Why we cannot see the tails of Schrödinger's cat

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Abstract

In collapse theories of quantum mechanics such as the GRW theory, the measurement result is represented by the post-measurement state which is still a superposition of different result branches, although the modulus squared of the amplitude of one result branch is close to one. This leads to the tails problem. In this paper, I present a new analysis of the tails problem of collapse theories, and suggest a more complete solution to the problem. First, I argue that the tails problem exists not only in collapse theories, but also in Everett's theory and even in Bohm's theory. Moreover, I point out that the tails problem has two levels: the physical and mental levels, which may be called the objective and subjective tails problems, respectively. One needs to analyze not only the connection between high modulus-squared values and macro-existence, but also the connection between these values and our experience of macro-existence. Second, I briefly review the existing solutions to the objective and subjective tails problems. I argue that although the objective tails problem may be solved more directly, one needs to further investigate the psychophysical connection in order to solve the subjective tails problem. Third, I analyze how the mental state of an observer is determined by her wave function in collapse theories. It is argued that the mental content of an observer is related to the amplitude of each result branch of the superposition she is physically in, and it may be composed of all possible results. Moreover, it is conjectured that the modulus squared of the amplitude of each result branch may determine the vividness of the part of the mental content containing the result. Finally, I argue that this vividness conjecture may help solve the subjective tails problem of collapse theories.

1 Introduction

Quantum mechanics is a very successful physical theory due to its accurate empirical predictions. The core of the theory is the Schrödinger equation and the Born rule. The Schrödinger equation is linear and it governs the time evolution of the wave function assigned to a physical system. The Born rule says that the result of a measurement on a physical system is definite but generally random, and the probability is given by the modulus squared of the wave function of the system. However, when assuming the wave function of a physical system is a complete description of the system, the linear Schrödinger equation is incompatible with the appearance of a definite measurement result. This leads to the measurement problem. Maudlin (1995) gave a precise formulation of the problem in terms of the incompatibility. Correspondingly, the three approaches to avoiding the incompatibility lead to three main solutions to the measurement problem, namely Bohm's theory, Everett's theory, and collapse theories. In these competing quantum theories, the measurement results are represented by different physical states. In particular, in collapse theories such as the GRW theory the measurement result is represented by the post-measurement state which is still a superposition of different result branches (although the modulus squared of the amplitude of one result branch is close to one). This leads to the tails problem (Pearle, 1989; Albert and Loewer, 1990; Lewis, 1995), which is essentially to explain why high modulus-squared values are macro-existence determiners, as well as the determiners of our macro-experience. It seems that this problem has not been solved in a satisfactory way (see McQueen, 2015 and references therein). In this paper, I will present a new analysis of the tails problem, and suggest a more complete solution to the problem.

This paper is organized as follows. In Section 2, I first introduce Maudlin's conventional formulation of the measurement problem, and then introduce two new formulations of the problem which lays more stress on the result assumption and psychophysical connection, respectively. It is pointed out that the three main solutions to the measurement problem, namely Bohm's theory, Everett's theory and collapse theories, correspond to three different representations of the measurement result and three different forms of psychophysical connection. In Section 3, I introduce the tails problem, which may be regarded as the remnant of the measurement problem. First, I note that the problem exists not only in collapse theories as usually thought, but also in Everett's theory and even in Bohm's theory (with a different form). Next, I point out that the tails problem has two levels: the physical and mental levels, which may be called the objective and subjective tails problems, respectively. In order to solve the problem, one needs to analyze not only the connection between high modulus-squared values and macro-existence, but also the connection between these values and our experience of macro-existence. Third, I argue that collapse theories may avoid the structured tails problem.

In Section 4, I briefly review the existing solutions to the objective and subjective tails problems. I suggest a more complete solution to the objective tails problem, and point out that in order to solve the subjective tails problem one needs to know the psychophysical connection of an observer. In Section 5, I analyze how the mental state of an observer is determined by her wave function (or, more precisely, by the physical state represented by the wave function of her brain) in collapse theories. It is argued that the mental content of an observer is related to the amplitude of each result branch of the superposition she is physically in, and it may be composed of all possible results. Moreover, it is conjectured that the modulus squared of the amplitude of each result branch may determine the vividness of the part of the mental content containing the result. In Section 6, I argue that this vividness conjecture may help solve the subjective tails problem of collapse theories. Conclusions are given in the last section.

2 The measurement problem

According to Maudlin's (1995) formulation, the measurement problem of quantum mechanics originates from the incompatibility of the following three claims:[1]

(C1). the wave function of a physical system is a complete description of the system;

(C2). the wave function always evolves in accord with a linear dynamical equation, e.g. the Schrödinger equation;

(C3). each measurement has a definite result.

The proof of the inconsistency of these three claims is familiar. Suppose a measuring device M measures the x-spin of a spin one-half system S that is in a superposition of two different x-spins $1/\sqrt{2}(|up\rangle_S + |down\rangle_S)$. If (C2) is correct, then the state of the composite system after the measurement

[1]Maudlin (1995) called this problem the problem of outcomes.

must evolve into the superposition of M recording x-spin up and S being x-spin up and M recording x-spin down and S being x-spin down:

$$1/\sqrt{2}(|up\rangle_S |up\rangle_M + |down\rangle_S |down\rangle_M). \tag{1}$$

The question is what kind of state of the measuring device this represents. If (C1) is also correct, then this superposition must specify every physical fact about the measuring device. But by symmetry of the two terms in the superposition, this superposed state cannot describe a measuring device recording either x-spin up or x-spin down. Thus if (C1) and (C2) are correct, (C3) must be wrong.

It can be seen that there are three direct solutions to the measurement problem thus formulated. The first solution is to deny the claim (C1), and add some hidden or additional variables and corresponding dynamics to explain the appearance of definite measurement results. A well-known example is Bohm's theory (Bohm, 1952). The second solution is to deny the claim (C3), and assume the existence of many equally real worlds to accommodate all possible results of measurements (Everett, 1957; DeWitt and Graham, 1973). In this way, it is also possible to explain the appearance of definite measurement results in each world including our world. This approach is called Everett's interpretation of quantum mechanics or Everett's theory. The third solution is to deny the claim (C2), and revise the Schrödinger equation by adding certain nonlinear and stochastic evolution terms to explain the appearance of definite measurement results. Such theories are called collapse theories (Ghirardi, 2016).[2]

In order to facililate my later analysis, I will introduce another two formulations of the measurement problem. In the first formulation, the measurement problem originates from the incompatibility of the following three assumptions:

(B1). the linear dynamics: the wave function of a physical system always evolves in accord with a linear dynamical equation, e.g. the Schrödinger equation;

(B2). the Born rule: after each measurement the probability of obtaining a particular result is given by the modulus squared of the wave function of the measured system;

(B3). the standard result assumption: the measurement result is represented by the wave function of the measuring device.[3]

It can be seen that by a similar proof as above these three assumptions are incompatible (see also Maudlin, 1995). This formulation highlights the important role of the result assumption in causing the measurement problem. In order to lead to the measurement problem, the wave function of a physical

[2]Certainly, there are also other solutions to the measurement problem, such as a recently suggested solution which denies both the claims (C1) and (C2) (Gao, 2017a).

[3]Strictly speaking, when the measuring device is entangled with the measured system after the measurement, the measuring device (by itself) will not have a wave function, and the composite system will be in an entangled state. In this case, the state of the measuring device is sometimes referred to as an improper mixture of different result states. For simplicity I always say the wave function of the measuring device (or the observer) in this paper. Note also that this result assumption is used in the standard formulation of quantum mechanics, and thus it may be called the standard result assumption.

system is not necessarily a complete description of the system, and it is only required that the wave function of the measuring device represents the measurement result.

However, this formulation of the measurement problem is still not precise. It has been realized that the measurement problem in fact has two levels: the physical level and the mental level, and it is essentially the determinate-experience problem (Barrett, 1999; Gao, 2017b). The problem is not only to explain how the linear dynamics can be compatible with the appearance of definite measurement results obtained by physical devices, but also, and more importantly, to explain how the linear dynamics can be compatible with the existence of definite experiences of conscious observers. After all, what we are sure of is that we as observers obtain a definite result and have a definite mental state after a measurement, while we are not sure of what physical state this mental state corresponds to. However, this mental aspect of the measurement problem is ignored in the above physicalistic formulations, which define the problem at the physical level. Recently Gao (2017b) introduced the mentalistic formulation of the measurement problem which defines the problem at the mental level and lays more stress on the psychophysical connection. In the formulation, the measurement problem originates from the incompatibility of the following three assumptions:

(A1). the linear dynamics: the wave function of a physical system always evolves in accord with a linear dynamical equation, e.g. the Schrödinger equation;

(A2). the Born rule: after each measurement the probability of obtaining a particular result is given by the modulus squared of the wave function of the measured system;

(A3). the standard psychophysical connection: the mental state of an observer is (uniquely) determined by her wave function.[4]

The proof of the inconsistency of these assumptions is also similar to the above proof. Suppose an observer M measures the x-spin of a spin one-half system S that is in a superposition of two different x-spins, $1/\sqrt{2}(|up\rangle_S + |down\rangle_S)$. If (A1) is correct, then the physical state of the composite system after the measurement will evolve into the superposition of M recording x-spin up and S being x-spin up and M recording x-spin down and S being x-spin down:

$$1/\sqrt{2}(|up\rangle_S \, |up\rangle_M + |down\rangle_S \, |down\rangle_M). \tag{2}$$

If (A3) is also correct, then the mental state of the observer M will be determined by this superposed wave function.[5] Since the mental states cor-

[4]In this paper, I will always say "the wave function of an observer" for simplicity, and this denotes "the wave function of the brain of an observer", and when the brain of the observer is entangled with another system, this denotes the entangled state of the composite system. Certainly, it is extremely difficult to know what the wave function or physical state of an observer is, let alone its connection with the mental state of the observer. But, as we will see, what I will consider below is a very simple situation, in which the connection between the physical state and the mental state is assumed for each result branch of a post-measurement superposition, and what I need to analyze is just what the connection is for the whole superposition.

[5]Note that (A3) is incompatible with the existence of many worlds. In the many-worlds theory, a post-measurement wave function corresponds to many observers, and the

responding to the physical states $|up\rangle_M$ and $|down\rangle_M$ differ in their mental content, the observer M being in the superposition (2) will have a mental content different from the mental content of M being in each branch of the superposition by the symmetry of the two branches. In other words, the result that M obtains is neither x-spin up nor x-spin down when she is physically in the superposition (2). While according to (A2), after the measurement the result that M obtains is either x-spin up or x-spin down with the same probability 1/2. Therefore, (A1), (A2) and (A3) are incompatible.

By these two new formulations of the measurement problem, we can look at the three main solutions of the problem from a new angle. Since the assumption (A2), namely the Born rule, has been verified by experiments with great precision, and even an approximate form of (A2) is enough for leading to the incompatibility, denying (A2) is not an option. Then, the solution to the measurement problem must deny either the assumption (A1) or the assumption (A3) or both. Denying the assumption (A1) is the same as denying the claim (C2) in Maudlin's (1995) original formulation, which means that the Schrödinger equation must be revised. This corresponds to collapse theories. In this case, the measurement result is represented by the wave function of the measuring device, and the mental state of an observer is determined by her wave function. Denying the assumption (A3) means that the mental state of an observer is determined not by her wave function

mental state of each observer is (uniquely) determined not by her wave function, but by the corresponding branch of her wave function.

but by another physical state. If the physical state is additional variables or hidden variables, then the solution will be hidden variable theories, a typical example of which is Bohm's theory. If the physical state is a decoherent branch of the wave function, then the solution will be Everett's theory. Finally, denying both the assumptions (A1) and (A3) will lead to a particular form of collapse theories. I will not discuss it in this paper. In other words, I will suppose that (A3) is a basic assumption of collapse theories in this paper.[6]

To sum up, the three main solutions to the measurement problem, namely Bohm's theory, Everett's theory and collapse theories, correspond to three different result assumptions and three different forms of psychophysical connection. In fact, there are only three types of physical states that may determine the mental state of an observer, which are (1) the wave function in collapse theories, (2) certain branches of the wave function in Everett's theory, and (3) other hidden variables such as particle configuration in Bohm's theory.[7]

[6]The reason is that in collapse theories such as the GRW theory it is usually assumed that the wave function of a system is a complete description of the physical state of the system. Then by the principle of psychophysical supervenience, the mental state of an observer will be naturally determined by her wave function. If assuming the mental state of an observer is determined by a branch of her wave function such as the high-amplitude branch, then we will need an additional explanation (see further discussion below).

[7]Here I do not consider the many-minds theory, in which no physical state determines the mental state of an observer, and the principle of psychophysical supervenience is violated.

3 The residual tails problem

No matter what physical state represents the measurement result, the state should be definite in some objective sense and also correspond to a definite mental state whose content contains only the result. For example, after a measurement, the state of the pointer of a measuring device which represents the measurement result should indicate a definite position, and when an observer observes the pointer she should also obtain a definite record corresponding to the position. Only if the above solutions to the measurement problem satisfy this requirement, can they claim to really solve the measurement problem.

Let us first see the collapse theories. In collapse theories such as the GRW theory, the preferred bases, which represent definite measurement results and correspond to definite mental states of an observer, are not position eigenstates or other localized states with compact support, but localized states which spread out to infinity or have infinitely long tails in space. This leads to the so-called tails problem; one needs to explain in what sense the position of an object such as the pointer of a measuring device is definite when its wave function has infinitely long tails in space, being a superposition of different position eigenstates.[8] Besides, in collapse

[8]It should be noted that the formulation of the tails problem given here is somewhat different from the original formulation of the problem given by Albert and Loewer (1990) and Lewis (1995). In the original formulation, the problem originates from the incompatibility between the eigenvalue-eigenstate link, the collapse dynamics and the existence of definite measurement results, and it is also called the bare tails problem (Wallace, 2008; McQueen, 2015).

theories, the post-measurement state is still a superposition of the preferred bases, although one of which has high amplitude close to one. Thus, besides the above familiar tails problem at the physical or objective level, there is also a tails problem at the mental or subjective level, which is to explain how an observer has a definite mental state such as recording a definite result when her wave function is a superposition of different result branches, in which the modulus squared of the amplitude of one result branch is close to one.

It has been argued that there is a stronger version of the tails problem for collapse theories, the so-called structured tails problem (Cordero, 1999; Wallace, 2008; Maudlin, 2010; McQueen, 2015). The latest formulation of the problem, which is given by McQueen (2015), is as follows. For a post-measurement state in collapse theories, if the structure of the high-amplitude branch determines a pointer with a definite position, then so do the structures in the low-amplitude branches or tails. This is because these branches are structurally isomorphic (or at least relevantly structurally similar). Nothing about low modulus-squared value can suppress this isomorphic structure. Therefore, collapse theories will have an Everettian many-worlds ontology.

There are several responses to the structured tails problem, which seem to be not wholly satisfactory (see McQueen, 2015 for a review). In my view, the main issue with the above conditional formulation of the structured tails problem is in the antecedent “the structure of the high-amplitude branch determines a pointer with a definite position”. As noted in the previous section, collapse theories do not assume (as a fundamental postulate) that a branch of the post-measurement wave function, either high-amplitude or low-amplitude, determines the measurement result such as the pointer with a definite position; rather, they assume (as a fundamental postulate) that the whole post-measurement wave function determines the measurement result. This means that in collapse theories not only the structures of the result branches of the post-measurement wave function, but also the amplitudes of these branches, may both determine the measurement result. Thus it is possible that collapse theories may avoid the structured tails problem. However, as McQueen (2015) correctly pointed out, collapse theories do need to explain (based on the above assumption) why the high-amplitude branch *actually* determines the measurement result or why the tails can be ignored in determining the result. I will analyze this tails problem of collapse theories in the next section.

Next, let us see Everett’s theory. In Everett’s theory, the post-measurement state is a superposition of many decoherent result branches, which are quasi-classical states reasonably localized in position and momentum with infinitely long tails in space. These decoherent result branches, called worlds, are the preferred bases, which represent definite measurement results and correspond to definite mental states of an observer. Again, since the preferred bases are not position eigenstates or localized states with compact support, but localized states which have infinitely long tails in space, Everett's theory, like collapse theories, also has a tails problem at the physical or objective level. However, since each decoherent result branch correspond to a world in which an observer has a definite mental state, it seems that there is no tails problem at the mental or subjective level for Everett's theory.[9]

Finally, let us see Bohm's theory. In Bohm's theory, the measurement result is not represented by the post-measurement wave function or any decoherent result branch of it, but represented by the relative configuration of Bohmian particles (i.e. positions of all of the particles relative to one another) after the measurement, which is always definite. Then, although each result branch has infinitely long tails in space, the measurement result represented by the particle configuration is definite in any case, and thus, unlike collapse theories and Everett's theory, Bohm's theory has no tails problem as defined above. However, it seems that there is also a problem induced by the tails of the result branches, a problem about what the result is. Since every result branch has tails spreading out to infinity, any two result branches will overlap in space. Then, when the particle configuration is in the overlaping region of two result branches and the amplitudes of the

[9]Due to the imperfectness of decoherence, a more careful analysis may be needed here. For example, the decoherent branch corresponding to each world may be also a superposition of different preferred bases that correspond to definite mental states of an observer.

two branches are the same there, it cannot be determined, even in principle, which result the particle configuration represents. It seems that this may be a more serious problem than the tails problem.[10]

To sum up, the tails problem is universal in the sense that it exists not only in collapse theories (as usually thought), but also in Everett's theory. Moreover, it is arguable that the problem may also exist in Bohm's theory, though in a different form. The tails problem can be regarded as the remnant of the measurement problem. Only after solving it can one claim to really solve the measurement problem. In the next section, I will analyze how to solve the tails problem of collapse theories.

4 A suggested approach to solving the tails problem

As I noted above, the tails problem of collapse theories has two levels: the physical and mental levels, which may be called the objective and subjective

[10]Note that the Born rule requires that there should exist a one-to-one correspondence from the relative configurations of Bohmian particles to the result branches of a post-measurement superposition in Bohm's theory. The reason is as follows. A measurement is an interaction between the measured system and the measuring device. In Bohm's theory, this interaction is described by the Schrödinger equation with a potential term whose concrete form is determined by the interaction. As a result, the different values of the measured quantity are correlated with the different branches of the post-measurement wave function. Then, different measurement results must be represented, first of all, by the different result branches of the post-measurement wave function. For example, in the Stern-Gerlach experiment which measures the spin of a particle, the measurement is realized by the spin-magnetic field interaction, which is described by the Schrödinger equation for the wave function. Thus the measurement result being spin-up or spin-down is encoded in a result branch of the post-measurement wave function. This necessiates the above requirement of the Born rule.

tails problems, respectively. It is arguable that breaking the tails problem down into an objective problem and a subjective problem seems necessary. The objective tails problem is to explain in what sense the position of an object such as the pointer of a measuring device is definite when its wave function is a superposition of different position eigenstates and has infinitely long tails in space. This problem may be solved more directly, e.g. solved by "a proposal for how to use language" (Lewis, 2006). However, the subjective tails problem, which is to explain how an observer has a definite mental state such as recording a definite result when the wave function of her brain is a superposition of different result branches (in which the modulus squared of the amplitude of one result branch is close to one), cannot be solved directly in this way. Even though we may *say* that the position of the brain of an observer is definite in some sense when its wave function has infinitely long tails in space, we cannot conclude only by this that her mental state is also definite. We still need to know the psychophysical connection in order to account for the definite mental state of the observer, as the *actual* physical state, the wave function of her brain, is still a superposition of different result branches (or in other words, there are still brains in the tails).

There are several suggested solutions to the objective tails problem, such as the fuzzy link (Albert and Loewer, 1996; Lewis, 2003, 2006), the accessible mass density link (Ghirardi, Grassi and Benatti, 1995), the mass density simpliciter link (Monton, 2004), and the flashy link (Lewis, 2006; Tumulka, 2018). According to Monton (2004), the mass density simpliciter link seems to provide a satisfying solution to the problem.[11] In this solution, the reason macroscopic objects appear highly localized is that most all of their mass (as defined by the solution) is concentrated in a small region of space, the region where the object appears to be localized. In particular, the pointer of a measuring device indicates a definite position in the sense that most all of the mass of the pointer is concentrated in the position.

However, it is arguable that the solution to the tails problem provided by the mass density simpliciter link is not complete. As argued by Myvold (2018) recently, resorting merely to the mass density is not enough for solving the objective tails problem. The reason is that objects being what they are also depend on the realistic features of the wave function that go beyond the mass density it represents. For example, the electromagnetic properties of ordinary objects such as the pointer of a measuring device are crucial to their being what they are. A pointer-shaped region of high mass density composed entirely of electrically neutral matter would not hold its shape except momentarily, nor would it reflect light, which would pass right through it without resistance. In other words, the other properties of the objects

[11] It has been argued that the mass density link does not solve the tails problem when adopting a structural or functional account of the measuring device (see Maudlin, 2010; McQueen, 2015). As argued in the last section, however, this argument seems not valid for collapse theories. The reason, again, is that in collapse theories it is the whole post-measurement state, not any branch of it, that represents the measurement result, which means that the measurement result may be determined not only by the structure in each branch but also by the amplitude of each branch. This permits the possibility of the mass density link or a more complete version of it (see below) solving the tails problem.

represented by the wave function are an integral part of the ontology, and we may need to resort to the complete ontic state in space to solve the objective tails problem in general. Therefore, a more complete solution to the objective tails problem may be something like this: no matter what entity and ontic state the wave function represents, the reason macroscopic objects appear highly localized is that most all of the entities is concentrated in the small region where the object appears to be localized, and these entities may be massive and also charged when the involved objects are massive and charged.

As to the subjective tails problem, there are also a few discussions in the literature, although they seem to be not wholly satisfying. Ghirardi, Grassi and Benatti (1995) first suggested that the low-density matter in the tails is inaccessible, that is, observers cannot directly measure it. They proposed the accessible mass density link, according to which only accessible mass density is real. Monton (2004) further proposed the mass density simpliciter link, and pointed out that in order to solve the tails problem, a certain assumption about the psychophysical connection needs to be made. He suggested that since the masses of particles in a brain are concentrated in the appropriate regions of space, it is reasonable to assume that the appropriate mental states are determined by or supervene on those mass concentrations. Then, although each of the particles in the brain is located in an unbounded region of space, the corresponding mental state of the brain is still definite.

Although these suggestions are reasonable, they also have several potential problems. The first problem is still the incompleteness problem. As noted above, it is the wave function, not the mass density, that is a complete description of the physical state. Then by the principle of psychophysical supervenience, the mental state of an observer will be naturally determined by her wave function. If assuming the mental state of an observer is determined only by the mass concentration, not by other aspects of the underlying physical state such as charge concentration etc, then we will need an additional reasonable explanation, which is still wanting in the literature. As a result, a more complete version of Monton's assumption (for now) will be that the mental state is determined by the high-amplitude branch, that is, the mental state is determined by the physical state that is described by the high-amplitude branch, no matter what state the physical state is.

Second, the above suggestions do not tell us what the mental state of a brain is when the brain is in a superposition of two of the same mass concentrations or two branches with the same amplitude. It seems that they cannot provide a definite answer to this question. Since there is no unique mass concentration or high-amplitude branch in this case, one cannot determine what the mental state is according to the above suggestions. This may be not a problem of the above suggestions, since it seems unnecessary to analyze such an equal superposition in order to solve the tails problem. However, since the dynamical collapse of a wave function is purely random, there is a non-zero probability that the above superposition holds for a time longer than the normal conscious time of an observer, and thus it is still necessary to answer what mental state is generated by a brain being in the superposition. Moreover, as we will see later, answering the question of how the mental state of an observer is determined by her wave function may help solve the subjective tails problem.

Last but not least, one still needs to justify the claim that the mental state is determined by the high-amplitude branch in the final analysis. As noted previously, collapse theories only assume that the mental state of an observer is determined by her wave function, which is the complete description of her physical state. Thus the tails problem is still there; one needs to explain why the mental state is *actually* determined by the high-amplitude branch, without depending on the tails.

To sum up, in order to solve the subjective tails problem of collapse theories once and for all, one needs to know how the mental state of an observer is determined by her wave function. I will try to answer this question in the next section.

5 How is the mental state of an observer determined by her wave function?

Let us first consider an observer M being in a definite post-measurement state:

$$|1\rangle_P \, |1\rangle_M \,, \tag{3}$$

where $|1\rangle_P$ is the state of a pointer being centered in positions x_1, and $|1\rangle_M$ is the state of the observer M who observes the pointer being in positions x_1. As noted before, $|1\rangle_M$ is a quasi-classical state reasonably localized in position and momentum with infinitely long tails in space.[12] Since this preferred basis is assumed to corrrespond to a definite mental state, there is no subjective tails problem here.[13] Moreover, if there are a range of wave functions that correspond to the same definite mental state (in which the observer observes the pointer being in the same position), and these preferred bases include all post-collapse states, then the subjective tails problem will be solved for collapse theories. However, it seems more probable that, as usually thought, the preferred bases do not include all post-collapse states in collapse theories. In this case, some post-collapse states are superposi-

[12]If this is not true, then Everett's theory will be wrong. For in Everett's theory the decoherent branches are just such quasi-classical states.

[13]Why there are preferred bases (which correspond to definite mental states) and exactly what they are is the preferred basis problem. My following analysis is independent of how to solve the preferred basis problem, which is beyond the scope of this paper.

tions of different preferred bases. Then, in order to solve the subjective tails problem, we still need to answer the question of how the mental state is determined by a superposition of different preferred bases in collapse theories.

Consider an observer M being in the following superposition:

$$\alpha \left|1\right\rangle_P \left|1\right\rangle_M + \beta \left|2\right\rangle_P \left|2\right\rangle_M , \tag{4}$$

where $\left|1\right\rangle_P$ and $\left|2\right\rangle_P$ are the states of a pointer being centered in positions x_1 and x_2, respectively, $\left|1\right\rangle_M$ and $\left|2\right\rangle_M$ are the states of M who observes the pointer being in positions x_1 and x_2, respectively, and α and β, which are not zero, satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. The question is: What does M observe when her mental state is determined by the above superposition?[14]

In order to solve the subjective tails problem, we need to consider only how the mental content of M is determined by the amplitude of each result branch of the superposition she is physically in.[15] First of all, it can be seen that the mental content of M is related to the the amplitude of each result branch of the superposition she is physically in. When $|\alpha|^2$=1 and $|\beta|^2$=0, M will observe the pointer being in position x_1. When $|\alpha|^2$=0 and $|\beta|^2$=1,

[14]Note that I will not use the bare theory-style reasoning to analyze this question. The main reason is that although it can be applied to collapse theories with tails, it will lead to some well-known problems, such as the empirical incoherence problem and the problem of no account of statistical results (Barrett, 1999: 116-8). Moreover, if the bare theory is true, then the collapse of the wave function will be not needed to solve the measurement problem.

[15]How the mental content of an observer is determined by the phase of each result branch of her wave function will be analyzed elsewhere.

M will observe the pointer being in position x_2. When $\alpha = \beta = 1/\sqrt{2}$, by the symmetry of the two result branches, the mental content of M will be neither the content of observing the pointer being in position x_1 nor the content of observing the pointer being in position x_2.[16]

Next, it can be seen that the mental content of M does not contain the content of observing the pointer being in another position x_3 that is different from x_1 and x_2. When $|\alpha|^2$=0 or $|\beta|^2 = 0$, the mental content of M does not contain the content of observing the pointer being in position x_1 or x_2. Then, similarly, the mental content of M does not contain the content of observing the pointer being in another position x_3, since the amplitude of the corresponding term $|3\rangle_P |3\rangle_M$ is exactly zero in her wave function. This means that when $|\alpha|^2 \neq 0$ and $|\beta|^2 \neq 0$, the mental content of M has four possibilities: (1) it is empty, or it contains neither the content of observing the pointer being in position x_1 nor the content of observing the pointer being in position x_2; (2) it contains only the content of observing the pointer being in position x_1; (3) it contains only the content of observing the pointer being in position x_2; and (4) it contains both the part of the content of observing the pointer being in position x_1 and the part of the content of observing the pointer being in position x_2. These possibilities may depend

[16]Note again that in collapse theories it is assumed that the mental state of an observer is (uniquely) determined by her wave function, not by a branch of the wave function, and there is always one observer with the unique mental content throughout a quantum measurement with many possible results. By contrast, in the many-worlds theory, this assumption is rejected, and when $\alpha = \beta = 1/\sqrt{2}$ there are two mental contents, one corresponding to observing x_1 and the other corresponding to observing x_2.

on the values of $|\alpha|^2$ and $|\beta|^2$.

Finally, I will analyze how the mental content of M is determined by the amplitude of each branch of the superposition she is physically in. I will resort to the assumption of the continuity of psychophysical connection. It says that the mental properties of a system change continuously with the changes of its physical properties. Concretely speaking, the change of a mental property of a system depend on the changes of its physical properties, and the dependence is represented by a continuous function, in which the physical properties are described by independent variables, and the mental property is described by a dependent variable. This continuity assumption can be regarded as an extension of the principle of psychophysical supervenience, which requires that the mental properties of a system cannot change without a change in its physical properties.[17] Although the continuity assumption is stronger than the principle of psychophysical supervenience (the former implies the later, while the later does not imply the former), it is consistent with our experiences and experimental observations and seems also reasonable; if the continuity assumption is wrong, then either the changes of the mental properties of a system do not depend on the changes of its physical properties (which violates the principle of psychophysical supervenience) or the dependence is discontinuous (which seems lack of a reasonable

[17]The standard definition of supervenience is that a set of properties A supervenes on another set B in case no two things can differ with respect to A-properties without also differing with respect to their B-properties (see McLaughlin and Bennett, 2014). The principle of psychophysical supervenience is in accordance with this definition.

explanation).

By the continuity assumption, when $|\alpha|^2 \approx 1$ (or $|\alpha|^2$ is arbitrarily close to 1), the mental content of M still contains the content of observing the pointer being in position x_1. If when $|\alpha|^2 \approx 1$ the mental content of M does not contain the content of observing the pointer being in position x_1, then the psychophysical connection will be discontinuous when the physical state changes from $|\alpha|^2 = 1$ to $|\alpha|^2 \approx 1$; when $|\alpha|^2 = 1$, the mental content of M is the content of observing the pointer being in position x_1, while when $|\alpha|^2 \approx 1$, the mental content of M does not contain the content of observing the pointer being in position x_1. Similarly, when $|\beta|^2 \approx 1$, the mental content of M still contains the content of observing the pointer being in position x_2.

Now the question is: when does the mental content of M not contain the content of observing the pointer being in position x_1 or x_2? It seems that the threshold value of $|\alpha|^2$ or $|\beta|^2$ may be any value in the set $[0, 1)$ when there is no further restriction. Since the situations are the same for $|\alpha|^2$ and $|\beta|^2$, the threshold values for them should be the same, which can be denoted by ϵ. There are two cases. The first case is $\epsilon \geq 1/2$. In this case, when $|\alpha|^2 > \epsilon$, we have $|\beta|^2 < \epsilon$, and the mental content of M contains only the content of observing the pointer being in position x_1. Similarly, when $|\beta|^2 > \epsilon$, we have $|\alpha|^2 < \epsilon$, and the mental content of M contains only the content of observing the pointer being in position x_2. While when $|\alpha|^2 \leq \epsilon$ and $|\beta|^2 \leq \epsilon$, the mental content of M is empty. In particular, if $\epsilon = 1/2$, then when $|\alpha|^2 = 1/2$ and $|\beta|^2 = 1/2$, the mental content of M is empty.

The second case is $\epsilon < 1/2$. In this case, when $|\alpha|^2 < \epsilon$, we have $|\beta|^2 > \epsilon$, and the mental content of M contains only the content of observing the pointer being in position x_2. Similarly, when $|\beta|^2 < \epsilon$, we have $|\alpha|^2 > \epsilon$, and the mental content of M contains only the content of observing the pointer being in position x_1. While when $|\alpha|^2 > \epsilon$ and $|\beta|^2 > \epsilon$, the mental content of M contains both the part of the content of observing the pointer being in position x_1 and the part of the content of observing the pointer being in position x_2.

It seems that the most natural threshold value is $\epsilon = 0$.[18] I will mainly analyze this case below for simplicity, and the analysis also applies to other threshold values.

In the case of $\epsilon = 0$, when $|\alpha|^2 \neq 0$ and $|\beta|^2 \neq 0$, the mental content of M contains both the part of the content of observing the pointer being in position x_1 and the part of the content of observing the pointer being in position x_2. Then, how do these two parts constitute the whole mental content of M? Obviously the constitution depends on the values of $|\alpha|^2$ and $|\beta|^2$. In the extreme cases, when $|\alpha|^2$=0 or $|\beta|^2 = 0$, the mental content of M

[18] If $\epsilon \neq 0$, then since the form of psychophysical connection is supposed to be a law of nature, ϵ will be a new constant of nature, while its appearance is in want of an additional reasonable explanation. By comparison, it seems more natural to assume $\epsilon = 0$. In this case, there exists no such a new constant of nature, and no additional explanation is needed either. Here it is worth noting that for a particular human brain there may exist a practical nonzero threshold value (like the resolution limit of the human eye), and when $|\alpha|^2$ or $|\beta|^2$ is smaller than the threshold value, the mental content does not contain the content of observing the pointer being in position x_1 or x_2. Since this threshold value is determined by the concrete condition of the brain, its existence is understandable. See the next section for further discussion.

does not contain the content of observing the pointer being in position x_1 or x_2, while when $|\alpha|^2 = 1$ or $|\beta|^2 = 1$, the mental content of M is the content of observing the pointer being in position x_1 or x_2. This suggests that there exists a mental property that is determined by the modulus squared of the amplitude, and by the continuity assumption, when the modulus squared of the amplitude changes continuously, the mental property also changes continuously.

Then, what property is this mental property? Here is a seemingly reasonable conjecture. Since when the modulus squared of the amplitude of a result branch, $|\alpha|^2$ or $|\beta|^2$, changes from 1 to 0, the corresponding mental content changes from visible to invisible, it seems natural to conjecture that the mental property determined by the modulus squared of the amplitude is a certain property of visibility of mental content or vividness of conscious experience, and the larger the modulus squared of the amplitude, the higher the vividness of the corresponding part of the mental content. For example, when $|\alpha|^2$ is close to 1 the part of the mental content of M observing the pointer being in position x_1 has almost the highest vividness, while when $|\alpha|^2$ is close to 0, the part of the mental content of M observing the pointer being in position x_1 has almost the lowest vividness. In particular, when $|\alpha|^2 = |\beta|^2 = 1/2$, the part of the mental content of M observing the pointer being in position x_1 and the part of the mental content of M observing the pointer being in position x_2 have the same intermediate vividness.

It is worth pointing out that the above mental property, the so-called vividness of mental content, if it exists, is purely a quantum property in the sense that it is determined by the modulus squared of the amplitude of a result branch of a post-measurement superposition. A classical observer or an observer being in a definite result state would not have this mental property; the modulus squared of the amplitude is either zero or one for her.[19] Moreover, since the collapse time is usually extremely short for observers like us, it is very likely that we have never had a conscious experience with intermediate vividness, such as observing both x_1 and x_2 with intermediate vividness. However, it seems that no principles prohibit an observer different from us such as an observer with a smaller brain having such a conscious experience in collapse theories.[20]

To sum up, I have argued that the mental content of an observer is related to the amplitude of each result branch of the superposition she is physically in, and it may be composed of every result (when the threshold

[19]However, a classical observer may have a similar property of vividness of conscious experience, which is determined by the particular structure and state of her brain. By comparison, the above property of vividness originates from the form of psychophysical connection, and it is independent of a particular brain.

[20]Here a deep question arises: when an observer has a conscious experience with intermediate vividness, can she put it into her physical memory or report it to another observer? If assuming a reductionist view regarding the mind-body problem, then one could argue that this is impossible; the linear quantum dynamics cannot give rise to the experimental report "I saw both x_1 and x_2 with intermediate vividness". In this case, the vividness conjecture, like the bare theory, will also have the empirical incoherence problem. On the other hand, if assuming a dualist view regarding the mind-body problem, then this problem may be avoided and the vividness conjecture may be testable in principle. But that will require that some mental property should introduce certain definite nonlinearity into the quantum dynamics, which seems to be an astonishing hypothesis. Since this issue is irrelevant to the solution of the subjective tails problem given in the next section, I will discuss it in more detail in another paper.

value is zero). Moreover, it is conjectured that the modulus squared of the amplitude of each result branch may determine the vividness of the part of the mental content containing the corresponding result.

6 Solving the subjective tails problem

It can be seen that the above analysis of how the mental state of an observer is determined by her wave function may help solve the subjective tails problem for collapse theories. In particular, if assuming the modulus squared of the amplitude of each result branch determines the vividness of the corresponding result in the whole mental content, then the subjective tails problem may be solved.

First, consider the case of $\epsilon = 0$, namely the case of the threshold value being zero. In this case, when the modulus squared of the amplitude of a result branch is close to zero, the part of the mental content corresponding to the branch will have almost the lowest vividness. It is conceivable that below a certain small threshold value an ordinary human observer will not be able to be consciously aware of the corresponding mental content. In other words, as noted before, for a human observer M there may exist a practical threshold value $\epsilon_p > 0$, then if $|\alpha|^2$ or $|\beta|^2$ is larger than zero but smaller than ϵ_p, the corresponding part of content of observing the pointer being in position x_1 or x_2 will not appear in the actual mental content of M. Then, even though in collapse theories the post-measurement state of an observer is still a superposition of different result branches, if only the sum of the squared modulus of the amplitudes of the low-amplitude branches or tails is smaller than ϵ_p, the observer will be consciously aware of only the part of the mental content corresponding to the branch with high amplitude, and those branches with low amplitudes will have no corresponding parts of content appearing in the mental content of the observer. This may provide a possible solution to the subjective tails problem of collapse theories.

Next, consider the case of $\epsilon > 0$. It can be seen that in this case the subjective tails problem may be solved more easily, and the larger the threshold value, the more easily the problem can be solved. In this case, when the modulus squared of the amplitude of a result branch is smaller than ϵ, the part of the mental content corresponding to the branch will not appear in the mental content of the observer. Then, if the sum of the squared modulus of the amplitudes of the low-amplitude branches of the post-measurement state of an observer is smaller than ϵ, the observer will be unable to be consciously aware of the parts of the mental content corresponding to these branches. This may solve the subjective tails problem without resorting to the vividness conjecture. Certainly, if the sum is larger than ϵ, then we still need the above argument for the case of $\epsilon = 0$ to solve the subjective tails problem.

Ghirardi, Grassi and Benatti (1995) have suggested that observers cannot directly measure the low-density matter in the tails, and thus they are inaccessible or invisible for observers. Moreover, Monton (2004) has further suggested that the mental state supervenes on the high-density matter. These suggestions are consistent with the above analysis, which may provide a more complete solution to the subjective tails problem.

7 Conclusions

It has been a debated issue how to solve the tails problem of collapse theories. The problem is essentially to explain why high modulus-squared values are macro-existence determiners, as well as the determiners of our macro-experience. In this sense, the tails problem has two levels: the physical and mental levels, which may be called the objective and subjective tails problems, respectively. In this paper, I give a new analysis of the tails problem of collapse theories, and suggest a more complete solution to it. In particular, I analyze how the mental state of an observer is determined by her wave function in order to solve the subjective tails problem. It is argued that the mental content of an observer is related to the amplitude of each result branch of the superposition she is physically in, and it may be composed of all possible results. Moreover, it is conjectured that the modulus squared of the amplitude of each result branch may determine the vividness of the part of the mental content containing the result. Finally, I argue that the vividness conjecture may help solve the subjective tails problem of collapse theories.

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