# Thumper the Infinitesimal Rabbit: A Fictionalist Perspective on Some "Unimaginable" Model Systems in Biology

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## Abstract

Fictionalists believe that scientific models are about model systems that are imaginary. Weisberg has claimed that fictionalism is indefensible because many scientific models are about model systems that are unimaginable. According to a certain account of imagination, what Weisberg says is plausible. According to another, more defensible account of imagination, it is not. I discuss these issues within the context of an allegedly unimaginable model system in ecology, but the conclusions I draw are more general. I then describe how fictionalism should be recast in order to deal with Weisberg's critique.

# 1 Introduction

I assume a *model* is a set of propositions that is expressed as a mathematical formalism and that is interpreted by some person to be about some thing. The system that is precisely described by the mathematics of the model is the *model system.* Not all models are mathematical, to be sure. But this paper is only about mathematical models.

Some proponents of *fictionalism* have argued that we should think of model systems as imaginary. Godfrey-Smith refers to these systems as "imagined concrete things" (2006, 734-735). Frigg calls them "imagined physical systems" (2010, 253). Toon says models "prescribe imaginings about" a model system (2010, passim).

Michael Weisberg (2013) has claimed this view won't work, since some models are about model systems that are "unimaginable." As one example, Weisberg discusses models in ecology that are *ordinary differential equations* (ODEs). Weisberg claims we can't imagine organisms coming in non-integer values. But, when one uses an ODE to model the growth of a population of organisms, population size is treated as a continuous, not discrete, quantity. Thus, Weisberg concludes, the model system can't be imagined, and fictionalism must be false.

Whether Weisberg's anti-fictionalist argument succeeds depends on what sense of "imagination" the fictionalist is committed to. If the fictionalist believes that to imagine a model system requires one to have a visual representation of it, then what Weisberg says is plausible. And indeed, some fictionalists, at least some of the time, do appear to treat imagination as a primarily visual

activity (e.g., Godfrey-Smith 2006; Levy 2015).

But fictionalists need not be—in fact, *should not be*—committed to such a restrictive account of imagination. Imagination can be non-visual and, importantly for our purposes, one can imagine what is conceptually impossible. In other words, fictionalism *is* compatible with the sorts of model systems that animate Weisberg's critique, so long as fictionalists adopt a sufficiently robust account of what imagination is, along with a more careful account of the way in which model systems are imaginary. Below, I show how this can be done.

Why does this debate matter? Many if not most philosophers of science think of a scientific theory as a collection of models. But there are a number of competing accounts of what models are and how modeling works. Fictionalism, the new kid on the block, has become a prominent way to think about these issues, and so it is important to assess whether Weisberg's anti-fictionalist argument succeeds. While I'll show that Weisberg's argument ultimately fails, along the way we'll see that his argument makes clear which formulations of fictionalism will not work, and which will. This is essential information for us fictionalists as we further develop the approach.

## 2 The Problem of Unimaginable Model Systems

In this section I'll present Weisberg's argument against the fictionalist claim that model systems are imaginary. I have added a few details that render the argument more filled in, but the argument is by and large what one finds in Weisberg (2013, chapter 4.4).

The concepts we have of certain organisms appear to entail that these organisms come in only non-negative integers.[1] True, RABBIT does not have a clean set of necessary and sufficient conditions like TRIANGLE and BACHELOR do, but RABBIT *does* seem to entail that a population cannot have, say, 3.1528 rabbits. Couched in the language of possible worlds, we might say there is a conceptually possible world about which "Sally owns one rabbit" is true, though no conceptually possible world about which "Sally owns 3.1528 rabbits" is true. To be sure, if we put Sally's rabbits in a meat grinder, we can treat the resultant quantity as continuous. And perhaps we might think of a rabbit embryo as in some sense a "partial" rabbit. But the models that Weisberg discusses are about living rabbits, out of the womb. Such rabbits are, it seems, necessarily discrete.

Indeed, we might insist that 3.1528 rabbits is not just conceptually impossible, but also unimaginable. After all, there is undoubtedly a *sense* in which I cannot imagine such a collection of rabbits. I cannot conjure a visual representation of it, for example.

This line of reasoning seems to underlie Weisberg's claim that there are models in science whose corresponding model systems are "unimaginable." We see this in his discussion of the Lotka-Volterra model. In its standard form, Lotka-Volterra is a system of ODEs that describes the relative abundance of predators and prey in some ecosystem (equations 1-2) (Murray 2002, 79). Suppose that the prey are rabbits and that the predators are foxes. Equation

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[1]Some might say rabbits come in only non-negative integers as a matter of "metaphysical necessity." I will describe the issue in terms of "conceptual necessity," since my hunch is that it is our concepts that are doing the work here, not mind-independent metaphysical truths.

1 says the rate of change of the abundance of rabbits ($x$) is a function of the rate at which they reproduce ($a$) and the rate at which they are eaten by foxes ($b$). Equation 2 says the rate of change of the abundance of foxes ($y$) is a function of the rate at which they reproduce ($c$) and the rate at which foxes die ($d$).

$$\frac{dx}{dt} = ax - bxy \tag{1}$$

$$\frac{dy}{dt} = cxy - dy \tag{2}$$

The key point is that equations 1-2 treat the populations of rabbits and foxes as continuous quantities; the relative abundance of each species is drawn from a segment of the reals. A consequence of this is that, given a specific setting of the parameters, we can show that at some point in time there will be, for instance, 3.1528 rabbits in the population.

According to Weisberg, this is where the fictionalist gets into trouble. The fictionalist wants to maintain that equations 1-2 are about an "imaginary" system. But one can only imagine rabbits and foxes coming in non-negative integers: "For a fictionalist, a model of predation has to be composed of concrete populations of discrete and distinct individuals" (2013, 62). Equations 1-2, in contrast, are what Weisberg calls a "population-level" model—there are no discrete, distinct individuals—and such population-level models "can't be imagined in their entirety," which "rules out the possibility of equating such models with imagined fictional scenarios" (Ibid., 63).

This is not Weisberg's only argument against fictionalism, but it strikes

me as the most compelling.[2] It also has broad reach. The same argument will apply to any model in which one tracks infinitesimal changes in the size of some population if that population is composed of organisms that are necessarily discrete. There are *many* such models in biology.

# 3 Imagination and Conceptual Impossibility

Weisberg says very little about what he means by "imagination" or what it is for something to be "unimaginable." Based on what he does say, however, Weisberg appears to equate imagination with visual representation. For instance, he writes, "Even highly idealized models of the reactions of simple molecules consist of potential energy surfaces in state spaces of high dimensionality. No chemist can *hold this picture in her mind* and hence cannot directly reason about the imagined system" (2013, 69, my emphasis). Likewise, Weisberg seems to maintain that since a scientist cannot visualize a continuous population of rabbits—she cannot "hold this picture in her mind"—many ODEs in ecology and evolutionary biology are about model systems that are similarly unimaginable.

As Odenbaugh (2015) notes, Weisberg has adopted a quite narrow account of imagination. In fact, the type of imagination that Weisberg describes is sometimes referred to as "imagistic imagination" in order to distinguish it from other, non-visual forms of imagination (Van Leeuwen 2013; Salis and Frigg, forthcoming). A moment's reflection reveals this distinction to be sensible: we can imagine the *smell* of a campfire; what dog food might *taste* like; and

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[2]For a critical discussion of these other arguments, see Odenbaugh (2015).

what it would *feel* like to have one's tongue pierced. We can also imagine the content of a conversation we will have with a friend, in which case what we have access to are phonological representations (i.e., "inner speech").

Moreover, imagination often involves adopting an attitude toward a proposition, and such "propositional imagination" need not have a visual component either. I'll say that to propositionally imagine that $P$ is to act or reason as though $P$ were true while bracketing one's actual beliefs about $P$'s truth-value. (As with some other fictionalists, I will say that to imagine that $P$ is also to "pretend" or "make-believe" that $P$. The latter term is used in Kendall Walton's work, from which fictionalists take a great deal of inspiration (see, e.g., Walton [1990]).) So construed, one can imagine what is false, what is true, or what has a truth-value that is unknown.

What is important for our purposes is that one can imagine that $P$ without visualizing a world about which $P$ is true. For instance, if someone tells me to imagine that all tuna are mortal and that, moreover, Chucky is a tuna, I myself don't need to visualize any tuna before concluding that Chucky is mortal. This is even more obviously the case when someone tells me to imagine that all $T$s are $M$ and that $c$ is a $T$. Aphantasiacs' ability to voluntarily create visual imagery is diminished if not lacking entirely, but they are perfectly capable of doing logic or, for that matter, understanding a piece of literature (Zeman, Dewar, and Della Sala 2015).

Of particular relevance to the present discussion is whether we can imagine what is conceptually impossible. I agree with Weisberg, or at least I am willing to grant the point, that we cannot *visualize* what is conceptually impossible

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(e.g., an object that is both $A$ and $\neg A$; a squared circle; 3.1528 rabbits). But it does not follow from this that we cannot *imagine* these things.

In fact, it seems clear that we *can* imagine what is conceptually impossible.[3] Often, sentential expressions of this type of imagination have the form of a subjunctive or counterfactual conditional. Consider the following examples:

(a) "Were both $A$ and $\neg A$ true, then I could prove that $B$ is true."

(b) "If circles are squares, then Euclid had it all wrong."

(c) "If Hobbes had squared the circle, then he would have been a famous mathematician."[4]

(d) "If Hobbes had squared the circle, then the storming of the Bastille would have happened in June, rather than July."

The antecedents of these conditionals refer to what is conceptually impossible, but we can imagine (i.e., make-believe or pretend) they are true. After all, that is how we know that the consequents follow from the antecedents in the first three sentences, but not the fourth (see Priest [2016] for the same argument). Yet, unless the reader has some powers of visualization unknown to me, it seems we do not visualize the antecedents of these conditionals when we imagine their truth. Hence, not all imagination is visual, and, more to the point, we can imagine what is conceptually possible despite that we cannot visualize it.

So an account of imagination that treats imagination as a purely imagistic

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[3]In their taxonomy of forms of imagination, Salis and Frigg (forthcoming) refer to this type of imagination as a form of "counterfactual reasoning." Goodman (2006a, 2006b) would call it a form of "suppositional imagination."

[4]This example is adapted from Priest (2001, 66).

affair will fail. Which account of imagination will succeed? There is no consensus on this topic, in either philosophy or cognitive science. I have suggested, however, that propositional imagination works as follows: We "bracket" our actual belief about a proposition's truth-value while, simultaneously, provisionally assuming the proposition's truth and inferring its consequences. Applied to sentences (a)-(d), to assess the truth of each conditional we act as though the antecedent were true and then determine whether the consequent is thereby rendered true or at least plausible. Sometimes, as with (a), this inference seems deductive. But this is not always so. For instance, (c) seems to rely on causal inferences about the sociology of mathematics.

This account of propositional imagination is similar to Nichols and Stich's account of pretense (Nichols and Stich 2003, chapter 2). According to their account, an act of pretense involves manipulating premises in a "Possible World Box" (PWB). One imagines a possible world about which the premises are true, then infers the consequences. Crucially, the truth-values assigned to the premises in the PWB are "quarantined" from the rest of the cognitive system, so in the PWB we can treat as true what we might otherwise believe is false or unknown. This seems to be how propositional imagination works too. That's how we can reason about circles being squares without *actually* believing they are.

Now let's apply all of this to the fictionalist idea that model systems are "imaginary." Recall, I assume a model is a set of propositions.[5] When a modeler considers a model, she adopts a particular *attitude* toward these proposi-

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[5]I am certainly not the first to do so. See, for instance, Thomson-Jones (2006).

tions; namely, she imagines (or pretends or make-believes) they are true and, then, infers the consequences. (In the case of mathematical models, such inference often *will* be deductive.) According to the version of fictionalism I favor, imagination is involved here not because the modeler visualizes a corresponding model system, but because the modeler's belief that the propositions that constitute the model are true is provisional. Often if not typically, in fact, the modeler knows that various assumptions in the model are *false*—no population is infinitely large, or perfectly well-mixed, and so on—but she pretends they are true nevertheless.

When the role of imagination in modeling is construed in this manner, there is no basis to claim Lotka-Volterra, or other models that treat discrete organisms as infinitesimals, are unimaginable. A model system is imaginable if one can act or reason as though the propositions that constitute the corresponding model are true. And we clearly *can* act as reason as though the propositions that constitute Lotka-Volterra are true, since we can derive their mathematical consequences. That the model system cannot be visualized is irrelevant.

Before moving on, there are two further issues I should address. First, perhaps Weisberg does not believe visual representation is necessary for imagination, but rather that imagination requires a more general, "perception-like" experience. Since we can have no such perception-like experience of, say, a continuous population of rabbits, the argument would go, such a model system is thereby unimaginable. (I thank an anonymous reviewer for bringing this possibility to my attention.)

While this is a possible reading of Weisberg, it does not help his argument, for the simple reason that propositional imagination need not be perception-like in some broader sense either. For instance, I can imagine that all $F$s are $G$—all Flubs are Garb, say—while failing to have a perception-like experience of this fact, visual or otherwise.

Second, I should bring to the foreground another difference between Weisberg's account of modeling and my own. Weisberg describes mathematical models as "mathematical structures" that a modeler "interprets" or "construes" in a particular manner (2013, 29, 39). This is different from what I say modeling involves. A "construal," for Weisberg, establishes "relations of denotation between the model and real-world targets" (Ibid, 39). That is, a modeler interprets the mathematical expressions as being about a real world system. In contrast, according to my account, and to the fictionalist approach in general, a mathematical model is about an imaginary system that can then be compared to a real world system, if one wishes.

This last point raises an interesting epistemological question for the fictionalist: If model systems are imaginary, fictional worlds, then how do we compare these model systems to the real world? This is of course not the place to discuss this issue in full; I wish mainly to draw attention to its existence. But one plausible view is that the entities, interactions, and states of many fictional systems can be "close enough" to a real world system that comparisons between the two are sensible. Take the above ODEs. In the real world, animals are discrete and their evolutionary dynamics are stochastic, but biologists routinely use models that construe organisms as continuous and their dynamics

as deterministic, particularly when population size is large. The thinking here is that the stochastic behavior of a large population of discrete organisms is "close enough" to that of an infinite population of continuous organisms that a comparison between the two is justified. What counts as "close enough" in these and other cases is of course unclear, and presumably will vary with the goals the modeler has.
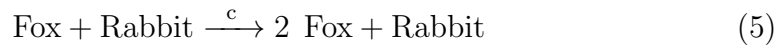
# 4   Imagination and Probability

The above discussion was focused on Weisberg's claim that ODEs can result in unimaginable model systems, but I have not addressed the other allegedly unimaginable models that Weisberg describes. These include "aggregate models...ensemble models, probabilistic models, high-dimensional models, and others" (2013, p. 63). I do not have the space to work through each of these independently. But in general, note that, once we recognize that imagination includes more than imagistic imagination, it is unclear on what basis Weisberg can claim these models are "unimaginable." If one can pretend or make-believe the models are true, then they are imaginable in the sense that matters to the fictionalist.

But I *will* explicitly discuss what Weisberg says about probabilistic models, since such models are quite common in biology, the primary focus of this paper.

Weisberg's argument that probabilistic models are unimaginable is based around his argument that "probabilistic interactions" are unimaginable. The idea is this: When you imagine, for instance, that a prey dies, you can only imagine that *single* death, which, according to Weisberg, is not sufficient for

you to imagine the death as a probabilistic event. As Weisberg explains, "Any given fictional scenario will be a single instantiation of the probabilistic interactions. But how can a single instantiation actually represent the probability?" (2013, 63). Weisberg then claims that, in order to imagine that the event is probabilistic, you must imagine that the animal is both alive and dead, which, he claims, cannot be done.[6]

To assess Weisberg's argument, let's consider a concrete example. In fact, let's consider the stochastic version of the deterministic Lotka-Volterra model that I described above. In the stochastic version, we have a finite set of discrete rabbits and a finite set of discrete foxes. The "reactions" in this system are as follows: at a rate equal to $a$, one rabbit gives birth to another rabbit; at a rate equal to $b$, when a fox and a rabbit meet, the rabbit dies; at a rate equal to $c$, when a fox and a rabbit meet, the fox reproduces; and at a rate equal to $d$, a fox dies. We can write this in the formalism common in systems biology (Wilkinson 2012, chapter 6).

$$\text{Rabbit} \xrightarrow{\;a\;} 2\ \text{Rabbit} \tag{3}$$

$$\text{Fox} + \text{Rabbit} \xrightarrow{\;b\;} \text{Fox} \tag{4}$$

$$\text{Fox} + \text{Rabbit} \xrightarrow{\;c\;} 2\ \text{Fox} + \text{Rabbit} \tag{5}$$

$$\text{Fox} \xrightarrow{\;d\;} \emptyset \tag{6}$$

The above describes a probabilistic system—rabbits and foxes reproducing

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[6]I am referring to a passage in which Weisberg claims "no individual can be both alive or dead" (2013, 63). Clearly, Weisberg meant to say "alive and dead."

and dying according to a continuous time Markov process. Now, suppose the next event in the time evolution of this system is that a rabbit dies. The model itself tells us the event is probabilistic; on this I am confident Weisberg and I agree. But, as mentioned, Weisberg says we cannot imagine this event as probabilistic because doing so would require us to imagine that the rabbit is both "alive and dead."

Let's set aside whether it is possible to imagine contradictions, and instead focus on Weisberg's claim that to imagine an event as probabilistic requires one to imagine that the event both occurred and did not occur (i.e., both $e$ and $\neg e$). This claim is incorrect. Indeed, the occurrence of two mutually exclusive events would violate the Kolmogorov axioms, since the probability of their joint occurrence is 0. Rather, to imagine an event as probabilistic is to imagine that the event occurred but that it was possible for the event to have not occurred (i.e., both $e$ and $\Diamond \neg e$). Applied to the present case, to imagine a single rabbit death as probabilistic is to imagine that the rabbit died but that some other event could have occurred instead. Even on the (problematic) supposition that imagination requires visualization, imagining counterfactuals of this sort does not seem particularly difficult. Our capacity to do so is what makes the literary genre of alternate history possible, after all. Philip K. Dick's *The Man in the High Castle* is about an America in which the Axis powers won World War II, rather than the Allies. The story is disturbing precisely because it portrays a world that is *easy* to imagine.

# 5 Making Fictionalism Safe for Formal Models

Where does all of this leave fictionalism? Some fictionalist work has assumed, or at least implied, that visualization is a central if not necessary feature of imagination. For instance, Godfrey-Smith says, "An imaginary population is something that, if it was real, would be a flesh-and-blood population" and that fictional worlds are similar to "entities like Sherlock Holmes' London, and Tolkien's Middle Earth" (2006, 735). Godfrey-Smith appears to characterize these "imaginary populations" imagistically. What we imagine when we consider a formal model in biology is a system of organisms or genes that are like the real thing; these model systems "would be concrete if they were real" (Ibid., 735). Levy is more explicit. As he explains, "Imagining typically involves having a visual or other sensory-like mental state—a 'seeing in the mind's eye'" (2015, 785).

Weisberg's argument shines a spotlight on the difficulties that any account of modeling will face if imagination is treated as central to modeling and visualization is regarded as a necessary component of imagination. Weisberg's criticism is therefore relevant not just to fictionalism, but to other accounts of modeling in which visualization is taken to play an essential role (see, e.g., Morgan 2012, chapter 3). Many models in science are simply too outré for us to visualize, and those that we *can* visualize constitute only a fraction—in some fields, a small fraction—of the models that people are interested in.

Fortunately for the fictionalist, imagination need not be yoked to visualization in this way. To my mind, the core feature of fictionalism is the way it

construes the attitude a modeler adopts toward a model. A modeler engages in a game of counterfactual make-believe when considering a model. She brackets her actual beliefs about the truth-values of the various propositions that constitute the model, then pretends all the propositions of the model are true in order to infer the model's consequences. A willingness to provisionally imagine to be true what we might otherwise know to be false is a noteworthy similarity between modeling and reading fiction. This component of fictionalism is defensible, once we appreciate the scope of imagination.

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