Closing the superdeterminism loophole in Bell's theorem

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Abstract

It is well known that there is a freedom-of-choice loophole or superdeterminism loophole in Bell's theorem. Since no experiment can completely rule out the possibility of superdeterminism, it seems that a local hidden variable theory consistent with relativity can never be excluded. In this paper, we present a new analysis of local hidden variable theories. The key is to notice that a local hidden variable theory assumes the universality of the Schrödinger equation, and it permits that a measurement can be in principle undone in the sense that the wave function of the composite system after the measurement can be restored to the initial state. We propose a variant of the EPR-Bohm experiment with reset operactions that can undo measurements. We find that according to quantum mechanics, when Alice's measurement is undone after she obtained her result, the correlation between the results of Alice's and Bob's measurements depends on the time order of these measurements, which may be spacelike separated. Since a local hidden variable theory consistent with relativity requires that relativistically non-invariant relations such as the time order of spacelike separated events have no physical significance, this result means that a local hidden variable theory cannot explain the correlation and reproduce all predictions of quantum mechanics even when assuming superdeterminism. This closes the major superdeterminism loophole in Bell's theorem.

In 1964, based on the Einstein-Podolsky-Rosen (EPR) argument [1], Bell derived an important result that was later called Bell's theorem [2]. It states that certain predictions of quantum mechanics cannot be accounted for by a local hidden variable theory. However, there are a few loopholes in Bell's theorem, most notably the freedom-of-choice loophole or superdeterminism loophole [3-5]. Since no experiment can completely rule out the possibility of superdeterminism, including recent cosmic Bell test experiments [6,7], a local hidden variable theory can in principle be constructed to explain the quantum correlations in Bell-type experiments [8]. In this paper, we will try to close the superdeterminism loophole in Bell's theorem by a new theoretical analysis. The key idea is to find certain stronger quantum correlations. These correlations depend on the time order of spacelike separated events, while a local hidden variable theory consistent with relativity cannot explain such correlations even when superdeterminism is assumed.

Consider a usual EPR-Bohm experiment. There are two observers Alice and Bob who are in their separate laboratories and share an EPR pair of spin 1/2 particles in the spin singlet state:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2). \tag{1}$$

Alice measures the spin of particle 1 at angle a, and Bob measures the spin of particle 2 at angle b. These two measurements can be spacelike separated. Each measurement result is $+1$ or -1, corresponding to spin up or spin down. Then we can calculate the probabilistic correlation function $E(a,b)$ for Alice's and Bob's measurement results according to the Born rule, which is $E(a,b) = -cos(a-b)$. In particular, in the EPR anti-correlation case of $b = a$, we have $E(a,b) = -1$, which means that when Alice's result is $+1$, Bob's result is -1, and vice versa.

It has been known that a local hidden variable theory can explain the EPR anti-correlations, as well as the quantum correlations in Bell-type experiments when assuming superdeterminism. Then, can we close the superdeterminism loophole? If yes, how? As we will see, the key is to notice that although a hidden variable theory, local or nonlocal, adds hidden variables and their dynamics to quantum mechanics, it does not revise the Schrödinger equation for the wave function and the Born rule.[1] The universality of the Schrödinger equation permits that the time evolution of the wave function can be reversed in principle, e.g. the wave function of the measured system and the measuring device after a measurement can be restored to the initial one. In particular, a local measurement such as Alice's measurement on particle 1 can be undone locally in the sense that the wave function of the composite system after the measurement can be restored to the initial one by a local interaction with Alice and particle 1.[2]

Now consider a variant of the above EPR-Bohm experiment in which Alice's measurement on particle 1 can be undone locally.[3] First, suppose in

[1]Note that if admitting the collapse of the wave function, then it will be unnecessary to introduce hidden variables to solve the measurement problem.

[2]By comparison, if Alice's measurement on particle 1 collapses the wave function of the two particles including particle 2, then the wave function of all systems after Alice's measurement cannot be restored to the initial one by a local reset operation.

[3]There have been some discussions about such reset operations in the literature [9-15].

the laboratory frame (in which Alice's and Bob's laboratories are at rest), Bob first measures the spin of particle 1 at angle b and obtains his result, then Alice measures the spin of particle 2 at the same angle and obtains her result, and finally Alice's measurement is undone. This can be formulated as follows.

$$\begin{aligned} & U_A^{1\dagger} U_A^1 U_B^2 \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2) \left|ready\right\rangle_A \left|ready\right\rangle_B \\ & = U_A^{1\dagger} \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 \left|\uparrow\right\rangle_A \left|\downarrow\right\rangle_B - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2 \left|\downarrow\right\rangle_A \left|\uparrow\right\rangle_B) \end{aligned} \tag{2}$$

In this case, Alice's and Bob's measurement results will be anti-correlated. For example, when Bob's measurement result is $+1$, Alice's measurement result must be -1 with certainty.

Next, suppose in the laboratory frame Alice first measures the spin of particle 1 at angle $a = b$ and obtains her result, then Alice's measurement is undone, which restores the wave function of Alice and the particles to their initial state, and finally Bob measures the spin of particle 2 at the same angle and obtains his result. This can be formulated as follows.

$$\begin{aligned} & U_B^2 U_A^{1\dagger} U_A^1 \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2) \left|ready\right\rangle_A \left|ready\right\rangle_B \\ = \quad & U_B^2 U_A^{1\dagger} \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 \left|\uparrow\right\rangle_A - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2 \left|\downarrow\right\rangle_A) \left|ready\right\rangle_B \\ = \quad & U_B^2 \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2) \left|ready\right\rangle_A \left|ready\right\rangle_B \\ = \quad & \frac{1}{\sqrt{2}} (\left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 \left|\downarrow\right\rangle_B - \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2 \left|\uparrow\right\rangle_B) \left|ready\right\rangle_A \end{aligned} \tag{3}$$

In this case, Alice's and Bob's measurement results will be not precisely anti-correlated; when Alice's measurement result is $+1$, Bob's measurement result cannot be -1 with certainty. The reason is that if Bob's measurement result is -1 with certainty, then if Alice makes her second measurement her result will be $+1$ with certainty, which further means that if Alice's measurement and the reset operation are repeated a large number of times, Alice's results will be all $+1$, which violates the Born rule; the Born rule requires that the results of Alice's repeated measurements should be $+1$ and -1 with roughly equal frequency.

It can be seen that the correlation between Alice's and Bob's measurement results depends on the time order of these two measurements, which

Note that the reset operation only needs to restore the wave function of relevant systems. Even if there exist hidden variables of these systems and they are changed by Alice's measurement, the reset operation needs not restore them. This point is important for our later analysis.

may be spacelike separated. For measurements on an ensemble of the above EPR pairs of particles, when Bob makes a measurement before Alice's measurement and the reset operation, Alice's and Bob's measurement results are precisely anti-correlated, while when Bob makes a measurement after Alice's measurement and the reset operation, Alice's and Bob's measurement results are not precisely anti-correlated.

The existence of a stronger correlation that depends on the time order of spacelike separated measurements can be seen more clealy from the following experiment. Suppose in the laboratory frame, Alice first measures the spin of particle 1 at angle a and obtains her result, then Alice's measurement is undone (which restores the wave function of Alice and the particles to their initial one), and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then Alice's second measurement is undone, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at the same angle $b = a$. According to the Born rule, the probability distribution of Alice's results is $P(+1) = 1/2$ and $P(-1) = 1/2$.

Measurements	1	2	3	4	5	6	...
Results	$+1$	-1	-1	$+1$	-1	$+1$	...

Table 1: Alice's results when Bob finally measures

Now suppose in the laboratory frame, Bob first measures the spin of particle 2 at angle $b = a$, and then Alice measures the spin of particle 1 at angle a and obtains her result, and then Alice's measurement is undone, and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then Alice's second measurement is undone, and this process in Alice's side repeats a large number of times. In this case, according to the Born rule, the probability distribution of Alice's results is either $P(+1) = 0$ and $P(-1) = 1$ (when Bob's result is $+1$) or $P(+1) = 1$ and $P(-1) = 0$ (when Bob's result is -1).

Measurements	1	2	3	4	5	6	...
Results (when B $= +1$)	-1	-1	-1	-1	-1	-1	...

Measurements	1	2	3	4	5	6	...
Results (when B $= -1$)	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	...

Table 2: Alice's results when Bob first measures

It can be seen that the statistics of the results of Alice's repeated measurements are correlated with Bob's measurement choice, and the correlation depends on the time order of Bob's measurement and Alice's measurements, which may be spacelike separated. When Bob makes a measurement after Alice's measurements, Alice will obtain two different results, spin up and spin down, with roughly equal frequency, while when Bob makes a measurement before Alice's measurements, Alice will always obtain the same result, either spin up or spin down.

In a local hidden variable theory (consistent with relativity), relativistically non-invariant relations such as the time order of spacelike separated events have no physical significance, and thus the correlation between the statistics of Alice's results and Bob's measurement choice does not depend on the time order of Bob's measurement and Alice's measurements when these measurements are spacelike separated. This means that a local hidden variable theory is not consistent with quantum mechanics, whether it assumes superdeterminism or not. In other words, although a local hidden variable theory may explain the quantum correlations in Bell-type experiments when assuming superdeterminism, it cannot explain the stronger correlations here, which depend on the time order of spacelike separated events. This will close the major superdeterminism loophole in Bell's theorem.[4]

Note that at the end of the second experiment, all of Alice's measurement results are erased. Thus, the statistics of Alice's results can only be calculated from a theory, and it cannot be found by experiment. This is consistent with the no-signaling theorem; the correlations cannot be used for superluminal signalling. However, the correlations is enough for closing the superdeterminism loophole in Bell's theorem. The statistics of the results of Alice's repeated measurements can be properly defined and also precisely predicted by a local hidden variable theory and quantum mechanics. The result we have derived above is just that a local hidden variable theory cannot produce the statistics of Alice's results and thus cannot reproduce all predictions of quantum mechanics (even when assuming superdeterminism).

Finally, it is interesting to note that superdeterminism is untenable when the underlying dynamics is continuous, whether the theory is consistent with relativity or not. Superdeterminism admits the relation of temporal precedence but does not use it to explain the correlation between the results of Alice's repeated measurements and Bob's measurement choice. Rather, it resorts to the common cause in the past to explain the correlation. Then, if the underlying dynamics is continuous, the correlation will depend on the time difference between Alice's measurements and Bob's measurement continuously, which means that when the time difference is close to zero, the correlation will be close to a limit. But this contradicts the predictions of quantum mechanics. No matter how small the time difference, the correlation for the case of Alice's measurements preceding Bob's measurement and the correlation for the case of Bob's measurement preceding Alice's mea-

[4]This result may also close other loopholes in Bell's theorem, such as the retrocausality loophole. For a further analysis see [15].

surements are always greatly different; when Alice's measurements precedes Bob's measurement, Alice will obtain two different results, spin up and spin down, with roughly equal frequency, while when Bob's measurement precedes Alice's measurements, Alice will obtain the same result each time, either spin up or spin down.

To sum up, we propose a variant of the EPR-Bohm experiment with reset operactions that can undo measurements. We find that according to quantum mechanics, when Alice's measurement is undone after she obtained her result, the correlation between the results of Alice's and Bob's measurements depends on the time order of these measurements, which may be spacelike separated. Since a local hidden variable theory consistent with relativity requires that relativistically non-invariant relations such as the time order of spacelike separated events have no physical significance, this result means that a local hidden variable theory cannot explain the correlation and reproduce all predictions of quantum mechanics even when superdeterminism is assumed. This closes the superdeterminism loophole in Bell's theorem.

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