The Non-Equivalence of Einstein and Lorentz

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Abstract

In this paper, I give a counterexample to a claim made in (Norton [2008]) that empirically equivalent theories can often be regarded as theoretically equivalent by treating one as having surplus structure, thereby overcoming the problem of underdetermination of theory choice. The case I present is that of Lorentz's ether theory and Einstein's theory of special relativity. I argue that Norton's suggestion that surplus structure is present in Lorentz's theory in the form of the ether state of rest is based on a misunderstanding of the role that the ether plays in Lorentz's theory, and that in general, consideration of the conceptual framework in which a theory is embedded is vital to understanding the relationship between different theories.

1 Introduction

The existence of distinct but empirically equivalent theories is seen as a threat to the scientific realist. This is because empirically equivalent theories are often thought to be epistemically equivalent - that is, the evidence supports both theories equally - and therefore the choice between them is underdetermined by the evidence. Many realists have responded to this challenge by arguing that empirical equivalence is not sufficient for epistemic equivalence. In particular, empirically equivalent theories may not be equally confirmed by the evidence. One notable advocate of such a position is Glymour ([1977]) who argues that spacetime theories which are empirically equivalent can receive different degrees of confirmation - if one spacetime theory has structure over and above the other, then that theory will be less supported by the evidence because it makes additional, unnecessary ontological claims. This avoids the problem facing the realist since if one theory is epistemically preferred to other empirically equivalent theories, then there is no underdetermination of theory choice.

There is another route for the realist. This is to argue that the cases the anti-realist gives as examples of underdetermined theories do not in fact correspond to distinct theories, but to one theory formulated in different ways, and therefore the underdetermination is dissolved. Norton ([2008]) follows this line, by arguing that instead of thinking, as per Glymour, that the theory with more structure is confirmed to a lesser degree, this extra structure should instead be interpreted as surplus structure. In this paper, I first present Norton's argument for why this allows us to treat empirically equivalent theories as theoretically equivalent. I then argue that despite being a case that Norton uses to support his argument, the case of Lorentz's ether theory and Einstein's theory of special relativity does not fit his framework because of the particular role that the ether plays in Lorentz's theory which means that it cannot be regarded as surplus, and therefore the issue of underdetermination remains. Finally, I look at some consequences of my argument for understanding the change from Lorentz's theory to Einstein's, as well as the comparison of theories more generally.

2 Norton's Argument

Norton's argument is the following:

P1. Theories that are empirically equivalent will have structure in common.

P2. Consider two empirically equivalent theories, $T1$ and $T2$. There are two cases:

  a) $T1$ and $T2$ are structurally equivalent.

  b) $T1$ and $T2$ are structurally inequivalent, but one (say $T1$) has extra structure over the other ($T2$) which can be regarded a surplus.

P3. Theories are equivalent when they are structurally equivalent.[1]

C. Therefore, in either of the two cases above, the theories can be regarded as formulations of a single theory.

For case (a), the conclusion follows simply from $P3$. For case (b), the conclusion follows because if we regard the extra structure in $T1$ as surplus and therefore don't take it to be significant in terms of the content of the theory, then the two theories can be seen as structurally (and therefore, by $P3$, theoretically) equivalent, since they have

[1]There is a wide literature on theoretical equivalence and Norton is not clear where he stands in this. In particular, Norton suggests that equivalence is given both by 'intertranslatability' and by structural equivalence. However, these are often not thought of as compatible views - the former is a syntactic relation between theories, using notions such as definitional equivalence, which (Glymour [1970], [1977]) endorses, while the latter is a relation between the mathematical structure of theories, which has been made more precise using the tools of category theory (see, for example, (Halvorson [2016]; Weatherall [2016a], [2016c], [2017])). Since Norton wants to use comparison of mathematical structure to determine whether theories are equivalent, I take him to be endorsing a form of structural, rather than definitional, equivalence.

the same physically significant structure. One immediate concern is when one is able to treat some structure as being surplus. Informally, I take some structure to be surplus if it plays no role in the theory of which it is part, so that it can be removed without changing the empirical content of the theory. This means that it is not sufficient that there is another theory without this structure - it must be that this structure is 'doing no work' in the theory at hand.[2] Later, I will discuss how one can refine this informal notion.

$P1$ is not particularly controversial since it makes a weak claim and it doesn't seem unreasonable to think that empirically equivalent theories will at least have some structure in common. However, the role of this premise is just to provide motivation for the stronger claim in $P2$ that there are only two cases of empirically equivalent theories. The identification of theoretical equivalence with structural equivalence given in $P3$ is more controversial. First, in order to compare the structure of two theories, one first has to characterise the structure of the theories. Norton is assuming that there is a natural way to do this. Second, there are different ways of comparing the structure of theories and depending which criteria one uses, one might reach different conclusions regarding which theories are equivalent. Finally, one might think that structural equivalence is not a strong enough criterion for theoretical equivalence - one might also want to preserve something over and above structure, such as representational content.[3]

While the task of characterising and comparing the structure of theories in the general case is problematic, for the examples I look at in this paper I take there to be a relatively natural way of doing so. In particular, I will look at theories that are characterised in terms of their spacetime structure, which can be compared by looking at their automorphism groups. Instead, I will argue that the main problem with Norton's argument is that the two cases of empirically equivalent theories he considers, (a) and (b) above, do not cover all cases of empirically equivalent theories. I will present an example of two theories that are empirically equivalent, where there is a clear sense in which one has more structure than the other, but where it is not the case that the theory with more structure has surplus structure and therefore they cannot be interpreted as equivalent. This is the case of Lorentz's ether theory and Einstein's theory of special relativity.[4]

3 Background to Lorentz's and Einstein's Theory

In order to understand Lorentz's and Einstein's theory, I will consider how Maxwell's equations can be interpreted in different spacetime structures. The spacetime structures that I will be concerned with are Galilean spacetime, Newtonian spacetime, and Minkowski spacetime. Galilean spacetime consists of $(M, \nabla, t_{ab}, h^{ab})$, a manifold isomorphic to $\mathbb{R}^4$ with a derivative operator, a temporal and spatial metric.[5] This corresponds to a foli-

[2] Of course, this might be something that is only seen with hindsight - until one is able to recognise that the theory can be formulated without this structure, it may not be clear that this structure is not playing any role in the theory.

[3] See (Nguyen [2017]; Coffey [2014]) for views on theoretical equivalence that aim to go beyond formal accounts.

[4] The theory of special relativity as considered in this paper also makes use of the contributions from Minkowski and others, but I will continue to refer to it as 'Einstein's theory'.

[5] The operator ∇ is a flat derivative operator on $\mathbb{R}^4$ compatible with t_{ab} and h^{ab}. Both t_{ab} and h^{ab} are smooth, symmetric fields on M, where t_{ab} has signature $(1, 0, 0, 0)$ and h^{ab} has signature $(0, 1, 1, 1)$, and they obey the orthogonality condition $h^{ab}t_{bc} = 0$. For details, see (Malament [2012], 4.1).

ation of spacetime into simultaneity hypersurfaces which each have the structure of Euclidean space. Newtonian spacetime consists of $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$, which adds to Galilean spacetime a privileged frame of reference represented by a timelike vector with fixed 4-velocity ξ^a. Minkowski spacetime consists of (M, g_{ab}), where M is the manifold $\mathbb{R}^4$ and g_{ab} is a complete, flat and smooth metric with signature (1,3).[6] The metric g_{ab} gives Minkowski spacetime 'lightcone structure'.

It is also important to consider the symmetries of each spacetime - these are given by automorphisms of the spacetime. The automorphisms of a spacetime are diffeomorphisms which preserve the structure on the spacetime. So, for example, the automorphisms of Galilean spacetime, $(M, \nabla, t_{ab}, h^{ab})$, are diffeomorphisms which preserve t_{ab}, h^{ab}, and ∇. These are the Galilean transformations, given by spatial and time translations, rotations, reflections, and Galilean velocity boosts. The automorphisms of Newtonian spacetime, $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$, are then diffeomorphisms which preserve the Galilean spacetime structure, but also preserve ξ^a. This corresponds to spatial and time translations, rotations and reflections[7], but not Galilean velocity boosts. The automorphisms of Minkowski spacetime are the Lorentz transformations, which includes translations, rotations and reflections, as well as Lorentz boosts, which all preserve g_{ab}.[8]

In Minkowski spacetime, Maxwell's equations admit the following, observer independent formulation, where the electromagnetic field is given by the smooth, anti-symmetric tensor field F_{ab}, and the charge-current density is given by the vector field J^a:[9]

$$\nabla_{[a}F_{bc]} = \mathbf{0} \tag{1}$$

$$\nabla_n F^{na} = J^a \tag{2}$$

To understand how the electric and magnetic fields are related to F_{ab} , we must consider the way in which they appear to different observers. In Minkowski spacetime, the electric field relative to a observer O at a point p with four-velocity v^a is given by:

$$E^a = F^a_{\ n} v^n \tag{3}$$

The magnetic field is given by:

$$B^a = \frac{1}{2}\epsilon^{abcd} v_b F_{cd} \tag{4}$$

$\epsilon_{a_1 \ldots a_n}$is an orientation tensor, which gives a choice of 'handedness' or right-hand rule. It follows from (3) and (4) that the electric and magnetic fields are always orthogonal to v^a. We also need to define the electric charge density

[6]For details, see (Malament [2012], 2.1).

[7]As long as these rotations and reflections are about ξ^a.

[8]Sometimes the term 'Lorentz transformations' is used to refer just to Lorentz boosts, but I will use the phrase to refer to all transformations which are automorphisms of Minkowski spacetime.

[9]The demonstration in this section follows that of (Weatherall [2016b]) and (Malament [2012], 2.6).

μ, and 3-current density j^a relative to O. These are given by:

$$\mu = J^a v_a \tag{5}$$

$$j_a = h^a_{\ b} J^b \tag{6}$$

h_{ab} is the spatial projection tensor at the point determined by v^a. We can also define F_{ab} in terms of the electric and magnetic fields:

$$F_{ab} = 2E_{[a}v_{b]} + \epsilon_{abcd}v^c B^d \tag{7}$$

Relative to a choice of constant timelike vector field v^a ($\nabla_a v^b = 0$), Maxwell's equations can then be given in the following form, where D is the derivative induced on hypersurfaces orthogonal to v^a (see (Malament [2012], 1.10)):

$$\begin{aligned} D_b B^b &= \mathbf{0} \\ \epsilon^{abc} D_b E_c &= -v^b \nabla_b B^a \end{aligned} \tag{8}$$

$$\begin{aligned} D_b E^b &= \mu \\ \epsilon^{abc} D_b B_c &= v^b \nabla_b E^a + j^a \end{aligned} \tag{9}$$

Equations (8) and (9) correspond to Equations (1) and (2) respectively.

It is important to note that if one considers a different observer in Minkowski spacetime, represented by a different choice of timelike vector, Maxwell's equations in the form of (8) and (9) still hold. This is a consequence of the fact that the equations are invariant under the Lorentz transformations, which are automorphisms of Minkowski spacetime. Einstein's theory of special relativity takes spacetime to have the structure of Minkowski spacetime, and hence this is the interpretation of Maxwell's equations in Einstein's theory.

If we try to interpret these equations in Galilean spacetime, problems arise because the spacetime symmetries are given by the Galilean transformations, where Galilean boosts preserve the spacelike vectors and not the timelike vectors, rather than the Lorentz transformations, where Lorentz boosts preserve neither spacelike nor timelike vectors. This means that if we consider a new observer, related to the first by a Galilean boost, Maxwell's equations in the form of (8) and (9) do not hold except for the case of constant electric and magnetic fields, with no sources.[10] Therefore, we cannot successfully interpret Maxwell's equations in Galilean spacetime.

In order to overcome this problem, we can adopt a privileged state of motion in the framework of Galilean spacetime and state that Maxwell's equations hold only in this frame of reference. But we can understand the adoption of a privileged state of motion as corresponding to moving to Newtonian spacetime, $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$. In this setting, ξ^a can be taken to represent the 'ether state of rest'.

[10]For proof, see (Weatherall [2016b]).

We can now understand Maxwell's equations by substituting ξ^a for v^a in equations (8) and (9), and define F_{ab} as:

$$F_{ab} = 2E_{[a}t_{b]} + \epsilon_{abcd}\xi^c B^d \tag{10}$$

Here, $E_a = h^{ab}E_b$. We then recover Maxwell's equations in the following form:

$$\nabla_{[a}F_{bc]} = \mathbf{0} \tag{11}$$

$$(\xi^a\xi^n - h^{an})(\xi^b\xi^c - h^{bc})\nabla_b F_{cn} = J^a \tag{12}$$

We can see explicitly that the ether state of rest, represented by ξ^a, is required to make sense of Maxwell's equations in Newtonian spacetime.

Using the resources of Newtonian spacetime, we can also define an inverse Minkowski metric:

$$g^{ab} = \xi^a\xi^b - h^{ab} \tag{13}$$

In Minkowski spacetime, F_{ab} and F^{ab} are related as follows:

$$F^{ab} = g^{ac}g^{bd}F_{cd} \tag{14}$$

and so equation (12) can be written in Minkowski spacetime as:

$$g^{an}g^{bc}\nabla_b F_{cn} = J^a \tag{15}$$

This makes explicit that the metric appears the same way in both formulations of Maxwell's equations. This gives the sense in which they 'say the same thing'.

I take Lorentz's theory to correspond to Maxwell's equations interpreted in Newtonian spacetime. Lorentz of course would not have thought of his theory in this way, since he did not have the resources to do so. However, it allows us to see clearly the connection between the introduction of the ether state of rest with the endorsement of the Newtonian picture of space and time.

This gives the basis of both theories. But we need to know more than Maxwell's equations to adjudicate between them. In particular, we need to consider the behaviour of *matter*. In Einstein's theory, the laws governing matter are Lorentz invariant. But Lorentz was born in the Newtonian tradition, where the Galilean invariant Newton's Laws govern matter. So the reason we would want to endorse Lorentz's theory over Einstein's is if we characterize inertial frames of reference by the Galilean frames of reference - and this in turn would be if we believe that Newton's Laws hold in all frames.[11] If inertial frames of reference are the Galilean ones then, by

[11] It might also be if we think that there is a notion of absolute simultaneity, but it seems the reason we would want to hold this is if it is implied by the laws.

the argument above, we are forced to adopt Newtonian spacetime. But this theory on its own - a combination of Newton's Laws and Maxwell's equations with moving observers characterized by Galilean frames of reference - is inconsistent with experimental results, since it implies that we would be able to detect motion with respect to the ether, which several experiments failed to detect. What Lorentz did was construct a theory that maintained the Newtonian conception of space and time but was consistent with these experiments. He introduced a set of coordinate transformations, under which Maxwell's equations remained invariant. This is called his 'theorem of corresponding states':[12]

> If there is a solution of the source free Maxwell equations in which the real fields E and B are certain functions of $\mathbf{x_0}$ and t_0, the coordinates of S_0 and the real Newtonian time, then there is another solution of the source free Maxwell equations in which the fictitious fields E' and B' are those exact same functions of $\mathbf{x}'$ and t', the [auxiliary] coordinates [in] S and the local time in S. (Janssen [1995], 3.3.3)[13]

The frame S_0 is the rest frame of the ether, and the frame S is the Galilean frame with coordinates $\mathbf{x}$ and time t, distinct from the auxiliary coordinates $\mathbf{x}'$ and t', defined as:[14]

$$x' = \gamma(x_0 - vt_0) \qquad y' = y_0 \qquad z' = z_0 \qquad t' = \gamma[t_0 - (v/c^2)x_0]$$

The factor γ is defined as $\sqrt{1 - v^2/c^2}^{-1}$. What this theorem says is that Maxwell's equations hold in terms of the coordinates $\mathbf{x}'$ and t' when they hold in the ether rest frame. This is just a mathematical theorem. In order to have any empirical consequences, Lorentz had to add a physical assumption to the effect that the fields E' and B' are those fields that are actually produced in the Galilean frame S. This is obtained with what is called the 'generalised contraction hypothesis':

> If a material system, with a charge distribution that generates a particular electromagnetic field configuration in S_0, a frame at rest in the ether, is given the velocity v of a Galilean frame S in uniform motion through the ether, it will rearrange itself so as to produce the configuration of particles with a charge distribution that generates the electromagnetic field configuration in S that is the corresponding state of the original electromagnetic field configuration in S_0. (Janssen [1995], 3.3.3)

In other words, corresponding states - states related by the coordinate transformations given in the theorem of corresponding states - physically transform in to one another. It is important to emphasise that this is a physical transformation that happens in Galilean frames, since Lorentz held on to the Galilean transformations as being the true coordinate transformations between observers. The coordinates $\mathbf{x}'$ and t' are auxiliary coordinates that

[12] In this paper I am concerned with the final version of Lorentz's theory, in which the theorem of corresponding states holds exactly, since it is this theory that can be seen as empirically equivalent to Einstein's theory. However, Lorentz previously proved a first-order version of the theorem of corresponding states, one that was valid as long as he neglected quantities smaller than the ratio $\frac{v}{c}$. For details, see (Janssen [2017]).

[13] In the original quote, the coordinates $\mathbf{x}'$ are mistakenly attributed to the frame S, whereas they should be the fictitious coordinates, as is added here. This is made clear in (Janssen [2017]).

[14] Originally, Lorentz included a factor on the right hand side of these equations to stand for a possible transverse deformation, but this was set equal to 1 in 1904 for dynamical reasons. These coordinate transformations are what we recognise now as a Lorentz boost.

represent the coordinates in which Maxwell's equations hold, not the true spacetime coordinates $\mathbf{x}$ and t of the moving frame. This means that while S is the true frame of reference for moving observers, the electromagnetic field that is produced in S is the one that is related to S_0 by the coordinate transformations above (rather than the Galilean transformations taking one from S_0 to S). From these two posits, Lorentz was able to account for the negative result of any ether drift experiment.

It turns out that what is needed to ensure that the generalised contraction hypothesis holds is that all laws, including those governing matter, are Lorentz invariant. Of course, Lorentz needed to derive that this was true, or at least justify that it was the case in order to have a compelling theory.[15] The important point is that for Lorentz, this fact about the Lorentz invariance of all laws is a coincidence - it is because of the behaviour of matter when it moves through the ether that it holds. To ensure full empirical equivalence with Einstein's theory, we also need to assume that the rods and clocks of moving observers register the auxiliary coordinates and local time that is made reference to in the theorem of corresponding states. In other words, moving Galilean frames appear 'as if' they are Lorentzian frames, those that have the variables $\mathbf{x}'$ and t' as their true space-time coordinates.

In what follows, when I speak of Lorentz's theory I will take it to be that which is empirically equivalent to Einstein's. Lorentz himself did not endorse this theory until after 1905, when Einstein published his theory.[16]And as mentioned before, this way of looking at Lorentz's theory in terms of Newtonian spacetime is an anachronistic way of doing so. However, I think that it captures the essential features of Lorentz's theory, and it is in this form that is particularly philosophically interesting, since we can compare Lorentz's and Einstein's theory directly in terms of their spacetime structure.

4 The Ether State of Rest is Not Surplus

Before I argue that Lorentz's theory does not have surplus structure, it is instructive to first present a case where Norton's argument does apply neatly. I will use the case of Newtonian gravitation set in Galilean and Newtonian spacetime respectively. As in the last section, Galilean spacetime consists of $(M, \nabla, t_{ab}, h^{ab})$, while Newtonian spacetime consists of $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$, which adds to Galilean spacetime a privileged observer represented by a timelike vector with fixed 4-velocity ξ^a, which can be interpreted as what Newton thought of as 'absolute space', giving a state of absolute rest, and therefore a notion of absolute velocity.[17] From the way we have presented these spacetime structures, it is clear that Newtonian spacetime simply consists of Galilean spacetime + X, where X is some structure - namely, the structure corresponding to ξ^a. Therefore, if we regard ξ^a as surplus structure, then we can regard the two theories as structurally and thus, according to Norton, theoretically equivalent.

A more rigorous way of seeing that Newtonian spacetime has more structure than Galilean spacetime is by

[15] See (Janssen [1995], [2009]) for such dynamical arguments that Lorentz gave.

[16] Poincaré earlier suggested that observers will register the 'local time' of Lorentz's theory rather than the true Newtonian time, but Lorentz only accepted this later (Janssen [1995], 3.5.4).

[17] Although the ether state of rest in Lorentz's theory plays an analogous role to absolute space in Newtonian Gravitation, it should not be identified with it. This is because the ether need not be said to be 'absolutely' at rest, it merely needs to be *one* state of motion in which Maxwell's equations hold.

comparing the automorphism groups of both structures. In this, I follow Barrett ([2015]), who suggests the following criterion of structural comparison:

(SYM*) A mathematical object X has more structure than a mathematical object Y if Aut(X)$\subset$Aut(Y).[18]

Since Newtonian spacetime consists of Galilean spacetime + ξ^a, the automorphisms are more restrictive than those of Galilean spacetime. As mentioned in the previous section, the automorphisms of Newtonian spacetime correspond to spatial and time translations, rotations and reflections, as for Galilean spacetime, but not Galilean velocity boosts. So it is clear that Aut(*Newt*) $\subset$Aut(*Gal*) where *Newt* and *Gal* correspond to Newtonian and Galilean spacetime respectively. Newtonian spacetime therefore has more structure than Galilean spacetime by (SYM*). And this 'more' structure clearly corresponds to that structure ξ^a, picking out a privileged frame of reference. Therefore, if we regard ξ^a as surplus structure, we are in effect equivocating between models in Newtonian spacetime related by a Galilean velocity boost, and returning something which is structurally equivalent to Galilean spacetime. Consequently, Norton's argument goes through, and these two theories - Newtonian gravitation set in Newtonian and Galilean spacetime - can be regarded as equivalent.

Turning to the case of Lorentz's and Einstein's theory, Barrett ([2015]) shows that under his standard of comparison (SYM*), Minkowski spacetime has strictly less structure than Newtonian spacetime. That is, the automorphism group of Newtonian spacetime is a strict subset of the automorphism group of Minkowski spacetime.[19] We therefore seem to have a case where Norton's argument should apply, and Newtonian spacetime can be interpreted as having surplus structure resulting in the equivalence of Lorentz's and Einstein's theory. Norton himself introduces this example, suggesting that the surplus structure in Lorentz's theory is the ether state of rest. This is clear in the following quote:

> The mainstream of physics found Lorentz's view implausible and decided that his ether state of rest was physically superfluous structure, so that the two theories were really just variants of the same theory. (Norton [2008], p.36)[20]

However, by comparing the structure of both theories, it is clear that the ether state of rest is not the surplus structure that Norton is looking for. This is because the ether state of rest corresponds to that privileged reference frame ξ^a, which if removed gives back Galilean and not Minkowski spacetime, in which (as shown in Section 3) the laws of electromagnetism can not be formulated. In other words, simply removing the ether state of rest from Lorentz's theory neither results in a sensible theory, nor one that is structurally equivalent to Einstein's theory.

Another way of seeing this is in terms of the fact that Galilean boosts map timelike vectors to other timelike ones while preserving the spacelike vectors, whereas Lorentz boosts map both timelike and spacelike vectors

[18]The idea behind this criterion is that the automorphism group gives the symmetries of a mathematical object, and fewer symmetries indicates that there is more structure to be preserved, since it implies that there is some map which preserves all the structure of one object, but not of the other.

[19]This can be seen intuitively by the fact that one can write the Minkowski metric in terms of structure from Newtonian spacetime (Equation 13), and so transformations which preserve the structure of Newtonian spacetime will preserve the Minkowski metric, but the converse is not (necessarily) true - see (Barrett [2015]) for proof.

[20]To be clear, I am not objecting to the historical claim made here - that after the introduction of special relativity, the ether was regarded as superfluous. What I am objecting to is the claim implied by this quote that this example fits Norton's thesis about the equivalence of theories.

to different timelike and spacelike vectors. So Galilean boosts preserve the simultaneity structure of Newtonian spacetime, while Lorentz boosts do not. Consequently, while both Galilean and Minkowski spacetime are ways of removing the preferred state of rest from Newtonian spacetime and therefore the notion of absolute velocity, Galilean spacetime corresponds to doing just that, whereas Minkowski spacetime corresponds to something different - it removes the preferred state of rest, while also modifying the notion of simultaneity. This results in a completely different spacetime structure with a different interpretation.

So why might Norton have thought that the ether state of rest is surplus structure? One reason is that, like absolute space, it is in principle undetectable in Lorentz's theory. However, it is undetectable for a different reason than absolute space, and this difference corresponds to the reason why absolute space, and not the ether state of rest, can be regarded as surplus structure.

This can be seen by considering the symmetry argument that one can give for why we ought to regard absolute space as being surplus. Given a model of Newtonian Gravitation in which the laws hold, a transformation that adds a constant velocity to all particles gives another model in which the laws also hold and which is observationally equivalent to the original. But under such a velocity boost in Newtonian spacetime, the element which corresponds to absolute space - the distinguished vector ξ^a - is not invariant, and therefore solutions to Newton's laws related by a constant velocity boost correspond to distinct, but observationally indistinguishable, state of affairs. In particular, what is undetectable is precisely the state of rest represented by ξ^a. Therefore, the state of rest plays no role in the empirical consequences of the theory, and ought to be regarded as superfluous. And indeed, we can equivocate between models related by a Galilean velocity boost by removing the state of rest, which takes us to Galilean spacetime.

In Lorentz's theory, the situation is more subtle. In the rest frame of the ether, the laws governing matter are Newton's laws. Since Newton's laws are invariant under the Galilean transformations, this implies that inertial observers are related by Galilean boosts. But under a Galilean velocity boost, we do not get a solution to Maxwell's equations as a function of the Galilean coordinates $\mathbf{x}$ and t. So such a transformation does not map solutions to solutions, as it did in the previous example. But we do get an observationally equivalent state of affairs in the following sense - as a result of dynamical effects (the interaction of matter with the ether), Maxwell's equations (and all other laws) hold as a function of the auxiliary coordinates $\mathbf{x}'$ and t' which are the coordinates that moving observers measure, and so observers are unable to tell whether they are at rest or moving with respect to the ether.[21]

Therefore, although we can conclude that the ether is undetectable, this is not for the same reason as absolute space is in Newtonian gravitation, since Galilean velocity boosts in Lorentz's theory do not take solutions to solutions (as they do in Newtonian gravitation). Importantly, in Lorentz's theory, the ether is actually required in order to ensure that it is undetectable, since it is the interaction with the ether that provides the physical underpinning

[21] One might respond that if we consider a Lorentz velocity boost rather than a Galilean velocity boost, then we do get an analogous argument - such a boost maps solutions of Maxwell's equations to solutions and yet the ether state of rest differs. However, it is not clear that we can say that such models are observationally equivalent, since in Lorentz's theory such a transformation does not have physical meaning, other than when coupled to Galilean inertial observers.

for the generalized contraction hypothesis, and thus the claim that moving observers can never detect the presence of the ether. This makes clear that it is not undetectability on its own that makes something surplus. The ether is in principle undetectable in Lorentz's theory but it plays an ineliminable role in the theory, and therefore it cannot be thought of as surplus.

5 Does Lorentz's Theory Have Surplus Structure?

I have argued that Norton ([2008]) was wrong to think that one can simply treat the ether state of rest as surplus in Lorentz's theory, thereby demonstrating the equivalence of Lorentz's and Einstein's theory. This is both because removing it does not give Minkowski spacetime, and because it plays an essential role in the conceptual framework of the theory. However, this does not rule out that there are other ways that one can think of Lorentz's theory as having surplus structure. One might argue that if we think of Minkowski spacetime as a substructure of Newtonian spacetime, which we can do on the basis that Aut(*Newt*)⊂Aut(*Mink*), then we can think of Minkowski spacetime as resulting from equivocating between those models related by a Lorentz velocity boost. From this perspective, one can regard Minkowski spacetime as a result of simply removing some structure from Newtonian spacetime, rather than as a result of removing the ether state of rest and radically changing the other structure.

To make this precise, using the fact that one can define g^{ab} in terms of ξ^a and h^{ab} (Equation 13), one can regard g^{ab} as also being part of the structure of Newtonian spacetime. That is, one can think of Newtonian spacetime as consisting of $(M, \nabla, t_{ab}, h^{ab}, \xi^a)$, but also consisting of structure that can be defined from these, which includes g^{ab}. Then, one can remove all of this structure except g^{ab} to give back Minkowski spacetime, including the tensor fields t_{ab}, h^{ab} and ξ^a. This suggests that there is in fact a clear sense in which one can isolate structures in Lorentz's theory which can be removed to give back Einstein's theory, and it includes the ether state of rest. From this perspective, it seems that Lorentz's theory does have surplus structure and can be seen as equivalent to Einstein's theory.

The problem with this argument is that, as argued before, in the context of Lorentz's theory it is the structure of Galilean spacetime and the ether state of rest that plays an essential role in the explanations that the theory provides. Therefore, even though there is a sense in which one can move from Newtonian spacetime to Minkowski spacetime by removing some structure, one is only able to get from Lorentz's theory to Einstein's theory by changing the understanding of the notions of simultaneity, distance and duration, as well as the explanations of phenomena. In particular, since the field g^{ab} does not have physical meaning on it's own in Lorentz's theory, it must be understood as derivative of other structure in Newtonian spacetime - specifically ξ^a and h^{ab} - which do have a physical interpretation in this theory. Therefore, by removing this other structure, the interpretation of g^{ab} also has to change. Accordingly, Lorentz's and Einstein's theory cannot be regarded as equivalent by simply treating the extra structure in Newtonian spacetime as surplus, since this does not result in a sensible theory without changes in interpretation and explanation.

This is different to the case of Newtonian gravitation, where we do not need to reinterpret anything in the theory set in Newtonian spacetime when moving to Galilean spacetime. All the explanations of phenomena do not need the structure ξ^a, even in the context of the theory in Newtonian spacetime.[22] This is clear from the fact that Newton's second law in Newtonian spacetime can be expressed as:

$$F^a = mv^n \nabla_n v^a \tag{16}$$

We do not need a notion of spatial length for timelike vectors, which the structure ξ^a gives, to make sense of this equation, and therefore the structure ξ^a is not essential to understanding the laws. This is unlike Maxwell's equations in Lorentz's theory which require the structure ξ^a and h^{ab} (Equation 12), and so one needs a new understanding of Maxwell's equations when one loses reference to this structure.[23] In particular, since the frames of reference that correspond to moving observers changes when moving from Lorentz's to Einstein's theory, so too does the understanding of the relationship between Maxwell's equations and moving observers. In the Newtonian gravitation case, Newton's laws are understood in both theories as holding in all Galilean frames of reference, which are the reference frames of uniformly moving observers, and so no such new understanding is required.

This highlights the problem with simply looking at the formal structure of theories for the purpose of comparison - it doesn't take into account the role that the structure is playing in the theories at hand. In the case of Lorentz's theory, although there is a strict sense in which one can move from Newtonian spacetime to Minkowski spacetime by removing some structure, this does not mean that Lorentz's theory has surplus structure, since the structure that can be removed from Newtonian spacetime plays an essential role in the conceptual framework of Lorentz's theory, where moving observers are given by the Galilean transformations and the interaction with the ether provides the physical underpinning for the generalized contraction hypothesis.

One might respond that what is really at issue here is the meaning of surplus structure, and that in arguing that Lorentz's theory does not have surplus structure, I have implicitly refined the informal sense of surplus structure given in Section 2. In particular, I have taken the meaning to be that it can be removed to get an empirically equivalent theory, without changing the physical interpretation of the other parts of the theory and the explanations that the theory provides. That is, structure is surplus only if it plays no role in deriving and explaining the empirical predictions of that theory. However, the argument that Lorentz's theory *does* have surplus structure because one can remove some structure from Newtonian spacetime to get Minkowski spacetime relies on a weaker notion of surplus structure to mine. On this weaker notion, structure is surplus if it can be removed to get an empirically equivalent theory, even if that means changing the understanding of the theory. In other words, this notion takes some structure to be surplus if it is playing no role in the formalism of the theory, so that the laws can be written without reference to it.

[22]This doesn't mean that the structure ξ^a *cannot* be used in the explanations the theory provides. Indeed, many people have taken Newton himself to think that for there to be absolute acceleration, there must be absolute space giving a notion of absolute velocity, but we now recognise that this is not needed.

[23]Equation 12 is written explicitly in terms of ξ^a and h^{ab}, but even if it is written in terms of the field g^{ab} when this field is understood as also being part of Newtonian spacetime, the definition/understanding of this field requires the structure ξ^a and h^{ab} in Lorentz's theory.

It is not clear which notion Norton ([2008]) is assuming. He says that structure is surplus if it is 'unnecessary for the recovery of the observational consequences (of a theory)' (p.35). But this could either mean unnecessary in as much as the laws can be formulated in that theory without explicit reference to this structure, or it could mean unnecessary in as much as it plays no role in that theory in the derivation and physical understanding of the laws. The former corresponds to the weaker notion of surplus structure, whereas the latter corresponds to the notion I have taken in this paper. If Norton is assuming the weaker notion, then his argument would go through - Lorentz's and Einstein's theory can be regarded as equivalent since one can remove structure from Newtonian spacetime to get Minkowski spacetime. However, this implies that theories can be treated as equivalent even though the explanations they provide for empirical phenomena are radically different. In particular, it means that the theory 'Lorentz's theory with the structure ξ^a, h^{ab} etc. not taken seriously' can be thought of as a version of Lorentz's theory that is equivalent to Einstein's theory. But this theory doesn't make sense without changing the interpretation of the rest of the theory, and so under this weaker notion of surplus structure, Norton is committed to thinking that it still makes sense to label a theory 'Lorentz's theory' when the whole conceptual framework of the theory has changed.[24]

On the notion of surplus structure I have taken in this paper, no such problem arises - two theories are equivalent only if one can remove some structure from one to get the other without needing to change the explanations that the theory provides. I think that this captures better what it means for two theories to be equivalent since it takes into account the theory as a whole and not just the particular way of formulating it mathematically, and therefore that Norton's argument should be based on this notion.

There is also a stronger view of surplus structure that one could take, that some structure is surplus if it plays no role at all, either in determining and explaining the empirical consequences of the theory, or in what the theory says about what objects exist. On this stronger view, absolute space (given by the structure ξ^a) in the theory of Newtonian gravitation set in Newtonian spacetime would be regarded as essential, since it is taken seriously in the theory as telling one something about the world, namely that there is a preferred state of rest. Therefore, Norton's argument wouldn't go through even for this case. I have not taken this stronger view since it does not distinguish the case where some structure is only playing a role in the metaphysics of a theory, from the case where some structure is playing a role both in the metaphysics, and in the explanations of a theory. I have argued that this is the distinction between the case of Newtonian gravitation set in Newtonian and Galilean spacetime, and the case of Lorentz's and Einstein's theory, and by treating them analogously one loses sight of the interesting differences between these cases.

[24] On the other hand, the theory of Newtonian gravitation set in Newtonian spacetime with absolute rest not taken seriously arguably does make perfectly good sense without changing the understanding of the rest of the theory, since all the explanations within the conceptual framework of the theory can be done without absolute rest.

6 Consequences

6.1 Earman's principle

Earman ([1989], Chapter 3) argues that there is an important methodological principle that ought to determine what the structure of spacetime is:

(Earman's Principle) The symmetries of the dynamical laws should be the same as the symmetries of spacetime.

In order to understand this principle, we must define what it means to be a symmetry of the dynamical laws and a symmetry of spacetime. As before, symmetries of spacetime correspond to the automorphisms of the spacetime structure in question. The dynamical symmetries are not so easy to specify precisely - they correspond to a set of transformations between solutions of the laws of motion. However, it cannot be any such transformation, since this would imply that a map between arbitrary solutions counts as a symmetry.[25] It must be a transformation that preserves the physically significant structure. But a problem arises: how do we know what the physically significant structure is, other than through the spacetime symmetries? In the context of spacetime theories, some people have argued that there is a relatively natural way of picking out the dynamical symmetries - the idea is to distinguish between the fields that represent spacetime structure from the fields which represent the matter content of spacetime, and then define a dynamical symmetry in terms of diffeomorphisms which act only on the matter fields and which map solutions to solutions (see (Earman [1989]; Pooley [2013])). Whether this will work for all spacetime theories is not clear, but it suffices for the cases at hand.[26]

The argument Earman ([1989]) gives for all dynamical symmetries being spacetime symmetries is that a dynamical symmetry without a spacetime symmetry would indicate some surplus structure in the structure of spacetime. This is because there would be two models of the theory which correspond to distinct spacetime possibilities (they are non-isomorphic) which nonetheless are observationally equivalent. For example, the theory of Newtonian gravitation in Newtonian spacetime has Galilean velocity boosts as a dynamical symmetry but not as a spacetime symmetry. This indicates that Newtonian spacetime has surplus structure.

The argument for all spacetime symmetries being dynamical symmetries is that if the laws are generally covariant, then it follows from a transformation being a spacetime symmetry that it is also a dynamical symmetry. As we saw in Section 3, it is not possible to interpret the generally covariant version of Maxwell's equations in a spacetime structure - Galilean spacetime - which has a spacetime symmetry which is not a dynamical symmetry.

Applying this to the case of Lorentz's and Einstein's theory, we find the following. In Einstein's theory, there is a perfect match between spacetime and dynamical symmetries - both are the Lorentz symmetry transformations.

In Lorentz's theory, the spacetime symmetries are the Newtonian ones while the dynamical symmetries are the Lorentz transformations, since the laws in Lorentz's theory are invariant under these transformations. Hence all

[25] See (Belot [2013]) for a more detailed argument against such a notion of symmetry.

[26] There have also been attempts to generalise this argument to cover all kinds of theories, for example by Dasgupta ([2015]) and North ([2009]). However, it becomes even more difficult to specify the dynamical symmetries in the general case.

spacetime symmetries are dynamical symmetries, but the group of dynamical symmetries is larger than the group of spacetime symmetries. If we follow Earman, this would imply that Lorentz's theory has surplus structure. The problem with Earman's argument in this case is that it doesn't take in to account how the dynamical symmetries are interpreted in Lorentz's theory. This is similar to the reasoning given previously for why the ether is undetectable but not surplus in Lorentz's theory: Lorentz took moving bodies to be characterised by the Galilean transformations, since Newton's Laws, which were held to govern matter, are Galilean invariant. But since Maxwell's equations do not hold in these frames, a dynamical explanation was needed to explain their observations. The interaction between moving bodies and the ether was meant to provide this explanation. The resulting laws that did hold in these frames of reference were forced (in accordance with negative results from ether drift experiments) to be the Lorentz invariant ones.

So while it is true that the laws are invariant under the Lorentz transformations, this is a result of dynamical effects in Galilean frames of reference, and not because Lorentzian frames correspond to the frames of reference for moving observers. Consequently, Lorentz's theory does fail the test of symmetry matching in the sense that the laws are invariant under a wider range of transformations than the spacetime structure. However, this is not due to Lorentz's theory having surplus structure, it is due to the peculiarity of Lorentz's theory that all laws transform in the same way in Galilean frames of reference. In other words, the Lorentz transformations only have meaning in Lorentz's theory when combined with Galilean frames of reference and the dynamical effects of the ether, and therefore one needs to maintain the structure of Newtonian spacetime in order to make sense of this dynamical symmetry in the context of Lorentz's theory.

The only sense of surplus structure that Earman's argument can be working with in this example is the weaker sense that was discussed above - that structure is surplus if it plays no role in the formalism, so that the laws can be written without it. In other words, what motivates Earman's principle in this case is just that the formulation of the laws does not require all of the structure of Newtonian spacetime (it only requires the Minkowski subset), even though the explanations in Lorentz's theory do require them.

This example shows how Earman's principle, by making reference only to the formalism and not to the interpretation of theories, misses subtleties in comparing theories. While Lorentz's theory does fail the test of symmetry matching, this has to do with the whole conceptual framework on which the theory rests, and therefore one cannot say that the problem with Lorentz's theory is simply that the spacetime and dynamical symmetries don't match, and that this means that the theory has excess structure which can be excised to give a theory working in a simpler spacetime structure. In the framework of Lorentz's theory, this 'excess' structure is not superfluous.

6.2 What makes Einstein's theory superior to Lorentz's theory?

If we want to maintain that Einstein's theory is a distinct, superior theory to Lorentz's, then we have to provide reasons for why Einstein's theory is to be preferred epistemically. The arguments given in this paper imply that it cannot be because Einstein's theory doesn't have surplus structure, whereas Lorentz's theory does.

One reason is that Einstein's theory has *less* structure, since Minkowski spacetime has strictly less structure than Newtonian spacetime. Therefore, by Occam's razor, we ought to prefer Einstein's theory, since the same laws can be written in a weaker spacetime structure. However, there are reasons to be unsatisfied with this argument. First, it doesn't explain historically why Einstein's theory was preferred - the comparison of spacetime structure has only been made relatively recently. But even without considering the historical reason, simplicity of spacetime structure is not all that we might care about in a theory - there are other theoretical virtues such as unification and explanation. It is also only one source of simplicity in a theory.

It might be that the simpler structure of Minkowski spacetime and the matching of symmetries in Einstein's theory means that other theoretical virtues are met. Michel Janssen argues that one virtue of Einstein's theory over Lorentz's is explanatory virtue - two observationally equivalent states of affairs which receive very different explanations in Lorentz's theory receive a unifying explanation in Einstein's theory.[27] For example, consider the case of a moving magnet inducing a current in a stationary wire (with respect to the ether). This receives a different explanation in Lorentz's theory from the case of the same wire moving through the magnetic field of the magnet when stationary, even though the current produced is the same. This is because in one case the magnet is stationary with respect to the ether whereas in the other the wire is, and so they correspond to distinct situations. On the other hand, in Einstein's theory, these two cases are understood as the same situation looked at from different perspectives, and so they do not have distinct explanations.[28] Additionally, in Lorentz's theory the Lorentz invariance of all dynamical laws is in need of explanation on a case by case basis - it is just a coincidence that all matter behaves in the same way when moving through the ether. On the other hand, in Einstein's theory the Lorentz invariance of the laws is explained by the Minkowski metric - Lorentz invariance is a symmetry of the spacetime, and so the fact that the laws are the same in Lorentzian frames of reference can just be thought of as reflecting 'default spatio-temporal behaviour' (Janssen [2009]). This argument suggests that the matching of symmetries in Einstein's theory means that the dynamics can now be explained by the spacetime structure, which it couldn't be in Lorentz's theory.

The problem with this argument is that it takes the arrow of explanation to be from spacetime structure to dynamical laws. Some people have argued[29] that this gets things the wrong way round - dynamics should explain spacetime structure. On this view, it is not the case that Einstein's theory is superior because it allows the dynamics to be explained by the spacetime structure. However, this view also takes Einstein's theory to have higher explanatory power, since in Lorentz's theory the mismatch between dynamical and spacetime symmetries means that the spacetime structure cannot be explained by the dynamics.[30] Therefore, we can maintain the explanatory virtue of Einstein's theory over Lorentz's theory while remaining agnostic as to the arrow of explanation between dynamics and spacetime structure.

[27] See (Janssen [2002a], [2002b], [2009]). The connection to Earman's principle is made clear in (Balashov and Janssen [2003], pp. 341-2).

[28] This example is used in (Einstein [1905]).

[29] See, for example, (Brown [2005]; Brown and Pooley [2006]).

[30] In fact, on this view a theory such a Lorentz's is not even coherent, since it maintains that the dynamical symmetries underwrite the spacetime symmetries and so there cannot be a mismatch between them. I think that this goes too far and that Lorentz's theory should be treated as a plausible theory, but I will not discuss this further here.

There is also an epistemological virtue to Einstein's theory - in Lorentz's theory, we are unable in principle to observe the ether state of rest, whereas in Einstein's theory, we have no analogous undetectable structure. This is not sufficient on it's own since, as highlighted before, the fact that the ether is undetectable does not mean that isn't essential to the theory. However, both the epistemological and explanatory benefits of Einstein's theory give reason to think that Einstein's theory is not only theoretically simpler because it has less structure, but also *conceptually* simpler.

I do not want to claim that these are the ultimate reasons why special relativity was or should be preferred over Lorentz's theory, but I give them to demonstrate that despite Lorentz's theory not having surplus structure (given the conceptual framework of the theory), there are still pragmatic benefits to special relativity that aren't seen purely in formal analysis.[31] This is contrary to what Acuna ([2014]) argues - that these pragmatic virtues are tenuous, and that Earman's principle can be used to argue for the superiority of Einstein's theory on purely physical grounds. However, he takes the violation of Earman's principle to reflect that Lorentz's theory has surplus structure. For example, he says:

> The violation of the symmetry principles in Lorentz's theory results in that the ether - in connection with the privileged ether rest frame and the Newtonian structure of space-time - becomes suspicious of representing nothing physical. The fact that special relativity does not postulate entities or structures that are dubious in this sense makes it a physically and ontologically simpler theory than Lorentz's. That is, the comparative simplicity of Einstein's theory is not a merely pragmatic virtue, but it reflects more solid ontological foundations. (Acuna [2014], p.294)

I have argued that if taken to mean that the ether is superfluous to Lorentz's theory, this view is mistaken, and therefore we need to incorporate pragmatic reasons which look not only to the formal structure of theories, but also how they are interpreted, in order to say why Einstein's theory is preferred. In particular, while it may be that theories in which spacetime and dynamical symmetries match are superior to those in which they don't, this is not because it indicates that there is no surplus structure - instead, having a weaker spacetime structure (which still supports the laws) must reflect not only that the laws can be formulated without the extra structure, but that there are pragmatic benefits to the theory, such as explanatory or epistemological advantages.

7 Conclusion

In this paper, I have given an example of two empirically equivalent theories - Lorentz's ether theory and Einstein's theory of special relativity - where the former has strictly more structure than the latter, but where one cannot regard this theory as having surplus structure and therefore they cannot be treated as equivalent theories.

[31] Other historical reasons one might give for why special relativity was accepted include that Lorentz's theory was meant to be part of a theory of everything whereas Einstein's theory didn't commit itself to a specific theory of everything - rather, it took Lorentz invariance to be a constraint on such a theory (see (Janssen [2009])). This meant that the introduction of quantum theory did not hinder the progress of Einstein's theory, while making the compatibility with Lorentz's theory more problematic. The four-dimensional approach that Minkowski introduced also helped to simplify special relativity significantly, whereas there was no such simplification of Lorentz's theory until much later.

The reason for this is that although the spacetime structure of Einstein's theory is the result of removing some structure from the spacetime structure of Lorentz's theory, these parts of Lorentz's theory play an essential role in the conceptual framework of the theory, and thus removing them does not result in a sensible theory without changes in interpretation and explanation. Therefore, contra Norton, this is not an example of one theory formulated in two different ways, where one formulation simply has surplus structure that the other doesn't have.

This highlighted the following. First, one cannot infer from something being unobservable, or absent in an empirically equivalent theory, to it being surplus - one has to recognise the role that object/structure plays in the theory of which it is part. Second, one cannot simply look at the formal structure of theories to tell us what is problematic or which theory is to be preferred - one has to consider how the theories at hand can be interpreted and the conceptual framework in which they are embedded. In the case of Lorentz's and Einstein's theory, ignoring these points can lead to mistakenly thinking that the source of problems in Lorentz's theory is the presence of surplus structure.

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