A tautological interpretation of Gödel's ontological proof

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ABSTRACT: Gödel's ontological proof is interpreted in a logically clear and sensible way without empirical and theological implications - rendering it mostly tautological interpretation-wise. Gödel's ontological argument thus cannot be said to prove existence of God. The real value of Gödel's ontological proof lies on the modal collapse consequence.

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1 Introduction

What I would like to demonstrate here is that there are ways to interpret Gödel's ontological proof that have formal logical and mathematical values, other than controversial interpretations of proving some God. In fact, under the new interpretation, Gödel's ontological proof will be shown to be largely obvious and tautological. The real value of the proof lies on its modal collapse outcome, not on other theorems.

2 Gödel's ontological proof

First, formal axioms of Gödel's ontological proof in higher-order modal logic presented by Dana Scott[1] as a refinement of Gödel's original axioms:

Ax. 1. $(P(\varphi) \wedge \Box \forall x(\varphi(x) \Rightarrow \psi(x))) \Rightarrow P(\psi)$
Ax. 2. $P(\neg\varphi) \Leftrightarrow \neg P(\varphi)$
Th. 1. $P(\varphi) \Rightarrow \Diamond \exists x\, \varphi(x)$
Df. 1. $G(x) \Leftrightarrow \forall\varphi(P(\varphi) \Rightarrow \varphi(x))$
Ax. 3. $P(G)$
Th. 2. $\Diamond \exists x\, G(x)$
Df. 2. $\varphi \text{ ess } x \Leftrightarrow \varphi(x) \wedge \forall\psi\,(\psi(x) \Rightarrow \Box \forall y(\varphi(y) \Rightarrow \psi(y)))$
Ax. 4. $P(\varphi) \Rightarrow \Box P(\varphi)$
Th. 3. $G(x) \Rightarrow G \text{ ess } x$
Df. 3. $NE(x) \Leftrightarrow \forall\varphi(\varphi \text{ ess } x \Rightarrow \Box \exists y\, \varphi(y))$
Ax. 5. $P(NE)$
Th. 4. $\Box \exists x\, G(x)$

We will assume Axiom K and B of modal logic (for proving Th. 1 to Th. 4) and Axiom T (for proving complete modal collapse from weaker modal collapses)[2].

3 Analysis of the proof

The fundamental question surrounding this proof is how we should interpret P. Of course it is possible to attach any meaning to P and G as some would say, but this is largely missing the mark. The real problem is how plausible and natural provided interpretations of P and G are. We may say P is probability-like or phone-like or what, but no one would say that these interpretations make sense. Thus valid criticisms of Gödel's ontological proof either provide sensible interpretations of P or deny axioms. It is the former - providing an alternative sensible interpretation of P - that I will carry out. It will also provide a natural motivation behind the axioms.

In a way, the strategy for the sensible interpretation is obvious: we take cues from the theorems derived - especially Th. 1, because it is the only theorem presented above that involves only P, not G.

Th. 1 says that if a φ satisfies P, then there must be y in at least one world that satisfies φ. Thus, we say that P is about predicate φ having satisfying y in at least one world. With this partial reading of P, a part of Ax. 1 comes to make sense: if $\varphi(y) \Rightarrow \psi(y)$ in some world, then if $\varphi(y)$ in some world, then ψ may satisfy P.

What then requires an additional definition of P is that Ax. 1 requires that in order for $\psi(x)$ to be confirmed to satisfy P from φ satisfying P, $\varphi(x) \Rightarrow \psi(x)$ for all x, not just for $x = y$, and this holds for all world. Thus, a predicate satisfies P if its satisfaction is a logical consequence of satisfaction of a predicate that is known to satisfy P, and that such an antecedent-consequent relationship holds for all worlds. Ax. 4 suggests that if a predicate satisfies P, then this must be the case for all worlds. These suggest the following reading of P:

Criterion 1. In order for a predicate φ to satisfy P: $P(\varphi)$, φ must have at least one satisfaction in some world: $\Diamond \exists x \, \varphi(x)$, but this does not have to be the case in all worlds - and such fact is known to all worlds: $P(\varphi) \Rightarrow \Box P(\varphi)$.

But it is possible that some predicates that satisfy Criterion 1 do not satisfy P. Criterion 1 is considered a necessary condition, as part of defining P, for a predicate to satisfy P. This is an important remark, because of Ax. 2, as will be explained below.

Criterion 2. If $\forall x \, \varphi(x) \Rightarrow \psi(x)$ for all worlds, and φ satisfies P, then ψ satisfies P.

Criterion 3. By Ax. 2: only one of φ or $\neg\varphi$ must satisfy P. And at least one of them has to satisfy P.

These criteria provide one mean of constructing P that satisfy Ax. 1, Ax. 2 and (by assuming) Ax. 4: pick one predicate φ that has one instance of satisfaction in some world - let us pick one satisfying element y and use it as a reference, and label predicates that are consequences of satisfaction of φ for all x and all worlds as satisfying P, along with φ itself. This by procedure satisfies Ax. 1 and Ax. 4. Ax. 2 can easily be satisfied, if we relax the construction mechanism: we simply choose predicates ψ that has $\psi(y)$ in a reference world, meaning y satisfies ψ in the reference world.

Let us use this definition of P: it checks whether a predicate can be satisfied by element y in some reference world. With this definition, Ax. 3 means $G(y)$ in some reference world, which is tautological by definition of $G(x)$ in Df. 1. Ax. 4 is really an assumption: it

means that all worlds know how the reference world behaves with regards to element y. It is somewhat like common knowledge in game theory.
Now Ax. 5: that necessary existence NE must be chosen to satisfy $P(NE)$. It may seem that we could simply set $\varphi = NE$ and forget about Ax. 5. But because of the way P is defined, this means: $\Diamond \exists x\, NE(x)$.
The main question thus becomes: is there an object of necessary existence in at least some world? Once we accept this, we arrive at Th. 4: for any world, some z that satisfies "all predicates with satisfying element y in the reference world" exists.
In other words, we can form the new definition for P : first, identify some reference world W where an element of necessary existence exists, and call the element y. Label all predicates that can be satisfied by y in W. This definition satisfies all axioms, arriving at exact same theorems. And interpreted this way, the theorems (Th. 1 to Th. 4) are largely boring - they are by the definition true.

3.1 Summary

We provided an interpretation into P (so-called "positiveness") such that $P(\varphi)$ is true if $\varphi(y)$ is true for pre-given (pre-fixed) element y in some pre-fixed reference world. $G(x)$ (so-called "God-like") thus refers to x having all properties (satisfying predicates) that pre-fixed y satisfy in the pre-fixed reference world.

Df. 1 (which is the only definition involving either P or G), Ax. 1, Ax. 2 and Ax. 4 become tautological by the above interpretation. Ax. 3 is also tautological as far as element y does exist and satisfies at least one predicate. But Ax. 5 is required to finally prove Th. 4, which says that for all worlds, there exists element x that satisfies all properties that y satisfy in the reference world. Thus we have to ask whether an object of necessary existence exists in at least one possible world. Furthermore, Ax. 5 dictates that our y is an object of necessary existence.

4 The actual value of the proof is modal collapse

What is not boring, then, is the complete modal collapse outcome[1] that is the theorem of the axioms provided. So far the definition does not directly provide hints that the complete modal collapse has to occur, but it has to be. We just asserted that there exists an object of necessary existence, and we eliminated needs for modal logic. The definition for P is largely innocuous, so would not have provided much power.
Would this be the reason to deny an object of necessary existence? The question is in fact far more complicated than what may be initially suggested - note that we have not provided some concrete definition of what "element" or "object" means. Instead of ordinary objects, an object may be some truth. We may say that $NE(T_s)$ represents necessary existence of some truth. There is nothing so far in the axioms and definitions that prevent such interpretations. Analytic philosophy has spawned massive studies based on ideas of necessary truth, so, that we can arrive at complete modal collapse outcome somewhat easily is a startling discovery, even when acknowledging that definitions of necessary do differ from one philosopher or logician to another.

In this form, Gödel's ontological proof gains another life other than proving God - it provides means of reflections for current progress of analytic philosophy, gives directions and hints for what definitions we should adopt and what conclusions we should support.

References

[1] C. Benzmüller and B. W. Paleo, *Automating gödel's ontological proof of god's existence with higher-order automated theorem provers*, in *Proceedings of the Twenty-first European Conference on Artificial Intelligence*, ECAI'14, (Amsterdam, The Netherlands, The Netherlands), pp. 93–98, IOS Press, 2014, DOI.

[2] C. Benzmüller and B. W. Paleo, *The Ontological Modal Collapse as a Collapse of the Square of Opposition*, pp. 307–313. Springer International Publishing, Cham, 2017.