Is there density matrix realism?

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

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Abstract

In a recent paper ["Quantum Mechanics in a Time-Asymmetric Universe: On the Nature of the Initial Quantum State", The British Journal for the Philosophy of Science, 2018], Chen uses density matrix realism to solve the puzzles of the arrow of time and the meaning of the quantum state. In this paper, I argue that density matrix realism is problematic, and in particular, it is inconsistent with the latest results about the reality of the wave function.

Recently Chen (2018) proposed an original and ingenious idea to solve the puzzles of both the arrow of time and the meaning of the quantum state. The idea is based on the so-called density matrix realism, according to which the ontic state of an isolated system such as the universe as a whole is represented by an (impure) density matrix, not by a wave function (see also Anandan and Aharanov, 1998; Dürr et al, 2005; Maroney, 2005). In this paper, I will argue that density matrix realism is problematic, and in particular, it is inconsistent with the latest results about the reality of the wave function.

In quantum mechanics, a density matrix is usually used to represent either a system with a random wave function ("statistical density matrix") or a system that is entangled with another system ("reduced density matrix"). According to density matrix realism, the density matrix is fundamental, directly representing the actual ontic state of an isolated system such as the universe as a whole. Let's use a simple example to illustrate what this means (Maroney, 2005). Suppose the wave function of an isolated system is random, being $|0\rangle$ with probability p_0 or $|1\rangle$ with probability p_1, where $p_0 + p_1 = 1$. For the system whose wave function is $|0\rangle$, if an observable A is measured, the probability of obtaining the result a is $|\langle 0 | a\rangle|^2$. Similarly, for the system whose wave function is $|1\rangle$, the probability is $|\langle 1 | a\rangle|^2$. According to density matrix realism, the ontic state of each system, no matter its wave function is $|0\rangle$ or $|1\rangle$, is represented by the density matrix $\rho = p_0 |0\rangle \langle 0| + p_1 |1\rangle \langle 1|$, and the probability of obtaining the result a is always $p(a) = p_0 |\langle 0 |a\rangle |^2 + p_1 |\langle 1 |a\rangle |^2$.

Here is a major argument for density matrix realism (Maroney, 2005). When the wave function of an isolated system is random, such as being $|0\rangle$ with probability p_0 or $|1\rangle$ with probability p_1, it is impossible to detect any difference between the ensemble constructed out of such wave functions (which is described by a statistical density matrix) and an ensemble of states where the statistical results of measurements upon every individual system is given by probabilities that come from a fundamental density matrix equal to the statistical density matrix, such as $p(a)$. This kind of indistinguishability is also the basis of the empirical equivalence between a theory with a fundamental wave function and a theory with a fundamental density matrix, such as the usual Bohmian mechanics (BM) and the density matrix Bohmian mechanics or W-BM in brief (Dürr et al, 2005). Moreover, the decomposition of a general density matrix is not unique either; even if we assume an ensemble of systems is constructed from individual pure states, we are unable to determine which set of pure states is involved.

The indistinguishability of statistical density matrix and fundamental density matrix, as well as the non-uniqueness of decomposition of a general density matrix, seem to provide a strong support for density matrix realism. If assuming two things which are indistinguishable (e.g. the above two ensembles) must be the same, it will be unreasonable to require that individual systems be described by pure states, rather than density matrices. Furthermore, if assuming something which cannot be measured (e.g. a particular decomposition of a density matrix) does not exist, then an ensemble of systems cannot be constructed from individual pure states, which means that each individual system in the ensemble must be described by a fundamental density matrix.

However, this view of reality is arguably heuristic and limited. It is possible that two different things cannot be distinguished due to the laws of nature. It is also possible that something which cannot be measured does exist. These are issues of ontology. In general, when we have an empirically successful theory, the quantities in the theory do not necessarily directly represent the state of reality. This is what Einstein's worry about quantum mechanics; the wave function in the Schrödinger equation might not directly represesnt the ontic state of a single quantum system such as an electron. In order to find whether the wave function in quantum mechanics or the density matrix in W-BM is real for individual systems, the empirical success of a theory is not enough, and we need a general and rigorous approach to address the issue.

In recent years such an approach has been proposed, which is called the ontological models framework (Harrigan and Spekkens, 2010; Leifer, 2014). The framework has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that an isolated system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ.[1] In general, a wave function ψ corresponds to a probability distribution $p(\lambda|\psi)$ over all possible ontic states λ, and the probability distributions corresponding to two different wave functions may overlap. In a ψ-ontic (ontological) model, the ontic state of a physical system uniquely determines its wave function, and thus the wave function is a property of the system. While in a ψ-epistemic (ontological) model, there are at least two wave functions which are compatible with the same ontic state of a physical system. In this case, the wave function represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system.

In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we also need a rule of connecting the underlying ontic states with the results of possible measurements. This is the second assumption of the ontological models framework, which says that if a measurement is performed on a system, the behaviour of the measuring device will be determined only by the ontic state of the system, along with the physical properties of the measuring device. For a (projective) measurement M, this assumption means that the ontic state λ of a physical system determines the probability $p(a|\lambda, M)$ of different results a for the measurement M on the system. The consistency with the predictions of quantum mechanics then requires the following relation: $\int p(a|\lambda, M)p(\lambda|\psi)d\lambda = p(a|\psi, M)$, where $p(a|\psi, M) = |\langle\psi\,|a\rangle\,|^2$ is the Born probability of a given M and ψ.

In the following, I will argue that the ontological models framework can help determine whether the ontic state of an isolated system such as the universe as a whole is represented by an (impure) density matrix or a wave function, and density matrix realism is not true in the framework.

First of all, there are already several important ψ-ontology theorems, such as the Pusey-Barrett-Rudolph theorem or PBR theorem (Pusey, Barrett and Rudolph, 2012), Hardy's theorem (Hardy, 2013), and the Colbeck-Renner theorem (Colbeck and Renner 2012, 2017). These theorems prove, under certain auxiliary assumptions, that the wave function of an isolated system is a property of the system. For example, the PBR theorem proves the reality of the wave function under the preparation independence assumption, which says that multiple systems can be prepared such that their ontic states are uncorrelated, e.g. the ontic states of two isolated systems are uncorrelated. Since the PBR theorem resorts not to the dynamics but only to the prepare-and-measure experiments, and the preparation indepen-

[1] This is somewhat different from the original assumption, which says that if a physical system is prepared such that quantum mechanics assigns a pure state or wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state.

dence assumption holds true not only in the three main realistic quantum theories, namely BM, collapse theories, and Everettian quantum mechanics, but also in their density matrix variants such as W-BM (see Chen, 2018), the wave function is real in these theories. In other words, the ontic state of an isolated system is represented by a wave function, not by an (impure) density matrix.

One might want to revise these quantum theories to make the wave function unreal by dropping the auxiliary assumptions. However, it can also be proved that density matrix realism is not true in the ontological models framework without resorting to any auxiliary assumptions. Consider a system with a random wave function $|\psi_i\rangle$ with probability p_i, where $|\psi_i\rangle$ are orthogonal states. If density matrix realism is true, then the ontic state of the system, no matter what its wave function is, will be the same, represented by the fundamental density matrix $\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$. But it is a simple result of the ontological models framework that different orthogonal states correspond to different ontic states, and this result can be proved based solely on the two fundamental assumptions of the framework (Leifer, 2014).

Here it is worth noting that in W-BM the density matrix is not complete and there are additonal variables (i.e. the positions of Bohmian particles), but this does not influence the above result. If density matrix realism is true, then the complete ontic state of the above system may be still the same for two orthogonal states, namely the additonal variables may assume the same values for these two states. Thus, even when the density matrix is not complete as in W-BM, density matrix realism is not consistent with the ontological models framework either.

Density matrix realism admits the existence of the ontic state of an isolated system, and thus it accepts the first assumption of the ontological models framework. This means that one must drop or revise the second assumption of the ontological models framework in order to save density matrix realism. The second assumption says that when a measurement is performed on a system, the behaviour of the measuring device, especially the probability of different results for the measurement, is determined by the ontic state of the system, along with the physical properties of the measuring device. It seems that no realists would like to drop this fundamental assumption. If this assumption is dropped, then it seems that the Born probabilities in quantum mechanics will be inexplainable ontologically. This is unsatisfactory. Note that this explanatory deficiency is more obvious for the distinguishability of orthogonal states. If different orthogonal states correspond to the same ontic state as assumed by density matrix realism, then their distinguishability cannot be explained.

Finally, one might think that the above analysis for an isolated system may not hold true for the universe as a whole, and density matrix realism at the universal level may be compatible with the reality of the wave function for an isolated subsystem of the universe. But this is arguably not true. First, the universe as a whole is a perfectly isolated system. Then, if the wave function of an isolated system is real, the wave function of the universe must be real too. In other words, if only the universe as a whole has an ontic state, this state can be described (at least partly) by a wave function. One may object that since the second assumption of the ontological models framework concerns measurements, while nothing exists outside the universe and no measurements can be made on it, the results obtained based on the framework may be not valid for the universe as a whole. However, this is a misunderstanding. The reason is that the measurements involved in the second assumption of the ontological models framework are not necessarily actual. The assumption is essentially about the connection between the ontic state of an isolated system and the Born rule. If only the Born rule is universally valid and the universe as a whole has an ontic state, then this assumption will apply to the universe, and the results obtained based on the framework will be valid for it.

Next, if the ontic state of every isolated subsystem of the universe is described by a wave function, and in particular, if the ontic state of every entangled composite subsystem of the universe is described by a wave function, then it is arguable that the ontic state of all these subsystems as a whole should be also described by an (entangled) wave function. In other words, the ontic state of the universe as a whole should be described by a wave function. For example, suppose the wave function of each isolated system in the universe is a field (Albert, 1996, 2013, 2015), then since the universe is composed of these fields, its ontic state must be also a field, the largest field.

Third, the above conclusion is also supported by an analysis of the effective density matrix in W-BM (Dürr et al, 2005; Chen, 2019). Let A be a subsystem of the universe including N particles with position variables $x = (x_1, x_2, ..., x_N)$. Let $y = (y_1, y_2, ..., y_M)$ be the position variables of all other particles not belonging to A. Then the subsystem A's conditional density matrix at time t is defined as the universal density matrix $W_t(x, y, x', y')$ evaluated at $y, y' = Y(t)$:

$$w_t^A(x, x') = W_t(x, y, x', y')|_{y,y'=Y(t)}. \tag{1}$$

If the universal density matrix can be decomposed in the following form:

$$W_t(x, y, x', y') = \rho_t(x, x')\chi_t(y, y') + w_t(x, y, x', y'), \tag{2}$$

where $\chi_t(y, y')$ and $w_t(x, y, x', y')$ are functions with macroscopically disjoint supports, and $Y(t), Y(t)$ lie within the support of $\chi_t(y, y')$, then $w_t^A(x, x') = \rho_t(x, x')$ (up to a multiplicative constant) is A's effective density matrix at t. This corresponds to the effective wave function in BM, namely the Bohmian analogue of the usual wave function in the standard formulation of quantum mechanics.

Now, if the universal density matrix is impure and has a very general form as assumed by Chen (2018), then the effective density matrix of each quasi-ioslated system will be also impure in general. But this is inconsistent with the reality of the wave function for the subsystems of the universe. Note that for a special universal density matrix, it seems possible that the effective density matrices of some quasi-isolated systems may be pure. But if the effective density matrix of every quasi-isolated system in the universe is pure, which means that every quasi-isolated system in the universe is actually described by a wave function, then the theory will be BM, not W-BM, and in this case, the universal density matrix will be also pure and reduced to the universal wave function.

To sum up, I have argued that density matrix realism is problematic, and in particular, it is inconsistent with the latest results about the reality of the wave function.

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