On the measurement process in Bohmian mechanics (reply to Gao)

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This note clarifies some technical and conceptual details about the description of the measurement process in Bohmian mechanics and responds to a recent manuscript by Shan Gao (2019), wrongly claiming that Bohmian mechanics is inconsistent and doesn't solve the measurement problem.

1 Introduction

Bohmian mechanics is a quantum theory that grounds the predictions of standard quantum mechanics in an ontology of point particles and two precise dynamical laws: the Schrödinger equation for the wave function, and the guiding equation in which the wave function enters to define a velocity field for the particles. For advocates of the theory, one of the main virtues of Bohmian mechanics is that it allows for a physical description of the measurement process that avoids the infamous measurement problem and illuminates the meaning and status of the usual quantum formalism. On the flipside, this has occasionally raised the ambition of critics to deny these otherwise well-established accomplishments. A recent example is a manuscript by Shan Gao (2019), claiming to derive a "contradiction" in the Bohmian description of the measurement process.

Since Gao's objections seem to exemplify a variety of misunderstandings about the Bohmian theory – some more subtle than others – I want to use it as an opportunity to discuss several technical and conceptual details about the measurement process that are rarely spelled out explicitly in the literature. In Section 3, I will then address Gao's argument in detail and explain why it is incorrect.

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2 Measurements in Bohmian mechanics

A prototypical measurement in Bohmian mechanics is an interaction between a system S and a measurement device D resulting in one of several macroscopically discernible configurations of D ("pointer positions") which are correlated with certain possible quantum states of S. Schematically, the interaction between the measured system and measurement device is such that, under the Schrödinger evolution,

$$\varphi_i \Phi_0 \overset{\text{Schrödinger evolution}}{\longrightarrow} \varphi_i \Phi_i \ , \tag{1}$$

where the wave function Φ_0 is concentrated on pointer configurations corresponding to the "ready state" of the measurement device, and Φ_i are concentrated on configurations indicating a particular measurement result, e.g., by a pointer pointing to a value on a scale, a point-like region of a detector screen being darkened, a detector clicking or not clicking, etc. The Schrödinger time evolution is linear, so that the superposition

$$\varphi = c_1\varphi_1 + c_2\varphi_2, \qquad c_1, c_2 \in \mathbb{C}, \qquad |c_1|^2 + |c_2|^2 = 1,$$

leads to

$$\varphi\Phi_0 = (c_1\varphi_1 + c_2\varphi_2)\Phi_0 \overset{\text{Schrödinger evolution}}{\longrightarrow} c_1\varphi_1\Phi_1 + c_2\varphi_2\Phi_2. \tag{2}$$

At this point, standard quantum mechanics is hit by the measurement problem (Maudlin, 1995a). In Bohmian mechanics, however, the system is described not only by the wave function but also by the actual spatial configuration $(X, Y) \in \mathbb{R}^m \times \mathbb{R}^n$ of measured system and measurement device, given by the positions of their constituent particles. It thus has a well-defined configuration at all times, regardless of whether or not its wave function is in a superposition.

For illustrative purposes, we assume that Φ_1 is concentrated on a region $\mathrm{L} \subset \mathbb{R}^n$ of the configuration space of D corresponding to the pointer of the measurement device pointing to the left, while Φ_2 is concentrated on a region $\mathrm{R} \subset \mathbb{R}^n$ of the configuration space of D corresponding to the pointer pointing to the right. Obviously, the two regions are disjoint, i.e. $\mathrm{L} \cap \mathrm{R} = \emptyset$. By assumption, Φ_1 and Φ_2 must be well localized in the respective regions (otherwise, the measurement device is no good), i.e., almost zero outside. This implies, in particular,

$$\int_{\mathrm{L}} |\Phi_1|^2 \, \mathrm{d}^n y \approx 1, \quad \int_{\mathrm{L}} |\Phi_2|^2 \, \mathrm{d}^n y \approx 0 \tag{3a}$$

$$\int_{\mathrm{R}} |\Phi_1|^2 \, \mathrm{d}^n y \approx 0, \quad \int_{\mathrm{R}} |\Phi_2|^2 \, \mathrm{d}^n y \approx 1. \tag{3b}$$

As Gao (2019) rightly points out, it's not realistic to assume that Φ_1 and Φ_2 have compact support in L, respectively R, i.e., that they are precisely zero outside.[1] Hence the "≈" in the above equations. The better these pointer states are localized, the better the approximation. In practice, this will depend on the details of the experiment, such as the makeup of the measurement apparatus, and the strength and duration of its interaction with the microscopic system.

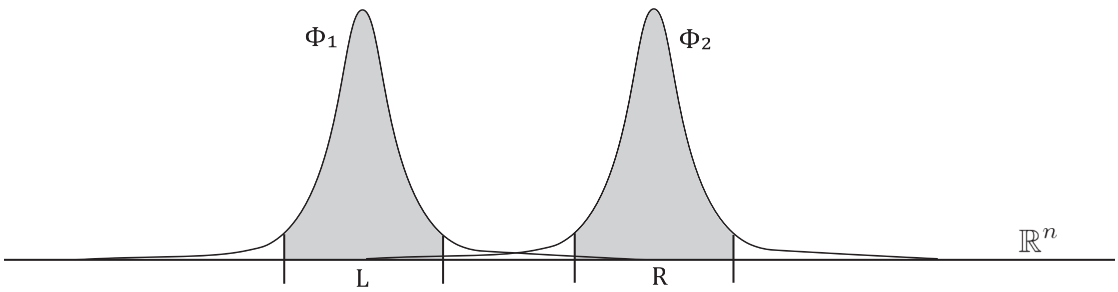

Figure 1: Sketch of the pointer wave functions on configuration space.

Now, according to Bohmian mechanics, the probability for the pointer *actually* pointing to the left is:

$$\begin{aligned}\mathbb{P}(Y \in \mathrm{L}) &= \int_{\mathbb{R}^m \times \mathrm{L}} |c_1 \varphi_1 \Phi_1 + c_2 \varphi_2 \Phi_2|^2 \, \mathrm{d}^m x \mathrm{d}^n y \\ &= |c_1|^2 \int_{\mathbb{R}^m \times \mathrm{L}} |\varphi_1 \Phi_1|^2 \mathrm{d}^m x \mathrm{d}^n y \\ &+ |c_2|^2 \int_{\mathbb{R}^m \times \mathrm{L}} |\varphi_2 \Phi_2|^2 \mathrm{d}^m x \, \mathrm{d}^n y \\ &+ 2 \operatorname{Re}\Big(c_1 c_2 \int_{\mathbb{R}^m \times \mathrm{L}} (\varphi_1 \Phi_1)^* \varphi_2 \Phi_2 \mathrm{d}^m x \, \mathrm{d}^n y \Big) \approx |c_1|^2 .\end{aligned} \tag{4}$$

The final approximation follows from eq. (3a) (together with the Cauchy-Schwarz inequality $|\int_L \Phi_1^* \Phi_2| \leq \sqrt{\int_L |\Phi_1|^2} \sqrt{\int_L |\Phi_2|^2}$). Similarly, the probability of the pointer pointing to the right is $\mathbb{P}(Y \in \mathrm{R}) \approx |c_2|^2$. If φ_1 and φ_2 are eigenstates of some quantum observable, $|c_1|^2$ and $|c_2|^2$ are the statistical predictions of standard quantum mechanics for an *ideal* measurement. The better the pointer states Φ_1 and Φ_2 are localized in disjoint regions of configuration space, the closer the measurement is to "ideal".

[1] It is also not realistic to assume, as he does, that the pointer wave functions evolve freely. Usually, there will be some potential keeping the pointers in place, and decoherence, through interactions with the environment, leading to further localization (see Remark 4 below).

Since a realistic measurement is not quite ideal, we see that there is also a very small, yet non-zero, probability that the final pointer position is inconclusive, e.g. because it remains roughly in the ready state, or because the measurement device is blown into pieces. Tough luck, measurements can fail.

2.1 Remarks and Observations

1. After the measurement (assuming it was not destructive), the system S will be guided by the effective wave function $\varphi_1(x)\Phi_1(Y) + \varphi_2(x)\Phi_2(Y)$. If the pointer actually points left (let's say), i.e. $Y \in \mathrm{L}$, we have $\Phi_2(Y) \approx 0$ and hence (after normalization) the *effective wave function* φ_1 describing the System S after the measurement. This is the effective collapse in Bohmian mechanics.[2]

2. In many papers – including some of my own – it is said with regard to eq. (4) that we are integrating "over the support of Φ_1". This is indeed a little abuse of mathematical language. For non-idealized situations, one should read the statement like a physicist, not like a mathematician, namely as saying: we integrate over a region of configuration space – here L – that contains almost the entire L^2-weight of Φ_1 (and which corresponds to configurations in which the pointer points to the left). Let's call this the *FAPP[3]-support*.

3. If the pointer after the measurement is actually pointing to the left (let's say), i.e. $Y \in \mathrm{L}$, then the contribution of Φ_2 to the Bohmian guiding field at Y will be negligibly small, as well – provided the tails are "well-behaved". This justifies the statement that the configuration of the measurement device is effectively guided by the wave packet Φ_1 only. "Well-behaved" means that not only Φ_2 itself but also its gradient – more precisely $\mathrm{Im}\,\nabla_y \Phi_2$ – is very small outside the FAPP-support (think, for instance, of Gaussian tails). This assumption is commonly made in physics, and well justified in general. Moreover, the contribution of the other branch will further diminish (even for not so well-behaved tails) as *decoherence* is progressing through interactions with the environment, leading in effect to the situation of the effective collapse for the pointer states (see Remark 4).

4. Suppose we go one step further and consider a "measurement of the pointer position" by another system E. You may think of an "observer" looking at the measurement device, resulting, ultimately, in a particular particle configuration in her brain, though I prefer a camera or some other system under no suspicion of

[2] See Dürr et al. (2013, ch. 2) for more details.
[3] For All Practical Purposes

consciousness. In any case, the spatial resolution of such an observation can easily be finer than the localization of the initial pointer wave functions, thus leading to a Schrödinger evolution of the form

$$\Phi_i \longrightarrow \sum_j \Phi_{ij}\Psi_j,$$

where $\sum_j \Phi_{ij} = \Phi_i$, and the Ψ_j are well localized in disjoint regions of the configuration space of E (corresponding to different "record" configurations of the observing system). This then leads to decoherence and localization (by effective collapse) of the apparatus wave function into one of the wave packets $\frac{\Phi_{ij}}{\|\Phi_{ij}\|_2}$.

Hence, clearly, the precision of an observation of the pointer position is not limited by the spread of the pointer states Φ_i prior to observation (contrary to what Gao (2019) seems to suggest). In particular, the apparatus wave function can effectively collapse into wave functions other than Φ_1 or Φ_2.

Notably, we didn't even have to consider a second "measurement" – environmental decoherence, if only by scattering of air molecules, photons, etc., occurs everywhere and all the time (unless one takes very special precautions to prevent it), leading to localization of the macroscopic wave function.

If some of these points are rarely spelled out in detail in the "Bohmian" literature, then because they involve arguments that are (or at least should be) fairly standard in physics. No deep foundational issues are hiding behind the mathematical details here. If there's a lesson to learn, then that serious physics is a bit messier and a bit more subtle than the sterile operator-formalism of quantum mechanics reveals.

3 What Gao's objection gets wrong

So, if not mathematical nitpicking, what is the point of the objection formulated by Shan Gao (2019)? My best attempt at a reconstruction of his argument goes as follows:

i) The possible measurement results are first and foremost given by the pointer states Φ_1 and Φ_2. The role of the Bohmian particle configuration Y is to pick out one of the two results.

ii) The particle configuration picks out one of the two results by ending up in the support[4] of either Φ_1 or Φ_2.

[4]The support is the smallest (closed) set in which the function is not zero.

iii) Since Φ_1 and Φ_2 do not have disjoint supports, there is a non-zero probability that the pointer configuration ends up in their overlap and hence doesn't pick out either Φ_1 or Φ_2 as the measurement result. Therefore, Bohmian mechanics doesn't solve the measurement problem.

    In Gao's words: "This means that there is no one-to-one correspondence from the particle configurations of a measuring device to the result wave functions of the device or the measurement results." (p. 3)

This objection is based on a superficial understanding of Bohmian mechanics and misses its mark for several reasons. Let's start to unpack them.

Premise i) seems to come from Brown and Wallace (2005), who claim to identify this "Result Assumption" in the second part of David Bohm's 1952 paper.[5] Here, I am not interested in an exegesis of Bohm's original work (and I lack the historical competence to provide one). I believe that Brown and Wallace are reading too much into an innocuous statement, but it's possible that Bohm had not yet appreciated the implications of his theory in full. What I can unequivocally say is that this "Result Assumption" plays no role in the modern understanding of Bohmian mechanics (that has been further developed by Bell, and Dürr, Goldstein, Zanghì, among others). Instead, one should to take the ontological commitment to particles seriously and say that *the pointer configuration is the measurement result.*

More precisely, the complete physical state of the measurement apparatus – as with any other subsystem in Bohmian mechanics – is described by a pair (Y, Φ), where $Y \in \mathbb{R}^n$ is the particle configuration and $\Phi \in L^2(\mathbb{R}^n)$ the (effective or conditional) wave function.[6] This state is always uniquely determined by the deterministic laws defining Bohmian mechanics (plus initial conditions). Hence, there cannot be any ambiguity or inconsistency in the predictions of the theory. In practice, of course, we cannot know the exact initial conditions and care only about a *coarse-grained* description of the measurement outcome – e.g. "pointer left" ($Y \in \mathrm{L}$) or "pointer right" ($Y \in \mathrm{R}$). This then leads to the statistical analysis sketched in Chapter 2.

Note that, according to Bohmian mechanis, the wave function of a closed system (in particular, the universal wave function) always evolves unitarily; in and by itself, it is thus insensitive to different measurement results. However, the particle configuration of the apparatus – and then, notably, its environment if we take it into account – will determine which branch of the wave function is guiding the system and which decoherent

[5] Bohm (1952) writes: "[T]he packet entered by the apparatus variable y determines the actual result of the measurement, which the observer will obtain when he looks at the apparatus." (p. 182)

[6] See Dürr et al. (2013, ch. 2) for the difference between effective and conditional wave function.

branches can be ignored. Particle configurations, in other words, determine the collapsed effective wave function at the end of the experiment. To this end, the separation of the superposed wave packets on configuration space doesn't need to be perfect, they don't need to have literally disjoint support as explained in the Remarks above. Hence, Gao's premise ii) – to the extent that it is even meaningful – is wrong, as well. As is the still common misunderstanding that particle configurations are irrelevant to, or provide no information about, the wave functions of subsystems (cf. Maudlin (1995b)).

Coming back to the "Result Assumption", the reason why it would indeed be a rather silly assumption to make is that it leaves open the critical question, how and why and it what sense a particular wave function is supposed to "correspond to a measurement result" – or any concrete physical fact at all. This question is really at the core of the measurement problem (and many other problems in quantum mechanics), and different quantum theories provide different answers (though I don't know which one answers it in a way consistent with Gao's analysis).

Gao seems to assume that the pointer states Φ_1 and Φ_1 – which he calls "result wave functions" – are somehow an observationally "preferred basis", a distinguished set of quantum states that correspond to the possible observations or measurement results. This assumption is a) not part of Bohmian mechanics b) poorly motivated (what physical law or principle distinguishes these wave functions and their connection to particular measurement results?) and c) not borne out by any quantum theory I can think of. Any quantum theory (applicable on macroscopic scales) seems to agree that the apparatus wave function can evolve, decohere, and/or (in whatever sense) collapse into wave functions other than Φ_1 or Φ_2 (cf. Remark 4). For this reason alone, the assertion that there are exactly two possible outcomes of the measurement, corresponding precisely to Φ_1 or Φ_2, seems incorrect or at least arbitrary – and not just in Bohmian mechanics.

Some quantum theories – in particular, modern versions of Everettian quantum mechanics – try to relate the wave function to objects and events in physical space (like measurement devices showing indicating a measurement result) by some sort of functional analysis in terms of internal degrees of freedom of the wave function or quantum state. This seems to be the basis of the discussion of Brown and Wallace (2005), but it is *not* how the Bohmian theory connects to the physical world.

Bohmian mechanics is a theory about particles moving in physical space. The empirical content of the theory lies in the spatio-temporal configuration of matter, constituted by particles. The role of the wave function is first and foremost to determine how the particles move, and also (though this is a theorem rather than an additional postulate) to describe typical statistical distributions in ensembles of subsystems.

In the present case, the wave packet Φ_1 (Φ_2) "corresponds" to the pointer pointing left (right) in the sense that it is well-localized in a region of configuration space whose points realize a pointer configuration pointing to the left (right) and thus assign very high probability to the respective pointer position. This (and only this) is also what justifies common notations such as $|\text{left}\rangle$ and $|\text{right}\rangle$ for the pointer states. Nonetheless, Φ_1 may be consistent with a pointer actually pointing to the middle or even to the right, and Φ_2 may be consistent with a pointer actually pointing to the middle or even to the left. So again, it is unclear what Gao has in mind when he insists that "the whole result wave function [not just a truncated part] ... corresponds to the result" (p. 2). If the relevant result is "pointer left" or "pointer right", the statement seems incorrect.

The most charitable reading of Gao's objection is that the final pointer position "left" ($Y \in \mathrm{L}$) or "right" ($Y \in \mathrm{R}$) is not *perfectly* correlated with the quantum states φ_1 or φ_2 of the measured system. Indeed! The pointer states Φ_1 and Φ_2 having a finite overlap means precisely that the detector is not perfect in this sense. And this is an utterly realistic limitation of measurements that more sophisticated quantum measurement formalisms capture, as well (under "non-ideal measurements", see e.g. Albert and Loewer (1993); Bacciagaluppi and Hemmo (1994) for a philosophical discussion).[7] Hence, if the objection here is that Bohmian mechanics contradicts the predictions of standard quantum mechanics, it is based on a questionable – and I would say wrong – understanding of what "'standard" quantum mechanics actually predicts. Ironically, Gao rightly points out that the assumption of pointer states with disjoint supports is an unrealistic idealization but doesn't seem to realize that the very same idealization lies behind the usual von Neumann measurements of textbook quantum mechanics.

Finally, I also have to warn against thinking of the quantum states φ_1 or φ_2 – even if they are eigenstates of some relevant observable – as corresponding to pre-existing properties of the system that the measurement is supposed to reveal. This idea, which is conclusively dispelled by Bohmian mechanics, lies behind many of the alleged paradoxes of quantum mechanics or misguided talk about "quantum logic" (Bell, 2004, chs. 17, 23). Thus, contrary to what Gao seems to assume (p. 5), there is no "measured quantity" with pre-existing values that the Bohmian particle configuration registers (cf. Norsen (2014) for the particular example of spin measurements). Bohmian particles have a position and nothing else,[8] while different wave functions or quantum states have to be understood through their dynamical role for the particle motion.

[7] A more extreme, but very important, case are so-called weak measurements, in which the possible pointer states overlap a lot, thus providing very little information from a single measurement event but also affecting the state of the measured system as little as possible (cf. e.g. Wiseman (2007)).

[8] We can leave open the status of dynamical parameters such as mass and charge.

John Bell (2004) summarized this important insight brilliantly:

> "[I]n physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency." (p. 166)

4 Conclusion

Possibly the most basic mistake committed by many critics is to think of Bohmian mechanics essentially as standard quantum mechanics plus an *ad hoc* addition of particle positions to solve the measurement problem. In fact, the measurement formalism of quantum mechanics *reduces* to Bohmian mechanics as an effective statistical description of the fundamental microscopic theory. Simply put, Bohmian mechanics is to textbook quantum mechanics what Hamiltonian mechanics is to thermodynamics. There would thus be a lot more to learn by studying the measurement process from a Bohmian point of view: the status of Born's rule (Dürr et al., 2013, ch.2), the role of observables (Dürr et al., 2013, ch.3), the meaning of the no-hidden-variables theorems (Lazarovici et al., 2018) – all this and more is clarified by Bohmian mechanics.

What Bohmian mechanics doesn't provide – and what a serious physical theory shouldn't provide, as we learned, in particular, from Bell – are postulates about "observables", "measurements", "measurement results", etc. The theory describes what is going on in the world, and we have to analyze the theory to know what it predicts for a particular physical situation, what we can measure and how, and whether these predictions match our empirical evidence.

One of the many intellectual harms done by operational quantum mechanics is that this way of doing serious physics is no longer common ground.

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