The Logic of Complementarity

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"We must, in general, be prepared to accept the fact that a complete elucidation of one and the same object may require diverse points of view which defy a unique description."

Niels Bohr, 1929

Abstract

This paper is the sequel of a previous one where we have introduced a paraconsistent logic termed paraclassical logic to deal with 'complementary propositions' [17]. Here, we enlarge upon the discussion by considering certain 'meaning principles', which sanction either some restrictions of 'classical' procedures or the utilization of certain 'classical' incompatible schemes in the domain of the physical theories. Here, the term 'classical' refers to classical physics. Some general comments on the logical basis of a scientific theory are also put in between the text, motivated by the discussion of complementarity.

1 Introduction

J. Kalckar, the editor of volume 6 of Bohr's Collected Works [11, pp. 26-27], suggested that the first reference to the notion of complementarity is to be found in a manuscript written by Bohr on July 10, 1927, where we read that "... the theory exhibited a duality when one considered on one hand the superposition principle and on the other hand the conservation of energy and momentum (...) Complementary aspects of experience that cannot be unified into a space-time picture on the classical theories" (cf. *ibid.*, pp. 26-27). Roughly speaking, the idea involves something like this: complementarity means the possibility of unifying aspects which cannot be put together from a 'classical' perspective.

Kalckar also keeps off the view claimed by some writers who have sustained that Bohr was motivated by sources outside physics, like the readings of Kierkegaard or of the Danish philosopher H. Høffding. According to Kalckar, the very origins of such an idea came from physics itself and, to reinforce his claim, he recalls L. Rosenfeld's words, which interest also us here: "Bohr's conception of complementarity in quantum mechanics is not the expression of a 'specific philosophical position', but an inherent *part of the theory* which has the same validity as its formal aspect and is inseparable from it." (*apud ibid.*, p. 28, italics ours).

It should be remarked that it seems to exist a discrepancy between Rosenfeld and Bohr in what concerns the way of understanding complementarity. This difference may justify Bohr's refuse to accept that von Weizsäcker had described 'the logic os complementarity', as we shall see below. Apparently, Bohr envisaged his ideas on complementarity as forming part of a general epistemological principle, which could guide us not only in physics (from which the ideas really came), but in any other field of science as well; as he said, "... the lessons taught us by recent developments in physics regarding the necessity of a constant extension of the frame of concepts appropriate for the classification of new experiences leads us to a general epistemological attitude which might help us to avoid apparent conceptual difficulties in other fields of science as well" [8]. In other words, we might say that, according to Bohr, complementarity may be viewed as a kind of a general regulative methodological principle. On the other hand, there are positions sustained by people like Rosenfeld (and von Weizsäcker), who see such ideas as making part of the (physical) theory itself. What is the difference?

The difference lies in what we consider as a meta-theoretical principle of science and what is to be considered as a strict principle of a particular (say, axiomatized) scientific theory. The former may be viewed as a meta-principle, while the latter is something to be 'internalized' within the object language of the theory itself. In what follows, we shall try to push this distinction a little bit in relation to the concept of complementarity. This is of particular importance for, as we shall see below, the very idea of complementarity resembles that of the existence of contradictions; keeping it as a meaning principle, it seems easier to understand how it may help us in accepting that "[t]he apparently incompatible sorts of information about the behavior of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as 'complementary' to each other" [8].

In this paper, we shall consider how 'complementary ideas' can be seen from both perspectives, that is, as standing both for a general regulative meaning principle and also as a law that can be internalized in the language of the theory proper. As we shall see, although resembling contradictions (but see a way of better specifying them below), the concept of complementary propositions can be put within a certain object language (so keeping it as an 'inherent part of the theory' as Rosenfeld has claimed) without risk of trivializing the whole theory. This will enable us to discuss also the role played by logic in the context of the physical sciences.

We begin firstly by describing the main features connected with the idea of complementarity. We note that there is no general agreement among historians and philosophers (and even among physicists) about the precise meaning of Bohr's Principle of Complementarity (henceforth, PC), what makes the historical analysis quite problematic [1], [24], [25]. Even so, after revising some of the main references made by Bohr himself and by various of his commentators on complementarity, we arrive at a characterization of 'complementary propositions' from a strict logical point of view (that is, as defined in a suitable formal language). Then, we shall sketch the main norms of the logic of such propositions, by evidencing that it is a kind of paraconsistent logic, termed *paraclassical logic* (see [15], [32]). Nonetheless, complementarity, for us, also encompasses meta-theoretical meaning principles imposing some limitations on theories; in addition, it sanctions the use of incompatible approaches in physics. Complementarity, as a meaning principle, plays the role of a kind of normative rule.

Secondly, we insist that the relevance of this kind of study is neither merely historical nor an exercise of logic. In addition to the necessity of a philosophical distinction between meaning principles and strict physical laws, we believe that this discussion has a profound philosophical significance also in showing some of the relationships that there exist between certain non-classical logics and the empirical sciences, in particular to physics. Of course, although in this paper we neither have pursued the historical details on complementarity in deep, though we have mentioned some of the main references one finds in the literature, nor have investigated the logical system we propose in all its formal aspects (a task we hope to accomplish in the near future), we hope to make clear the general underlying idea of the paper. It was and continue partially motivated by Bohr's own way of accepting both the particle and the wave pictures of reality. We believe that the understanding of a wide field of knowledge, like quantum physics, may gain in much if we accept a pluralistic view according to which there are several and eventually non equivalent ways of looking at it (perhaps some of them based on non-classical logics), each one being adequate from its particular perspective, and showing details which cannot be seen from the other points of view, analogously to the different drawings of an engineer in descriptive geometry, *à la* Monge, of a given object.

2 Complementarity

The concept of 'complementarity' was introduced in quantum mechanics by Niels Bohr in his famous 'Como Lecture', in 1927, although the basic ideas go back to 1925 [3], [11]. The consequences of his view were fundamental, particularly for the development of the Copenhagen interpretation of quantum mechanics and constitutes, as it is largely recognized in the literature, one of the most fundamental contributions to the development of quantum theory (see also [1], [24], [25]).

In this section we make clear in what sense we understand the word 'complementarity'. The quotations taken from Bohr and from other important commentators aim at to reinforce our view, although we are of course aware that a few isolated quotations cannot provide evidence for the full understanding of concepts, especially regarding the present (and difficult) case. Even so, we hope we can convince the reader that complementarity can be interpreted as a more general principle related to 'incompatibility' in some sense (the 'sense' being explained in the next sections) than to some kind of impossibility of 'simultaneously measuring'.

In what concerns this point, we remark that we find Bohr speaking about complementary concepts which cannot be used *at the same time* (as we see in several of his papers listed in our references [11, p. 369]). Though this way of talking should be viewed as a way of speaking, for it stands for situations which, according to Bohr himself, demand specific analysis; as he says, "[o]ne must be very careful, therefore, in analyzing which concepts actually underly limitations" (ibid., p. 370). Really, there are several ways of looking at complementarity. Pauli, for instance, claimed that "[if] the use of a classical concept excludes of *another*, we call both concepts (...) *complementary* (to each other), following Bohr" ([29, p. 7], quoted in [19, p. 33]). By the way, J. Cushing also stressed his own view, in saying that "[w]hatever historical route, Bohr did arrive at a doctrine of mutually exclusive, incompatible, but necessary classical pictures in which any given application emphasizing one class of concepts *must* exclude the other" (op. cit., pp. 34-5).

This idea of complementary propositions as 'excluding' each other (what appears to mean something like 'incompatibility') is reinforced by Bohr himself in several passages, as the following ones:

> "The existence of different aspects of the description of a physical system, seemingly incompatible but both needed for a complete description of the system. In particular, the wave-particle duality." (*apud* [22, p. 370])
>
> "The phenomenon by which, in the atomic domain, objects exhibit the properties of both particle and waves, which in classical, macroscopic physics are mutually exclusive categories." (ibid., pp. 371-2)
>
> "The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively." [3, p. 566]
>
> "The apparently incompatible sorts of information about the behavior of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as 'complementary' to each other." ([8, p. 291]; [31, p. 31])

> ”Information regarding the behaviour of an atomic object obtained under definite experimental conditions may, however, according to a terminology often used in atomic physics, be adequately characterized as *complementary* to any information about the same object obtained by some other experimental arrangement excluding the fulfillment of the first conditions. Although such kinds of information *cannot be combined into a single picture* by means of ordinary concepts, they represent indeed equally essential aspects of any knowledge of the object in question which can be obtained in this domain.” ([9, p. 26], *apud* [31, p. 31], second italic ours).

E. Scheibe also says that

> ”. . . which is here said to be ‘complementary’, is also said to be ‘apparently incompatible’, the reference can scarcely be to those classical concepts, quantities or aspects whose *combination* was previously asserted to be characteristic of the classical theories. For ‘apparently incompatible’ surely means incompatible on classical considerations alone.” [31, p. 31]

In other words, it is perfectly reasonable to regard complementary aspects as *incompatible*, in the sense that their *combination* into a single description may lead to difficulties. But in a theory grounded on standard logic, the conjunction of two theses is also a thesis; in other words, if α and β are both theses or theorems of a theory (founded on classical logic), then $\alpha \wedge \beta$ is also a thesis (or a theorem) of that theory. This is what we intuitively mean when we say that, on the grounds of classical logic, a ’true’ proposition cannot ‘exclude’ another ’true’ proposition. In this sense, the quantum world is rather distinct from the ‘classical’, for although complementary propositions are to be regarded as acceptable, their conjunction seems to be not.

This corresponds to the fact that, in classical logic, if α is a consequence of a set Δ of statements and β is also a consequence of Δ, then $\alpha \wedge \beta$ (α *and* β) is also a consequence of Δ. If β is the negation of α (or vice-versa), then this rule implies that from the set of formulas Δ we deduce a contradiction $\alpha \wedge \neg\alpha$ (or $\neg\beta \wedge \beta$). In addition, when α and β are in some sense incompatible, $\alpha \wedge \beta$ constitutes an impossibility.

Therefore, as we shall show below, part of a natural procedure to surmount the problem is to restrict the rule in question. But before that, let us make some few additional remarks on complementarity.

3 Recent results

As it is well known, Bohr and others like P. Jordan and F. Gonseth have suggested that complementarity could be useful not only in physics but in other areas as well, in particular in biology and in the study of primitive cultures (see [25, pp. 87ff], where still other fields of application, like psychology, are mentioned). Although these applications may be interesting, they are outside the scope of this paper. Keeping within physics, it should be recalled that in 1994 Englert et al. argued that complementarity is not simply a consequence of the uncertainty relations, as advocated by those who believe that “two complementary variables, such as position and momentum, cannot simultaneously be measured to less than a fundamental limit of accuracy”, but that

> ”(...) uncertainty is not the only enforce of complementarity. We devised and analyzed both real and thought experiments that bypass the uncertainty relation, in effect to ‘trick’ the quantum objects under study. Nevertheless, the results always reveal that nature safeguards itself against such intrusions –complementarity remains intact even when the uncertainty relation plays no role. We conclude that complementarity is deeper than has been appreciated: it is more general and more fundamental to quantum mechanics than is the uncertainty rule.” [21]

If Englert et al. are right, then it seems that the paraclassical logic we shall describe below may in fact be useful.

Recently (1998), some experiments developed in the Weizmann Institute in Israel indicated that the Principle of Complementarity has been verified also for fermions (electrons) [12]. Through nano-technology devices created in low-temperature scales, the scientists developed measuring techniques which have enabled them to show that in a certain two-slit experiment, the wave-like behaviour occurs when the possible paths a particle can take remain indiscernible, and that a particle-like behaviour occurs when a 'which-path' detector is introduced, determining the actual path taken by the electron. These recent experiments show that the ancient intuitions and some *Gedankenexperimente* performed by Bohr and others were in the right direction, so sustaining Bohr's position that complementarity is in fact a characteristic trait of matter. So, to accommodate this idea within a formal description of physics is in fact an important task.

A still more recent (2001) 'experimental proof' of Bohr's principle came from Austria, where O. Nairz and others have reported that Heisenberg uncertainty principle, which is closely related to complementarity, was demonstrated for a massive object, namely, the fullerene molecule C_{70} at a temperature of 900 K. In justifying their work, they said that ”[t]here are good reasons to believe that complementarity and the uncertainty relation will hold for a sufficiently well isolated object of the physical world and that these quantum properties are generally only hidden by technical noise for large objects. It is therefore interesting to see how far this quantum mechanical phenomenon can be experimentally extended to the macroscopic domain” [28].

This apparently opens the road for the acceptance of the validity of complementarity also in the macroscopic world. The analysis of these applications should interest not only physicists and other scientists, but philosophers as well. We believe that Bohr's intuitions that complementarity is a general phenomenon in the world deserves careful examination in the near future. But let us go back to logic.

4 Logics of Complementarity

The expression 'logic of complementarity' has been used elsewhere to designate different logical systems, or even informal conceptions, which intended to provide a description of Bohr's ideas of complementarity from a 'logical' point of view.

As a historical remark, we recall that some authors like C. von Weizsäcker, M. Strauss and P. Février have already tried to elucidate Bohr's principle from such a logical point of view (cf. [23], [25, pp. 377ff], [33]). Jammer mentions Bohr's negative answer to von Weizsäcker's attempt of interpreting his principle, and observes that this should be taken as a warning for analyzing the subject (ibid. p. 90). As shown by Jammer, Bohr explained that his rejection was due to his conception that "[t]he complementary mode of description (...) is ultimately based on the communication of experience, [quoting Bohr] 'which has to use the language adapted to our usual orientation in daily life' "; Jammer continues by recalling that, to Bohr, "objective description of experience must always be formulated in [quoting Bohr again] 'plain language which serves the needs for practical life and social intercourse' " ([25, p. 379]. These points reinforce our emphasis that Bohr ascribed to complementarity the role of a meaning principle. So, maybe Bohr's rejection of accepting a 'logic of complementarity' could be due to the discrepancies (or 'divergent conceptions') related to the way of understanding complementarity. In his 1966 book, Jammer also suggested something analogous [24, p. 356].

Another tentative of building a 'logic of complementarity' was P. Février's. She began by considering Heisenberg uncertainty relations not simply as something which can be derived in the formalism of quantum theory, but attributed to them a distinctive fundamental role as being the very basic principle on which quantum theory should be built on. She distinguished (yet not explicitly) between propositions which can and which cannot be composed. The last are to stand for complementary propositions; in her logic, a third value is used for the conjunction of complementary propositions to mean that their conjunction is 'absolutely false'. Other connectives are presented by the matrix method, so that a 'logic of complementarity' is proposed, yet not detailed in full ([23]; for further details on her system, see [18]).

Strauss' logic is based on his conception that the complementarity principle "excludes simultaneously decidability of incompatible propositions" [25, p. 356]; then, he proposed a different theory in which conjunctions and disjunctions of complementary propositions were to be excluded. So, in a certain sense, although described in probabilistic terms, we may say that his intention was to develop a logic in which two propositions, say α and β (which stand for complementary propositions) may be both accepted, but not their conjunction $\alpha \wedge \beta$ (op. cit., p. 335). It is interesting to remark that Carnap declared that

Strauss' logic was 'inadvisable' [13, p. 289]. Today, by using another kind of paraconsistent logic, termed Jaśkowski logic, we think that perhaps Strauss' position can be sustained.

Leaving these historical details aside, we shall proceed as follows. After introducing the concept of a theory which admits a *Complementarity Interpretation* (to use Jammer's words –see below), we shall argue that under a plausible definition of complementarity, the underlying logic of such a theory is *paraclassical* logic. In the sequel we shall sketch the main features of this logic.

5 Complementarity theories

Bohr's view provides the grounds for defining a very general class of theories, which we have elsewhere termed 'complementarity' ($\mathcal{C}$-theories; see [17]). Here, we generalize the concept of a $\mathcal{C}$-theory, by defining 'complementarity theories with meaning principles' (termed $\mathcal{C}_{mp}$-theories), more or less paraphrasing Carnap (but without any compromise with his stance), in which some meta-rules are considered in order we know about the possibility of accepting (or not accepting) certain propositions. Before characterizing these theories, let us see how some authors read complementarity; this will guide our definition of $\mathcal{C}_{mp}$-theories.

To begin with, let us quote Max Jammer:

> "Although it is not easy, as we see, to define Bohr's notion of *complementarity*, the notion of *complementarity interpretation* seems to raise fewer definitory difficulties. The following definition of this notion suggests itself. A given theory T admits a complementarity interpretation if the following conditions are satisfied: (1) T contains (at least) two descriptions D_1 and D_2 of its substance-matter; (2) D_1 and D_2 refer to the same universe of discourse U (in Bohr's case, microphysics); (3) neither D_1 nor D_2, if taken alone, accounts exhaustively for all phenomena of U; (4) D_1 and D_2 are mutually exclusive in the sense that their combination into a single description would lead to logical contradictions.
>
> "That these conditions characterize a complementarity interpretation as understood by the Copenhagen school can easily be documented. According to Léon Rosenfeld, (...) one of the principal spokesmen of this school, complementarity is the answer to the following question: What are we to do when we are confronted with such situation, in which we have to use two concepts that are mutually exclusive, and yet both of them necessary for a complete description of the phenomena? "Complementarity denotes the logical relation, of quite a new type, between concepts which are mutually exclusive, and which therefore cannot be considered at the same time –that would lead to logical mistakes– but which nevertheless must both be used in order to give a complete description of the situation." Or to quote Bohr himself concerning condition (4): "In quan-

> tum physics evidence about atomic objects by different experimental arrangements (...) appears contradictory when combination into a single picture is attempted." (...) In fact, Bohr's Como lecture with its emphasis on the mutual exclusive but simultaneous necessity of the causal (D_1) and the space-time description (D_2), that is, Bohr's first pronouncement of his complementarity interpretation, forms an example which fully conforms with the preceding definition. Borh's discovery of complementarity, it is often said, constitutes his greatest contribution to the philosophy of modern science." [25, pp. 104-5]

Jammer's quotation is interpreted as follows. We take for granted that both D_1 and D_2 are sentences formulated in the language of a complementary theory T, so that items (1) and (2) are considered only implicitly. Item (3) is understood as entailing that *both* D_1 and D_2 are, from the point of view of T, *necessary* for the full comprehension of the relevant aspects of the objects of the domain; so, item (3) is asserted on the grounds of a certain meaning principle; so, we take both D_1 and D_2 as 'true' sentences, that is, $T \vdash D_1$ and $T \vdash D_2$. Important to remark that here the concept of truth is taken in a syntactical way: a sentence is true in T if it is a theorem of T, and false if its negation is a theorem of T. If neither the sentence nor its negation are theorems of T, then the sentence (so as its negation) is said to be independent.

Item (4) deserves further attention. Jammer (loc. cit.)says that 'mutually exclusive' means that the "combination of D_1 and D_2 into a single description would lead to logical contradictions", and this is reinforced by Rosenfeld's words that the involved concepts "cannot be considered at the same time", since this would entail a "logical mistake". Then, we informally say that *mutually exclusive*, *conjugate propositions*, or *complementary propositions*, are sentences which lead (by classical deduction) to a contradiction; in particular, their conjunction yields a contradiction.

So, following Jammer and Rosenfeld, we shall say that a theory T admits complementarity interpretation, or that T is a $\mathcal{C}$-theory, if T encompasses 'true' formulas α and β (which may stand for Jammer's D_1 and D_2 respectively) which are 'mutually exclusive' in the above sense, for instance, that their conjunction yields to a strict contradiction if classical logic is applied. In other words, if $\vdash$ is the symbol of deduction of classical logic, then, α and β being complementary, we have $\alpha, \beta \vdash \gamma \wedge \neg\gamma$ for some γ of the language of T (see [27, pp. 34-5]).

The problem with this characterization of complementarity is that if the underlying logic of T is classical logic, then T, involving complementary propositions in the above sense, is contradictory or inconsistent. Apparently, this is precisely what Rosenfeld claimed in the above quotation. Obviously, if we intend to maintain the idea of complementary propositions as forming part of the theory and being expressed in the object language without trivialization, one solution (perhaps the only one) is to employe as the underlying logic of T a logic such that the admission of both α and β would not entail a strict contradiction (i.e., a formula of the form $\gamma \wedge \neg\gamma$). One way to do so is to modify the classical concept of deduction, obtaining a new kind of logic, called *paraclassical* logic, as we shall do in what follows.

That kind of logic is the underlying logic of what we have termed complementary theories; here we call complementary theory or $\mathcal{C}_{mp}$-theory, a $\mathcal{C}$-theory with meaning principles. For instance, as we have seen, Heinsenberg uncertainty relations were taken by Février as the starting point for quantum physics. According to her, these relations should not be a simple result obtained within the formalism of quantum theory,[1] but should be the *base* of quantum mechanics. Meaning principles, as we have said before, are here understood as assumptions which sanction either some restrictions of 'classical' procedures or the utilization of certain 'classical' incompatible schemes in the domain of scientific theories. The word 'classical' refers to classical physics (Bohr strongly believed that all the discourse involving quantum phenomena should be done in the language of classical physics).

6 The underlying logic of $\mathcal{C}_{mp}$-theories

We shall restrict our explanation to the propositional level, although it is not difficult to extend our system to encompass quantifiers (and set theory) as it would be necessary if we intend to construct a possible logical basis for physical theories. Let us call C an axiomatized system for the classical propositional calculus. The concept of deduction in C is taken to be the standard one; we use the symbol $\vdash$ to represent deductions in C (see [27]). Furthermore, the formulas of C are denoted by Greek lowercase letters, while Greek uppercase letters stand for sets of formulas. The symbols $\neg$, $\rightarrow$, $\wedge$, $\vee$ and $\leftrightarrow$ have their usual meanings, and standard conventions in writing formulas will be also assumed without further comments.

Definition 1 *Let Γ be a set of formulas of C and let α be a formula (of the language of C). Then we say that α is a (syntactical) P-consequence of Γ, and write $\Gamma \vdash_{\mathsf{P}} \alpha$, if and only if*

(P1) $\alpha \in \Gamma$*, or*

(P2) α *is a classical tautology, or*

(P3) *There exists a consistent (according to classical logic) subset $\Delta \subseteq \Gamma$ such that $\Delta \vdash \alpha$ (in classical logic).*

We call $\vdash_{\mathsf{P}}$ the relation of P-*consequence.*

Definition 2 P *is the logic whose language is that of C and whose relation of consequence is that of P-consequence. Such a logic will be called* paraclassical.

It is immediate that, among others, the following results can be proved:

[1] For a derivation of Heisenberg relations within the Hilbert space formalism, see [30, pp. 59ff].

Theorem 1

1. If α is a theorem of the classical propositional calculus C and if Γ is a set of formulas, then $\Gamma \vdash_{\mathsf{P}} \alpha$; in particular, $\vdash_{\mathsf{P}} \alpha$.

2. If Γ is consistent (according to C), then $\Gamma \vdash \alpha$ (in C) iff $\Gamma \vdash_{\mathsf{P}} \alpha$ (in P).

3. If $\Gamma \vdash_{\mathsf{P}} \alpha$ and if $\Gamma \subseteq \Delta$, then $\Delta \vdash_{\mathsf{P}} \alpha$ (The defined notion of P-consequence is monotonic.)

4. The notion of P-consequence is recursive.

5. Since the theses of P (valid formulas of P) are those of C, P is decidable.

Definition 3 *A set of formulas Γ is P-trivial iff $\Gamma \vdash_{\mathsf{P}} \alpha$ for every formula α. Otherwise, Γ is P-non-trivial. (Similarly we define the concept of a set of formulas being trivial in C).*

Definition 4 *A set of formulas Γ is P-inconsistent if there exists a formula α such that $\Gamma \vdash_{\mathsf{P}} \alpha$ and $\Gamma \vdash_{\mathsf{P}} \neg\alpha$. Otherwise, Γ is P-consistent.*

Theorem 2

1. If α is an atomic formula, then $\Gamma = \{\alpha, \neg\alpha\}$ is P-inconsistent, but P-non-trivial.

2. If the set of formulas Γ is P-trivial, then it is trivial (according to classical logic). If Γ is non-trivial, then it is P-nontrivial.

3. If Γ is P-inconsistent, then it is inconsistent according to classical logic. If Γ is consistent according to classical logic, then Γ is P-consistent.

A semantical analysis of P, for instance a completeness theorem, can be obtained without difficulty [15]. We remark that the set $\{\alpha \wedge \neg\alpha\}$, where α is a propositional variable, is trivial according to classical logic, but it is not P-trivial. Notwithstanding, we are not suggesting that complementary propositions should be understood necessarily as pairs of contradictory sentences. This is made clear by the following definition:

Definition 5 (Complementarity Theories or $\mathcal{C}_{mp}$-theories) *A $\mathcal{C}$-theory is a set of formulas T of the language of C (the classical propositional calculus) closed by the relation of P-consequence, that is, $\alpha \in T$ for any α such that $T \vdash_{\mathsf{P}} \alpha$. In other words, T is a theory whose underlying logic is P. A $\mathcal{C}_{mp}$-theory is a $\mathcal{C}$-theory subjected to meaning principles.*

Of course the definition of a $\mathcal{C}_{mp}$-theory is a little bit vague. However, for instance in the case of a meaning principle that introduces restrictions in the acceptable statements of the theory, the hypothesis and axioms used in deductions have to satisfy such restrictive conditions. For instance, if a meaning principle of a theory T is formulated as Heisenberg Uncertainty Principle, this circumstance will impose obvious restrictions to certain statements of T.

Theorem 3 *There exist $\mathcal{C}$-theories and $\mathcal{C}_{mp}$-theories that are inconsistent, although are* P*-non-trivial.*

Proof: Immediate consequence of Theorem 2. □

Finally, we state a result (Theorem 4), whose proof is an immediate consequence of the definition of P-consequence. However, before stating the theorem, let us introduce a definition:

Definition 6 (Complementary Propositions) *Let T be a $\mathcal{C}_{mp}$-theory (in particular, a $\mathcal{C}$-theory) and let α and β be formulas of the language of T. We say that α and β are T-complementary (or simply* complementary*) if there exists a formula γ of the language of T such that:*

1. $T \vdash_{\mathsf{P}} \alpha$ and $T \vdash_{\mathsf{P}} \beta$

2. $T, \alpha \vdash_{\mathsf{P}} \gamma$ and $T, \beta \vdash_{\mathsf{P}} \neg\gamma$ (in particular, $\alpha \vdash_{\mathsf{P}} \gamma$ and $\beta \vdash_{\mathsf{P}} \neg\gamma$).

Theorem 4 *If α and β are complementary theorems of a $\mathcal{C}_{mp}$-theory T and $\alpha \vdash_{\mathsf{P}} \gamma$ and $\beta \vdash_{\mathsf{P}} \neg\gamma$, then in general $\gamma \wedge \neg\gamma$ is not a theorem of T.*

Proof: Immediate, as a consequence of Theorem 2. □

In other words, T is inconsistent from the point of view of classical logic, but it is P-non-trivial.

It should be emphasized that our way of characterizing complementarity does not mean that complementary propositions are always contradictory, for α and β above are not necessarily one the negation of the other. However, as complementary propositions, we may derive from them (in classical logic) a contradiction; to exemplify, we remark that 'x is a particle' is not the direct negation of 'x is a wave', but 'x is a particle' *entails* that x is not a wave. This reading of complementarity as not indicating strict contradiction, as we have already made clear, is in accordance with Bohr himself; let us quote him once more to reinforce this idea. Bohr says:

> "In considering the well-known paradoxes which are encountered in the application of the quantum theory to atomic structure, it is essential to remember, in this connection, that the properties of atoms are always obtained by observing their reactions under collisions or under the influence of radiation, and that the (...) limitation on the possibilities of measurement is directly related to the apparent contradictions which have been revealed in the discussion of the nature of light and of the material particles. In order to emphasize that we *are not concerned here with real contradictions*, the author [Bohr himself] suggested in an earlier article the term 'complementarity'." [6, p. 95] (italics ours).

Let us give a simple example of a situation involving a $\mathcal{C}_{mp}$-theory. Suppose that our theory T is a fragment of quantum mechanics admitting Heisenberg relations as a meaning principle and having as its underlying logic paraclassical logic. If α and β are two incompatible propositions according to Heisenberg's principle, we can interpret this principle as implying that α entails $\neg\beta$ (or that β entails $\neg\alpha$). So, even if we add α and β to T, we will be unable to derive, in T, $\alpha \wedge \beta$. Analogously, Pauli's Exclusion Principle has also an interpretation as that of Heisenberg's.

As we said before, the basic characteristic of $\mathcal{C}_{mp}$-theories is that, in making P-inferences, we suppose that some sets of statements we handle are consistent. In other words, $\mathcal{C}_{mp}$-theories are closer to those theories scientists *actually* use in their everyday activity than those theories with the classical concept of deduction. In other words, paraclassical logic (and paraconsistent logics in general) seems to fit more accurately the way scientists reason when stating their theories.

7 The paralogic associated to a logic

As we noted in [17], the technique used above to define the paraclassical logic associated to classical logic can be generalized to other logics $\mathcal{L}$ (including logics having no negation symbol, but we will not deal with this case here), as well as the concept of a $\mathcal{C}_{mp}$-theory. More precisely, starting with a logic $\mathcal{L}$, which can be seen as a pair $\mathcal{L} = \langle \mathcal{F}, \vdash \rangle$, where $\mathcal{F}$ is an abstract set called the set of formulas of $\mathcal{L}$ and $\vdash \subseteq \mathcal{P}(\mathcal{F}) \times \mathcal{F}$ is the deduction relation of $\mathcal{L}$ (which is subjected to certain postulates depending on the particular logic $\mathcal{L}$) [2], we can define the $\mathsf{P}_{\mathcal{L}}$-logic associated to $\mathcal{L}$ (the 'paralogic' associated to $\mathcal{L}$) as follows.

Let $\mathcal{L}$ be a logic, which may be classical logic, intuicionistic logic, some paraconsistent logic or, in principle, any other logical system. By simplicity, we suppose that the language of $\mathcal{L}$ has a symbol for negation, $\neg$. Then,

Definition 7 *A theory based on $\mathcal{L}$ (an $\mathcal{L}$-theory) is a set of formulas Γ of the language of $\mathcal{L}$ closed under $\vdash_{\mathcal{L}}$ (the symbol of deduction in $\mathcal{L}$). In other words, $\alpha \in \Gamma$ for every formula α such that $\Gamma \vdash_{\mathcal{L}} \alpha$.*

Definition 8 *An $\mathcal{L}$-theory Γ is $\mathcal{L}$-inconsistent if there exists a formula α of the language of $\mathcal{L}$ such that $\Gamma \vdash_{\mathcal{L}} \alpha$ and $\Gamma \vdash_{\mathcal{L}} \neg\alpha$, where $\neg\alpha$ is the negation of α. Otherwise, Γ is $\mathcal{L}$-consistent.*

Definition 9 *An $\mathcal{L}$-theory Γ is $\mathcal{L}$-trivial if $\Gamma \vdash_{\mathcal{L}} \alpha$ for any formula α of the language of $\mathcal{L}$. Otherwise, Γ is $\mathcal{L}$-non-trivial.*

Then, we define the $\mathsf{P}_{\mathcal{L}}$-logic associated with $\mathcal{L}$ whose language and syntactical concepts are those of $\mathcal{L}$, except the concept of deduction, which is introduced as follows: we say that α is a $\mathsf{P}_{\mathcal{L}}$-*syntactical consequence* of a set Γ of formulas, and write $\Gamma \vdash_{\mathsf{P}_{\mathcal{L}}} \alpha$ if and only if:

(1) $\alpha \in \Gamma$, or

(2) α is a provable formula of $\mathcal{L}$ (that is, $\vdash_{\mathcal{L}} \alpha$), or

(3) There exists $\Delta \subseteq \Gamma$ such that Δ is $\mathcal{L}$-non-trivial, and $\Delta \vdash_{\mathcal{L}} \alpha$.

For instance, we may consider the paraconsistent calculus $\mathcal{C}_1$ [14] as our logic $\mathcal{L}$. Then the paralogic associated with $\mathcal{C}_1$ is a kind of 'para-paraconsistent' logic.

It seems worthwhile to note the following in connection with the paraclassical treatment of theories. Sometimes, when one has a paraclassical theory T such that $T \vdash_{\mathsf{P}} \alpha$ and $T \vdash_{\mathsf{P}} \neg\alpha$, there exist *appropriate* propositions β and γ such that T can be replaced by a classical consistent theory T' in which $\beta \to \alpha$ and $\gamma \to \neg\alpha$ are theorems. If this happens, the logical difficulty is in principle eliminable and classical logic maintained.

8 Logic and physics

When we hear something about the relationships between logic and quantum physics, we usually tend to relate the subject with the so called 'quantum logics', a field that has its 'official' birth in Birkhoff and von Neumann's well known paper from 1936 (see [20]). This is completely justified, for their fundamental work caused the development of a wide field of research in logic. Today there are various 'quantum logical systems', although they have been studied specially as pure mathematical systems, far from applications to the axiomatization of the microphysical world and also far from the insights of the forerunners of quantum mechanics (for a general and updated account on this field, see [20]).

Of course an axiomatization of a given empirical theory is not always totally determinate, and the need for a logic distinct from the classical as the underlying logic of quantum theory is still open to discussion. In fact, the axiomatic basis of a scientific theory depends on the several aspects of the theory, explicitly or implicitly, appropriate to take into account its structure. So, for example, Ludwig [26] studies an axiomatization of quantum mechanics based on classical logic. All the stances, that of employing a logic like paraclassical logic (or other kind of system as one of those mentioned above), and that of Ludwig, are in principle acceptable, since they treat different perspectives of the same domain of discourse, and different 'perspectives' of a domain of science may demand for distinct logical apparatuses; this is a philosophical point of view radically different from the classical. As we said in another work (see [18]), the possibility of using non-standard systems in the foundations of physics (and in general of science) does not necessarily entail that classical logic is wrong, or that (in particular) quantum theory *needs* another logic. Physicists probably will continue to use classical (informal) logic in the near future. But we should realize that other forms of logic may help us in the better understanding of certain features of the quantum world as well, not easily treated by classical devices, such as complementarity. Only the future of physics will perhaps decide what is the better solution, a decision that involves even pragmatic factors.

To summarize, we believe that there is no just one 'true logic', and distinct logical (so as mathematical and perhaps even physical) systems, like paraclassical logic, may be useful to approach different aspects of a wide field of knowledge like quantum theory. The important point is to be open to the justifiable revision of concepts, a point very lucidly emphasized by Niels Bohr, who wrote:

> "For describing our mental activity, we require, on one hand, an objectively given content to be placed in opposition to a perceiving subject, while, on the other hand, as is already implied in such an assertion, no sharp separation between object and subject can be maintained, since the perceiving subject also belongs to our mental content. From these circumstances follows not only the relative meaning of every concept, or rather of every word, the meaning depending upon our arbitrary choice of view point, but also we must, in general, be prepared to accept the fact that a complete elucidation of one and the same object may require diverse points of view which defy a unique description. Indeed, strictly speaking, the conscious analysis of any concept stands in a relation of exclusion to its immediate application. The necessity of taking recourse to a complementarity, or reciprocal, mode of description is perhaps most familiar to us from psychological problems. In opposition to this, the feature which characterizes the so-called exact sciences is, in general, the attempt to attain to uniqueness by avoiding all reference to the perceiving subject. This endeavour is found most consciously, perhaps, in the mathematical symbolism which sets up for our contemplation an ideal of objectivity to the attainment of which scarcely any limits are set, so long as we remain within a self-contained field of applied logic. In the natural sciences proper, however, there can be no question of a strictly self-contained field of application of the logical principles, since we must continually count on the appearance of new facts, the inclusion of which within the compass of our earlier experience may require a revision of our fundamental concepts." [6, pp. 212-213]

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