Direct and indirect empirical statuses compared to the Newtonian and Leibnizian interpretations of theoretical symmetries in physics

Valeriya CHASOVA
Centre de philosophie des sciences et sociétés
Institut supérieur de philosophie
Université catholique de Louvain

The study of the ontology of theoretical symmetries in physics has long being the question of choice between the usual Newtonian and Leibnizian interpretations. These were saying respectively that a theoretical symmetry corresponds to unobservable changes or is a mere mathematical surplus. But recently the notions of direct empirical status (DES) and indirect empirical status (IES) were introduced in addition. DES is usually defined as a status that a theoretical symmetry has when it is matched with an empirical symmetry in the world. IES is usually described as an empirical status opposed to DES and exemplified by the fact for theoretical symmetries to entail conservation laws via Noether's theorems. In this contribution I provide a framework for thinking of the ontology of theoretical symmetries in terms of their components, describe with its help the usual Newtonian and Leibnizian interpretations of theoretical symmetries, compare with them the ontological significance bestowed on theoretical symmetries by DES and IES, and generalise these empirical statuses further. I also pay a particular attention to whether global/local distinctions matter to any of the ontological interpretations and statuses considered, and the conclusion is that they do not.

Introduction

Virtually any theory of physics has symmetries, so it must be explained what are they doing there. And in parallel there is a debate between scientific realists and anti-realists as to whether various theoretical elements have ontological significance. If one clarifies within it whether theoretical symmetries have ontological significance, this would tell whether theoretical symmetries are so common because they are ontologically significant or by another reason.

Until recently just two main positions towards the ontological significance of theoretical symmetries were being discussed. On the one hand, there was a "Leibnizian" interpretation of theoretical symmetries saying that they are a mere mathematical surplus. On the other hand, there was a "Newtonian" interpretation saying that theoretical symmetries may correspond to unobservable changes in the world. However, the latter interpretation was heavily criticised because of postulating absolute space and similar entities. And while the former interpretation was dominating, it was not satisfactory either, be it from the explanatory or from the realist perspective.

But since Kosso [2000] two new ontological notions come into play: direct empirical status (DES) and indirect empirical status (IES) of theoretical symmetries. So, are these competing with the two previous interpretations, and what do they tell us about the link between theoretical symmetries and the world?

Below I present a simple framework for speaking of symmetries and of the ontology of theoretical symmetries in terms of their components; show that it applies well to the two usual ontological interpretations of theoretical symmetries; explain within the same framework how DES and IES differ from these interpretations; tell how to generalise DES and IES further; and explain in parallel why global/local distinctions do not matter for my topic.

A framework for symmetries and the ontology

Nowadays a symmetry is usually defined as an invariance under a transformation. This definition covers well both ordinary and scientific uses of the term of symmetry. However, a symmetry transformation often not only preserves something but also changes something else. This may and in fact does matter for the ontology. So to be able to track this I take a *symmetry* to have as its *constituents* a transformation, an invariant part and sometimes also the differing parts whose differences are induced by the transformation. I also call items linked by a transformation *states*, so a *symmetry* can alternatively be defined as consisting of a transformation plus two to some extent identical states.

I call something *physical* if it occurs in the world. There can be physical symmetries, in particular. Example is a physical symmetry constituted by the fact that the initial disposition of chess figures is invariant under a horizontal reflection passing through the centre of a chessboard, although the colours of the figures are switched under this reflection. Here the reflection is a physical symmetry transformation, relative positions of the figures are the invariant part of the physical states and the figures' colours are the differing parts of the physical states.

I call something *theoretical* if it occurs in a theory, particularly in a theory belonging to physics (it is somewhat confusing not to call this 'physical', but let it be so). There can be theoretical symmetries as well. Examples are symmetries of equations of motion. There a theoretical transformation usually consists in changing the values of some variables, the differing parts of the theoretical states are the values of these variables before and after the transformation and the invariant part of the theoretical states is the form of equations of motion.

Any component of something theoretical will be called *a theoretical element*. Examples of theoretical elements are the constituents of a theoretical symmetry, namely the theoretical transformation, the invariant part of the theoretical states and their differing parts, but also any components of any of these constituents. *Observational consequences* are theoretical elements which can be interpreted as predictions about what can be observed and which follow from other theoretical elements which cannot be so interpreted; the latter theoretical elements are called *theoretical underpinnings*. In particular, it is possible to distinguish observational consequences (void or not) and theoretical underpinnings within each constituent of a theoretical symmetry. I call theoretical transformations and symmetries *observationally complete* when observational consequences of transformations are void, i.e.when transformations change at most some or all of the theoretical underpinnings while leaving all the observational consequences of the states invariant, and *observationally incomplete* when some of the latter are (also) changed. The term '*gauge symmetries*' from the literature on symmetries often designates observationally complete theoretical symmetries.

Analogously to theoretical elements, observational consequences and theoretical underpinnings one can define *physical features* as parts of the world, *observable features* as physical features which are, were, will be or could be observed and *unobservable underpinnings* as physical features which are not observable features and which underlie the latter. *Observable (in)completeness* for physical transformations and symmetries can be defined as the observational (in)completeness for theoretical transformations and symmetries provided we replace theoretical underpinnings by unobservable underpinnings and observational consequences by observable features.

Put simply, *the ontology of science* is the study of whether given theoretical elements or their collections have proper physical counterparts. For observational consequences *proper physical* *counterparts* are relevant observable features, for theoretical underpinnings these are relevant unobservable underpinnings and for observationally (in)complete theoretical symmetries these are relevant observably (in)complete physical symmetries.

If a theoretical element has a proper physical counterpart, I say that this element is *ontologically significant*. For example, a prediction of a lightning by a theory is an observational consequence which is ontologically significant if it can be matched with an observable lightning in the world. Likewise, values of the electromagnetic potential distributed across spacetime are theoretical underpinnings which are ontologically significant if there is a corresponding unobservable electromagnetic potential field in the world. Meanwhile, as we will see empirical symmetries leading to DES are specific observationally incomplete physical symmetries, so they could make ontologically significant some theoretical symmetries.

An ontological interpretation of a theoretical symmetry should say of different components of a theoretical symmetry whether these are ontologically significant. Thus it may say so for the constituents of a theoretical symmetry, or for observational consequences and theoretical underpinnings of each of these constituents, or for some other theoretical elements distinguished within a theoretical symmetry.

The usual interpretations

Newton and Clarke in his correspondence [1717] with Leibniz thought that putative physical spatiotemporal transformations on the whole universe could transform absolute space and time, i.e.constitute observably complete physical symmetries. By contrast, Leibniz thought that as these putative physical transformations would not change anything observable, there is also nothing unobservable they could transform. So Leibniz would not associate observationally complete theoretical spatiotemporal transformations on the whole universe and the theoretical changes they induce with any proper counterparts, while Newton and Clarke could do so. I call these two positions respectively *the Leibnizian* and *the Newtonian interpretations of theoretical symmetries*, because the theoretical transformations and changes they concern can be thought of as constituents of observationally complete theoretical symmetries.

These two interpretations, and especially the Leibnizian interpretation, still remain popular nowadays for observationally complete theoretical symmetries. As various global/local distinctions are sometimes claimed to be relevant to DES and IES, it is worth mentioning that none of them determines whether a theoretical symmetry is observationally complete and thus subject to the Newtonian and Leibnizian interpretations. Indeed, whether *global* means 'universal' or 'expressible using parameters' or 'effectively uniform', and the negation of whatever of this qualifies as *local*, spatial translations on the whole universe come out as global and electromagnetic potential transformations of classical electromagnetism come out as local and yet both give rise to observationally complete theoretical symmetries.

My framework allows to delimit the domains of application of the Newtonian and Leibnizian interpretations more clearly by saying that they primarily apply to observationally complete theoretical symmetries given that they originally concerned observationally complete theoretical transformations. Also, my framework is better than the usual definition of symmetry for expressing these interpretations, because they concern the differences induced by a transformation in addition to a transformation itself, and only my framework takes these differences into account. Besides, my framework allows to measure *the strength of an interpretation* by the number of constituents of a theoretical symmetry to which the interpretation assigns the ontological significance. And so it shows that in this sense both the Leibnizian and the Newtonian interpretations are rather weak. Firstly, this is because both can be read as concentrating on the ontological significance of the transformations and of the differing parts alone, in which case they are compatible with the negation of the ontological significance for the invariant part. Secondly, this is because the Newtonian interpretation can be read as assigning merely possible but not actual counterparts to the transformations and the differing parts. So it is compatible for instance with supposing the absolute space to be actually immutable, in which case among all possible distributions of values of absolute positions which entail the same correct predictions of what we observe, and hence are linked by theoretical symmetry transformations, only one distribution will have an actual counterpart in the world.

Are DES and IES stronger alternatives to the usual Leibnizian and Newtonian interpretations, and do they concern the same theoretical symmetries? I will next try to clarify this within the same framework.

DES

DES is a status that a theoretical symmetry has when it is matched with an empirical symmetry. It was introduced by Kosso [2000] and discussed by Brading and Brown [2004], Healey [2009] and Greaves and Wallace [2014], among others.

Empirical symmetry is usually defined as kind of observationally incomplete physical symmetry exemplified by Galileo's ship, Einstein's elevator, Faraday's cage and 't Hooft's beam-splitter cases. *Galileo's ship case* consists in that the happenings within a ship are invariant for an observer within it under the physical boost of the ship with respect to the shore. *Einstein's elevator case* consists in particular in that phenomena within an elevator are invariant for an observer within it whether the elevator is in free float or in free fall, as well as whether the elevator is accelerating upwards or located on the surface of a massive body. *Faraday's cage case* consists in that the electromagnetic experiments within a cage are invariant for an observer within it under the physical charging of the cage with respect to the ground (manifested by the appearance of observable sparkles on the outer boundaries of the cage). *'t Hooft's beam-splitter case*, on Kosso's and my view (as opposed to Brading and Brown's plus Greaves and Wallace's), consists in that in a double-slit experiment the interference pattern is invariant under some changes of the setup, such as the addition of two half-wave plates behind the slits or of a half-wave plate behind one slit plus a shielded solenoid in between the slits (like in the Aharonov-Bohm effect). Given these cases, I roughly define *empirical symmetries* as physical symmetries whose invariant and differing features are dynamical and relational. I further concentrate on empirical symmetries which are identifiable, i.e.can be established, and so restrict my attention to empirical symmetries which are observably incomplete, i.e.whose physical states have non-void observable differences (as in the four cases just given).

The literature on DES usually takes DES to be stronger than the Leibnizian interpretation defined above, but does not explain this in detail. Also, the dominating position there is that only global but not local theoretical symmetries have DES, where *global* usually means 'expressible using parameters' or '(effectively) uniform'. I wish to explain why I think that the latter position is wrong and what this tells us about the ontological significance bestowed by DES.

Firstly, as DES is based on a matching between a theoretical symmetry and an empirical symmetry, I distinguish between two approaches to establishing DES. *The theoretical approach* consists in starting with a theoretical symmetry and seeking which empirical symmetry matches with it, *the empirical approach* consists in starting with an empirical symmetry and seeking which theoretical symmetry matches with it. Also, I take that the notion of matching relevant to the theoretical approach is the instantiation of theoretical symmetries by empirical symmetries, and the notion of matching relevant to the empirical approach is the representation of empirical symmetries by theoretical symmetries. Besides, I require either matching to hold constituent-wise, i.e.so that transformations match with transformations, differences with differences and invariances with invariances. From this it follows that an empirical symmetry is guaranteed to be matched with a theoretical symmetry, because it is presumably always possible to represent all the constituents of an empirical symmetry constituent-wise, while a theoretical symmetry may match with an empirical symmetry, a physical symmetry or nothing in the world, because it is not guaranteed that whatever there is in a theory is in the world and forms an empirical symmetry there. Therefore, by the definition of DES the latter matching characteristic of the theoretical approach does not necessarily leads to DES while the former matching characteristic of the empirical approach does. So I prefer the empirical approach over the theoretical approach, because it is DES that I wish to study.

Besides, I distinguish between two kinds of DES: *the observational DES* only concerns matchings between observational consequences of theoretical symmetries and observable features of empirical symmetries, while *the ontological DES* also concerns matchings between their theoretical underpinnings and unobservable underpinnings. In the empirical approach theoretical symmetries with the observational DES are those whose theoretical underpinnings entail observational consequences suitable for describing observable features of empirical symmetries. According to my proof which I omit here, these observational consequences can be exhibited by theoretical symmetries whether their transformations are global or local in the senses of the global/local distinction relevant to the discussion of DES. Therefore, both global and local theoretical symmetries can have the observational DES contrary to most of the literature on DES.

As identifiable empirical symmetries are observably incomplete and their observable differences have to be represented by differences in observational consequences, theoretical symmetries with the observational DES in the empirical approach will necessarily be observationally incomplete. So theoretical symmetries with this DES are not the same as the observationally complete theoretical symmetries primarily concerned by the Leibnizian and Newtonian interpretations. Besides, the observational DES in the empirical approach is stronger than the Leibnizian and Newtonian interpretations in terms of constituents having the ontological significance, because for the former all the constituents of a theoretical symmetry, namely the transformation, the invariant part and the differing parts of theoretical states, are inconditionally matched with actual proper counterparts. In terms of components the interpretation of theoretical symmetries following from the observational DES is thus actually the strongest there can be.

But on a more nuanced level the strongest interpretation would be that which inconditionally assigns actual counterparts to both the observational consequences and the theoretical underpinnings of each of the components. As the observational DES is about the observational consequences alone, this last interpretation could only be achieved if we pass to establishing the ontological DES instead. By definition of the latter DES, symmetries having it are a subset of symmetries having the former DES, hence establishing the ontological DES would be easier if only global symmetries had the observational DES. But my proof mentioned above says that local symmetries can have the observational DES too, therefore as much global as local symmetries may have the ontological DES. Moreover, my proof implies that which of the theoretical symmetries with the observational DES have the ontological DES depends on the ontological significance of observationally complete theoretical symmetries. This means that the analysis of DES brings us back to asking of the latter symmetries whether the Leibnizian, the Newtonian or some stronger interpretation should be adopted for them.

IES

IES is mentioned in the literature on DES as an empirical status opposed to DES and exemplified by the fact for theoretical symmetries of actions to entail conservation laws via Noether's theorems. To my knowledge no-one has analysed IES as such, but there exists a huge literature on Noether's theorems since their formulation by Noether in [1918], summarised in particular in a book by Kosmann-Schwarzbach [2011]. This literature mostly tells what Noether's theorems, their generalisations and related results are. What has to be added is the study of them as as providing an instance of IES.

Both Noether's theorems start with postulating symmetries of actions under variations of dependent or independent variables which do not necessarily vanish on the boundary of the region of integration. Variations can be global or local according to whether they are expressible using a finite number of essential parameters or functions respectively. Noether's first theorem says that if the variations are global and relevant equations of motion are satisfied, continuity equations ("differential conservation laws") follow. Under appropriate boundary conditions these can further lead to conservation laws proper ("integral conservation laws"). Noether's second theorem links local variations with other conservation expressions which hold identically, i.e.independently of whether any equations of motion are satisfied.

What one could infer from this is that only global symmetries of actions have IES. However, Brading and Brown [2000] explain that if the satisfaction of equations of motion is added to Noether's second theorem, then the same conservation laws can be derived as those following from Noether's first theorem. Moreover, they explain that these conservation laws can be obtained from local symmetries of action independently of whether these are assumed to have a global subgroup. For me this is shows that as much global as local symmetries of actions can have IES. The situation is thus analogous to that which obtains with the observational DES as well as with the Newtonian and Leibnizian interpretations, namely neither is sensitive to whether a theoretical symmetry is global or local. We may however ask next whether there is a strengthening of IES analogous to the ontological DES which only a subset of symmetries of actions would possess. This may well be the case, but again there does not seem to be any obvious reason to credit only global symmetries with that stronger IES.

Another aspect to analyse here is the relationship between symmetries of actions and conservation laws. The two seem to be on a par, because Noether's theorems work as much in the direction from symmetries of actions to relevant conservation laws or expressions as the other way round. However, this does not prevent Lange [2007] from arguing that symmetries are more important than conservation laws in that context and Romero-Maltrana [2015] from defending the opposite view. As to myself, I think the following. Firstly, on the physical side we seem to have observably incomplete symmetries here, namely conservation phenomena understood as invariances of some observable features coupled to observable (measurable) changes in time. Secondly, what is most directly linked to this on the theoretical side are theoretical conservation laws, so it is primarily these laws that should get the ontological significance if relevant conservation phenomena obtain. Thirdly, if what entails these laws is to have any ontological significance by virtue of the laws having it, then this ontological significance should be spread not only to theoretical symmetries of actions but also to equations of motion, as only the combination of these theoretical elements allows to derive theoretical conservation laws. On the other hand, by Hamilton's principle equations of motion are themselves the consequence of those symmetries of actions which concern variations of dependent variables vanishing on the boundary of the region of integration. Therefore, postulating that equations of motion hold is not an independent premiss of Noether's and Brading and Brown's results but rather an indication that the symmetries of actions concerned comprise the symmetries of actions under the variations just mentioned. This may be used to argue that only symmetries of actions and not them plus equations of motion considered separately are to get the ontological significance in our case. Fourthly, even if symmetries of actions get the ontological significance as just stated, it is unclear which of their components are to get it more precisely, and thus how strong is IES as compared to DES, the Newtonian and the Leibnizian interpretations. Perhaps, however, it is precisely the hallmark of IES that it is not able to tell us exactly what in theoretical symmetries is to receive the ontological significance, given its indirectness.

Generalisations of DES and IES

To fully understand DES and IES it is not enough to study the matchings of theoretical symmetries with empirical symmetries and with conservation laws (or rather conservation phenomena). This is because the latter matching is usually said to be just an example of IES. Therefore, there must also be other examples of IES, and so one may ask whether there are other examples of DES too. DES, however, is usually not defined as being exemplified by the matching involving empirical symmetries but simply as arising from this matching. This suggests that the usual notions of DES and IES are actually on different levels of generality. So, the DES to which IES is usually opposed is not the usual DES but rather some more general DES of which the usual DES is an example. To account for this I will designate the usual IES as IES+ and the more general DES it is opposed to as DES+. *DES+* is thus an empirical status exemplified by the DES based on the matching of some theoretical symmetries with empirical symmetries, while *IES+* is the empirical status opposed to DES+ and exemplified by the IES based on the fact for some theoretical symmetries to entail conservation laws via Noether's theorems.

So far we have thus only understood DES and IES, but what about DES+ and IES+? One way to understand the latter statuses is to seek for examples of DES+ and IES+ other than DES and IES. However, it is unclear how to do so, because it is unclear which similarity to DES and IES is relevant in our context. Another way is to note that both DES+ and IES+ are empirical statuses, and so can be characterised by saying what an empirical status is in general and how it can be direct or indirect. This has the advantage of defining DES+ and IES+ more precisely than through examples, so let us explore this way while allowing to recur to the examples already known, namely DES and IES, for an additional help when needed.

Arguably, *the empirical status of theoretical symmetries* in general is just their matching with the empirical level. And if the latter is the level of observable features, this matching will involve observational consequences in the first place. We know next that for DES+ the relevant matching is exemplified by the observational DES, where observational consequences of theoretical symmetries are matched with observable features of empirical symmetries. And for IES+ the relevant example of matching involves observational consequences of the theoretical conservation laws derivable from theoretical symmetries and conserved observable features in the world. This shows that an empirical status is *direct* when it is observational consequences of theoretical symmetries which primarily ensure the matching, and *indirect* when it is observational consequences of something derivable from theoretical symmetries which primarily ensure the matching. Therefore, *DES+* and *IES+* can be defined as matchings with the empirical level of theoretical symmetries themselves and of theoretical underpinnings (together with their observational consequences) derivable from theoretical symmetries, respectively.

As a further step we can generalise the DES+/IES+ distinction itself into a gDES/gIES distinction ('g' standing for 'generalised') by allowing empirical statuses to apply to arbitrary theoretical elements rather than theoretical symmetries. Some of the justifications are as follows: firstly, IES+ anyway concerns theoretical elements derivable from theoretical symmetries more than theoretical symmetries themselves; secondly, as DES+ and IES+ allow various physical features to be matched with the theoretical elements they are concerned with, it is worth exploring what happens if we allow various physical features to be matched with various theoretical elements too. Following this way, we get the definitions of *gDES* and *gIES* from those of DES+ and IES+ by replacing 'symmetries' by 'elements' throughout.

Here is what this leads to: Firstly, the theoretical underpinnings whose observational consequences ensure an instance of IES+ then get a gDES. Secondly, while it is less clear whether the same theoretical symmetries can ever have both DES+ and IES+, the same theoretical elements may well have both gDES and gIES. Thirdly, if DES+ and IES+ are able to clarify the ontology of theoretical symmetries, gDES and gIES may do so for any theoretical elements whatsoever. Fourthly, as global/local distinctions usually concern theoretical transformations and symmetries alone (although elsewhere I have extended them to theoretical states as well), generalising from these to arbitrary theoretical elements makes these distinctions even less likely to be of any importance in our context. Fifthly, whatever can be said of gDES and/or gIES in general will carry back to DES+ and/or IES+ as well as to DES and/or IES.

To illustrate the latter, note first that we can define analogues of the observational DES and of the ontological DES for the other empirical statuses we have discussed: *observational empirical statuses* are those dealing with matchings of observational consequences alone and *ontological empirical statuses* are those dealing with matching of theoretical underpinnings in addition. Consider next a collection of theoretical elements TE'', TE', TE such that TE'' entails TE', TE' entails TE and TE has observational consequences matching to some phenomena. Here TE has the observational gDES and TE'' with TE' have the observational gIES, but do they have the ontological varieties too? That is, do theoretical underpinnings of TE'', TE', TE have proper counterparts in the world if TE's observational consequences agree with phenomena?

A way to answer positively actually comes from the No Miracle Argument (NMA). The NMA says that the truth of empirically successful theories is the best explanation of their empirical success, and so the correct one. In other words, the NMA infers from something about explanations that the matching of observational consequences with phenomena establishes the existence of proper counterparts of theoretical underpinnings of these observational consequences. Here theoretical underpinnings are usually understood in a very comprehensive way, so that in our case this would include not only TE but also TE' and TE''. So given the NMA TE would have the ontological gDES and TE' with TE'' would have the ontological gIES. Thus, carrying this back to DES and IES, in particular theoretical symmetries with the observational DES or IES would get their ontological varieties too.

Perhaps this would be fine if we had a single theoretical element or a single chain of theoretical elements per any observational consequence. But usually we have a radical underdetermination between elements or chains instead, as demonstrated in the context of DES and IES respectively by my proof and by Brading and Brown's results. Taking all elements to have ontological statuses would then lead to a proliferation of entities and would often result in incompatible ontologies. To avoid this we should apply the NMA or other arguments to the same effect selectively, if at all. One possibility is then to restrict such applications to the level of TE by leaving TE' and TE'' out. In this case the ontological gIES is not assigned but only the ontological gDES is. But as shown by my analysis of DES, the assignment of the ontological DES depends on the interpretation of observationally complete theoretical symmetries. So, at least for DES the restriction to TE is not sufficient to obtain a definite ontology, and perhaps this holds for gDES more generally. On the other hand, not leaving TE' and TE'' out would strengthen a kind of scientific realism to which empirical statuses may lead, for it is the assignment of the ontological significance to theoretical elements more remote from successful observational consequences that is usually at stake in scientific realists' and anti-realists' debates. But in this case the assignment of the ontological gIES would have to be reassessed in addition to the assignment of the ontological gDES.

Conclusion

A rating in terms of increased strength goes from the Leibnizian interpretation to the Newtonian interpretation, the observational DES and the ontological DES, but the first two usually concern observationally complete theoretical symmetries while at least the third usually concerns observationally incomplete theoretical symmetries. The classification and extension of DES+, IES+, gDES and gIES is less clear, but their ontological varieties are anyway stronger than their observational varieties and more general statuses are worth studying so that to carry back to more specific ones. Global/local distinctions seem not to matter for all the interpretations and statuses discussed.

References

Brading K., Brown H. [2000], "Noether's theorems and gauge symmetries", *arXiv,* https://arxiv.org/abs/hep-th/0009058 .

Brading K., Brown H. [2004], "Are gauge symmetry transformations observable?", *The BJPS*, Vol. 55, No. 4, pp. 645-665.

Clarke S. [1717], *A Collection of Papers, Which passed between the late Learned Mr. Leibnitz, and Dr. Clarke, in the years 1715 and 1716*, available online at http://www.newtonproject.ox.ac.uk/view/texts/normalized/THEM00224 .

Greaves H., Wallace D. [2014], "Empirical consequences of symmetries", *The BJPS*, Vol. 65, pp. 59-89.

Healey R. [2009], "Perfect symmetries", *The BJPS*, Vol. 60, pp. 697-720.

Kosmann-Schwarzbach Y. [2011], *The Noether's Theorems: Invariance and Conservation Laws in the Twentieth Century*, Springer.

Kosso P. [2000], "The empirical status of symmetries in physics", *The BJPS*, Vol. 51, No. 1, pp. 81-98.

Lange M. [2007], "Laws and meta-laws of nature: Conservation laws and symmetries", *SHPMP*, Vol. 38, pp. 457-481.

Noether E. [1918], "Invariante Variationsprobleme", *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, pp. 235-257. Translated by Tavel M. A. as Noether E. [1971], "Invariant variation problems", *Transport Theory and Statistical Physics*, Vol. 1, No. 3, pp. 183-207. Also translated as Noether E. [2011], "Invariant variational problems", in Kosmann-Schwarzbach Y. [2011], *The Noether's Theorems: Invariance and Conservation Laws in the Twentieth Century*, Springer, pp. 3-22.

Romero-Maltrana D. [2015], "Symmetries as by-products of conserved quantities", *SHPMP*, Vol. 52, pp. 358-368.