Machian Comparativism about Mass

Niels C.M. Martens*

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Abstract

Absolutism about mass within Newtonian Gravity claims that mass ratios obtain in virtue of absolute masses. Comparativism denies this. Defenders of comparativism promise to recover all the empirical and theoretical virtues of absolutism, but at a lower 'metaphysical cost'. This paper develops a Machian form of comparativism about mass in Newtonian Gravity, obtained by replacing Newton's constant in the law of Universal Gravitation by another constant divided by the sum over all masses. Although this form of comparativism is indeed empirically equivalent to the absolutist version of Newtonian Gravity—thereby meeting the challenge posed by the comparativist's bucket argument—it is argued that the explanatory power and metaphysical parsimony of comparativism (and especially its Machian form) are highly questionable.

*RWTH Aachen University, Institute for Theoretical Particle Physics and Cosmology, Sommerfeldstrasse 16, 52074 Aachen, Germany, martens@physik.rwth-aachen.de, +49 241 80 27054, Orcid:0000-0002-2839-1387

1 Absolutism vs. Comparativism

Dimensionful physical determinables appear to have two kinds of determinates (Dasgupta [2013]). Take mass[1], which is the central determinable in this paper: objects with mass appear to have a determinate monadic[2] property, let us call this their absolute mass, but they also stand in determinate mass relationships with one another. (Here we focus on mass relations that are scale-independent—see (Baker [2013b]) for a justification—which under suitable constraints—detailed below—can be interpreted as mass ratios.) How do these two determinates relate, metaphysically speaking? *Absolutism about mass* claims that the determinate mass relations obtain in virtue of the determinate absolute masses (Armstrong [1978, 1988]; Mundy [1987]; Lewis [1986]). *Comparativism about mass* denies this (Russell [1903]; Ellis [1966]; Field [1980, 1985]; Bigelow *et al.* [1988, 1990]; Arntzenius [2012]; Dasgupta [2013]; Eddon [2013]; Baker [2013b, 2013a]). Comparativism is often held in conjunction with views espousing the redundancy of absolute masses: they are merely conventional or even meaningless altogether. Although such claims may certainly seem in the spirit of comparativism, it is important to note that these claims are distinct and go beyond the core of comparativisim, as its definition is strictly speaking silent on the status of absolute masses. For instance, *regularity comparativism* is a form of comparativism that is nevertheless *realist* about absolute masses, i.e. it admits the empirical meaningfulness of objective, non-conventional absolute masses (Martens [2017a, 2019]), recovering them from a purely comparativist Humean mosaic (i.e. a spatiotemporal Humean mosaic including fundamental mass relations). I have argued against regularity comparativism elsewhere [2017b], as well as against the dominant comparativist proposal in the literature [2019], i.e., Dasgupta's [2013] comparativism. Hence, in this paper I develop a more viable form of comparativism about mass, within the context of Newtonian Gravity (NG): *Machian comparativism.* Besides improving upon Dasgupta's proposal, it does save the spirit of comparativism (*contra* regularity comparativism), that is *anti-realism about absolute masses*: absolute masses are not empirically meaningful. In other words, an *Active Leibniz Mass Scaling* (Martens [2019])—a uniform scalar multiplication of each of the absolute mass magnitudes of all the particles, *ceteris paribus* (i.e. whilst keeping everything else unchanged)—does not generate an empirically distinct evolution of the system, and thus constitutes nothing more than an identity operation as far as comparativism is concerned. The paper is brought to a close by detailing the extent to which Machian comparativism is still unsatisfactory, despite improving upon the other available comparativist proposals.

In the next section I will define absolute masses and mass relations more precisely. §3

[1]Given the distinct conceptual roles of inertial and gravitational mass it would be interesting to consider whether one could be an absolutist about one and a comparativist about the other (as defined below). Here we follow the literature by not distinguishing between inertial and gravitational mass.

[2]Dasgupta [2013] defines absolute masses as intrinsic properties. However, regularity comparativism (Martens [2017b]) grounds absolute masses in a complete four-dimensional mosaic of spatiotemporal relations and fundamental mass ratios, making them as extrinsic as they could possibly be, whilst leaving their monadicity untouched. The Higgs mechanism for generating inertial masses in the Standard Model of Particle Physics seems to similarly generate monadic masses that are nevertheless extrinsic, as the mass emerges from an interaction with, or immersion in the Higgs field (Bauer [2011]). I thus side with Roberts [unpublished] in taking the crucial distinction between absolute masses and mass relations to be that the former are monadic properties, and the latter dyadic properties (i.e. binary relations). In the context of absolutism and comparativism about other determinables, say distance, we may have to adjust this: absolutism about distance will concern binary relations, and comparativism about distance will concern 4-place relations.

introduces the main argument for comparativism: comparativism promises to recover all the empirical and theoretical virtues of absolutism, but at a lower 'metaphysical cost'. The subsequent three sections focus on empirical virtues: I cash out the requirement of empirical equivalence with absolutism as comparativism's ability to generate the correct set of empirically possible worlds (§4), and show that a Machian form of comparativism does manage to satisfy this requirement (§6), despite the comparativist's bucket argument to the contrary (§5). However, things look much less peachy when I turn to the explanatory virtue and supposed metaphysical parsimony of (Machian) comparativism in §7.

2 Absolute Masses & Mass Ratios

Let us be a bit more precises about the notion of absolute masses.[3,4] The determinate physical magnitudes falling under one physical determinable comprise a set of monadic properties with cardinality $2^{\aleph_0}$. On the elements of this set we place two structures: 1) a total order, and 2) an associative concatenation structure which is to be interpreted as addition—it fixes where the sum of several magnitudes 'fits into the total order'.[5] In the case of mass, the first structure is associated (operationally speaking) with comparing the masses of two objects by putting one massive object on each scale of a balance. Adding the second structure corresponds to adding several massive objects on each scale of the balance. In the case of electric charge these structures generate a totally ordered group, since electric charge can be neutral or negative, thereby providing an identity element and an inverse for every element in the set. In the case of mass this generates a totally ordered semi-group. It does not form a totally ordered group since there is no inverse (no negative masses) and no identity element (no zero mass, at least not in NG).

As it stands, these structures render all absolute masses qualitatively identical. Then, in order for the laws to know in each possible world which determinate magnitude they are 'latching onto', the determinate magnitudes need to be endowed with non-qualitative, transworld identities (i.e. these identities are required for the forces to be well-defined, in the sense of uniquely matching up instances of initial conditions, including masses, with, say, accelerations; cf. fn.23). (Primitive) non-qualitative, transworld identity of properties is usually referred to as quidditism.[6]

Perhaps surprisingly, the mass ratios of the comparativist have a similar structure. They also consist in a set of determinate properties, again with cardinality $2^{\aleph_0}$. These

[3]Here I draw heavily upon earlier work [2019].

[4]It is crucial to not confuse absolute masses (nor mass relations) as defined in this paper with the numerical quantities (times a conventional unit) that may be used to represent them (Martens [2017a], §1.1; [2019]; Field [1980]; Sider [unpublished]).

[5]The constraints on mass relations required to prove the representation theorem mentioned below are exactly the constraints that ensure that the mass relations can be represented by monadic properties with these two structures.

[6]It is important to note, though, that while quidditism often (implicitly) refers to *determinable* properties, such as 'being massive', we are here concerned with transworld identification of *determinate* properties, namely absolute masses (eg. 'having *that* mass') and mass relationships (see below). (Arguably, determinables obtain in virtue of determinates, and hence determinate quidditism would seem to imply determinable quidditism, but what is primarily of importance in this paper is still just the quiddities of determinates.)

properties are however binary relations, rather than monadic properties. These deteminate properties are again 1) totally ordered, and 2) obey an associative concatenation structure, but this concatenation structure is to be interpreted as multiplication. It includes the relation '... is as massive as ...'—or in quantitative terms '... stands in a 1:1 mass ratio to ...'—which forms the identity element, and also includes an inverse for every mass ratio. Hence, the mass ratios form a totally ordered group. As with absolute masses, the comparativist mass ratios need to be endowed with non-qualitative, transworld identities, in order for the laws to know in each possible world which determinate mass relation they are 'latching onto'.[7]

It is easy to see that the mass relations of a system of objects are fully and uniquely determined by the absolute masses of those objects. The mass relations between each ordered pair of objects are uniqely represented by the ratio of quantities chosen to represent the absolute masses of the relata, which is always well-defined under the assumption of finite non-zero masses, and independent of the choice of units (i.e. quantities chosen to represent the absolute masses). Thus, in a world with 3 particles, with masses of 4 kg, 2 kg and 1 kg, the mass relations between object 1 and 2 and between 2 and 3 are in both cases 'twice as massive as', and the mass relation between object 3 and 1 is 'one fourth times as massive as'.

Does every complete set of mass relations between n massive objects also fully and uniquely determine the absolute masses of all n objects? No, as the following counterexample shows. Consider again a universe consisting of 3 particles. The mass relation between object 1 and 2 is (tentatively) 'twice as massive as', the relation between object 2 and 3 is (tentatively) 'twice as massive as', and the relation between 1 and 3 is (tentatively) 'ten times as massive as'. It is obvious that there is no distribution of absolute masses that satisfies this structure of mass relations—hence we were not even justified in interpreting them as mass ratios in the first place.

If a set of mass relations is sufficiently constrained—that is if every closed loop of mass relations combines (i.e. intuitively understood as 'multiplies') to the identity, that is to the relation 'is as massive as (itself)'—a *representation theorem* can be proven (Baker [2013b]). This means that every structure of relations so constrained can be instantiated by a distribution of absolute masses. Only then can they properly be understood as mass ratios! It is important to note that a distribution of intrinsic masses corresponding to a structure of mass relations is not unique: multiplying all absolute masses of a solution by a scalar provides a new solution. The representation theorem provides a homomorphism—not an isomorphism—from the distribution of absolute masses to the structure of mass relations.

Mathematically speaking such a constraint seems inconspicuous. Metaphysically speaking we may and should ask the comparativist what it is that ensures that this contraint applies, and why. Why would a comparativist world be so conspiratorial as to consist of a complicated structure that allows exactly for an instantiation by absolute masses? Piggy-backing on the absolutist framework, in which case the constraint is a mathematical necessity, is not allowed. We will return to this mystery in §7.2, and assume for now that the constraint is satisfied.

[7] The easiest way to see this is by realising that there is no qualitative distinction between a first set of binary relations that obey these structures, and a second set obtained from the first by squaring each relation—call this a Leibniz Mass Ratio Squaring. (I would like to thank Zee Perry for pointing this out to me.)

3 The Argument for Comparativism

The lure of comparativism is the promise that it can recover all the virtues of absolutism, both empirical and theoretical, but at a lower metaphysical cost (Dasgupta [2013]). The argument is analogous to the Occamist arguments that are usually invoked against absolute velocities in NG. Most of the discussions so far have focused on the issue of comparativist versions of NG being empirically adequate or not, or at least as (approximately) empirically adequate as absolutist NG.[8] It is important not to forget all other things that 'need to be equal', before the lower metaphysical cost favours comparativism. As far as empirical virtues are concerned, we might want comparativist NG to have the same modal consequences, that is allow and disallow the same set of physically possible worlds (up to empirical distinctness; explained below), as absolutist NG. This would be a sufficient (albeit not necessary (Martens [2017a], §4.4.2)) condition for comparativism being as empirically adequate as absolutist NG. On the theoretical side, we want to make sure that turning to a comparative metaphysics does not constitute a loss of, say, explanatory power.[9]

[8]Strictly speaking, (absolutist) NG is empirically inadequate and thus a false description of the actual world—as is the case for all of our current theories. On what Williams calls the standard account of theory interpretation—to give an interpretation of a physical theory is to answer the question "if this theory provided a true description of the [actual] world in all respects, what would the world be like?" ([unpublished], p.2)—the discussion in this paper would have no ramifications for the metaphysics of our actual world. In fact, we should not be drawing any metaphysical inferences from physics before we have our final theory. Instead, I intend the current debate and the results of this paper to be understood within William's alternative to the standard framework of theory interpretation, according to which interpreting a physical theory is to answer the question "given that this theory provides an approximately true description of our world, what is our world approximately like?" ([unpublished], p.2). The merely approximate truth of the theory of NG is cashed out by focusing only on a proper subset of the empirical data that needs to be accounted for, ignoring data at small length scales (quantum effects), at speeds comparable to the speed of light (relativistic effects), at small accelerations (MONDian effects), and in strong gravitational fields (generally relativistic effects). A more detailed discussion of how to understand the current debate in light of Newtonian Gravity not being the final theory is provided elsewhere (Martens [2017a], p.3-5; [2019]).

[9]Although Roberts [unpublished] ends up arguing that absolute masses have no explanatory power, he does agree that if they did have such power this would provide justification for absolutism even if comparativism is empirically adequate. In other words, he agrees that the 'all other things being equal' clause in the Occamist norm should at least quantify over explanatory power.

$\mathbf{P_{mod}}$	**Modal Adequacy/ Empirical equivalency:** Comparativism about mass in NG has the same modal consequences as absolutism—up to empirical distinctness (explained below). (It follows that comparativism is as **empirically adequate** as absolutism.)
$\mathbf{P_{exp}}$	**Explanatory Adequacy:** Comparativism is at least as explanatorily adequate as absolutism.
$\mathbf{P_{occ\text{-}par}}$	**Occamist norm:** All other things being equal (i.e. $P_{mod} \wedge P_{exp}$)[10], we should favour theories[11] that are metaphysically more parsimonious.
$\mathbf{P_{par}}$	Comparativism is metaphysically more parsimonious than absolutism.
C	Comparativism about mass in NG should be favoured over absolutism.

This argument is valid. Moreover, I will argue in §6.3 that Machian comparativism is empirically equivalent to absolutist NG (as opposed to Dasgupta's comparativism (Martens [2017a])). However, in §7 I will raise some issues that make the explanatory adequacy and metaphysical parsimony of any form of comparativism highly questionable, even slightly more so for the Machian version.

4 Empirical Equivalency via Possibility Checking

On one popular[12] approach to cashing out empirical adequacy—or, more broadly, empirical equivalence—one checks whether the theory generates the correct set of empirically possible worlds (Martens [2017a, 2019]). To every metaphysically possible world corresponding to a structure of mass relations, one would expect[13] there to correspond (via active Leibniz scalings) uncountably many possible 'absolutist worlds' with different absolute masses that are all nevertheless compatible with the mass structure in the corresponding 'comparativist world'. Comparativism should provide at least one metaphysically distinct (and dynamically allowed) possible world for each empirically distinct possible world allowed by absolutism. If the metaphysically distinct worlds that comparativism acknowledges fail to differentiate between those distinct empirical possibilities, then comparativism is wrong. If, on the other hand, the set of all the metaphysically distinct possible worlds acknowledged and dynamically allowed by comparativism contains all the empirically distinct possible worlds (that are dynamically allowed by absolutism), then we may opt for comparativism over absolutism based on an Occamist norm. More specifically, some form of the Leibnizian Principle of the Identity of Indiscernibles, which equates metaphysical parsimony with minimalisation of the number of metaphysically distinct possible worlds per associated empirically distinct possible world (see §7.4, as

[10]If one considers other extra-empirical criteria besides explanatory adequacy to be relevant, these would need to be included to make this a valid argument. In the discussions in this paper only explanatory power and metaphysical parsimony will play a role.

[11]For the sake of readability I will refer to absolutist and comparativist NG as two distinct theories. Perhaps it would be more appropriate to refer to them as different interpretations, or formulations or (metaphysical) characterisations of the same theory, but nothing in this paper depends on this choice of terminology.

[12]Cf. the analogous substantivalism–relationalism debate (Belot [1999, 2000]; Belot and Earman [2001]).

[13]Except, in a sense, in the case of regularity comparativism (Martens [2017b]).

well as the rest of §7 for other relevant notions of metaphysical parsimony).

Now in slightly more detail. The set of physically possible worlds according to absolutist Newtonian Gravity are obtained by solving all the possible initial value problems. Assuming the standard ideology[14] of distances, absolute velocities and masses, this would be to consider each possible combination of determinate magnitudes of these variables and parameters and evolve them using the absolutist laws. As absolutist NG is deterministic, each such initial value problem has a unique answer (modulo some well-known exotic counter-examples (Earman [1986]; Norton [2003])). If the comparativist laws are equally deterministic, the same protocol can be applied to comparativist intitial conditions and laws to obtain the corresponding set of physically possible worlds. Some of the absolutist models, despite representing metaphysically distinct possible worlds, might represent empirically equivalent possible worlds. For comparativism to be empirically equivalent to absolutism we would obviously not require it to generate a metaphysically equivalent counterpart—apart form the obvious lack of (relatively) fundamental absolute masses—for each of the absolutist models. Modal adequacy up to empirical distinctness suffices:

Empirical Equivalence of Theories: Theory T_2 (comparativism) is empirically equivalent to theory T_1 (absolutism) if and only if the laws of nature of theory T_2 pick out a set of dynamically possible worlds S_2 such that, 1) for *each* of the classes of empirically equivalent worlds that partition the set of dynamically possible worlds (S_1) picked out by theory T_1, S_2 includes a world empirically equivalent to the worlds in that class (i.e. *completeness*), and 2) all worlds that are empirically distinct from each of the absolutist solutions in S_1 are excluded from S_2 (i.e. *soundness*).

The required notion of empirical equivalence (and its counterpart, empirical distinctness) of *worlds* relevant to the current context will become clear in the next section.

Alternatives in the literature to the above 'possibility checking' approach to cashing out empirical equivalency—and by extension empirical adequacy[15]—focus either on mass scaling *symmetries* or absolute masses being *undetectable* (Martens [2019]). A comparativist using the symmetry approach will claim that Leibniz Scaling is a symmetry of Newtonian Gravity, and that models related by that transformation (therefore) represent the same possible world.[16] The Occamist norm is then explicated as a direct symmetry-to-(un)reality inference, for instance via Saunders' Invariance Principle: "only [magnitudes] invariant under exact symmetries are real" (Saunders [2007], p.1-2). The notion of symmetries of physical theories and a justification of their associated symmetry-to-(un)reality inference are notorious topics in the recent philosophy of physics literature, see for instance (Dasgupta [2016], and references therein), as well as (Møller-Nielsen [2017]; Read and Møller-Nielsen [2018]; Martens and Read [2018]; Martens [2018b]). Most importantly, Møller-Nielsen and Read [2018] argue that (epistemic) symmetries do not in fact allow us to *discover* that symmetry-related models are empirically equivalent; we need to know that from the start. The notion of symmetries will not provide a shortcut when checking whether a comparativist theory satisfies the above definition of empirical equivalency.

On the undetectability approach, empirical equivalence of comparativism to absolutism is equated to absolute masses being undetectable. The Occamist norm should then be interpreted as a razor against undetectable notions. I have elsewhere [2019] argued against

[14]Here I am using 'ideology' in the Quinean [1951] sense. Roughly, 'ontology' refers to the (primitive) objects, and 'ideology' to their (primitive) properties.

[15]Keeping in mind fn.8.

[16]But see fn.17.

this approach, to the extent that it differs from the possibility checking approach, for two reasons. Firstly, as mentioned, comparativism is compatible with absolute masses being detectable, as evidenced by regularity comparativism.[17] Secondly, since absolute masses are qualitatively identical (§2), we could never detect *which* specific absolute mass an object has—in the same non-modal sense that we can detect how many corners a tile has. We could only detect *that* a Leibniz Mass Scaling leads to empirically distinct possible world. This modal notion of detectability then boils down to nothing more than the possibility checking approach above. Thus, to the extent that the undetectability approach is distinct from the possibility checking approach—i.e., on a narrow, non-modal conception of detectability—it is problematic. To the extent that it is (not merely equivalent but literally) the same as the possibility checking approach—i.e., on the broader, modal conception of detectability—I suggest we refer to it, non-ambiguously, as the possibility checking approach.

5 A Challenge for Comparativism: the Comparativist's Bucket

The following argument, the *comparativist's bucket*[18], a simple instance of an inter-world Leibniz Mass Scaling derived from Baker [2013b], serves to challenge the empirical equivalence and by extension the empirical adequacy of comparativism (Martens [2019]). Consider a simple world, governed by the laws of Newtonian Gravity, with two equally massive particles a distance r apart, with a relative positive initial velocity v (such that there is zero angular momentum). How will this world evolve?

Whereas this description corresponds to a unique choice of initial variables and parameters for the comparativist, the absolutist will demand that more information is needed: this description is compatible with continuum infinitely many absolute masses (i.e. absolutist worlds differing by a Leibniz Mass Scaling). And, (s)he claims, this choice is important. For some choices of absolute masses the particles will escape each other and for other choices they will collide (or, more correctly, coincide)[19,20]—this topological criterion of particle coincidence[21] (i.e. trajectories intersecting or not) clearly being a sufficient condition[22] for empirical distinctness of worlds that both the absolutist and comparativist

[17]The symmetry approach faces a similar worry, as Leibniz Mass Scalings are, in some (complicated) sense, not symmetries of regularity comparativism.

[18]Newton's bucket similarly shows that the relationalist initial conditions, in particular relative accelerations, are insufficient for an empirically deterministic evolution of the system: absolute accelerations are empirically meaningful. An even closer analogy, using particles rather than a bucket, would be Skow's [2007] two-particle scenarios (discussed by Pooley [unpublished]) or Barbour's ([2000], Fig.13) three-particle scenarios. As all these scenarios are used to make the same point, and Newton's bucket is the most well-known of the three, I have chosen to adopt the name 'comparativist's bucket'.

[19]Any escape solution could always be turned into a collision solution by evolving the 'initial conditions' in the opposite temporal direction. In this paper we will only consider worlds from the initial time 'onwards', not from infinitely far into the 'past'.

[20] When two particles collide, this results in a singularity and the theory technically breaks down. At that point we would have to resort to a different branch of mechanics, which McKinsey, Sugar and Suppes [1953] refer to as 'impact mechanics'.

[21]See also Schlick's 'method of coincidences' ([1979/1922, 1974/1925, 1949]; Stevens [2014]).

[22]What would be a necessary condition for empirical distinctness of worlds—or a sufficient condition for empirical equivalence of worlds—as referred to in the definition of empirical equivalence of theories in §4? It may seem that all that is observable is shapes—i.e., distance ratios and angles (or in our one-dimensional example just distance ratios)—changing over time. Maybe then, worlds are empirically equivalent if and only if they agree on these observables. This is what Baker [2013b] seems to have in mind; Barbour and Bertotti's [1982, 2000] shape dynamics comes to mind as well. However, strictly

can agree on—depending on whether the following inequality is satisfied:[23]

$$v > v_e = \sqrt{\frac{2Gm}{r}}. \quad (1)$$

It is clear from this inequality that the evolution depends, deterministically, on the initial absolute masses of the particles, over and above their mass ratios. Once the initial masses are fixed, the corresponding absolutist initial value problem has a unique solution.

How does this pan out in the comparativist framework? This depends on the specific version of comparativism, but seems to be problematic either way: the comparativist laws will either fail to evolve this single set of comparativist initial determinates into a unique solution but allow instead for both escape and collision—that is indeterminism[24] (cf. Dasgupta's [2013] comparativism)—or they will provide a unique solution to the initial value problem, say collision, and thereby fail to generate the complete set of solutions. In other words, uniform variation of the allegedly redundant absolute masses—that is, they are not supposed to be an independent variable over and beyond the mass ratios and other non-mass variables—is, *ceteris paribus*, not supposed to lead to any empirical difference, but in this case it results in a dramatic empirical difference: particles escape each other instead of colliding. From the comparativist point of view, uniformly varying the redundant initial masses is supposed to amount to nothing more than the identity operation, but instead the evolution changes in an observable manner. Absolute masses are empirically meaningful; they are real! (In terms of the symmetry approach: a Leib-

speaking there are no rigid bodies in NG (let alone human observers) to use as rods to measure those distance ratios. Even if there were, their epistemic import would not be trivial. They themselves are complex physical systems, in particular physical systems with mass, and are therefore susceptible to Leibniz Mass Scalings. To treat them as primitive systems and use them to compare lengths across worlds would be to commit 'Einstein's sin' (Brown [2005]). (More on this elsewhere (Martens [2017a], §2).) If one is convinced by these worries, agreeing on particle coincidences is presumably both necessary and sufficient for two worlds to be empirically equivalent—such a topological criterion has no need for rods nor clocks. Note that regardless of whether one adopts the topological or the shape criterion, Machian comparativism satisfies the completeness (as well as the soundness) conditions in §4, and is thus empirically equivalent to absolutist NG (see the first problem in §6.2).

[23] When expressing Newtonian laws on paper, or inequalities derived from them, one is forced to represent them using symbols representing numerical quantities (times a conventional unit) representing determinate magnitudes (cf. kinematic comparativism (Martens [2017a, 2019])). Nevertheless, the law that is being expressed should be interpreted, at the most basic level, as a mapping from determinate magnitudes to other determinate magnitudes (cf. fn.4). For instance, referring to the determinates ostensively, a law might map *these* determinate masses, distances and velocities to *those* accelerations. See also (Martens [2017a], esp. §2.7).

[24] In this paper I am excluding indeterminism as a viable option (but see (Martens [2018a], p.606; [2017a], §3.2 & §4.4.1; Sider [unpublished]; Dasgupta [unpublished]) for further discussions), especially in light of the existence of a deterministic alternative: absolutism. (Absolute) mass is posited to provide predictive and explanatory power, as well as modal adequacy (i.e. completeness and soundess) (Martens [2018a]). If one were willing to give up the absolute mass scale at the cost of determinism and thereby predictive power, what would prevent one from getting rid of mass altogether (cf. Martens [2017b, 2018a])—rendering the absolutism–comparativism debate moot? Moreover, if a theory is indeterministic at the initial time, it is presumably indeterministic at all times (*pace* (Dasgupta [unpublished])). Thus, a two-particle world may start out with the two particles approaching each other, but a little while later they may exhibit escaping behaviour, and a little while later they may start approaching each other again. We would thus have a violation of soundness (at least on the shape criterion for empirical distinctness of worlds, cf. fn.22) (as well as a vivid illustration of lack of predictive power). Dasgupta's [2013] comparativism, for instance, exhibits this flaw (Martens [2017a], §4.4.1). One of the reasons Machian comparativism improves upon Dasgupta's comparativism is that it is deterministic.

niz Mass Scaling, interpreted as keeping the distances and velocities the same, is not a symmetry of the theory.)

6 Novel Comparativist laws

The previous section suggested that absolute masses are real, that is they are empirically meaningful. If you uniformly scale the masses, *ceteris paribus*, that is keeping the rest of the standard *ideology*—distances and velocities—the same, and then evolve according to the absolutist *laws*, then this will generate an evolution that is empirically distinct from that obtained without scaling. There are two obvious strategies to counter the claim that absolute masses are real: modify the ideology, or modify the dynamics. A simple example of chosing an alternative ideology: if you change the masses and keep the velocities the same this will change the momenta, but if you instead keep the momenta constant then this will change the velocities. I have argued elsewhere ([2017a], §4.2.2) against the strategy of choosing an alternative ideology. Dasgupta's comparativism is an instance of the second option. He re-interprets Newton's second law as relating ratios of forces, masses and accelerations, rather than absolute magnitudes. This modification is too radical and fails (Martens [2019]). Instead, I want to propose a more minor modification of the absolutist laws: Machian comparativist laws.

6.1 Varying Newton's constant

Perhaps the most popular response to the comparativist's bucket goes as follows:[25] the Leibniz Mass Scaling is ill-defined until we are told what happens to Newton's constant G when the masses change. Without knowing this there is no way of saying whether any observable differences will result. What if G were to change, or we decide to change it, exactly in the right way to compensate for any observable effect that might otherwise be thought to result from the change in masses?

One line of thought that may seem to motivate this response is as follows.[26] When we change units (and quantities)—a passive transformation—say we change 1 kg into 2.2 lb, why does the physics not change? Because we are used to concurrently changing the value (and unit) of Newton's constant. Why not do something similar when actively transforming the masses—that is when transforming the magnitudes rather than merely the quantities that represent them? Should we not change the (numerical) constant G as well? In fact, should we not change it in such a way that it exactly compensates for any observable effect that might otherwise be thought to result from the change in masses?

It is however plainly wrong that the *Active* Leibniz Mass Scaling is ill-defined until we tell a story about G. We want to know if changing the masses leads to an empirical difference, *ceteris paribus*. Clearly that means keeping the (standard, absolutist) laws—including their 'strength', as represented by parameters such as Newton's constant—the same. It is not at all surprising that if one were allowed to change (the strength of) the laws at will for each possible world one could get any (or at least many) of the evolutions

[25]Although I have never seen this in print—apart perhaps by Roberts [unpublished], who could be argued to subscribe to it—it comes up at every single conference. I would like to thank Erik Curiel in particular for insisting that this response is more worthwhile than I originally maintained.

[26]A second line of thought that may seem to motivate this response, in particular the idea that we are either 1) completely free to change G, or 2) perhaps even required to change G precisely to the extent as to compensate for changes in m, is the fact that we have empirical access to G and m only via the product $(G \cdot m)$ (Martens [2017a], §2.7 & §4.2.3).

one may have wanted. That is simply not an option within the rules of the game we are playing.

The above motivation was misguided for the following reason. When in common parlance we say 'changing units' (i.e. a *passive* Leibniz scaling), we in fact mean changing the [quantity · unit] combination, and we do so in such a way (say from $1kg$ to $2.21lb$) that the referent (i.e. the mass magnitude) does not change. In principle we would not even have to simultaneously change the [quantity · unit] combination of Newton's constant, but it provides calculational simplicity if the same units are chosen to match up across an equation. This is the only reason that a change in the unit (and the value) of the mass quantities is in practice always combined with a change in the unit (and the value) of Newton's constant. It would be a perfectly viable question to ask what would happen if we (uniformly) changed only the unit or only the values of the masses, whilst leaving both the referents of all units the same as well as the value and unit of Newton's constant. This would of course change the referents of the mass [quantity · unit] combinations, namely the mass magnitudes, and would thus correspond to an active scaling. And at that point we have no obvious reason to believe that either 1) the referent of (the [quantity · unit] combination of) Newton's constant must be changed as well, or 2) that other things that depend on the mass magnitudes, such as the trajectories, do not change. In summary, a passive Leibniz Scaling is *by intention* merely a change of representation—of one and the same state of affairs. Hence, it is on purpose that nothing changes 'out there'. It is thereby simply convenient to also change the units of Newton's constant. None of this has any implications for the well-definedness and relevance of active Leibniz Mass Scalings which do leave the referent of Newton's constant unchanged. And the comparativist's bucket applies exactly to those active rather than passive Leibniz Scalings.

Although I have argued that the above response fails, it does contain the seeds for a more serious response, albeit a more revisionary one: proposing a (syntactically) distinct theory. What if we modify the laws of nature (i.e. standard NG), in particular the gravitational law (i.e. the law 'out there', not just its mode of representation by the numerical constant G), such that in the actual world the absolutist law is an effective form of this more general, comparativist law? In particular, we may create this new law from the standard absolutist interpretation of Newton's law of gravitation by replacing—in the active sense, not just as a passive change of units (cf. fn.23)—G with something that depends on mass. Thus, Newton's 'constant' would not actually be a constant, but a variable—across possible worlds that is, not across time nor across space. If we choose the right function of mass, a uniform scaling of the masses will hopefully not produce any empirical difference after all. I propose $G = G(W) = \gamma/m_W$, where γ is a true constant (across possible worlds W) with mass dimension zero. The gravitational law becomes:

$$F_{grav,ij}(W) = \gamma \frac{m_i m_j}{r^2 m_W}., \tag{G3$_W$}$$

and the escape velocity inequality becomes:

$$v > \sqrt{\frac{\gamma}{r} \frac{m_i}{m_W}}. \tag{2}$$

We should then apply the Leibniz Mass Scaling to this new theory, by applying the *ceteris paribus* clause to the (standard) ideology and the laws of this new theory—this new law and this new inequality truly are invariant across possible worlds related by this scaling, not adjusted at will in each possible world to compensate for any unwanted empirically distinct evolution. The justification for this revisionary move was the hope that this revised theory would correspond to anti-realism about absolute masses—not some misguided motivation concerning a change of units; the proof of this truly novel (i.e. not just representationally, but syntactically, metaphysically and conceptually distinct) pudding, if any, was and will be in the eating.

6.2 The Alpha Mass

What is this m_W in $G = \gamma/m_W$ supposed to refer to? A first suggestion is that m_W is the mass of some privileged body, a primitive ur-mass, that serves as a standard across all possible worlds with respect to which all other intra-world mass comparisons can be made.[27,28] We may also call it the Alpha Mass, by analogy[29] with Neumann's Body Alpha: $m_W \equiv m_\alpha$. Neumann's [1870] response to the argument from inertial motion (i.e. Newton's bucket) was to privilege one body as moving inertially. All and only those other bodies in uniform rectilinear motion with respect to that Body Alpha are also moving inertially. However, as Newton [1962 [late 1660's]] pointed out in response to Descartes [1985/1644]—who had in effect been considering the 'fixed' stars to be Bodies Alpha 226 years before Neumann—in a world governed by Newtonian Gravity there are no inertially moving bodies, since gravity is an attractive, long-range force which can thus never be shielded-off. There cannot be a body that is the Body Alpha. The mass case is more promising though. Whereas the Body Alpha had to be inertially moving, it does not matter what the mass of Body Alpha is. All that matters is that any one massive object is privileged; it does not matter which object it is nor which mass it has. Thus, it may be the platinum alloy cube in Paris, but it need not be.

Four other problems do still arise. Firstly, it may seem that this response does not in fact succeed in generating the full set of empirically distinct worlds that are dynamically allowed by NG, but violates the completeness criterion. If we have one solution for a pair of two equally massive particles, say an escape (collision) solution, then varying the absolute masses *ceteris paribus* can not turn that solution into a collision (escape) solution. It seems that this theory rules out the possibility that two equally massive particles will collide (escape). However, an escape (collision) solution can still be turned into a collision (escape) solution by varying either the initial velocities or distances—although this of course means committing to absolutism about velocity or distance. The task for the comparativist was obviously not to generate the whole set of dynamically allowed solutions merely from applying (Active) Leibniz Mass Scalings to a first solution. Rather it requires us to range over all the possible initial determinates of the chosen ideology, and check whether that provides the desired set of worlds. In this case it does.

Secondly, even though there are some dynamically possible worlds containing the body with m_α whereas there are none containing Body Alpha, there will still be some that do

[27] Baker [2013a] considers this option, but ends up rejecting it.

[28] In Modified Newtonian Dynamics (MOND) (Milgrom [1983]), the new constant a_0, with dimensions of acceleration, seems to be similarly used to serve as a standard for acceleration comparisons. However, it would be incorrect to conclude from this that comparativism about acceleration is true of MOND. a_0 is not the acceleration of some privileged body, and thus not an "Alpha Acceleration" in the sense of Neumann's Body Alpha (see text), but a new constant of nature—a consequence of which is that MOND avoids the second and third objection discussed in the text. A Leibniz Acceleration Scaling would therefore change only the absolute accelerations a_i $(i = 1, ..., n)$ but not a_0, which implies a change in the ratios $\frac{a_0}{a_i}$ and thereby an empirical difference. I believe absolutism about acceleration to be true of MOND.

[29] I would like to thank John Norton for reminding me how interesting this analogy is.

not contain that privileged body either. In those worlds the law fails to refer. As Baker [2013a] puts it: the privileged massive object would have to exist with physical necessity, but that is an extremely implausible posit for a scientific theory to make.

Thirdly, having the law refer to one particular, privileged body or property violates a generally accepted view about laws of nature called 'generalism'. It claims that the world is fundamentally purely qualitative: a language or theory devoid of reference to particular individuals (such as m_α) suffices to give a complete and perspicuous description of fundamental reality (Møller-Nielsen [2015]). I will not discuss the merits of this view here—see (Saunders [2003]; Dasgupta [2009]; Pooley [unpublished]; Møller-Nielsen [2015]) for convincing arguments—but rest content with having pointed out this consequence of the m_α solution.

Fourthly, it is not clear how this significantly and positively differs from absolutism. The debate between comparativism and absolutism was only ever about a single degree of freedom: the absolute mass scale. Adding a privileged body m_α seems very much like adding something like a (relatively) primitive absolute mass scale. Even if it is supposed to be a different type of beast, its loss in metaphysical parsimony seems to be enough to counteract the gain the comparativist was supposed to obtain from dismissing a (relatively) fundamental absolute mass scale.

6.3 Summing over all the masses: Machian comparativism

In the substantivalism–relationalism debate about space, Mach's approach to inertial motion may be seen as an attempt at an improvement upon the Body Alpha response, by avoiding the requirement of there being one privileged body (or mass) in all possible worlds (on pain of failure of reference of the law) and the associated violation of generalism. Rather than choosing one specific body as a standard of inertial motion, inertial motion is abstracted from the bulk matter in the whole universe. By analogy, one might wonder whether m_W should refer to the sum over all the masses in the universe.[30] The corresponding *Machian gravitational law* is:[31]

$$F_{grav,ij} = \gamma \frac{m_i m_j}{r^2 \sum\limits_k m_k}. \tag{G3}$$

This would result in the following escape velocity inequality:

$$v > \sqrt{\frac{\gamma}{r} \frac{m_j}{\sum\limits_k m_k}}. \tag{3}$$

It is important to not take from this equation that the sum of all the absolute masses—which is an absolutist notion, unavailable to the comparativist[32]—plays a crucial role all by itself. Rewriting the escape velocity inequality as follows will make explicit that it

[30] Which is always well-defined if we assume a finite number of particles with finite mass.

[31] Especially since 'mass' is the topic of this paper, it is important to point out that the name 'Machian' is not intended to imply any analogy with Mach's treatment of (inertial) *mass*—reducing mass ratios to acceleration ratios. Connections between that analogy and comparativism are discussed elsewhere (Martens [2017b, 2018a]). In this paper the name derives purely from Mach's analogous treatment of inertial *motion*.

[32] Assuming that laws may refer to fundamental notions only. Regularity comparativism would be an exception to this (Martens [2017b]).

really is the sum of all mass ratios that matters:

$$v > \sqrt{\frac{\gamma}{r \sum_k \frac{m_k}{m_j}}}. \tag{4}$$

All worlds agreeing on all mass ratios (and initial spatiotemporal facts) will evolve in the same, deterministic way under the Machian gravitational law—the comparativist initial value problem now does have a unique solution. An Active Leibniz Mass Scaling does indeed not affect whether this escape velocity inequality is satisfied. (It is a symmetry of Machian comparativism!) Absolute masses are not empirically meaningful; they are not real! Thus, although no comparativist has proposed this theory—as far as I am aware—I believe it to be the most viable version of comparativism.

It may seem that this view has the following counter-intuitive consequences. If God were to suddenly add a massive particle or increase the mass of a massive particle, however far away from our two-particle system of interest, this would nevertheless reduce the force between these two particles. This adds a holistic flavour to comparativism. Similarly, the gravitational force between each pair of particles is weaker in a world W_2 that differs from a world W_1 solely by the addition of some massive particles somewhere. Beyond perhaps running counter to one's intuition, these two scenarios do not provide any empirical problem though. God has never suddenly increased the number of particles or their masses; if (s)he were to, that might indeed reduce the gravitational force, for all we know. Similarly, in the second case the 'strength of the law', represented by Newton's Constant, would still be constant within both worlds. Its constancy within the actual world is all the empirical data we have.[33]

However, it is worth repeating what was discussed in the context of the Alpha Mass: to ensure completeness, one needs to commit to absolutism about r (or v). Replacing G not with $\gamma / \sum_k m_k$ but with $\gamma \sum_i r_i / \sum_k m_k$ (*Doubly Machian Comparativism*) would violate completeness. Although it seems very much in the spirit of comparativism to be a comparativist about all determinables, comparativism about mass is logically consistent with absolutism about distance. There remains an arbitrary choice though between opting for the comparativism about mass and absolutism about distance combination, rather than the other way round. I will leave it to the reader to decide whether this is problematic.

7 Metaphysical Parsimony & Explanatory Power

The Machian form of comparativism is empirically equivalent to absolutist NG, and renders absolute masses unreal. So far so good. In this section I will however question the supposed explanatory adequacy and especially the metaphysical parsimony of (Machian) comparativism.

Advocates of comparativism tend to not argue for comparativism being more metaphysically parsimonious than absolutism—they take that for granted.[34] I take the naive

[33]Keeping in mind fn.8.

[34]It may seem that Dasgupta avoids the need for comparativism to be more metaphysically parsimonious than absolutism. He adopts the undetectability approach to empirical equivalence, and with it an Occamist norm that focuses explicitly on undetectability rather than metaphysical parsimony (§4): all other things being equal, we should favour theories that posit less undetectable structure. However, this 'all other things being equal' clause presumably also ranges over metaphysical parsimony, as Dasgupta

intuition behind this assumption to be the thought that absolutistism acknowledges both intrinsic masses and mass relations while comparativism only recognises the latter, and therefore comparativism has a 'lower metaphysical bill'. However, the absolutism–comparativism debate is a debate about relative fundamentality—do mass ratios obtain in virtue of more fundamental absolute masses—not about realism about absolute masses. It is consistent for the comparativist to 'acknowledge' absolute masses. But even if we were to focus only on realist absolutists and anti-realist comparativists, this intuition is mistaken. When an absolutist god builds the world (s)he 'pays' for intrinsic masses only. The mass relations emerge, that is they are facts that are true of this world in virtue of the fundamental intrinsic masses. They come free of charge.[35] (Similarly, a comparativist who is a realist about absolute masses—e.g. a regularity comparativist (Martens [2017b])—gets those absolute masses for free.) The comparativist god uses a completely different type of building block: mass relations. It is *prima facie* unclear which building block 'costs more', metaphysically speaking. The naive intuition did not require any (detailed) measure of metaphysical parsimony since it took the comparativist to require only a proper subset of the 'things' the absolutist needed. Since they in fact require distinct, non-overlapping[36] sets of fundamental 'things', a measure of metaphysical parsimony becomes relevant.

7.1 Quantitative & qualitative metaphysical parsimony

Do we care not about the number of types of 'things' our theory postulates, but only about the number of tokens or instantiations? Or do we aim for qualitative parsimony rather than quantitative parsimony: minimalisation of the number of types of things (entities, relations, spaces, laws, etc.), whilst getting an infinite stock of instantiations of each type for free (Lewis [1973]; Nolan [1997]; Baker [2003], Jansson and Tallant [forthcoming])? Or should we aim for a mix of both, and if so, how? And do we weigh each type equally?

I am skeptical that there is a rigorous answer to these questions in general, which is why I will not attempt to provide a rigorous measure of parsimony here. Nevertheless, as will be discussed in this subsection, in the specific case of comparativism both the quantitative and qualitative metaphysical parsimony are sufficiently questionable to at least shift the onus of proof to the comparativist. More importantly, in the next subsection I will point out a strange conspiracy implicit in the standard formulation of comparativism. Although I believe that under any relatively plausible measure of (qualitative) metaphysical parsimony the comparativist would lose all its (alleged) parsimonious advantage after having to introduce an additional structure to dissolve this conspiracy, a simple inference to the best explanation argument will also do to argue that the conspiracy defeats the comparativist.

With regards to quantitative parsimony, it seems that comparativism obviously loses out: for a world consisting of n particles, its ideology consists of $n(n-1)$ (equally) fundamental instantiations (of relations)—or perhaps even n^2 if we include fundamental mass relations between the particle and itself—whereas the absolutists needs only n

([2016], p.854; [2013]) seems to acknowledge. (Moreover, the undetectability approach—to the extent that it differs from the possiblity checking approach—and its associated Occamist norm are flawed (§4; Martens [2019]).)

[35] I side with Schaffer [2015], who replaces Occam's Razor—do not multiply entities without necessity—with Occam's Laser—do not multiply fundamental entities without necessity.

[36] Except perhaps for a comparativist committing to a fundamental m_α (§6.2), if this is basically the same as an absolute mass.

(equally) fundamental instantiations (of monadic properties). Comparing the qualitative parsimony is less straightforward. It seems the only way in which we can compare these different types of beasts is via their structure. As mentioned in §2, absolute masses are in fact structurally quite similar to mass relations. Both types of determinates are structured in two ways: a total order and a concatenation rule. Moreover, they both require transworld identities, quiddities, on top of that. Qualitatively speaking the metaphysical parsimony of both views are thus on a par. That is, if we consider generic forms of comparativism (say Dasgupta's); Machian comparativism actually requires additional structure.

To see this, recall that absolute masses are defined as a totally ordered semi-group, with the concatenation rule to be interpreted as addition. Under this structure, the sum over all the masses (or mass ratios) is well-defined. Comparativist mass relations, on the other hand, were defined as a totally ordered group, where the concatenation rule was to be interpreted as multiplication. Under this structure, sums over mass ratios are not yet defined.[37] Machian comparativism would thus require additional structure, namely an associative rule to be interpreted as addition.[38] Moreover, this rule is more complex, or at least of a different nature, in that it does not combine two (or more) mass relations to obtain another mass relation (i.e. a group concatenation rule), but it maps two (or more) mass relations to a number.[39] (The only benefit of this extra structure is that comparativist mass relations now do not require quiddities anymore, as the extra structure suffices for the Machian gravitational law to uniquely pick out mass relations in each possible world.)[40] The mass ratios now require a total order and two structures, whereas the absolute masses requires one structure less.[41] In sum, it is unclear why Machian comparativism would obviously be metaphysically more parsimonous than absolutism. The situation worsens in the next section, where another metaphysical cost for generic forms of comparativism is pointed out.

7.2 The conspiracy of mass relations

In §2 it was pointed out that, although any set of absolute masses *always* determines a unique set of mass ratios, a set of mass relations can only be interpreted as a consistent set of mass ratios if they satisfy a certain *constraint* (i.e. concatenating the relations of any closed loop needs to result in the identity). The mass relations conspire[42] in such a way as to allow for an interpretation as mass ratios, that is to allow for a representation by absolute masses (Figure 1.a) (albeit a non-unique one). To ensure/explain this behaviour it seems the comparativist needs constraining relations between the mass relations: meta-relations. Whereas the absolutist needs n absolute properties, the comparativist seems

[37]Such sums may be standardly associated with the real numbers, but neither absolute masses nor the comparativist's mass ratios are to be confused with the full structure of the real numbers (cf. §2, esp. fn.4).

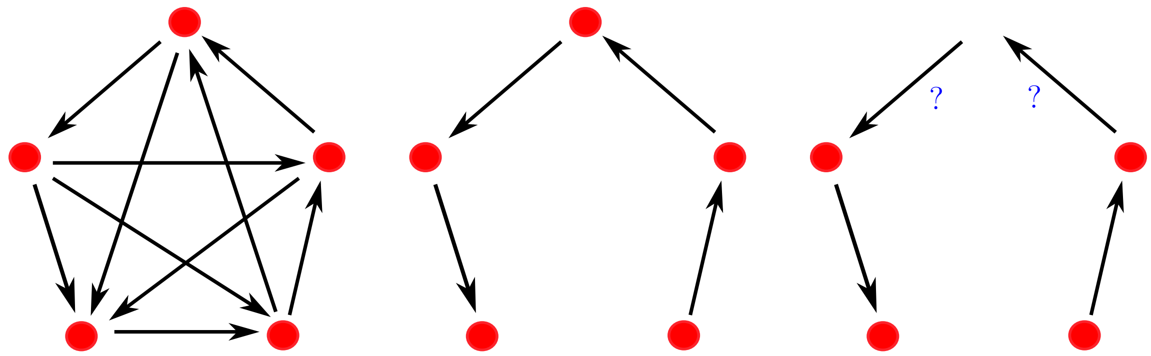
[38]There are no inverse or identity elements associated with this rule, since we do not have 0 mass ratios.

[39]This might lead to similar issues as those referred to in fn.4.

[40]The easiest way to see this is by realising that this structure is not invariant under Leibniz Mass Ratio Squarings anymore (cf. fn.7).

[41]That some of these rules lead to semi-groups rather than groups (cf. fn.38 & §2) is irrelevant, since the existence of identity and inverse elements is merely a fact that is true or false of a set plus a rule, not some extra structure that needs to be imposed.

[42]This conspiracy has recently been acknowledged as well by Roberts [unpublished], who traces it back to Russell [1903] and Armstrong [1988].



(a) Web of relations (b) Chain of relations (c) Removing a particle

Figure 1: Examples of five-particle worlds and their fundamental mass relations, illustrating (a) the overdetermination of mass relations and the corresponding conspiracy that is required for the relations to be interpretable as mass ratios (i.e. all 'loops' need to combine to the identity; not illustrated), (b) the Chain Comparativism response, and (c) the counter-response.

to not only need between $n(n-1)$ and n^2 relations, but in addition a number of meta-relations. This significantly reduces both the qualitative and quantitative parsimony of comparativism. Alternatively one might impose an additional law that ensures this constraint, but this also comprises a cost in (quantitative) parsimony.

This situation is closely related to two other *prima facie* conspiracies, which arise within the framework of relationalism about space. Arntzenius ([2012], Ch. 5) points out that the possible distances between objects are severly constrained. For instance, in Euclidean space, five particles cannot be equidistant; the distance between particle A and B cannot be shorter than the sum of the distances between A and C and B and C, etc. If distances are fundamental these constraints seem rather conspiratorial. Meta-relations between the distances are needed to ensure that all the constraints hold. What about the substantivalist framework? If the metric is interpreted as a mapping from all pairs of points to the real numbers, we obtain a similar overdetermination which requires constraints to avoid inconsistencies.[43] Instead the metric is to be interpreted locally: it maps 'neighbouring' spatial points to the real numbers. Distances between distant points or objects are then obtained by integrating over the infinitesimal distances (along the shortest path): no overdetermination, no meta-relations needed to constrain anything. The relationalist about space has no such local metric available, or at least not in theories such as Newtonian Gravity which consist of point-particles rather than a plenum: distances are only defined between (distant) material objects. The comparativist about mass has a similar problem: if the universe was filled with a (relative) density field, mass relations of distant objects could be obtained by multiplying a continuous chain of mass ratios. Here we are however treating Newtonian Gravity as a particle theory, not a field theory.

A second conspiracy arises when we attempt to interpret Newton's First Law within the relationalist framework about space. "[F]orce-free bodies ... conspire to move in straight lines at uniform speeds while being unable, by *fiat*, to communicate with each other" (Brown [2005], p.14-15). This is what Brown dubs the conspiracy of inertia. All force-free particles move as if they are following the affine structure of a single back-

[43] See (Belot [2011], p.25-27, and references therein).

ground space. In the relationalist framework particles do not live in such a space, and it is therefore a miracle that all the primitive, independent (relative) velocities of all the (pairs of) particles allow for a simple coordinate system—what we call the inertial coordinate system—in which the above mentioned conspiracy of inertia obtains. It seems the relationalist again needs to invoke some meta-structure to constrain these otherwise independent velocities.

Not only do all the meta-structures required to deal with these conspiracies make the comparativist/relationalist theory fatally unparsimonious—against $P_{\text{occ-par}}$ (p.6)—but, more strikingly, all these conspiracies follow the schema 'the relations conspire to behave exactly as if the absolutist/substantivalist theory were true'. The obvious move is to *infer to the best explanation*: absolutism (substantivalism) is correct. These conspiracies can be *no miracle*—this would constitute a substantive loss of explanatory power relative to the absolutist theory, against P_{exp} (p.6). If the absolutist type solutions to these problems dissolve these conspiracies trivially, this is sufficient reason to opt for absolutism, regardless of metaphysical parsimony.

7.3 Chain comparativism

The comparativist might respond by conceding that once we attribute to them a web of fundamental relations between each pair of objects then the overdetermination of mass relations suggests a weird conspiracy (if inconsistencies are to be avoided when attempting to interpret them as mass ratios), but they never in fact insisted on overdetermining these mass relations.[44] If we have the mass relations form a single chain, as in Figure 1.b, by having each particle be the relatum of only *two* fundamental mass relations (except for the particles at the beginning and end of the chain which each are the relatum of only one fundamental mass relation), the mass relations between all particles in the universe are determined without any overdetermination.[45] Not only does *chain comparativism* avoid the conspiracy of mass relations, it also seems quantitatively more parsimonious, and even qualitatively insofar as it does not require meta-relations. I will push back against this supposed gain in parsimony in the next subsection.

I am not aware of any chain comparativists. I read authors such as Field [1980] and Dasgupta [2013] to assume *web comparativism*, at least implicitly, and when Dasgupta ([2013], §3) discusses mass-counterpart theory even quite explicitly. Similarly I am not aware of any chain relationalists about space. Perhaps none of these authors has any problems moving to chain comparativism/relationalism. It would imply a certain holism. One of the basic assumptions in experimental physics is the assumption that one can do experiments that are isolated from the rest of the world (for all practical purposes). For instance, when we measure the force between two particles in a lab that are interacting gravitationally (for instance the two endpoints of the chain in Figure 1.b), we assume

[44]One might respond that it is simply in the essence of mass that all massive objects stand in equally fundamental mass relations to all other massive objects. In other words, it is in the essence of mass that all forms of comparativism must be web comparativism and cannot be chain comparativism. In the space case this may seem plausible: if points 1 and 2 stand in a distance relation, and points 2 and 3 stand in a distance relation, then points 1 and 3 must also instantiate a distance relation—this is just what it is to be a distance. It is not directly clear that this intuition is as obviously correct in the mass case.

[45]Whether such a move is desirable for the relationalist about space, as a response to the conspiracy pointed out by Arntzenius, is an interesting question which is nevertheless outside of the scope of this paper.

that the force only depends on those two particles. Chain comparativism would imply that the force between those two endpoints depends on the mass relation determined by the whole chain across the whole universe. It seems however that this would not lead to any different empirical predictions. For all we know the world is just like this—something that we have seen the Machian comparativist already commit to (§6.3). Similarly, it does not seem elegant that two particles are the relatum of only one mass relation, and the other particles of two mass relations. Again, this might just be the way the world is.

If we however consider the following intuition a valid intuition, chain comparativism faces a problem. It seems that if God would remove one massive particle at the far end of the universe, this should (for all practical purposes) not have any empirical effect on Earth. If chain comparativism were true, the removal of one such particle would break the chain into two chains (Figure 1.c). The mass relations between any pair of particles on different chains are then not defined. It is not even just the case that chain comparativism predicts that such a removal would have an effect on Earth: physics on Earth would become ill-defined.

7.4 Metaphysical parsimony at the level of worlds

In the spirit of the possiblity checking approach, why not adopt an inter-world version of the Occamist norm (of metaphysical parsimony), rather than the intra-world approach above? As suggested in §4, a Leibnizian-style Principle of the Identity of Indiscernibles may be applied to whole worlds, by equating metaphysical parsimony with minimalisation of the amount of metaphysically distinct possible worlds per empirically distinct possible world. In the case of regularity comparativism, the set of physically possible worlds is 'as large' as that of absolutism, as this view commits to realism about absolute masses. However, Machian comparativism, being anti-realist about absolute masses, indeed has a 'smaller' set of physically possible worlds.

It should be noted that the choice between this inter-world version of the measure of metaphysical parsimony and the intra-world version is not mutually exclusive. In fact, in light of the 'all other things being equal' clause in the argument for comparativism, we would require comparativism to come out favourably with respect to each of these measures, or at least with respect to the overall (i.e. both intra- and inter-world) parsimony. It is not obvious that comparativism is the winner here.

In light of this, although moving from web comparativism to chain comparativism indeed improves both the quantitative and qualitative intra-world parsimony, it comes at the cost of a reduced inter-world parsimony. There are many ways in which one could generate a 'chain world' corresponding to a single 'web world'. For a chain comparativist, choosing any single chain that connects all the particles in the world is an arbitrary choice. All of these choices are (empirically) equivalent, but nevertheless metaphysically distinct. This leads to a large proliferation of redundant possible worlds, whereas the whole point of comparativism was to do with no metaphysical distinctions without an empirical difference. Chain comparativism does not seem to do much better than absolutism in this regard.

Comparativists take it to be obvious that comparativism is metaphysically more parsimonious than absolutism, and therefore do not provide any detailed justification for this assertion. Comparing metaphysical parsimony is a tricky business, if at all possible. The issue is straightforward only in cases where one theory commits to a fundamental metaphysics that is a proper subset of the opposing theory's fundamental metaphysics.

This does not apply to the absolutism–comparativism debate. In this section I have considered the metaphysical parsimony of comparativism along the dimensions of quantitative vs. qualitative parsimony, and intra-world vs. inter-world parsimony. Along all these dimensions the parsimony of comparativism is questionable. In order to make their case, the onus is on the comparativist to tell us a proper story of why their view is so clearly more parsimonious.

8 Conclusion

This paper developed a Machian version of comparativism about mass in Newtonian gravity, obtained by actively replacing Newton's constant by another constant divided by the sum over all masses. This form of comparativism is deterministic, denies the reality of absolute masses, and is empirically equivalent to absolutist Newtonian Gravity, thereby meeting the challenge posed by the comparativist's bucket argument. In this sense Machian comparativism improves upon all other comparativist proposals on the market. However, all forms of (web) comparativism exhibit an unexplained conspiracy between the mass relations that allows them to be interpretable as mass ratios. Moreover, although comparativists take it to be obvious that comparativism is metaphysically more parsimonious than absolutism, parsimony is a tricky business. A rough investigation of comparativism's parsimony along the dimensions of quantitative vs. qualitative parsimony, and intra-world vs. inter-world parsimony, renders it highly questionable—even slightly more so for Machian comparativism. Although Machian comparativism delivers on recovering the empirical virtues of absolutism, all else is not equal.

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References

Armstrong, D. [1978]: *A Theory of Universals: Volume 2*, Cambridge: Cambridge University Press.

Armstrong, D. [1988]: 'Are Quantities Relations? A Reply to Bigelow and Pargetter', *Philosophical Studies*, **54**, pp. 305–316.

Arntzenius, F. [2012]: *Space, Time, & Stuff*, Oxford University Press.

Baker, A. [2003]: 'Quantitative Parsimony and Explanatory Power', *British Journal for the Philosophy of Science*, **54**, pp. 245–259.

Baker, D. J. [2013a]: 'Comparativism with Mixed Relations', Manuscript.

Baker, D. J. [2013b]: 'Some Consequences of Physics for the Comparative Metaphysics of Quantity', Manuscript.

Barbour, J. [2000]: *The end of time: The next revolution in physics*, Oxford: Oxford University Press.

Barbour, J. and Bertotti, B. [1982]: 'Machs Principle and the Structure of Dynamical Theories', in *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 382, pp. 295–306.

Bauer, W. [2011]: 'An Argument for the Extrinsic Grounding of Mass', *Erkenntnis*, **74**(1), pp. 81–99.

Belot, G. [1999]: 'Rehabilitating Relationalism', *International Studies in the Philosophy of Science*, **13**(1), pp. 35–52.

Belot, G. [2000]: 'Geometry and Motion', *British Journal for the Philosophy of Science*, **51**(4), pp. 561–595.

Belot, G. [2011]: *Geometric Possibility*, Oxford University Press.

Belot, G. and Earman, J. [2001]: 'Pre-Socratic Quantum Gravity', in C. Callendar and N. Huggett (*eds*), *Physics meets philosophy at the Planck scale: Contemporary theories in quantum gravity*, Cambridge University Press, chap. 10, pp. 213–55.

Bigelow, J. and Pargetter, R. [1990]: *Science and Necessity*, Cambridge: Cambridge University Press.

Bigelow, J., Pargetter, R. and Armstrong, D. [1988]: 'Quantities', *Philosophical Studies*, **54**, pp. 287–316.

Brown, H. R. [2005]: *Physical Relativity: Space-time structure from a dynamical perspective*, Oxford University Press.

Dasgupta, S. [2009]: 'Individuals: An Essay in Revisionary Metaphysics', *Philosophical Studies*, **145**(1), pp. 35–67.

Dasgupta, S. [2013]: 'Absolutism vs Comparativism about Quantity', in K. Bennett and D. W. Zimmerman (*eds*), *Oxford Studies in Metaphysics*, vol. 8, Oxford University Press, pp. 105–147.

Dasgupta, S. [2016]: 'Symmetry as an Epistemic Notion (Twice Over)', *The British Journal for the Philosophy of Science*, **67**, pp. 837–878.

Dasgupta, S. [unpublished]: 'Relationalism and the Limits of Possibility', Manuscript, Oct 2015.

Descartes, R. [1985/1644]: 'Principles of Philosophy', in *The Philosophical Writings of Descartes (vol. 1, pp. 207-252)*, Cambridge University Press Translated and edited by J. Cottingham, R. Stoothoff, and D. Murdoch. Excerpts reprinted in Huggett (1999).

Earman, J. [1986]: *A Primer on Determinism*, D. Reidel Publishing Company.

Eddon, M. [2013]: 'Quantitative Properties', *Philosophy Compass*, **8**(7), pp. 633–645.

Ellis, B. [1966]: *Basic Concepts of Measurement*, Cambridge: Cambridge University Press.

Field, H. [1985]: 'Can We Dispense with Space-Time?', in *PSA 2*.

Field, H. H. [1980]: *Science without Numbers: A Defence of Nominalism*, Oxford: Basil Blackwell.

Huggett, N. [1999]: *Space from Zeno to Einstein: Classic Readings with a Contemporary Commentary*, Cambridge, MA: MIT Press.

Jansson, L. and Tallant, J. [forthcoming]: 'Quantiative Parsimony: Probably for the Better', *British Journal for the Philosophy of Science*.

Lewis, D. [1973]: *Counterfactuals*, Cambridge: Harvard University Press.

Lewis, D. [1986]: *Philosophical Papers*, vol. ii Oxford University Press.

Martens, N. C. [2017a]: *Against Comparativism about Mass in Newtonian Gravity – a Case Study in the Metaphysics of Scale*, Ph.D. thesis, Magdalen College, University of Oxford.

Martens, N. C. [2017b]: 'Regularity Comparativism about Mass in Newtonian Gravity', *Philosophy of Science*, **84**(5), pp. 1226–1238.

Martens, N. C. [2018a]: 'Against Laplacian Reduction of Newtonian Mass to Spatiotemporal Quantities', *Foundations of Physics*, **48**(5), pp. 591–609.

Martens, N. C. [2018b]: 'Refining the Motivational Approach to Symmetry-to-(Un)reality Inferences, Case Study: Aharonov-Bohm Effect', Manuscript.

Martens, N. C. [2019]: 'The (Un)detectability of Absolute Newtonian Masses', *Forthcoming in Synthese*.

Martens, N. C. M. and Read, J. [2018]: 'Sophistry about symmetries?', Manuscript.

McKinsey, J., Sugar, A. and Suppes, P. [1953]: 'Axiomatic Foundations of Classical Particle Mechanics', *Journal of Rational Mechanics and Analysis*, **2**(2).

Milgrom, M. [1983]: 'A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis', *Astrophysical Journal*, **270**, pp. 365–370.

Møller-Nielsen, T. [2015]: *Symmetry, Indiscernibility, and the Generalist Picture*, Ph.D. thesis, Oriel College, University of Oxford.

Møller-Nielsen, T. [2017]: 'Invariance, Interpretation, and Motivation', *Philosophy of Science*, **84**(5), pp. 1253–1264.

Møller-Nielsen, T. and Read, J. [2018]: 'Redundant Epistemic Symmetries', Manuscript.

Mulder, H. and van de Velde-Schlick, B. (*eds*) [1979]: *Moritz-Schlick: Philosophical Papers*, vol. 11-I (1909-1922) of *Vienna Circle Collection Series* Reidel Publishing Company.

Mundy, B. [1987]: 'The Metaphysics of Quantity', *Philosophical Studies*, **51**(1), pp. 29–54.

Neumann, C. [1870]: *Ueber die Principien der Galilei-Newton'schen Theorie*, Leipzig: B.G. Teubner.

Newton, I. [1962 [late 1660's]]: 'De Gravitatione', in *Unpublished Papers of Isaac Newton: A Selection from the Portsmouth Collection in the university Library, Cambridge*, Cambridge University Press, pp. 123–146 Translated and edited by A.R. Hall and M.B. Hall. Excerpts reprinted in Huggett (1999).

Nolan, D. [1997]: 'Quantitative Parsimony', *British Journal for the Philosophy of Science*, **48**, pp. 329–443.

Norton, J. D. [2003]: 'Causation as Folk Science', *Philosophers' Imprint*, **3**(4), pp. 1–22.

Pooley, O. [unpublished]: *The Reality of Spacetime*,.

Quine, W. [1951]: 'Ontology and Ideology', *Philosophical Studies*, **2**, pp. 183–204.

Read, J. and Møller-Nielsen, T. [2018]: 'Motivating Dualities', *Synthese* Forthcoming.

Roberts, J. T. [unpublished]: 'A Case for Comparativism about Physical Quantities', .

Russell, B. [1903]: *The Principles of Mathematics*, W.W. Norton & Company.

Saunders, S. [2003]: 'Physics and Leibniz's Principles', in K. Brading and E. Castellani (*eds*), *Symmetries in Physics: Philosophical Reflections*, Cambridge: Cambridge University Press.

Saunders, S. [2007]: 'Mirroring as an A Priori Symmetry', *Philosophy of Science*, **74**, pp. 452–480.

Schaffer, J. [2015]: 'What Not to Multiply Without Necessity', *Australasian Journal of Philosophy*, **93**(4), pp. 644–664.

Schlick, M. [1949]: *Philosophy of Nature*, Philosophy Library, Inc. Translated by Amethe von Zeppelin.

Schlick, M. [1974/1925]: *General Theory of Knowledge*, Library of Exact Philosophy Series. Springer-Verlag Translated by Albert Blumberg, from the 2nd German Edition of *Allgemeine Erkenntnislehre*, Berlin.

Schlick, M. [1979/1922]: 'Space and Time in Contemporary Physics: An Introduction to the Theory of Relativity and Gravitation. See Mulder and Van de Velde-Schlick (Mulder and van de Velde-Schlick 1979, Ch.9, p. 207-269)', Translated by Henry Brose and Peter Heath from *Raum und zeit in der gegenwärtigen physik*, 4th edition, Berlin.

Sider, T. [unpublished]: 'The Tools of Metaphysics and the Metaphysics of Science', Draft of Sept 2018.

Skow, B. [2007]: 'Sklar's Maneuver', *British Journal for the Philosophy of Science*, **58**, pp. 777–786.

Stevens, S. [2014]: *The Dynamical Approach to Relativity as a Form of Regularity Relationalism*, Ph.D. thesis, Exeter College, University of Oxford.

Williams, P. [unpublished]: 'Scientific Realism Made Effective', http://philsci-archive.pitt.edu/12889/.