)$ worlds, the "nearest" (i.e. least index) $(A \vee B \vee C)$-world is an A world. When the agent learns that, actually, $A \vee D$ holds, the possible $(B \vee C)$ worlds lose salience; either they are to be dropped from the agent's model altogether or the model should be altered, and these worlds pushed "further away" from 0 than the $(A \vee D)$ worlds. (Recall, this is epistemic possibility we are working with.) Either way, the agent comes to regard $\neg D \to A$ as almost surely true; all of the nearest $\neg D$ worlds are A worlds.

Van Fraassen blames Lewis's modal realism for a refusal to admit updating of more concrete aspects of model structure (the nearness relation on possible worlds, in particular) upon revision. That isn't fair, though, because Lewis doesn't want to involve possible worlds at all in fixing the meaning of the indicative conditional. (He only wants that for the counterfactual conditional; he regards the indicative as having truth conditionals equivalent to the material conditional.) Lewis's complaint remains, and Van Fraassen (like any Thesis proponent who advocates for indicative conditional truth values) must simply bite the bullet on context sensitivity.

The Van Fraassen model must be tweaked in an obvious way whenever one's distribution over the "true" chances isn't concentrated at a point. Suppose for example that in Kaufmann's original example, the identity of the bag in play is completely determined by the answer to the following question:

Question. Let $a_n = 6^n + 8^n$. Determine the remainder upon dividing a_{83} by 49.

Suppose an agent understands the question perfectly well, and knows how to answer it, albeit not quickly. They've been told that the answer is either 7 or 35. If it is 7, Bag Y is before them. If it is 35, Bag X is before them. They assign the proposition that it is 7 probability $\frac{3}{4}$, so that is the probability they assign to Bag Y. So the situation is exactly as in Kaufmann's original question.

Or is it? If the answer to **Question** is 7, then it is necessarily 7. So if it is 7, there aren't any "possible worlds" where it's 35. If the agent applies the Van Fraassen model naively under her epistemic credences, however, she'll get a sample array of ersatz possible worlds, some of which will be 7-worlds, and some of which will be 35-worlds. That seems wrong. Instead, she should employ two separate functions from ordinal numbers to atoms. In one the implied sampling of the balls will be as they would be if Bag X were in play. In the other, the implied sampling will be as they would be if Bag Y were in play. One of the implied sampling distributions is apocryphal (the associated worlds aren't possible), though the agents doesn't know which. One model structure (the 7-world model) assigns (1) probability $\frac{1}{10}$. The other (the 35-world model) assigns (1) probability $\frac{9}{10}$. So the agent assigns (1) probability $\frac{3}{4}(\frac{1}{10}) + \frac{1}{4}(\frac{9}{10}) = \frac{3}{10}$; the local probability. If she were an ideal agent, she'd know that the answer to **Question** is in fact 35, and would assign (1) a probability of $\frac{9}{10}$.

What's different about Kaufmann's original setup? Arguably little. Perhaps now our agent *does* have ideal credences (which bag is in play it determined, say, by coin toss), and there are possible worlds of both types (Bag X worlds, and Bag Y worlds). What's to stop Kaufmann from claiming, however, that if the bag in play is Bag X (say) then all of the Bag X worlds are nearer than all of the Bag Y worlds? Nothing whatsoever. He can make that claim if he wants to. (It seems that he does.) That the claim is plausible is grounds for concluding that local interpretation *theorists* are on solid enough ground. (As solid, at least, as that of Van Fraassen 1976.)

On the other hand, Kaufmann's hypothesis that "many" speakers employ the local interpretation is supported by neither intuition nor empirical considerations. Kaufmann forged this hypothesis on the scant "evidence" that 9 out of 10 speakers he interviewed assigned (1) "low" strength of belief. But this result (*if* it can be consistently replicated, which we frankly doubt) is consistent with many hypotheses. It was premature to conclude that such speakers are theoretical local interpretivists. (And to think than an example like the one we criticized in Section 4 might work was wishful thinking.) Contrasting pairs such as (6) and (7) appear to indicate, to the contrary, that most were probably "fair weather" local interpretivists at best.

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