A Neo-Kantian Approach to the Standard Model

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Introduction

Let me begin with an admission: I am neither a Neo-Kantian myself nor a historian of philosophy. I became aware of Cassirer's work through my search for precedents for the kind of structural realism that Ladyman was developing, as captured in the slogan 'The world *is* structure'. As is now well-known, this differs from Worrall's form of structural realism in that the latter maintains 'All that we *know* is structure' and in his early writings, Worrall followed Poincaré in his insistence that the *nature* of the world, beyond this structure, was unknown to us. Subsequently he adopted a kind of agnosticism with regard to this 'nature' but in that earlier form we find certain Kantian resonances, which is not surprising of course, given its ancestry in Poincaré's work. One might initially think that the neo-Kantian would find Ladyman's collapse of 'nature' into 'structure' to be unfortunate but, of course, if one takes 'the world' of the realist to be the phenomenal world, with the noumena taken negatively and *not* regarded as the world of determinate but unknowable objects, in the way that Worrall conceives of it (and here I recognise that I am stepping into a minefield!) then there may not be such a chasm between these two views as might at first appear.

Having said that, and as I emphasised in my (2014), my intention was never to present Ladyman's form of structural realism as neo-Kantian (indeed, that would be close to oxymoronic!). Instead, my aim was two-fold: first, I wanted to bring back in to the light some of the history of structuralist thought about science that had been overshadowed by Russell's work. Typically, the latter's *Analysis of Matter* has been cited by both advocates and critics of structural realism (particularly with regard to the so-called Newman problem, widely but incorrectly perceived as a major issue for such a stance), with little or no appreciation of the fact that written as it was in 1926 and published in 1927, it contains only glimpses of the emerging theory of quantum mechanics. It was only subsequently that Eddington and, of course, Cassirer appreciated the significance of this new theory for the structuralist line of thought, particularly with regard to its impact on the notion of object. Incorporating this impact both protects their similar but crucially distinct accounts from the afore-mentioned criticisms (thus it enabled Eddington to sidestep the Newman problem in his debate with Braithwaite) and allows them to be further developed in fruitful ways that advocates of modern forms of structural realism have missed.

Secondly, and more provocatively, I wanted to present that history as a set of resources that the modern structuralist could appropriate. In my (2014) I described this – somewhat tongue in cheek – as a 'Viking approach', in the sense that history is viewed as there to be pillaged. Under the influence of kinder, gentler colleagues I've since come to refer to it as a 'toolbox approach' (cf. French and Mackenzie 2012), with the idea being that history can be seen as offering a toolbox of devices, moves and manoeuvres that one can draw on in the current context. Of course, that might be seen as sailing rather too close to a Whiggish attitude but in that case one can respond by asking how else can history be used to teach us?!

Certainly Cassirer's neo-Kantian stance towards quantum physics has a number of features that the structural realist might find attractive. Two of the most significant are, first, the ontological move away from objects that can be situated within a more general shift from substance that was driven by developments in modern physics—a shift in which not only Cassirer but also Eddington and Whitehead participated, in their very different ways. There is much yet to be said about this shift and how it shaped the responses of these different authors to the scientific developments of the time. In Cassirer's case, of course, the shift is from substance to function, in Eddington's it is to relational structures and in Whitehead's, famously, it is to process.

Secondly there is the prominent role given to high level physical 'principles', such as, crucially, *symmetry* principles within his overall structuralist conception. Again the emergence of such principles is a major feature of the development of modern physics but it is one that few commentators noted until comparatively recently. Cassirer was definitely 'ahead of the game' in this regard and it is perhaps in this respect that his framework meshes so well with the later developments that crystallised in the Standard Model. As is well known, and put bluntly, he suggested that this framework could be understood as a kind of non-hierarchical 'Parmenidean sphere', with such principles, laws and measurement outcomes all intertwined and mutually informing one another. As articulated within his classic work *Determinism and Indeterminism*, this represents a clear neo-Kantian view of quantum mechanics (see Cei and French 2009). One can obviously imagine it more or less straightforwardly extended to cover quantum field theory and since that underpins the Standard Model of modern physics, to the latter also. But in a sense, it already has been so extended; let me explain.

The Standard Model

At the core of the Standard Model lie certain symmetry principles. The importance of these in modern physics was noted above and has been well documented, not least in passages such as this, from Weinberg:

> Nature, like an enemy, seemed intent on concealing from us its master plan. … At the same time, we did have a valuable key to nature's secrets. The laws of nature evidently obeyed certain principles of symmetry, whose consequences we could work out and compare with observation, even without a detailed theory of particles and forces. There were symmetries that dictated that certain distinct processes all go at the same rate, and that also dictated the existence of families of distinct particles that all have the same mass. Once we observed such equalities of rates or of masses, we could infer the existence of a symmetry, and this we thought would give us a clearer idea of the further observations that should be made, and of the sort of underlying theories that might or might not be possible. It was like having a spy in the enemy's high command. (Weinberg 2011)

Here we have an expression of two of symmetry's crucial roles: the first is heuristic, in uncovering further observations and new theories (for one of the few explicit philosophically informed discussions of this heuristic role, see Post

1971). The second is metaphysical, in that symmetries are taken to dictate or govern the rate of certain processes, for example, or even the existence of certain kinds of particles. It is this latter aspect that I shall focus on here.

First let us begin with the grounds for Cassirer's core claim in *Determinism and Indeterminism*, namely that the appropriate lesson to be drawn from developments in quantum physics is not that we should abandon a causal view of the world, but rather that the notion of physical object must be cast aside. These grounds ultimately have to do with quantum statistics—that is, quantum mechanics as applied to assemblies of particles—at the heart of which sits the principle that quantum mechanics is 'permutation invariant'; or, in other words, that it does not matter to the relevant measurement outcomes whether the particles of the system are exchanged, or permuted, or not. As early as 1926 Born and Heisenberg noted the implication of this, to the effect that such particles could not be regarded as individual objects and this not only critically influenced Cassirer, but became part of the standard lore in the metaphysics of quantum mechanics (see French and Krause 2006).[1]

As a constraint this Permutation Symmetry divides up the Hilbert space into self-contained sectors, each corresponding to a certain fundamental kind of particle and yielding a particular form of quantum statistics, the two most well known being fermions, which obey Fermi-Dirac statistics and bosons, obeying Bose-Einstein statistics.[2] Thus, the most fundamental kinds in nature are 'dictated' by, or effectively drop out of, the action of this particular symmetry.[3]

Secondly, the underlying framework of the Standard Model is, of course, quantum field theory, so the second set of symmetries that needs to be considered are those of Minkowski space-time – the space-time of Special Relativity. These are the translations, rotations and 'boosts' that are captured mathematically by the Poincaré group and as Wigner famously showed, this yields a classification of all elementary particles, in terms of their mass and spin. Thus these fundamental properties also effectively drop out of this particular symmetry.

Finally, the Standard Model itself is fundamentally a gauge theory, represented mathematically by the group SU(3) x SU(2) x U(1) via which the relevant symmetries can be captured within the theory. This gauge-theoretic aspect refers to the way in which the Lagrangian of a system – which basically captures the dynamics of that system – remains invariant under a group of transformations, where the 'gauge' denotes certain redundant degrees of freedom of that Lagrangian. The generator of this group of transformations represents a field and when such a field is quantised, we get the so-called gauge bosons. Thus, consider electrodynamics, for example: here the relevant gauge symmetry group associated with the property of charge is labelled U(1) and the

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[1] As it turns out, it is not necessary to give up individuality in this context and hence the motivation for current forms of ontic structural realism is not this ,loss of individuality but rather that there is a kind of metaphysical underdetermination in play, in the sense that the physics cannot determine whether we should regard the particles as individual objects or non-individual objects.

[2] There are others, corresponding to so-called parastatistics.

[3] Thus Permutation Symmetry is represented mathematically by the permutation group, of course. It remains a source of puzzlement to me that Cassirer does not mention this in *Determinism and Indeterminism*, despite the fact that Weyl, for example, had emphasised its core role in the foundations of the new quantum theory.

requirement of gauge invariance yields a particular gauge boson, namely the photon. Thus the photon effectively drops out of the imposition of this further symmetry. This requirement can then be extended to the other forces in physics and so, for the weak nuclear force, we have the SU(2) symmetry group associated with isospin, a property of protons and neutrons, and the strong nuclear force associated with SU(3) which operates on the colour property of quarks. If we add the newly discovered Higgs boson associated with the breaking of the isospin symmetry of the unified electro-weak force and responsible for the acquisition of mass, we have a complete picture of the fundamental forces, gravity excepting of course.[4]

Now there has been a great deal written about the nature and role of such symmetry principles in modern physics (see, for example, the papers in Brading and Castellani (2003); also Brading and Castellani (2013); and for recent discussions, Greaves and Wallace (2014) and Friederich (2015)). A particularly apposite set of considerations—as far as this essay is concerned—can be found in Caulton (2015). Here he focuses on the dual aspect of the role of symmetries: on the one hand they are associated with certain well-known physical consequences. Consider the case of 'spin', which cannot be accommodated within classical physics but, as noted above, 'drops out' of the Poincaré symmetry. On the other hand, they are also associated with a kind of representational redundancy, as the examples of gauge and permutation symmetry illustrate. Take the latter: in classical statistical mechanics the permutation of two particles between two states, say, leads to a new arrangement that then must be counted in the combinatorial analysis that underpins Maxwell-Boltzmann statistics; but in quantum statistics, such a permutation is deemed not to lead to any new arrangement and thus the relevant 'count' is entirely different.[5] In this domain, the permutation is nothing more than a redundant feature of the representation and the constraint of permutation symmetry ensures that it is removed from effective consideration.

Caulton captures this distinction between those symmetries that have physical consequences and those that rule out certain features as redundant in terms of the labels 'analytic' and 'synthetic', drawing a comparison with analytic and synthetic propositions. Analytic symmetries on this view do not produce a different physical state and are true in all kinematically possible worlds. So, again, consider permutation symmetry in quantum physics: as noted above, this yields a division of Hilbert space into distinct sectors, corresponding to the different particle statistics. Thus a particle within the Bose-Einstein sector will behave as a boson and, it turns out, cannot escape to become a fermion, say. From the perspective of that boson, the other sectors (which include not only the Fermi-Dirac but also 'para-statistics') are all redundant theoretical structure.

By virtue of being true in all kinematically possible worlds, these analytic symmetries act as fixed points or constraints in any interpretation of a theory and determine the form of the representation relation between the relevant states as described mathematically and represented physically, via the partition

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[4] For structuralist accounts of the development of quantum gravity, see Rickles, French and Saatsi 2006.

[5] Thus, taking this case of two particles distributed over two states, in Maxwell-Boltzmann statistics we get four possible arrangements, but in Bose-Einstein statistics we get only three and in the Fermi-Dirac case, only one.

of the relevant space (the obvious example here being that of Hilbert space and permutation symmetry as just mentioned). The resulting constraints on a theory's representation relation amount to a precondition on that theory's formalism having empirical significance. Thus such symmetries cannot themselves be taken as empirical claims *per se*; rather they are the 'mediators' of the relationship between the mathematical formalism and the physical world (Caulton *ibid*.). In this regard, then, they serve a similar role to that of Reichenbach's constitutive a priori principles.

As is very well known, Reichenbach, in his early thought, distinguished between those synthetic apriori principles that are necessary and transcendent and those that are constitutive of our representations of the world. The latter, of course, are revisable. Putting things rather crudely, their constitutive role is cashed out in terms of underpinning the establishment of a form of coordination between some aspect of experience and our mathematico-scientific description of it. But of course, the delineation of that aspect arises in part via that coordination (Reichenbach 1965, p. 40). This resonates with a similar issue that arises within the structuralist framework: the appropriate mathematical description of symmetries such as permutation symmetry is group theory. But group theory requires an already available set of elements on which the group transformations act, in a sense. How can the structuralist shift focus from those elements to the structure embodied in the symmetry, or avoid reference to such elements entirely in favour of this structure, when the application of the relevant mathematics effectively begins by assuming such elements? This becomes a particularly acute issue in Cassirer's case: how can he push the notion of object away from centre stage, in favour of his Parmenidean sphere, when the mathematical description of the crucial general principles seems to demand that very notion? This was an issue faced by Poincaré and Weyl and Eddington and it crops up again and again. One way around it is to adopt the response of Poincaré himself and effectively take these elements to be no more than heuristic devices that enable the construction of the structural framework but which can then be discarded or, at least, downplayed in ontological terms (see French 2014 p. 66-68). Likewise, one can understand Reichenbach's coordination as proceeding iteratively: we begin with a set of elements – quantum particles say – that are initially conceived of as objects, in at least a minimal sense that permits the application of the relevant mathematics of group theory. Thus applying permutation symmetry, as encapsulated in the permutation group, we get the division between bosons and fermions (and other kinds if we are interested in parastatistics, as mentioned in footnote 2) and hence the relevant structure based kind distinction. But from this perspective, that distinction comes to be established on the basis of this Reichenbachian coordination.

Synthetic symmetries, on the other hand, do generate physical differences, as the afore-mentioned example of spin and Poincaré symmetry make clear. Caulton notes that these come in two kinds depending on whether we limit the *quantities* we want preserved or the *states* for which we demand preservation of values for the relevant physical quantities. Now, as science advances, we may find that certain symmetries generate no observable differences. These may be assimilated to the analytic kind, in two ways depending on the kind of synthetic symmetry involved. Thus, we may reduce the set of *quantities* we take to be physical, thereby reducing the set of natural properties and relations that according to the theory can be found in the world. Likewise, we may reduce the space of mathematical states taken to represent a physical state, again resulting in a reduction in the natural properties and relations, since the amount of possible worlds is decreased via which these may be distinguished (*ibid*.).

More importantly, perhaps, Caulton proposes the following meta-conventional principle: maximise the analytic symmetries of a theory, subject to empirical adequacy. This plays the role of Reichenbach's regulative apriori principles, used to eliminate universal forces in the case of relativity theory for example. Hence he proposes a two-stage process of theory construction: in the first stage, we establish representational connections between the theory and some aspect of the physical world, assuming the theory to be empirically adequate. In the second stage, we maximise the theory's analytic symmetries, taking advantage of the connections already established so as not to undermine the theory's empirical adequacy. If we adopt the Reichenbachian understanding of these symmetry principles, we can view this as a neo-Kantian approach to theory construction.

Mixing in a Little Metaphysics

Now, we shall return to this distinction between analytic and synthetic symmetries below, but turning our attention back to the physics, one might immediately question whether these symmetries are all there is to the Standard Model—if this is a model of elementary particle physics, where do the particles fit into the above schema? Or, to put it another way, how do the particles relate to the symmetries set out above? Here Cassirer has already suggested an answer, when he insists that we can talk about electrons, for example, 'only indirectly', '...not insofar as they themselves, as individuals, are given, but so far as they are describable as "points of intersection" of certain relations' (1936, p. 179; as cited in French and Ladyman 2003, which extends 'ontic structural realism' to QFT). But this invites the further question: *metaphysically speaking*, how should we conceive of the electrons, *qua* 'points of intersection—as derivative entities or as eliminated entirely? Adopting the first option amounts to a form of what has been called 'moderate' structural realism, while the second yields the 'radical' or 'eliminativist' form.

As for the metaphysical relationship between the relevant symmetries and the particles—described above in terms of the properties of the latter 'dropping out of' or being 'dictated by' the former— how is this to be captured? One option might be via some form of dependence relation, in the sense that these properties are understood to be *dependent* upon the relevant symmetries. Now, insofar as these properties are described by means of the mathematical representation associated with the relevant group, McKenzie has argued (2013 and 2017; see also Wolff 2012) that the nature of that relation is that of *inter-dependence*, formally between the relevant group and that associated representation which captures the particle properties, so adopting this metaphysical device is tantamount to adopting the moderate form of structural realism.

Alternatively, if one has eliminativist sympathies, one might conceive of the symmetries (and the associated laws) as *determinables* and the relevant properties as *determinates*, just as one conceives of red and shades or hues of that colour (French 2014 pp. 280-281). In this case there is no dependence but rather the relevant property is regarded as a more determinate feature of the symmetry *qua* determinable, just as crimson is a more determinate form of red. This captures the inherent modal character of these symmetries, in that they contain more possibilities than are actually realized—as noted above (fn 2), Permutation Symmetry offers more possibilities than the Bose-Einstein and Fermi-Dirac kinds that we observe in the actual world. These determinate kinds, any further properties that 'drop out' of the relevant symmetry group (suc as the Poincaré group for example) and also the concrete measurement results pertaining to these properties, can then be regarded as 'existential witnesses' (Wilson 2012) to the nature of the structure as a whole, via which it is effectively made actual. Using Cassirer's work as a toolbox, one can then conceive of the structure of the world from this perspective as an interlocking Parmenidean-like sphere of symmetries, laws, the properties they dictate and hence the measurement outcomes they yield, differentiated internally, as it were, by the determinable-determinate relationship as sketched here (French 2014 pp. 94).

A Cassirerean Framework for the Standard Model

My contention, then, is that the core ingredients of a neo-Kantian approach to the Standard Model can already be found in *Determinism and Indeterminism*, regarded as a historical toolbox as indicated above. Moving in the opposite direction to that of the above sketch, that is, from statements of the results of measurements to statements of laws and then to statements of the higher order principles, the moves involved represent 'decisive transformation[s]' (1936 p. 31). Thus, in the first case we have a move from immediate perceptual data to experimental observation, where the latter must be understood as a determination into which concepts of measure and number enter (what follows is taken from Cei and French 2009). From statements of measurement results to those of laws we shift from the 'space-time realm' in which individual facts are situated and thus from the 'here-now' to the 'if-then', or as I would say, from the determinate to the determinable. It is the laws that bring the particular into the whole via the notion of function and as noted in Cei and French (2009) there is a significant similarity between Cassirer's discussion of the form of laws and the kind of consideration that has motivated more recent forms of structuralism. Thus, he notes that once placed in this form, phenomena come to be established as 'enduring thoughts' (op. cit.., p. 38), giving as an example, Fourier's theory of heat which transcended the view of heat as a fluid via its mathematical description in terms of which the phenomena were represented as the results of 'purely geometrical relations'. It is this separation of the fundamental structure, as represented by the mathematical equations, from the underlying ontological commitments that was also noted by Poincaré and which motivated the likes of Worrall in his version of structural realism.

As Cassirer goes on to note, the endurance of this particular thought is further manifested in the deployment of Fourier's formulae by Heisenberg in his development of quantum mechanics, giving a nice example of what Cassirer calls the 'indwelling sagacity' (*Spürkraft*) of such forms. Here we see another commonality with more recent structuralist developments, as Saunders also notes this feature (1993), referring to it as the 'heuristic plasticity' of the equations as described mathematically and arguing that it is by means of this plastic mathematics that fundamental structural aspects of classical dynamics are isolated, become entrenched and are thereby preserved in subsequent developments. In particular, as Saunders notes, the group-theoretic features touched on above in particular provide '… over-arching abstract frameworks … within which one dynamical structure may be embedded in another' (*ibid*., p. 308), a comment with which Cassirer would surely agree.

From laws to statements of principle, or statements of third order, there is a further shift to a 'new dimension', involving what Cassirer refers to as an 'iridescent indeterminateness' (*ibid*., p. 51), or loss of subject of the principle, that allows them to function heuristically in the search for new laws. Unlike the latter, the principles do not refer directly to phenomena, but to '… the form of laws according to which we order these phenomena.' (*ibid*.). Such statements include those referring to symmetries, as well as causality of course. Crucially, they manifest a 'capacity for "synopsis"' (ibid.), which affords an overview of more than one physical domain, as manifested through their heuristic power.

These statements of measurement outcomes, laws and general principles mutually condition and support one another (*ibid*., p. 35) in a kind of 'reciprocal interweaving and bonding' (*ibid*.) that yields a '… functional coordination in which all the elements, all the determining factors of physical truth, uniformly participate' (*ibid*.).

Critically – and again this excites the modern day structuralist – Cassirer rejects any 'substantial carrier' within such a structure and insists that '… we do not need to posit objects as sundered beings-in-themselves behind these determinations' (*ibid*.). Here we see the further, more general, shift from objects to relations as the ground of objectivity in science; or as Cassirer put it, '[w]e are concerned not so much with the existence of things as with the objective validity of relations; and all our knowledge of atoms can be led back to, and depends on, this validity' (Cassirer 1936, p. 143). And in the quantum context, taking the 'conditions of accessibility' to be the 'conditions of the objects of experience', then '... there will no longer exist an empirical object that in principle can be designated as utterly inaccessible; and there may be classes of presumed objects which we will have to exclude from the domain of empirical existence because it is shown that with the empirical and theoretical means of knowledge at our disposal, they are not accessible or determinable' (*ibid*., p. 179). Thus, there are no epistemically inaccessible objects laying behind the structures which we can know; there is only the structure.

Of course, Cassirer was no realist, in the current sense and his neo-Kantianism is manifested, as is well known, in his rejection of the distinction between pure intuition and the understanding, in the sense that the pure intuition of space and time presents a kind of theatre in which the pure logic of understanding encounters the manifold of perception. Instead it is the notion of functional coordination that plays the relevant role, in that "[these] same basic syntheses upon which mathematics and logic rest, also govern the scientific structure of empirical knowledge and first enable us, by a fixed lawful ordering of phenomena to speak of its objective significance" (Cassirer 1907, p 45; quoted in English in Ryckman 1991, p. 65). Crucially for Cassirer, this established science and mathematics as having a common root in constructing the ideal concepts that order experience; in effect, mathematical and physical concepts are of the same kind.

Here we might note three points, the first two of which relate to a structuralist 'agenda': first, as already noted, this framework captures the roles of symmetry principles within modern science, where this includes not only their heuristic role but also the distinction suggested by Caulton above. Thus, the constitutive role of what Caulton calls 'analytic' symmetries is nicely captured here; consider, yet again, Permutation Symmetry. Although Cassirer, as I mentioned above, does not mention it explicitly in *Determinism and Indeterminism*, it is this that underpins the quantum statistics that he draws upon to reject quantum entities as substantive 'objects'. In effect, what we see here from the neo-Kantian perspective is the identification of representational redundancy which through the constitutive nature of this symmetry leads to the elimination of these objects in favour of a structural based objectivity. Secondly, this common root further underpins the comparison between electrons and geometrical points touched on above. Of course, the structural *realist* might want to resist any reduction of the physical to the mathematical that might be associated with such a commonality (see French 2014 Ch. 8). But for the structural idealist, the establishment of such a common root obviously yields a certain economy within the overall framework.

Thirdly, and finally, this commonality between mathematical and physical concepts captures the role of mathematics in science more generally. Consider the Standard Model again and one of its early successes: the prediction of the Ω^- particle. Very briefly, the history is as follows: the mathematical representation of the SU(3) symmetry in terms of group theory yielded a certain representation one node of which could not be identified with any then known particle. Given the physical interpretation of the rest of the framework and the known properties of the particles occupying the other nodes, physicists were able to predict the properties of this unknown particle, leading to its discovery. Now, the moves involved in this prediction have been presented as raising the following question: how is it that we can successfully reify mathematical structure in this way? (see for example, Bangu 2008). To some, this will seem to be part of the overall mystery of how highly abstract mathematics comes to be applied to concrete physical phenomena. The realist has one way of dissipating any sense of mystery by urging a focus on the physical reasons underpinning the deployment of such mathematics (see Bueno and French 2018). The neo-Kantian structuralist can offer an alternative account, building on Cassirer's work: it is through the conceptual activity of the mind that we obtain both mathematical concepts (such as those of group theory) and certain physical concepts (such as the associated symmetries). The successful prediction of the Ω^- can then be understood as the result of the above functional coordination of principles, laws and measurements in which all three uniformly participate as determining factors of the physical truth.

In the rest of this paper, I will consider two concerns: first, minor, that the structural realist cannot or should not appropriate Cassirer's framework in the manner suggested above; and second, major, that the downgrading of the role of intuition inherent to his neo-kantian vision raises obstacles to the extension of this vision to modern physics.

Realist Appropriation

With regard to the first, Mormann has objected to the appropriation of Cassirer's views by modern day structural realists, claiming that '... the structuralists of the 21st century try hard to play down the genuinely Neo- Kantian ingredients in Cassirer's thought that lead him to a structural interpretation of the new physics.' (Mormann preprint, p. 29). He goes on to argue that the appropriation of Cassirer's philosophy for structural realism '...is at variance with Cassirer's own interpretation of DI [*Determinism and Indeterminism*], according to which DI was a continuation and clarification of the account of scientific knowledge that he had formulated for classical physics some 25 years ago in SF ' (ibid., p. 34). And he concludes that '... Cassirer's idealist structuralism may have affinities not so much with some version of [structural realism], but rather with another species of structuralism, namely, the one which van Fraassen has characterized as „empiricist structuralism" (ES) ' (ibid., p. 31).

The central basis for this conclusion lies with the accusation that Cei and French (and before them, French and Ladyman 2003), ignore the role of measurement outcomes in their appropriation. Now, as should be obvious, that may be so for reasons having to do with the specific aims of those particular papers but in subsequent work that role has been considered, as the above summary should make clear (see again French 2014). Indeed, insofar as the structure presented by the relevant laws and symmetries is inherently modal in encapsulating more possibilities than are empirically observed, such measurement outcomes help underpin the crucial role of 'existential witnesses' as noted above. Crudely, it is via such measurements that we establish that certain properties allowed by the relevant symmetry are not in fact actualised.

More importantly, it is through consideration of the role of the measurement outcomes that Mormann relates Cassirer's framework to van Fraassen's 'empiricist structuralism', which, as is well-known, is grounded in the epistemic priority of measurement results and associated empirical sub-structures. However, leaving to one side the crucial point that Cassirer would have had no truck with giving measurement results such priority, given his view of the mutual conditioning of measurement, laws and general principles, the role he does assign to such results is compatible with the elimination of objects that the 'radical' form of structural realism advocates. The reason is obvious: what measurement gives us is not the object but only some determinate quantity. It is then a further metaphysical step to insist that such a quantity must 'inhere' in a substance or be bound together with other such quantities into a bundle constituting an object and of course those are steps that Cassirer would not take either.

But there is actually a broader and even more important point here, which is that the name of the game is not to show the *affinities* between Cassirer's neo-Kantian view and either ontic structural realism or empiricist structuralism—or at least not beyond the claim that all can be situated within a history of structuralist tendencies—but rather to simply appropriate certain moves and devices used by Cassirer for other aims. As I have noted above, there is the obvious concern that one cannot simply strip away such resources from their relevant context. Strictly this is correct: if one understands that resource in the terms in which it is originally presented, then those terms are such that they will indeed be bound to that original context. But again if context can never be peeled away, then what use will history be to us? Any device or claim becomes so context-bound it can serve as little more than a historical curiosity. However, I would argue that there is a sense in which we can pull such resources out of the relevant context and that even if by doing so we re-shape them in some way, yet they may be sufficiently similar as to be seen as sufficiently closely related but not so much that we must drag the associated interconnected meanings when we deploy them. Thus, '… we can take Cassirer's claims about the relative fundamentality of laws as compared to objects, and the shift in objectivity, and relate them to the debate over the metaphysical elaboration of structural realism without having to bring with them the associated claims about the ultimate grounding of such laws in mathematics, or the way that objectivity is constituted rather than given.' (French 2014, p. 100).

Capturing the Expansion in Possibilities

The second concern is, however, both more important and more subtle. Friedman has argued persuasively that the neo-Kantian downplaying of the faculty of pure intuition/sensibility in favour of pure *understanding* implies that it is not in fact the case that developments in modern physics can be accommodated 'without difficulty'. This of course goes to the heart of the suggestion articulated here, that Cassirer's framework can be extended to cover the Standard Model. At its core, Friedman's problem is one that has to do with the heuristics of theory change: how can the fundamental mathematical and physical concepts of a given theory give rise to— and be replaced by—those of its successor, given that by virtue of the context of the former such concepts are not inter-translatable with those of the latter? As a result of this lack of translatability, from the perspective of the context in which the original theory is embedded, the putative successor is not even conceivable. Thinking specifically of the example of General Relativity, Friedman claims that it is not just that '… Einstein's theory is not even *mathematically possible from the point of view of Newton's original theory*' but that it has to be shown that '… Einstein's new theory is *empirically or physically possible as well*'.

Let us consider this example in a little more detail. General Relativity incorporates two features that distinguish it from Newton's theory of gravity: the equivalance of inertial mass and gravitational mass (which Newton had kept distinct, grounding that distinction metaphysically) and the representation of space-time in terms of non-Euclidean geometry. Thus the theory required a genuine expansion of the space of intellectual possibilities (both mathematical and empirical). The problem then becomes one of explaining how such an expansion is possible—since before the expansion in question, from the Newtonian perspective, the new theory is not even physically possible (in Kantian terms, it is neither logically nor really possible*)*. According to Friedman, that intellectual space opened up with Einstein's formulation of the Principle of Equivalence which asserts the equivalence of gravitational mass and inertial mass. Together with the Riemannian manifolds on non-Euclidean geometry, this then acted as part of a constitutively a priori framework for the new theory.

But how was this expansion achieved? What was needed was '… a set of parallel developments in contemporaneous scientific philosophy to tie together the relevant innovations in mathematics and physics and thereby effect the necessary expansion in our physical or empirical possibilities' (Friedman 2008, p. 250). Einstein was only able to do this by '… *delicately situating himself within* *the earlier philosophical debate on the empirical and conceptual foundations of geometry between Helmholtz and Poincaré.* (ibid.) In other words, Einstein not only appreciated the heuristic force of the existence of the 'footprint' of the Principle of Equivalence within Newton's theory (see, again, Post op. cit.) which motivated his famous thought experiment, and was effectively handed the relevant mathematical tools by his friend Grossman, but he was also appropriately situated philosophically in the context of previous debates regarding the foundations of geometry.[6]

Given this, Friedman argues that what was needed was a more far-reaching revision of Kantian transcendental philosophy than Cassirer had suggested, in the sense of one that allows for such an expansion in the space of possibilities. Friedman draws on Kuhn's philosophy of science here, arguing that it is the incommensurability of certain theoretical terms in Newtonian and relativistic mechanics that blocks the accommodation of General Relativity within Cassirer's framework. As a result, he suggests, what is needed is a relativization of the Kantian apriori to a given scientific theory *in a given historical context* (or paradigm, if we remain within Kuhn's framework), which amounts to historicizing the very notion of transcendental philosophy itself. It is for this reason that General Relativity cannot be incorporated within transcendental philosophy "without difficulty," since the latter, at least as Kant originally proposed it, is unavoidably committed to the *a priori necessary* validity of both Euclidean geometry and the fundamental principles of Newtonian mechanics. (op. cit., pp. 250-251). As Friedman puts it:

'… whereas Euclidean geometry and the Newtonian laws of motion were indeed necessary presuppositions for the empirical meaning and application of the Newtonian theory of universal gravitation (and they were therefore constitutively a priori in this context), the radically new mathematical and physical framework consisting of the Riemannian theory of manifolds and the principle of equivalence defines an analogous system of necessary presuppositions in general relativity. Moreover, what makes the latter framework constitutively a priori in this new context is precisely the circumstance that Einstein was only able to arrive at it in the first place by self-consciously situating himself within the earlier tradition of scientific philosophy represented (especially) by Helmholtz and Poincaré…' (ibid., p. 251)

Recall what I just said (following Post op. cit.) that in the Newtonian framework the equivalence of inertial and gravitational mass is a kind of heuristically loaded footprint of Einstein's Principle of Equivalence: as far as Friedman is concerned, the force of this 'footprint' takes us from what was a contingent fact to what was, for Einstein, a constitutive principle.[7] How, within the neo-Kantian framework, can such a shift take place? It is not just a matter of

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[6] Einstein was not of course uniquely situated in this respect since as is well-known Hilbert also developed the field equations of General Relativity. Whether similar factors enter into this expansion of the space of possibilities from the perspective of Hilbert's situation is a further interesting question.

[7] Post (1971) referred to it as a 'footprint' of the new theory in the old and it would be interesting to consider how many such footprints in the history of science are indicative of such a transformation from contingency to constitutive.

tying the apriori to a particular theory but, according to Friedman and as just noted, that of relativising it to a particular context in which Einstein, in this case, was situated.

We can perhaps see what is in play here by considering one of our central symmetry principles discussed above, namely Permutation Symmetry, which falls on the side of the constitutively a priori in Caulton's division. With its articulation by Weyl and Wigner in particular, it can be considered one of the "system of necessary presuppositions" of quantum mechanics, forming part of the constitutively a priori framework, together with Hilbert space.[8] One could perhaps draw an analogy with the Principle of Equivalence above and see in the history of the emergence of such symmetries in physics a shift from a contingent fact (relating to the statistical behaviour of an assembly of quantum entities) to a constitutive principle (see Bueno and French 2018).

But then it is in this history that we can find the response to Friedman's problem of how the space of possibilities comes to be expanded. First of all, claims of incommensurability between theories have been widely attacked and dismissed in recent years and Kuhn's framework has crumbled under sustained criticism.[9] Secondly, but relatedly, if we look at the beginnings and ends of theoretical developments, we may well wonder how on earth scientists got from 'there' to 'here' and imagine that some precise set of circumstances had to be in place to effect the transition. So, we begin with Newtonian mechanics and end with General Relativity and we seem to have such a blatant disparity between the two that we are forced to ask how it was that something that was not even a mathematical possibility from the Newtonian point of view, comes to be a physical actuality? But if we track the individual moves made and the changes effected through the process, as Einstein elevated the empirical equivalence of inertial and gravitational mass into a principle and appropriated non-Euclidean geometry as a tool by which he could mathematically express his new theory, then we an begin to see how this expansion takes place, piecemeal fashion. Likewise, we can follow the moves made by Weyl, Wigner, von Neumann and others in the application of group theory to quantum mechanics and the concomitant elevation of certain expressions of symmetry (such as permutation symmetry but also including rotational symmetry) to the status of fundamental principles (see again Bueno and French 2018). These moves involve a complex but unmysterious intertwining of both mathematical and physical developments and, paraphrasing Friedman, we might perhaps say that Weyl and Wigner were only able to arrive at the introduction of group theory in quantum mechanics and the elevation of symmetry within modern physics by self-consciously situating themselves astride both sets of developments. By tracking this intertwining we can appreciate how Permutation Symmetry, for example, comes to be elevated to the status of a constitutive apriori principle as identified by Caulton.

Perhaps it was because the development of General Relativity was largely the work of one man (the afore-mentioned parallel research of Hilbert notwithstanding) that leads Friedman to try to solve the heuristic problem in

[8] Given Friedman's claim about Einstein above, it is interesting to note that in his later application of group theory to perception, Cassirer explicitly situates himself between Helmholtz & Poincaré (1944).

[9] Indeed, the kinds of heuristic moves hinted at here, involving symmetry principles and 'footprints' were originally articulated by Post as part of a general critique of Kuhn's work.

terms of Einstein personally situating himself within a particular philosophical debate. But then the following concern arises with regard to the way that Friedman frames the issue: given this shift in the status of the Principle of Equivalence, from contingent fact to constitutive principle, '… it is hard to see how the way in which Einstein self-consciously situated himself could be what *made the principle of equivalence constitutively a priori in the new paradigm.* ' (Chignell 2008, p. 258). Such constitutive principles must be accepted if we accept the theory itself. But then '… the fact that a particular principle is one of these presuppositions and thus constitutive *a priori relative to the theory in question appears to be a hard fact of transcendental logic.'* (ibid.) And it is difficult to see how such 'hard facts' can be made to obtain … *simply in virtue of the way that someone discovers them.'* (ibid.). In the case of symmetries and quantum mechanics, various personalities were 'in play', as just noted, with very different philosophical backgrounds and agendas (if they had any at all). As a solution to a historical problem, highlighting such personalities leaves in the dark the forces that lie behind the various moves that are made. Instead, by focusing on the small scale moves and manouevres that were undertaken at the time we can see how the expansion of possibility occurred, one step at a time.[10]

Curiously, Friedman himself doesn't discuss group theory in the context of quantum mechanics[11], as Corfield notes, '…in his book, the only pieces of mathematics we hear about are Euclidean geometry, the calculus, Riemannian geometry, Hilbert's Foundations of Geometry, then the possibility of quantum logic. ' (Corfield, preprint). Nevertheless, Friedman does argue that quantum theory falls prey to the Kuhnian critique, particularly if we focus on the contrast with classical physics, as he insists that here we lack the means 'to rationally bridge the gap between prerevolutionary and post-revolutionary conceptual landscapes" (Friedman 2001, p. 120). Again, however, this is too quick: recall what was said above about what Cassirer called the 'indwelling sagacity', or what Saunders referred to as the 'heuristic plasticity' of certain mathematical devices, such as Fourier analysis or the Poisson bracket, which can be seen as playing crucial roles in the early development of quantum theory.[12] Again, if we look closely at the moves actually made at the time we can see how to bridge the gap that Friedman's framing of the issue appears to reveal.[13]

Having said all of that, still the core issue remains: can Cassirer, with his rejection of pure intuition, accommodate these fundamental shifts? According to Friedman, ''[b]y rejecting Kant's original account of the transcendental schematism of the understanding with respect to a distinct faculty of sensibility in favour of a teleologically oriented 'genetic' conception of knowledge, Cassirer (and the Marburg School more generally) has thereby replaced Kant's constitutive a priori with a purely regulative ideal'' (Friedman 2000, p. 117). And in the absence of the former, Friedman claims, we cannot capture the transition

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[10] Of course, for the realist, the Principle of Equivalence and Permutation Symmetry are 'hard facts' insofar as they are part of the fundamental structure of the world.

[11] But then, as noted previously, Cassirer, despite invoking group theory in his consideration of the theory of perception, doesn't mention it in D&I.

[12] For a more recent account of how the structuralist can accommodate the shift from classical to quantum mechanics see Thebault (2016).

[13] In effect this amounts to a response to the Kuhnian basis of this framing.

from the theory of Newtonian gravity to General Relativity within a Kantian framework.

However, this is perhaps to give Cassirer short shrift. As noted here and highlighted by Ferrari (2012), Cassirer maintained a functional understanding of the apriori in terms of which, he argued, developments in modern science could be accommodated. With regard to both General Relativity and quantum mechanics, he attempted to preserve a Kantian stance in light of the impact of these theories on space-time and causality respectively. Thus the latter, as indicated above, effectively gets folded into the general requirement of lawfulness and the full impact of quantum mechanics is reoriented and directed at the notion of 'object' rather than causation. And it is as a result of the role of this lawfulness that we can claim that there is objectivity (rather than objects), understood functionally. As Ferrari notes, in a letter to Schlick, Cassirer emphasizes his view of the apriori '… not as a steady and definitively established complex of material intuitions or concepts, but only as a function, which is determined according to a law and therefore it remains the same regarding its direction and its form; nevertheless it can assume the most various developments in the progress of knowledge. ' (Cassirer 2009, p. 50). As a general principle, it can be understood simply in terms of the unity of nature, or the lawfulness of experience in general, but how this comes to be concretely specified in particular terms depends on the progress of science. And, recalling the Reichenbachian considerations touched on earlier, Ferrari also reminds us of a letter from Cassirer to Reichenbach in which he accuses the latter of presenting Kant's approach as in a 'very excessive opposition' to his (Reichenbach's) and that in fact, understood in the way that Cassirer proposes, the two are not so far apart.

Given this, let us now return to Friedman's claim that Cassirer has modeled his conception of the apriori on Kant's conception of regulative principles and thereby sees the progress of science as consisting of a converging sequence with what is apriori taken to be that which remains invariant throughout that process. Here one can question whether Cassirer's universal invariants of experience possess *no* constitutive role. As Ferrari notes, 'Space, time, causality and so on were the most general forms in which scientific experience became available and could be structured, without assuming any particular content as already embedded in such a priori invariants. ' (Ferrari 2012, p. 24). Of course, Cassirer took the distinction between constitutive and regulative principles to be weaker than in Kant's view, precisely because he viewed the development of scientific knowledge as a process involving the elimination of substantival concepts and the elevation of a functional picture. However, '… this signifies by no means that Cassirer transformed the constitutive and historical changeable, relativized a priori into a ''purely'' regulative one.' (ibid.). As Ferrari goes on to state, '…Cassirer emphasized that the proper task of the inquiry into the ''conditions of possibility of experience'' (that is not only scientific experience, but any kind of cultural experience as well as of symbolic form) can be seen in the individuation of constitutive principles making it possible that experience appears as a structured whole…' (ibid., pp. 24-25).

Thus it is not the case that the constitutive has been entirely replaced by the regulative. Indeed, granted that the distinction between constitutive and regulative principles was weaker for Cassirer than in the Kantian case, he understood them as mutually cooperating in the grounding of scientific experience. Given this, there is still space, as it were, within Cassirer's framework for a constitutive element to play a role and coupled with the historical point above regarding the steps actually made in moving from Newtonian theory to General Relativity, or from classical to quantum physics, we can accommodate the required expansion of the space of possibilities that Friedman illuminates. Indeed, if we regard Permutation Symmetry as a constitutive apriori principle in the way that Caulton suggests, we can see the excavation of the significance of this principle for quantum mechanics as an example of just such an expansion. And what motivated it was precisely the undermining by quantum statistics of the notion of 'object' that Cassirer claimed meshed so beautifully with Kant's philosophy, as he spelled out in detail in *Determinism and Indeterminism*.

Conclusion

As Ferrari suggests in the conclusion of his own paper (op. cit., p. 25), Cassirer can be regarded as a 'very modern' philosopher, not least in his approach to science. And as I have tried to indicate here, he can be regarded as a philosopher of modern science, in the form of quantum theory and the Standard Model. There seems to be no insurmountable obstacle to extending the framework laid down in *Determinism and Indeterminism* to the physics of the latter; nor, I would maintain, should there be any objection to appropriating aspects of this framework for realist purposes. Of course, whether the neo-Kantian stance or some form of structural realism is to be preferred in this context is another question entirely and one that I shall leave for another opportunity!

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