A simple proof that the many-worlds interpretation of quantum mechanics is inconsistent

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Abstract

The many-worlds interpretation of quantum mechanics is based on three key assumptions: (1) the completeness of the physical description by means of the wave function, (2) the linearity of the dynamics for the wave function, and (3) multiplicity. In this paper, I argue that the combination of these assumptions may lead to a contradiction. In order to avoid the contradiction, we must drop one of these key assumptions.

The many-worlds interpretation of quantum mechanics (MWI) assumes that the wave function of a physical system is a complete description of the system, and the wave function always evolves in accord with the linear Schrödinger equation. In order to solve the measurement problem, MWI further assumes that after a measurement with many possible results there appear many equally real worlds, in each of which there is a measuring device which obtains a definite result (Everett, 1957; DeWitt and Graham, 1973; Barrett, 1999; Wallace, 2012; Vaidman, 2014). In this paper, I will argue that MWI may give contradictory predictions for certain unitary time evolution.

Consider a simple measurement situation, in which a measuring device M interacts with a measured system S. When the state of S is $|0\rangle_S$, the state of M does not change after the interaction:

$$|0\rangle_S \, |ready\rangle_M \rightarrow |0\rangle_S \, |ready\rangle_M \, . \tag{1}$$

When the state of S is $|1\rangle_S$, the state of M changes and it obtains a measurement result:

$$|1\rangle_S \, |ready\rangle_M \rightarrow |1\rangle_S \, |1\rangle_M \, . \tag{2}$$

The interaction can be represented by a unitary time evolution operator, U. Then the above two processes can be formulated as follows:

$$U\left|0\right\rangle_S \left|ready\right\rangle_M = \left|0\right\rangle_S \left|ready\right\rangle_M . \tag{3}$$

$$U\left|1\right\rangle_S \left|ready\right\rangle_M = \left|1\right\rangle_S \left|1\right\rangle_M . \tag{4}$$

According to MWI, there is no world branching, and there is still one measuring device, namely the original one, after the above evolution.

Now suppose the measuring device M interacts with the system S being in a superposed state $\left|0\right\rangle_S + \left|1\right\rangle_S$. For simplicity I omit the nomalization factor $1/\sqrt{2}$. By the linear Schrödinger equation, the state of the composite system after the interaction will evolve into the following superposition:

$$\left|0\right\rangle_S \left|ready\right\rangle_M + \left|1\right\rangle_S \left|1\right\rangle_M . \tag{5}$$

That is:

$$U(\left|0\right\rangle_S + \left|1\right\rangle_S)\left|ready\right\rangle_M = \left|0\right\rangle_S \left|ready\right\rangle_M + \left|1\right\rangle_S \left|1\right\rangle_M . \tag{6}$$

According to MWI, there is world branching after this interaction, and the post-measurement state corresponds to two worlds, in each of which there is a measuring device which has a definite state, either being in the ready state or obtaining the result 1.[1]

In order to see whether MWI is consistent, let's analyze possible evolution of the above post-measurement state or the corresponding worlds. First, consider a unitary time evolution operator, U_A, which changes $\left|0\right\rangle_S \left|ready\right\rangle_M$ to $\left|1\right\rangle_S \left|1\right\rangle_M$ and $\left|1\right\rangle_S \left|1\right\rangle_M$ to $\left|0\right\rangle_S \left|A_0\right\rangle_M$:

$$U_A\left|0\right\rangle_S \left|ready\right\rangle_M = \left|1\right\rangle_S \left|1\right\rangle_M , \tag{7}$$

$$U_A\left|1\right\rangle_S \left|1\right\rangle_M = \left|0\right\rangle_S \left|A_0\right\rangle_M , \tag{8}$$

where $\left|A_0\right\rangle_M$ is a definite state of M. The first evolution of the measuring device is exactly the same as the evolution of the measuring device in (4); the state of the measuring device changes from $\left|ready\right\rangle_M$ to $\left|1\right\rangle_M$, and it obtains a definite result.

Then the unitary time evolution of the above post-measurement state is

$$U_A(\left|0\right\rangle_S \left|ready\right\rangle_M + \left|1\right\rangle_S \left|1\right\rangle_M) = \left|1\right\rangle_S \left|1\right\rangle_M + \left|0\right\rangle_S \left|A_0\right\rangle_M . \tag{9}$$

[1]Here I omit the environment terms in the evolution, which, in a more complete form, should be $U(\left|0\right\rangle_S + \left|1\right\rangle_S)\left|ready\right\rangle_M \left|ready\right\rangle_E = \left|0\right\rangle_S \left|ready\right\rangle_M \left|ready\right\rangle_E + \left|1\right\rangle_S \left|1\right\rangle_M \left|1\right\rangle_E$. This does not influence my following analysis. Besides, it is worth noting that in Wallace's (2012) formulation of MWI, worlds are emergent and their number after a measurement is not definite due to the imperfectness of decoherence.

By the linearity of the dynamics, the evolution of the two worlds are the same as the above two forms of evolution. For example, when the state of the measuring device in the first world changes from $|ready\rangle_M$ to $|1\rangle_M$, it obtains the result 1.

Now consider one of these unitary time evolution operators, U_N, for which $|A_0\rangle_M = |ready\rangle_M$. In other words, U_N changes $|0\rangle_S |ready\rangle_M$ to $|1\rangle_S |1\rangle_M$ and $|1\rangle_S |1\rangle_M$ to $|0\rangle_S |ready\rangle_M$. It is similar to the NOT gate for a single q-bit, and is permitted by the Schrödinger equation in principle (although it may hardly be realized in practical situations). Then the unitary time evolution of the above post-measurement state is

$$U_N(|0\rangle_S |ready\rangle_M + |1\rangle_S |1\rangle_M) = |1\rangle_S |1\rangle_M + |0\rangle_S |ready\rangle_M . \qquad (10)$$

Again, when the state of the measuring device in the first world changes from $|ready\rangle_M$ to $|1\rangle_M$, it obtains the result 1. Similarly, when the state of the measuring device in the second world changes from $|1\rangle_M$ to $|ready\rangle_M$, the result 1 is erased.[2] On the other hand, after the unitary time evolution the whole superposition does not change.

Therefore, MWI predicts that after the above unitary time evolution, the complete state of the composite system, which is described by the wave function of the system, does not change, while the states of the two worlds (either emergent or not) both change after the evolution. This is a contradiction. If the complete state of a system does not change, then every aspects of the system cannot change, including the state of each world the system comprises.[3]

There are three possible ways to avoid the contradiction. The first way is to deny that after the evolution the state of the composite system has not changed. This requires that the wave function of a system is not a complete description of the state of the system, and additional variables are needed to introduce to describe the complete state. In other words, the assumption of the completeness of the physical description by means of the wave function should be dropped. The second way is to deny that after the evolution the states of the worlds have changed. This is only possible when there is only one world. In other words, the assumption of multiplicity should be dropped. The third way is to deny the existence of the post-measurement state (6). In other words, the assumption of the linearity of the dynamics for the wave function should be dropped.

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[2]The possibility of undoing a measurement in a unitary quantum theory has been discussed recently (see Pusey, 2016; Healey, 2018).

[3]Note that the unitary time evolution cannot be implemented instantaneously, and the state of the composite system also changes *during* the evolution. But the key point is that the initial state and the final state of the system are the same, while the initial state and the final state of each world are different. It is this result that leads to the contradiction.

The above result can also be obtained by an analysis of a direct comparison between two systems being in the same wave function, such as two composite systems being in the same state $|0\rangle_S |ready\rangle_M + |1\rangle_S |1\rangle_M$. If the state $|0\rangle_S |ready\rangle_M + |1\rangle_S |1\rangle_M$ corresponds to two worlds according to MWI, then there will be two different ways of comparison. The first way is that a world of one system is compared with the world of the other system which has the same state. This corresponds to the simplest case of identity time evolution, for which there is no contradiction. The second way is that a world of one system is compared with the world of the other system which has a different state. This corresponds to the above case of the unitary time evolution U_N, for which there will be a contradiction. Note that if there were only one way of comparison, then the two worlds of each system would be identical, which means that there would be only one world.

To sum up, I have argued that the many-worlds interpretation of quantum mechanics gives contradictory predictions for certain unitary time evolution and thus it is inconsistent. In order to avoid the inconsistency, we must drop one of its three key assumptions: (1) the completeness of the physical description by means of the wave function, (2) the linearity of the dynamics for the wave function, and (3) multiplicity.

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