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THE GAUGE INTERPRETATION OF THE CONVENTIONALITY OF SIMULTANEITY



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In this work we will consider gauge interpretations of the conventionality of simultaneity as developed initially by Anderson and Stedman, and later by Rynasiewicz. We will make a critical reassessment of these interpretations in relation to the “tradition” as developed in particular by Reichenbach, Grünbaum, and Edwards. This paper will address different issues, including: the relation between these two gauge interpretations; what advantages or defects these gauge approaches might have; how “new” Rynasiewicz’s approach in relation to the previous ones is; how much of the gauge interpretation Rynasiewicz actually applies to deal with objections to the conventionality of simultaneity thesis. The conclusion is that the gauge interpretations, in their current formulation, do not provide a better “rationale” of the conventionality of simultaneity thesis that supersedes the “tradition”.

Keywords: conventionality of simultaneity; gauge interpretations; synchrony transformations; diffeomorphisms

1 – Introduction

The thesis that a certain arbitrariness occurs in the definition of simultaneity was developed by Reichenbach and later by Grünbaum. This is known as the conventionality of simultaneity thesis (see, e.g. Jammer 2006). In this paper, we will take a close look into a particular approach to this thesis, in which the analogy with the gauge freedom of electrodynamics as adopted in general relativity has a central role. We think of the adoption of a particular definition of simultaneity as “fixing the gauge” in a context of “gauge freedom” (i.e. in a context where different definitions seem to be possible). A particularly important notion in the approaches being considered in this paper is that of an (active) transformation, be it a boost or a diffeomorphism, that allows us to set a new definition of simultaneity from the definition initially given.

In section 2, we review briefly the conventionality of simultaneity thesis as developed by Reichenbach (1924, 1927), Grünbaum (1955, 1968), Edwards (1963), and Winnie (1970) – which we will refer to as the “tradition”. This provides the context in which Anderson and Stedman developed their gauge interpretation of the conventionality of simultaneity. In section 3, we analyze the work by Anderson and Stedman (1977). In section 4, we consider a recent gauge interpretation of the conventionality of simultaneity made by Rynasiewicz (2012). This section mainly intends to clarify how Rynasiewicz’s approach can be reframed in a way that, in our view, has fewer tensions than the ones present in Rynasiewicz’s paper. In section 5, we first address the issue of the relation of Rynasiewicz’s approach to the previous ones, in particular the approaches by Edwards (1963) and Anderson and Stedman (1977). Afterwards, we consider the extent to which the gauge

interpretation is necessary and actually applied by Rynasiewicz to address three objections to the conventionality of simultaneity thesis. We will argue that the gauge interpretation is not necessary in any of the cases. In our view, sections 3, 4, and 5 will clearly reveal three points: synchrony transformations (boosts) or diffeomorphisms play an excessive role in the gauge interpretations; a clear improvement over the “tradition” has not been developed; and, an “autonomous” argumentation against criticisms for accepting the conventionality of simultaneity remains absent. Section 6 will succinctly encapsulate – in a “nutshell” – our criticism of the gauge interpretations of the conventionality of simultaneity.

2 – The conventionality of simultaneity thesis

According to Reichenbach and Grünbaum, the definition of distant simultaneity in an inertial reference frame has a conventional element in it. This results from the fact that causality does not fix uniquely the relation between the time coordinates at different positions. Accordingly, the coordinate time can be defined by stipulating that $t_B = t_A + \epsilon(t_A - t_A)$, where $0 < \epsilon < 1$ (see, e.g., Reichenbach 1924, pp. 31-40; Grünbaum 1955, pp. 451-6). Here, t_A is the time reading at position A when a light signal is sent to position B; t_B is the time reading at B when the signal arrives; and t_A is the time reading at A when the signal reflected back at B arrives at A. Any value of ϵ between 0 and 1 is equally valid. Each choice of ϵ corresponds to a particular definition of coordinate time. Thinking in terms of clocks located at different positions in the inertial reference frame, this corresponds to a particular

definition of synchronization of distant clocks – we stipulate that two clocks are synchronized when verifying $t_B = t_A + \varepsilon (t_A - t_A)$ for the conventionally adopted ε . Einstein's definition of coordinate time – corresponding to the setting of a particular relation of distant simultaneity –, is made by setting $\varepsilon = 1/2$ (Einstein 1905, p. 142). In this case, the one-way speed of light in the positive direction of the x-axis is taken to be identical to the one-way speed of light in the negative direction of the x-axis.¹ For the general (non-standard) case, corresponding to $\varepsilon \neq 1/2$, the one-way speed of light is anisotropic. Grünbaum stressed this point:

The conventionality of simultaneity allows but does not entail our choosing the same value $\varepsilon = 1/2$ for all directions within every system. In each system this choice assures the *equality* of the one-way velocities of light in opposite directions by yielding *equal* one-way transit times [...] for equal distances. The ratio of these one-way transit times is $\varepsilon/(1 - \varepsilon)$, and therefore, in the case of $\varepsilon \neq 1/2$, these one-way times are *unequal*. But [...] no fact of nature [...] would be contradicted if we choose values of $\varepsilon \neq 1/2$ for each inertial system, thereby making the velocity of light different from c in both senses along each direction in all inertial systems. (Grünbaum 1968, p. 308)

In the general case, the one-way speed of light in the positive direction of the x-axis is $c_+ = c/2\varepsilon$ and the one-way speed of light in the negative direction of the x-axis is $c_- = c/2(1 - \varepsilon)$, where c is the constant two-way speed of light (see, e.g., Winnie 1970, p. 83). We can speak interchangeably of the conventionality of simultaneity, conventionality of synchronization, or the conventionality of the one-way speed of light.

The work by Reichenbach and Grünbaum was extended by Edwards (1963) who obtained the generalized Lorentz transformations between two inertial reference frames R and R' with any admissible value for ε (in the case of R) and ε' (in the case of R'). In this paper Edwards gave an example, directly in terms of the one-way speed of light, of an anisotropic space-time in which a non-standard definition of simultaneity was adopted; and while he did not make any reference to the line element or metric of an anisotropic space-time, he represented in figures the tilted light cone structures of anisotropic space-times.² If we adopt an anisotropic one-way speed of light, corresponding to a tilted light cone, there are no observable effects. Using Edwards' terminology, quantities that depend on the one-way speed of light are not directly observable (Edwards 1963, p. 489).

Edwards was not very explicit regarding the coordinates that correspond to a non-standard definition of simultaneity. However, when presenting his views in terms of the conventionality of the one-way speed of light, Edwards considered the case of light being sent from a point O to another point A (a distance x apart), which is reflected

back towards O . In this case, the total time for the process t_{OAO} is equal to $t_{OA} + t_{AO}$. The constant two-way speed of light c is related to the one-way speeds in the positive and negative directions of the x-axis by $2x/c = x/c_{OA} + x/c_{AO} = x/c_+ + x/c_-$ (Edwards 1963, p. 483). Using the expressions for c_- and c_+ , the expression corresponding to the definition of simultaneity for distant points $-t_B = t_A + \varepsilon (t_A - t_A)$ – can be transformed into what we might consider an expression that defines directly the coordinate time: $t_B = t_A + x/c_+$. Assuming that we set the time at O to zero, the expression $t_A = t_O + x/c_{OA}$ reduces to $t_A = x/c_{OA}$. Edwards considered this expression in a way closely related to the one employed here: he thinks in terms of time intervals (x/c_{OA} and x/c_{AO}), and here we show how these time intervals can be determined by considering explicitly the coordinate time in the non-standard definition of simultaneity. Edwards then went on to impose a causality condition that limits the possible values of the one-way speed of light (in any direction). This results in having $c/2 \leq c(\alpha, \beta, \gamma) \leq \infty$, where $c(\alpha, \beta, \gamma)$ is the one-way speed of light in the direction indicated by the direction cosines α, β , and γ . This is equivalent, in terms of Reichenbach's ε -definition, to having $0 \leq \varepsilon \leq 1$.

While it was not necessary, previous to the deduction of the generalized Lorentz transformations, Edwards gave the expression of the relation between the time coordinate t_i in the isotropic space-time and the time coordinate t in an anisotropic space-time: $t = t_i - (x/c)X$, where $X = 1 - c/c_+$ (Edwards 1963, p. 485).³ This is not a transformation within the same inertial reference frame, but between two inertial reference frames with different definitions of simultaneity. An inertial reference frame with a standard definition of simultaneity corresponds to the isotropic space-time, and an inertial reference frame with a non-standard definition of simultaneity corresponds to an anisotropic space-time. In Edwards' wording, we have two "coordinate systems representing these spaces" (Edwards 1963, p. 485). The expression $t = t_i - (x/c)X$ gives the "relationship between the readings of clocks" (Edwards 1963, p. 485), belonging to each inertial reference frame (see, e.g., Zhang 1997, pp. 11-3). This transformation was not used to deduce the generalized Lorentz transformation. However, Edwards referred to the possibility of applying it in a sort of heuristic derivation of the generalized Lorentz transformations (Edwards 1963, p. 486). Moreover, Edwards used this transformation to show that the generalized Lorentz transformations reduce to the standard Lorentz transformations when also making the transformation from the non-standard time coordinates to the standard ones (Edwards 1963, p. 487).⁴

In a closely related work, Winnie gave an alternative derivation of the generalized Lorentz transformations and made a systematic and important extension of what in Edwards' work was a brief reference to the fact that the time dilation formula depended on the adopted definition of simultaneity (Edward 1963, p. 489). Winnie showed that the mathematical

1 - Here, to simplify, we will only consider the (spatial) one-dimensional case.

2 - When adopting Einstein's standard definition of coordinate time, the line element is given by $ds^2 = (cdt)^2 - dx^2 - dy^2 - dz^2$ and the metric tensor is orthogonal (see, e.g., Zhang 1997, p. 27). When adopting a non-standard definition of coordinate time, the line element is given by $ds^2 = (cdt + qdx)^2 - dx^2 - dy^2 - dz^2$, where $q = 2\varepsilon - 1$, to which is associated a non-orthogonal metric tensor (see, e.g., Zhang 1997, pp. 82-5; Anderson et al. 1998, pp. 106 and 111). Making $ds^2 = (cdt + qdx)^2 - dx^2 - dy^2 - dz^2 = 0$ we obtain the expression describing the tilted light cone structure mentioned by Edwards (1963, pp. 488-9).

3 - Here, we show the formula for the simplified case of a non-standard synchronization along the x-axis.

4- For this purpose Edwards uses the expressions $t' = t_i' - (x'/c)X'$ and $t = t_i - (x/c)X$.

expressions for relative speed, time dilation, and length contraction depend on the adopted definition of simultaneity. In this way, Winnie generalized the kinematics of special relativity for any definition of simultaneity and showed how several mathematical expressions depend explicitly on ε . Like Edwards before him, Winnie deduced the generalized Lorentz transformations directly in terms of the relations between the coordinates (x, t) of an inertial reference frame with an ε -definition of simultaneity, and the coordinates (x', t') of another inertial reference frame in relative motion with an ε' -definition of simultaneity. Contrary to Edwards, Winnie did not mention first the transformation from the time coordinate corresponding to $\varepsilon = 1/2$ to the time coordinate for an $\varepsilon \neq 1/2$, nor did he use it to relate the generalized Lorentz transformations and the standard Lorentz transformations. Also, in a way similar to Edwards, while Winnie was not very explicit about the setting of coordinates depending on the adopted definition of simultaneity, he did show indirectly the expressions for the time coordinate for different definitions of simultaneity:

Let us consider a point P moving with a constant speed $\vartheta < c$ (where $\varepsilon = 1/2$) in the positive direction along the x-axis of a frame K. In particular, let us suppose that point P moves from A to B in frame K, and as point P coincides with A a light beam is sent from A to B ... Let the time (at B) of the arrival of the light-beam at B be equal to t_1 and the time (at B) of arrival of the point P at B be equal to t_2 . Now if a clock at B is synchronized with a clock at A using $\varepsilon = 1/2$, and the time at A of the departures of point P and the light-beam is set equal to zero, then clearly $t_2 = AB/\vartheta$ and $t_1 = AB/c$. Similarly, for arbitrary permissible values of ε , $t_2 = AB/\vartheta_\varepsilon$ and $t_1 = AB/c_\varepsilon$, where ϑ_ε is the speed of point P as determined by a particular choice of ε . (Winnie 1970, pp. 84-5)

$t_1 = AB/c$ and $t_1 = AB/c_+$ are the time coordinates in an inertial reference frame when adopting the standard or non-standard definitions of simultaneity (having set the initial time reading of the clock at the origin to zero).⁵

As we have seen, neither Edwards nor Winnie considered explicitly the setting of coordinates (t, x) when adopting a non-standard definition of simultaneity. This might seem to be a secondary issue. However, as we will see, when not explicitly accounting for the setting of standard and non-standard coordinates *and* considering the transformation from the standard to the non-standard coordinates (as, e.g., Edwards did), a misinterpretation of the conventionality of simultaneity thesis may arise.

5 - The spatial coordinates are the same (see, e.g., Zhang 1997, p. 13).

6 - To deduce what Anderson and Stedman call the synchronization or synchrony transformation we can consider the following expressions for the coordinate time: $t = t_0 + x/c$ (in Einstein's synchronization); $t' = t'_0 + x/c_+$ (in the non-standard synchronization). Setting the time at the origin (when the light is emitted) to zero (i.e. $t_0 = t'_0 = 0$), we have $t = t' + x(1/c - 1/c_+)$ (for details see, e.g., Zhang 1997, pp. 11-3). This expression leads to $ct' = ct + kx$, which is the expression we find in page 31 of Anderson and Stedman (1977). This expression together with $x' = x$ are the coordinate transformations between the standard and non-standard coordinate systems defined in the two inertial reference frames. It is important to notice that Anderson and Stedman do not make any reference to the expressions for the coordinate time with standard or non-standard definitions of simultaneity.

3 – The gauge interpretation of the conventionality of simultaneity

In a paper published in 1977, Anderson and Stedman addressed the conventionality of synchronization in terms of a gauge freedom in the special theory of relativity. Anderson and Stedman's approach was inspired by Møller's treatment, in general relativity, of the transformation of coordinates in a fixed reference frame. In classical electrodynamics, we have the so-called gauge freedom that results from the scalar and vector potentials $(\phi, \mathbf{A}) : A^\mu$ not being uniquely determined by the Maxwell-Lorentz equations. If we make a transformation of the potentials of the form $A_\mu' = A_\mu + \partial \Lambda / \partial x^\mu$, where $\Lambda(x)$ is an arbitrary scalar function, this leaves the Maxwell-Lorentz equations unchanged. We call these transformations gauge transformations and say that the theory is gauge invariant (see, e.g., Møller 1952, pp. 143-4; Barut 1964, pp. 92-5).

Møller considered the case of a coordinate transformation within a reference frame R: $x_i' = x_i'(x_k)$ and $x_0' = x_0'(x_\mu) = f(x_\mu)$, where the spatial coordinates x_i' are only function of the spatial coordinates x_k . In this case "the transformation simply implies another notation for the points of reference in R together with an arbitrary continuous change in the rate and setting of the coordinate clocks" (Møller 1952, p. 236). The gravitational field is left unchanged while the gravitational potentials are transformed in a way analogous to that of the gauge transformation of electromagnetic potentials. This led Møller to refer to this type of coordinate transformation, made within the same reference frame, as a gauge transformation (Møller 1952, p. 248). Anderson and Stedman considered what they called the special relativistic synchronization transformation as "the simplest nontrivial example" (Anderson and Stedman 1977, p. 29) of general relativistic gauge transformations (applied to the case of a flat space-time). Let us consider an inertial reference frame O^k in which the one-way speed of light is stipulated to be anisotropic. In the positive direction of the x-axis it is given by $c_+ = c/2\varepsilon = c/(1 - \kappa)$; and in the negative direction of the x-axis it is given by $c_- = c/2(1 - \varepsilon) = c/(1 + \kappa)$, where $\kappa = 2\varepsilon - 1$. And, let (t^o, x^o) be the coordinates in an inertial reference frame O^o in which we adopt the standard definition of simultaneity corresponding to choosing an isotropic one-way speed of light. According to Anderson and Stedman:

The introduction of one-way [speed of light] anisotropy ($O^o \rightarrow O^k$) is equivalent to altering the setting, though not the rate, of the coordinate clocks; if the coordinates of O^k are $[(t^k, x^k)] = X^\mu(\kappa)$, then $x^k = x^o$, $[ct^k = ct^o + x^o\kappa]$. We may write this more compactly using a synchrony transformation tensor $T^\mu_\nu : X^\mu(\kappa) = T^\mu_\nu(\kappa, 0) X^\nu(0)$. (Anderson and Stedman 1977, p. 31)⁶

Notice that the non-standard synchronization in an inertial reference frame is established through a particular coordinate transformation (the synchronization or synchrony transformation) made from the inertial reference frame O^o to another inertial reference frame O^k . We start with the coordinate system in O^o corresponding to a standard definition of simultaneity and transform to another coordinate system in O^k corresponding to a different definition of simultaneity. Applying Møller's terminology in the case of special relativity we might call this transformation a gauge transformation (Anderson and Stedman 1977, p. 29).^{7,8} In this approach we can see the choice of a particular synchronization as fixing the gauge (Anderson, Vetharaniam and Stedman 1998, p. 102).⁹

The synchrony transformation is the transformation considered by Edwards in his work just before the deduction of the generalized Lorentz transformations (Edwards 1963, p. 485).¹⁰ It is important to notice that here we depart in a crucial way from Møller's analogy with the gauge freedom of electrodynamics. In Møller's case, we are considering a coordinate transformation within the same reference frame; in Edwards' or Anderson and Stedman's cases, we are considering a transformation between two inertial reference frames with different stipulations of distant simultaneity.

The existence of a transformation between the coordinates corresponding to different definitions of simultaneity can be misinterpreted as a coordinate transformation within an inertial reference frame. In particular the adoption in special relativity of Møller's "gauge talk" in the context of general relativity might reinforce this misinterpretation, since Møller considered "gauge transformations" within the same reference frame.

In fact, there were authors that made the point that the conventionality of simultaneity might be nothing more than a trivial consequence of the coordinate freedom of the theory (see, e.g., Mittelstaedt 1977, Friedman 1977).¹¹ These authors' criticism can, in fact, be framed in terms of the synchrony transformation (which is the central feature of the gauge interpretation of the conventionality of simultaneity): we start with the Minkowski space-time in its isotropic formulation (Friedman 1977, pp. 417-8), or we consider an inertial reference frame with a coordinate system corresponding to the standard definition of simultaneity (Mittelstaedt 1977, p. 574). Then, using the synchrony/gauge transformation, we transform to another coordinate system adopted to *the same* time like geodesic (Friedman 1977, pp. 419-22), or *the same* inertial reference frame with a non-standard coordinate system K' (Mittelstaedt 1977, pp. 575-6). Addressing Winnie's

work on the thesis of the conventionality of simultaneity, Friedman remarked that:

Winnie's claim is that special relativity as formulated in ϵ -systems is equivalent to special relativity as formulated in inertial systems. It seems to me that there is one sense in which this claim is obviously true, but completely trivial. [...] The sense in which the equivalence claim is obviously true is that Minkowski space-time can be described equally well from the point of view of ϵ -coordinate systems as from the point of view of inertial coordinate systems. [...] Indeed, they are nothing but different coordinate representations of the same theory. (Friedman 1977, p. 421)

In a similar way, Mittelstaedt's view was that the conventionality of simultaneity is "almost equivalent to the free choice of the coordinate system K' in an inertial system" (Mittelstaedt 1977, p. 582). In both cases the authors do not take into account that previous to considering a coordinate transformation we must have already set a coordinate system in the first place. The issue of the conventionality of simultaneity occurs here, not on the subsequent transformation to another coordinate system.

The lack of an explicit treatment of the setting of coordinates with a standard or a non-standard definition of simultaneity in relevant works, like the ones by Edwards, Winnie, and Anderson and Stedman, does not help in avoiding this kind of misinterpretation. We can avoid, at least in part, the risk of making this misinterpretation of the conventionality of simultaneity thesis by considering the transformation as relating the coordinate systems of two different inertial reference frames with different definitions of simultaneity. That is, the coordinate transformation is not a transformation within one reference frame, it simply establishes the relation between the coordinate systems of two given inertial reference frames. We might, e.g., adopt Zhang's terminology when referring to a coordinate transformation between two different inertial reference frames (one with a standard and another with a non-standard definition of simultaneity) and speak of the "coordinate transformation between Einstein and Edwards frames" (Zhang 1997, p. 11).

Anderson and Stedman seem to think of the transformation as an active change of the setting of the clocks of the adopted inertial reference frame: "the introduction of one-way [speed of light] anisotropy ($O^o \rightarrow O^k$) is equivalent to altering the setting, though not the rate, of the coordinate clocks" (Anderson and Stedman 1977, p. 31). This in itself does not seem to be a difficulty, even if in this view the

7 - The adoption of this terminology in special relativity seems a bit awkward. In the case of general relativity the coordinates are also related to the field, and the analogy with the case of electrodynamics makes some sense. However, in special relativity the Minkowski space-time is not a field. In a more recent implementation of a gauge interpretation, Rynasiewicz treats the Minkowski space-time as "a special case of relativistic spacetimes in general" (Rynasiewicz 2012, p. 92). In fact, it seems that to justify in a more rigorous way the use of "gauge talk" in the context of special relativity, it is necessary to treat special relativity as a special case of general relativity for which we impose a "flatness condition" (see, e.g., Rickles 2008; Pooley 2017). This is an important issue on its own, which, however, is beyond the limited scope of this work.

8 - In Anderson and Stedman (1992) this approach is extended to the case in which the synchronization parameter κ is a function of position $\kappa(x)$.

9 - Making an analogy with electrodynamics we call the choice of one of the possible definitions of simultaneity a fixing of the gauge. In electrodynamics, depending on the application we are considering, we select different potentials (i.e. we fix the gauge differently) to make the mathematics simpler (see, e.g., Barut 1964, pp. 94-5).

10 - Edwards' expression is $t' = t - x/c (1 - c/c_+)$. Substituting $c/2\epsilon$ for c_+ we obtain $t' = t - x/c (1 - 2\epsilon)$, and from this $ct' = ct + \kappa x$.

11 - Here, we do not address these works as eventually representative of Mittelstaedt's or Friedman's views on this subject. We simply choose these works since they are historically representative (see, e.g., Jammer 2006, p. 254) of what we consider to be a misinterpretation of the conventionality of simultaneity thesis.

synchrony transformation is presented as actively changing/transforming a coordinate system. Mathematically, the situation is the same as that described above in terms of a transformation from an Einstein (isotropic) frame into an Edwards (anisotropic) frame. If fact, Anderson and Stedman seem to be thinking, in a way that is similar to what Rynasiewicz did later (Rynasiewicz 2012, p. 93), in terms of the (active) implementation of a new inertial reference frame with a non-standard definition of simultaneity by changing the setting of the clocks. However, by putting the synchrony transformation in the central stage regarding the issue of the conventionality of simultaneity, Anderson and Stedman do not address the “gauge fixing”, i.e. the stipulation of a particular definition of simultaneity (or, in other words, the initial setting of the clocks). Also, the “gauge freedom” – as a way to present the conventionality of simultaneity –, is addressed in terms of the application of the active synchrony transformation to the adopted coordinate system (which in the case of Anderson and Stedman is the standard one), not, e.g., in terms of the “freedom” of choosing between different definitions of simultaneity that are physically equivalent. This approach replaces considering, on an equal footing, different inertial reference frames with different definitions of simultaneity. In their work Anderson and Stedman misplace the locus of the conventionality of simultaneity, which is in the definition of simultaneity itself and the partial freedom in making this definition, and focus on the secondary issue of the mathematical relationship between physically equivalent inertial reference frames with different definitions of simultaneity.¹²

4 – Rynasiewicz's “new take”: the conventionality of simultaneity in terms of the action of an active diffeomorphism on a Minkowski space-time

Anderson and Stedman's view on the conventionality of simultaneity in terms of gauge transformations is based on a particular coordinate transformation, the synchrony transformation, between two inertial reference frames, which we might refer to as the synchrony boost.¹³ Rynasiewicz also makes a case for a gauge interpretation of the conventionality of simultaneity. In fact, Rynasiewicz starts, *de facto*, with the synchrony transformation, even if he makes his case in terms of the Lorentz local time $t' = t - \vartheta x/c^2$, where $\vartheta = c(1 - \alpha)/(1 + \alpha)$ with $\alpha = \varepsilon/(1 - \varepsilon)$, being ε the Reichenbach epsilon. With a little algebra we find that $\vartheta = (1 - 2\varepsilon)/c$, and from this we obtain $t' = t - (1 - 2\varepsilon)x/c$, which is the

synchrony transformation adopting a different convention for the signature of the metric. A difference between Rynasiewicz's and Anderson and Stedman's approaches is in how we interpret this mathematical expression. In the case of Anderson and Stedman we have an (active) boost; Rynasiewicz proposes to develop it in terms of the notion of an (active) diffeomorphism.

In philosophical discussions about space-time in relation to general relativity, one usually considers the triplet $\langle M, g, T \rangle$, where M is a four-dimensional manifold, g is a pseudo-Riemannian metric, and T is the energy-momentum tensor (see, e.g. Norton 1988, pp. 56-7). A diffeomorphism is a one-to-one, onto, and smooth map that assigns to each point of a manifold M another point of another manifold N with the same dimension (see, e.g., Wald 1984, p. 438). When we consider the action of the map in the same manifold we might specialize the terminology and call our map an automorphism (see, e.g., Stachel and Iftime 2005, p. 6). In what follows we will maintain a simplified terminology and just speak of “diffeomorphism” when considering a diffeomorphism from $M \rightarrow N$ or from $M \rightarrow M$.

In the case of a (passive) coordinate transformation there is no action on the points of the manifold: we simply label each point with different coordinates. In the case of an (active) diffeomorphism we swap points in the manifold maintaining the coordinate system (see, e.g., Norton 2005, pp. 84-5). A diffeomorphism d induces another map d^* that “carries” mathematical structures defined on the manifold M at each point p to the points dp belonging to N or M . In particular, the structures g and T are “carried” under d by d^* . In this way, under the action of a diffeomorphism we obtain a new triplet $\langle M, d^*g, d^*T \rangle$ or $\langle N, d^*g, d^*T \rangle$ (see, e.g., Norton 1988, pp. 56-7). We can think of (g, T) and (d^*g, d^*T) as physically equivalent solutions of Einstein's field equations (see, e.g., Wald 1984, p. 438).

As we have seen, Møller considered a coordinate transformation within the same reference frame as a gauge transformation. There is another tradition which applies the gauge analogy in the context of general relativity: we refer to the action of a diffeomorphism as a gauge transformation, and we speak of “gauge freedom” in general relativity due to the physical equivalence of diffeomorphically related solutions of the theory (see, e.g., Wald 1984, p. 438).

In Rynasiewicz's approach, the synchrony transformation is interpreted in terms of a diffeomorphism d applied to the Minkowski space-time $\langle E^4, \eta \rangle$, where E^4 is a manifold diffeomorphic to R^4 and η is a Minkowski metric on E^4 . We have: $t(dp) = t(p) - (1 - 2\varepsilon)x(p)/c = t(p) - \vartheta \eta x(p)/c^2$; $x(dp) = x(p)$; $y(dp) = y(p)$; $z(dp) = z(p)$. When applying the diffeomorphism d to the Minkowski space-time $\langle E^4, \eta \rangle$ we obtain, according to Rynasiewicz, a new Minkowski space-

¹² - The same criticism applies, mutatis mutandis, to the gauge interpretation in Anderson and Stedman (1992), in which there is a generalization to the case of a local gauge freedom.

¹³ - Here, “boost” is to be understood as an active action on an inertial reference frame. For example, we can regard the Lorentz boost as an active setting into relative motion of an inertial reference frame K' which is initially at rest in relation to another inertial reference frame K (see, e.g., Brown 2005, pp. 28, 30 and 34). The (active) synchrony boost corresponds to an active change of the setting of the clocks of an inertial reference frame, which corresponds to implementing a “new” inertial reference frame with a different coordinate time. It is in this sense that Anderson and Stedman apply the synchrony transformation. According to them, the transformation leads “to altering the setting, though not the rate, of the coordinate clocks” (Anderson and Stedman 1977, p. 31).

time $\langle E^4, d^*\eta \rangle$:

We have not simply switched from a coordinate chart (t, x, y, z) in which the simultaneity relation is standard to a chart $(t - \partial x/c^2, x, y, z)$ in which that relation is nonstandard. Rather we have used that transformation to induce an active point mapping of the Minkowski spacetime to a new Minkowski spacetime. (Rynasiewicz 2012, p. 93)

Rynasiewicz claims that the application of a diffeomorphism brings about a new Minkowski space-time with a different definition of simultaneity than that we started with (which in practice he takes to be the standard definition). This would circumvent the accusation that the conventionality of simultaneity results from a simple coordinate transformation (in the same inertial reference frame) and, therefore, is a trivial consequence of the coordinate freedom of physical theories.

Rynasiewicz claims that his “new take” on the issue of the conventionality of simultaneity, “has the merit of nailing the exact sense in which simultaneity is conventional” (Rynasiewicz 2012, p. 90). According to him, “It is conventional in precisely the same sense in which the gauge freedom that arises in the general theory of relativity makes the choice between diffeomorphically related models conventional” (Rynasiewicz 2012, p. 90). In general relativity $\langle M, g, T \rangle$ and $\langle M, d^*g, d^*T \rangle$ are physically equivalent; we have a gauge freedom in the sense that we can choose which particular “model” we want (we have the liberty to fix the gauge). In a somewhat related way, Rynasiewicz proposes to characterize the election between the diffeomorphically related space-times $\langle E^4, \eta \rangle$ and $\langle E^4, d^*\eta \rangle$, which are physically equivalent, in terms of a gauge freedom. The existence of diffeomorphisms is taken to be the crucial aspect that characterizes in a precise way the exact sense in which the definition of simultaneity is conventional.

In what follows we will analyze some of Rynasiewicz’s remarks in relation to this view. We will consider the following excerpts:

The original problem, however, was whether there is a unique way to draw in the light cones, given the Einstein frame-structure. (Rynasiewicz 2012, p. 93)

Choices of different standards of simultaneity are equivalent to active boosts under local time. Active boosts under local time preserve the antecedently given Einstein frame-structure but do not preserve light cone structure, and hence one-way speeds of light. (Rynasiewicz 2012, p. 94)

The synchronization problem is nothing more than a fixing of gauge for flat relativistic spacetime. (Rynasiewicz 2012, p. 94)

Let us consider the first excerpt. Rynasiewicz defines what he calls the Einstein frame-structure by following Einstein’s steps in his paper from 1905, where Einstein develops special

relativity:

Recall the very first steps Einstein takes in developing the kinematics of special relativity. [...] Initially, we are granted an inertial frame F . [...] Think of this as just an inertial fibration of an R^4 manifold. Next, Einstein specifies the use of rigid rods and Euclidean geometry in order to assign position coordinates to a material point at rest [...] think of this as the assignment of a Euclidean metric h_{ab} on the space F of fibres. Finally, Einstein populates his inertial frame with stationary clocks of identical constitution. [...] Thus, we have a temporal metric θ_A on each fibre $A \in F$. (Rynasiewicz 2012, pp. 90-1)

This is the Einstein frame-structure. At this point, we still do not have a coordinate time in the inertial frame, only local times at each fibre. It is here, as we have seen, the issue of the conventionality in the definition of simultaneity arises. Einstein defines the coordinate time, and subsequently the notion of distant simultaneity, by stipulating that the time a light signal takes when sent from fibre A to fibre B (as measured in “A-time” and in “B-time”) is equal to the time the light signal takes, when reflected from B , to arrive at A . Let t_A be the A-time when light is sent from A to B , t_B the B-time at which the light arrives at B and is reflected back towards A , and t'_A the A-time when the light signal arrives at A . We have $t_B - t_A = t'_A - t_B$. There seems to be no reason not to adopt a different stipulation according to Reichenbach’s prescription: $t_B = t_A + \epsilon(t'_A - t_A)$, where $0 < \epsilon < 1$. As we can see in Edwards (1963), each of these choices of coordinate time leads to a different light cone structure. In particular a definition of simultaneity for which $\epsilon \neq 1/2$, implies that the light speeds on the positive and negative direction of the x -axis are not identical. This leads to a tilting in the light cone structure. Rynasiewicz calls this the “original problem”: there does not seem to be a unique way to “draw in” the light cones as they seem to result from the conventional adoption of a particular definition of simultaneity. A question we can pose at this point is how does Rynasiewicz’s “new take” deal with the “original problem”? To start to address this question let us consider the second selected excerpt:

Choices of different standards of simultaneity are equivalent to active boosts under local time. Active boosts under local time preserve the antecedently given Einstein frame-structure but do not preserve light cone structure, and hence one-way speeds of light. (Rynasiewicz 2012, p. 94)

Rynasiewicz equates the “choices of different standards of simultaneity” with “active boosts under local time”, which he interprets as the action of a diffeomorphism on a Minkowski space-time $\langle E^4, \eta \rangle$. The diffeomorphism does not affect the Einstein frame-structure but it does change the light cone structure, corresponding according to Rynasiewicz to having a new Minkowski space-time $\langle E^4, d^*\eta \rangle$. As it is, Rynasiewicz’s approach seems to give a preferred position to the standard Minkowski space-time $\langle E^4, \eta \rangle$, and then equates the “choices of different standards of simultaneity” to a transformation into a new Minkowski space-time $\langle E^4,$

$d^* \eta \rangle$. We think this is not the case. Rynasiewicz seems to adopt the standard Minkowski space-time for simplicity, but he considers that the standard and non-standard Minkowski space-times are on an equal footing. Rynasiewicz mentions that “there is a complete symmetry between $\langle E^4, \eta \rangle$ and $\langle E^4, d^* \eta \rangle$ ” (Rynasiewicz 2012, p. 92); we think it is implicit in Rynasiewicz’s view that we could start with a Minkowski space-time with a non-standard definition of simultaneity and then consider a diffeomorphism to another space-time with another definition of simultaneity, standard or non-standard. However, we should take stock of the fact that we have to start already with a choice of simultaneity previous to applying the diffeomorphism to the Minkowski space-time $\langle E^4, \eta' \rangle$; that is, we need in the first place to complete the Einstein frame-structure with a definition of simultaneity – we need to “draw in” the light cone structure – so that we actually have a Minkowski space-time $\langle E^4, \eta' \rangle$. By nailing down the sense in which the notion of simultaneity is conventional in terms of the diffeomorphically related space-times, Rynasiewicz ignores a necessary element of what he calls the “original problem”: to address the “problem” of the uniqueness or not of the light cone structure, we need to consider the “construction” of a light cone structure “on top” of the Einstein frame-structure. When identifying the “choices of different standards of simultaneity” with the application of a diffeomorphism, Rynasiewicz does not deal with the definition of a Minkowski space-time $\langle E^4, \eta' \rangle$ by stipulating a light cone structure previous to any application of a diffeomorphism.

To further address what seems to be an excessive emphasis, in Rynasiewicz’s approach, on the notion of diffeomorphism and a lack of treatment of the completion of a Minkowski space-time from the Einstein frame-structure, let us consider the third selected excerpt:

The synchronization problem is nothing more than a fixing of gauge for flat relativistic spacetime. (Rynasiewicz 2012, p. 94)

We can rephrase “the synchronization problem” as “the non-uniqueness problem in the definition of simultaneity”; it is another terminology of what Rynasiewicz called the “original problem” of seemingly having a non-unique way to “draw in the light cone”. Rynasiewicz says that the non-uniqueness problem in the definition of simultaneity can be framed in terms of a fixing of the gauge in the Minkowski space-time. In the context of special relativity, “fixing the gauge”, as applied by Rynasiewicz, means choosing the definition of simultaneity between several equally valid possibilities, which for simplicity is usually the standard definition. Here, the conventionality of simultaneity is “nailed down” not in terms of the application of a diffeomorphism to an already constructed Minkowski space-time, but in terms of the completion of the Einstein frame-structure by a particular adoption of a definition of simultaneity (i.e. by fixing the gauge), in a context in which different definitions are equally valid (i.e. where we have gauge freedom). That is, the conventionality of simultaneity is addressed at the level of the constitution of the Minkowski space-time.

As it is, there seems to be a dissonance and even tension, among several of Rynasiewicz’s remarks. If we only take into account the second excerpt, the diffeomorphism has the central role, and the conventionality seems to result from the existence of different diffeomorphisms that can be applied to a selected Minkowski space-time. This part does not deal directly with what Rynasiewicz’s calls the “synchronization problem” or “original problem”, i.e. whether there is a unique way to “draw in” the light cones given the Einstein frame-structure. A diffeomorphism does not “draw in” a light cone; according to Rynasiewicz, it transforms an already given light cone. Also, the third excerpt seems to provide a different characterization of the conventionality of simultaneity in which the definition of simultaneity is presented as a fixing of the gauge.

We think it makes better sense if we change the “relative weight” of the notions employed and give a different narrative sequencing to Rynasiewicz’s remarks. It could be something along the following lines:

When stipulating different definitions of simultaneity – one standard and another non-standard – we arrive at two physically equivalent Minkowski space-times $\langle E^4, \eta \rangle$ and $\langle E^4, \eta' \rangle$. These are related by a diffeomorphism, i.e. $\langle E^4, \eta' \rangle = \langle E^4, d^* \eta \rangle$. Thinking in terms of gauge freedom, we have the liberty to complete the Einstein frame-structure with different light cone structures by fixing the gauge (i.e. by making a stipulation of simultaneity). Minkowski space-times with different light cone structures are diffeomorphically related; we can say that the diffeomorphisms “comprise the gauge freedom” of the theory (see, e.g. Wald 1984, p. 438). This means, in particular, that if we initially fix the gauge to the standard one (i.e. we adopt a Minkowski space-time with an isotropic light cone structure), we can, applying a diffeomorphism, cover the entire span of possible Minkowski space-times with anisotropic light cone structures, i.e. applying a diffeomorphism we obtain a new anisotropic Minkowski space-time physically equivalent to the one initially “constructed” when fixing the gauge.

We think that by reframing Rynasiewicz’s gauge interpretation along these lines we avoid (part of) the tensions present in the paper.¹⁴ The notion of diffeomorphism exits centre stage and we focus on how to “draw in the light cone” as the central aspect, which in a gauge interpretation can be seen as a “gauge fixing” in a situation where there is “gauge freedom”, i.e. where we can “draw in the light cone” in different ways that are physically equivalent.

The original or synchronization “problem” occurs when completing the Einstein frame-structure, by defining the light cone structure (i.e. by a particular fixing of the “gauge”), which corresponds to establishing a coordinate time. We notice that Minkowski space-times with different light cone structures are physically equivalent. We can address this, e.g., in terms of having a “gauge freedom” in the stipulation of the one-way speed of light. In this way, the original or synchronization problem can be characterized without making any reference to the notion of a diffeomorphism.

¹⁴ - That this is not completely the case will be seen at the beginning of the next section.

We can, however, characterize the change between different physically equivalent definitions of simultaneity in terms of the application of a diffeomorphism to a selected Minkowski space-time (corresponding to a particular definition of simultaneity). This is, so to speak, a partial approach to the original or synchronization problem as formulated in the first excerpt we have considered: interpreted as an active transformation of the light cone structure, the application of a diffeomorphism shows that we can produce different light cone structures. However, the application of a diffeomorphism does not enable to “draw in the light cones”, only to “redraw” them from an initially given light cone structure.

5 – The Novelty (or not) of Rynasiewicz's gauge interpretation and his response to objections to the conventionality of simultaneity thesis

Rynasiewicz's objective with his approach can be seen as two-fold. In his own words, his purpose is “to reconstruct rationally the sense in which Einstein claimed that simultaneity is conventional” (Rynasiewicz 2012, p. 94).¹⁵ Also, this should provide a “different rationale for taking seriously the conventionality of simultaneity than any that have appeared before” (Rynasiewicz 2012, p. 94). We can consider the following excerpt as an example of his “new” rationale:

There are certainly things about the conformal structure of Minkowski spacetime that are not conventional (e.g., that it's flat). But the choice between the many different representations of conformal structure on the manifold certainly is conventional, and what differs between those various representations is the one-way speed of light in a given chart adapted to the Einstein frame-structure. That is the exact sense in which simultaneity is conventional. (Rynasiewicz 2012, p. 94)¹⁶

As it stands, Rynasiewicz's approach does not seem to give

a “new rationale” regarding the issue of the conventionality of simultaneity. We know, at least, since Einstein's work from 1905 that the definition of simultaneity is a stipulation that does not bear on the factual content of the theory. Einstein established by definition that the time needed for light to travel from a point A to a point B is the same it takes to travel from B to A (Einstein 1905, p. 142). This corresponds to assuming that the propagation of light is isotropic. This is made more explicit in a later paper: “we shall now stipulate that the velocity of the propagation of light in vacuum from some point A to some point B is the same as that from B to A” (Einstein 1911, p. 345).¹⁷ With Reichenbach's and Grünbaum's work it becomes clear that we can adopt different definitions of simultaneity corresponding to different one-way speeds of light, and that they are physically equivalent. Paraphrasing Rynasiewicz we might say that what differs in the different definitions of simultaneity is the adopted one-way speed of light. In fact, the rationale contained in the above excerpt can be found, with a different terminology, basically in Edwards (1963): there are mathematical quantities that are directly observable and others not. Some of the non-observable mathematical quantities depend on the stipulation of the one-way speed of light. However, observable quantities do not depend on this stipulation. In this way, there is no observable difference when adopting an isotropic or an anisotropic speed of light – we can say that this choice is conventional. When adopting an anisotropic speed of light, the light cone structure is different from the one corresponding to the isotropic case. We have an anisotropic space-time. The light cone structure is the only difference between isotropic and anisotropic space-times (and we can transform from one case to the other by applying a transformation to the time coordinate).

It is true that Rynasiewicz further develops this approach using the terminology of gauge transformations and applying the mathematics of differential geometry, in a context of considering that “the case of distinct Minkowski spacetimes related by a diffeomorphism is just a special case of relativistic spacetimes in general” (Rynasiewicz 2012, p. 92). Rynasiewicz's use of the notion of “Einstein frame-structure” is particularly helpful in making the distinction between what changes and what does not change with a different stipulation of the one-way speed of light. In this

¹⁵ - Einstein mentioned that the synchronization of distant clocks was established by definition (Einstein 1905, p. 142), or by stipulation (Einstein 1911, p. 345). Einstein's definition of simultaneity rests on taking the one-way speed of light to be isotropic. In his book from 1917, he noticed that we are facing a logical circle, since “an examination of this supposition [regarding the one-way speed of light] would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle” (Einstein 1917, p. 272). Einstein never explored, as Reichenbach did, different definitions of simultaneity. However, we can consider Einstein as the forerunner of Reichenbach in his work on what become known as the conventionality of simultaneity thesis (see, e.g. Reichenbach 1920, pp. 9-10). In fact, Einstein subscribed to Reichenbach's development of his remarks. In a review of Reichenbach's 1927 book, Einstein emphasized Reichenbach's treatment of the conventionality of simultaneity: “special care has been taken to ferret out clearly what in the relativistic definition of simultaneity is a logically arbitrary decree and what in it is a hypothesis, i.e. an assumption about the constitution of nature” (cited in Jammer 2006, p. 191). When Rynasiewicz writes that his purpose is to provide a new rationale to the “sense in which Einstein claimed that simultaneity is conventional” (Rynasiewicz 2012, p. 94), he is in fact referring to what we have called the “tradition”, which was inspired by Einstein but actually developed by Reichenbach, Grünbaum, and followers.

¹⁶ - Returning to the issue of the previous section, notice how in this excerpt Rynasiewicz does not even characterize the conventionality in terms of “gauge fixing”: we have different representations of the conformal structure, which are all physically equivalent, and the only difference between them is how we “draw in” the light cone (corresponding to different one-way speeds of light). We are free to choose any one of them. This is, according to this excerpt, the exact sense in which simultaneity is conventional.

¹⁷ - Previous to special relativity, Poincaré had already mentioned that the notion of simultaneity involved assumptions and the isotropy of light was a stipulation (Poincaré 1898; see also Jammer 2006, pp.101-2).

way, Rynasiewicz's work can be seen as belonging to the same "tradition" as Edwards' 1963 essay.

We can also see Rynasiewicz's paper as part of another "tradition" initiated by Anderson and Stedman (also related to Reichenbach's, Grünbaum's, Edwards', and Winnie's works) that proposes to address the conventionality of simultaneity in terms of a gauge interpretation. Here, the use of the term "new" to characterize Rynasiewicz's work might make even less sense. In a way similar to what Rynasiewicz wrote later, in a review paper on the issue of the conventionality of simultaneity, Anderson, Vetharanim, and Stedman considered the possibility of having different definitions of simultaneity as a "gauge freedom" (Anderson, Vetharanim and Stedman 1998, p. 98). The authors used the term "re-coordinatization" to refer to a new coordinatization of time and associate it to a "gauge transformation" – the synchrony transformation (Anderson, Vetharanim and Stedman 1998, p. 98). Also, the authors called the choice of a particular definition of simultaneity a "gauge fixing" (Anderson, Vetharanim and Stedman 1998, p. 144).

If we consider the second of Rynasiewicz's excerpts considered above, it could equally be part of Anderson and Stedman's 1977 paper. As we have seen they made their case in terms of an (active) boost that "[alters] the setting, though not the rate, of the coordinate clocks" (Anderson and Stedman 1977, p. 31). Rynasiewicz's view of the action of diffeomorphisms is that:

Choices of different standards of simultaneity are equivalent to active boosts under local time [(i.e. the action of a diffeomorphism)]. Active boosts under local time preserve the antecedently given Einstein frame-structure but do not preserve light cone structure, and hence one-way speeds of light. (Rynasiewicz 2012, p. 94)

Rephrasing this sentence more in line with Anderson and Stedman's paper we might have something like this:

Choices of different standards of simultaneity are equivalent to active synchrony boosts. Active boosts preserve the antecedently given system of rods and clocks constituting an inertial reference frame but do not preserve the setting, though not the rate, of the coordinate clocks, and hence one-way speeds of light.

We have basically the same scheme with Anderson and Stedman (1977) and Anderson, Vetharanim and Stedman (1998) as that adopted by Rynasiewicz (2012); the difference is in the mathematics employed: where Anderson and co-workers employ a coordinate transformation between two inertial reference frames with a different coordinatization of time, interpreted as a boost, Rynasiewicz employs the notions of manifold and active diffeomorphism. By

fixing the gauge (i.e. by adopting a particular definition of simultaneity), we define the coordinate time in an inertial reference frame or we complete the Einstein frame-structure with a particular stipulation of light cone structure obtaining a Minkowski space-time. The gauge freedom of the theory can be seen by applying a gauge/synchrony transformation or a diffeomorphism so that we transform into a new inertial reference frame with a different gauge fixing (which we can see, e.g., as a different coordinate time) or a new Minkowski space-time with a different gauge fixing (i.e. a different light cone structure corresponding, e.g., to a different coordinate time).

While Rynasiewicz does not cite them, his work can be seen as a continuation of Edwards (1963) and Anderson and Stedman (1977). That his approach might not be as "new" as his author thinks is not, in our view, a defect in any way. In fact, we think that there are several virtues in Rynasiewicz's work. One of them is that it addresses and dispels the misinterpretation of the conventionality of simultaneity as due to a trivial coordinate transformation. The use by Rynasiewicz of terms like "active boost" or "active point mapping" stresses that we are not considering a passive coordinate transformation within the same inertial reference frame, but that we face an altogether different situation. In fact, Rynasiewicz's forcefully argues against the view that the conventionality of simultaneity is nothing but a trivial consequence of coordinate freedom, by stressing that when applying a diffeomorphism we have not "simply switched" between coordinate systems but that the transformation is applied to "induce an active point mapping of the Minkowski spacetime to a new Minkowski spacetime" (Rynasiewicz 2012, p. 93). As we have seen, it is not necessary to make reference to the notion of diffeomorphism to present this view. It can be made in terms almost identical by referring to the synchrony boost. More than this, we can dispel this misinterpretation simply by considering what Rynasiewicz called the synchronization or original problem. By considering the "drawing in" of the light cone (or, equivalently, the establishing of a coordinate time) it is clear that we are dealing with a coordinatization of space-time, not with a transformation of coordinates.

Rynasiewicz also considers other objections to the conventionality of simultaneity thesis. Here, we will not go into details regarding these objections, neither will we make an analysis of Rynasiewicz's response to them. To our purpose, it is only necessary to see how Rynasiewicz articulates his response to the objections. The point is how much of the gauge interpretation plays a crucial role in Rynasiewicz's defense of the conventionality of simultaneity thesis in light of these objections.

Regarding the objection that the adoption of an anisotropic speed of light results from adding a further gratuitous

structure to the Minkowski space-time (Friedman 1983, p. 312), and because of that we should adopt the Minkowski space-time with the standard definition, Rynasiewicz notes that:

This would be true if we attempted to define simultaneity only after having fixed the light cone structure on spacetime. But as we have seen, the problem of defining simultaneity is equivalent to the problem of determining how to draw the light cones in spacetime after having been given an Einstein frame-structure on the spacetime. (Rynasiewicz 2012, p. 93)

Here, Rynasiewicz makes his point in terms of what he called the synchronization or original problem. We deal with the objection by making clear that we do not have an isotropic Minkowski space-time to start with but the Einstein frame-structure. At this point, there is no fixed light cone structure, we have to define it. The anisotropic case does not have any more structure than the isotropic case, both are defined in equivalent ways, by “drawing in” the light cones in space-time. Rynasiewicz makes his case without using the notion of “diffeomorphism” or “gauge freedom”. In fact, he does not even need to make an explicit reference to the notion of “gauge fixing”.

A similar situation occurs regarding another objection to the conventionality of simultaneity thesis. Malament (1977) made a strong case against the conventionality thesis in terms of a uniqueness theorem that would show that the standard definition of simultaneity is the only one compatible with causal relations of special relativity. According to Rynasiewicz:

Malament’s result, however, has simply been misapplied to the problem of the conventionality of simultaneity. For the result presupposes that we first fix the light cone structure, i.e., the one-way speeds of light, and then ask what simultaneity relations are definable from this. And obviously the standard relation is uniquely definable from this. The original problem, however, was whether there is a unique way to draw in the light cones, given the Einstein frame-structure. If we draw the cones in with anisotropic one-way velocities, that also defines a unique, but non-standard simultaneity relation. (Rynasiewicz 2012, p. 93)

Again, Rynasiewicz makes his case without an explicit resort to his gauge interpretation. The three objections are all dealt with in the same way: we only have the Einstein frame-structure at the beginning. By “drawing in” the light cone and completing a Minkowski space-time we have established a coordinate time and defined a notion of distant simultaneity (and it turns out that the “drawing in” of the light cone is not uniquely determined). According to Rynasiewicz, the three objections result from a misunderstanding of this fact.

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We see that the three objections considered by Rynasiewicz can be addressed in terms of the “drawing in” of the light cone structure. The gauge interpretation has no crucial role in how Rynasiewicz addresses the objections. In fact, as mentioned, the central role given to the synchrony boost or the diffeomorphism in gauge interpretations has the danger of hiding the synchronization problem, i.e. the “drawing in” of the light cone in a context in which there seems to be no unique way to do it.

6 – Conclusion: why we find gauge interpretations of the conventionality of simultaneity unconvincing

In this paper, we made a critical reassessment of gauge interpretations of the conventionality of simultaneity. We have considered the historical locus of the gauge interpretation, Anderson and Stedman paper from 1977, and a recent work by Rynasiewicz from 2012. While Rynasiewicz believes that he offers a “new take” on the issue, we cannot agree with this view. Rynasiewicz’s work is too close to previous efforts, in particular Edwards’ paper and the gauge interpretation of Anderson and Stedman, to be considered “new”.

Rynasiewicz’s gauge interpretation seems to share with the previous work by Anderson and Stedman what we might call a structural problem. By “nailing down” the exact sense in which the definition of simultaneity is conventional in terms of the application of a diffeomorphism to a Minkowski space-time, it pushes into the background the issue of the “fixing of the gauge” in a context in which there seems to be a “gauge freedom”, i.e. it does not address how to “draw in” the light cones and the partial freedom to do it. This is, *mutatis mutandis*, the case of Anderson and Stedman’s gauge interpretation. Simply we adopt a somewhat different mathematical framework, and, as we have seen, instead of considering “gauge transformations” in terms of active diffeomorphisms we consider them in terms of active boosts (or synchrony transformations). In this way, in our view, the gauge interpretations of the conventionality of simultaneity, do not, in their present formulation, provide a better understanding or a better “rationale” of the conventionality of simultaneity than the “traditional” formulation of Reichenbach, Grünbaum, and others. This is the case even for our tentative reframing of Rynasiewicz’s gauge interpretation in terms of the equating of “gauge fixing” with the “drawing in” of the light cone structure and considering this “gauge fixing” in a context of “gauge freedom” as the central theme. This reframing of Rynasiewicz’s position avoids (partially) the tensions existing in his text, in which we find contradictory remarks with different emphasis on different notions (the application of diffeomorphisms; the

synchronization or central problem, i.e. the non-uniqueness in “drawing in” the light cone; the “drawing in” of the light cone as a gauge fixing). However, calling “gauge fixing” to the completion of the Einstein frame-structure by stipulation a particular light cone structure does not provide any deeper “insight” into the issue of the conventionality of simultaneity. It only provokes us to go astray by using the gauge terminology of “gauge freedom”, “gauge fixing” and particularly “gauge transformation” or “diffeomorphism”. In fact, reframing Rynasiewicz’s position in terms of the “gauge fixing” does not completely “wipe out” tensions that are present in his remarks. As mentioned, Rynasiewicz even writes that “the exact sense in which simultaneity is conventional” (Rynasiewicz 2012, p. 94), is that we can choose

between the many different representations of conformal structure on the manifold ... and what differs between those various representations is the one-way speed of light in a given chart adapted to the Einstein frame-structure. (Rynasiewicz 2012, p. 94)¹⁸

We see from this excerpt that there is no need for, and Rynasiewicz does not employ, any “gauge talk” to present “the exact sense in which simultaneity is conventional”. This characterization of the conventionality of simultaneity is basically the one we can find in the works of Reichenbach, Grünbaum, and, particularly, Edwards, whose work made explicit that the different definitions of simultaneity correspond to different light cones, without being any observational difference arising with the adoption of a different one-way speed of light (i.e. the adoption of a different light cone structure). We see that regarding Rynasiewicz’s purpose of “[reconstructing] rationally the sense in which Einstein claimed that simultaneity is conventional” (Rynasiewicz 2012, p. 94), his approach does not go beyond the “tradition”. In fact, as we have seen it can lead to difficulties due to the central role that was given to the notion of a diffeomorphism.

Regarding the other related objective of his work, which was to provide a “different rationale for taking seriously the conventionality of simultaneity than any that have appeared before” (Rynasiewicz 2012, p. 94), the defense made by Rynasiewicz does not, in fact, depends on his gauge interpretation, so it does not provide a better “rationale” than the one given by the “tradition”. Of the three objections to the conventionality of simultaneity thesis considered by Rynasiewicz, only the first is dealt with by making reference to the notion of a diffeomorphism. This, as mentioned, is unnecessary, and the objection can be addressed in terms of the “drawing in” of the light cone. In fact, as we have

seen, Rynasiewicz addresses the other two objections in this way, not using directly the gauge interpretation. If as Janis remarks, Rynasiewicz made “a vigorous defense of conventionality” (Janis 2014), it is our view that if this is so, it would be more a merit of the author than a consequence of the use of the gauge interpretation. In conclusion, as they stand, at the present time, the gauge interpretations of the conventionality of simultaneity do not seem to supersede the “tradition” as formulated by Reichenbach, Grünbaum, and others.

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¹⁸ - To see another example of tension in the work, we can contrast this excerpt with a previous one in which Rynasiewicz mentions that his approach (in terms of a privileged role for the notion of a diffeomorphism), “has the merit of nailing the exact sense in which simultaneity is conventional” (Rynasiewicz 2012, p. 90). According to him, “It is conventional in precisely the same sense in which the gauge freedom that arises in the general theory of relativity makes the choice between diffeomorphically related models conventional” (Rynasiewicz 2012, p. 90).

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