ADAMS' THESIS AND THE LOCAL INTERPRETATION OF CONDITIONALS

ABSTRACT. Adams' Thesis states that the probability of a conditional is the probability of the consequent conditional on the antecedent. S. Kaufmann introduced a rival "local" method for calculating strength of belief in a conditional that, according to a claimed majority, squares better with intuition in some circumstances. He also gave a wagering example purporting to show that it is sometimes "advantageous to follow the local interpretation". We challenge the intuitions, expose an error in the example and critique Kaufmann's semantics for the local interpretation.

1. LOCAL PROBABILITIES FOR CONDITIONALS

Stefan Kaufmann (2004) introduces a method for predicting "strength of belief" (of a purported majority of speakers) in some conditionals. It (the method) is intended as a complement to (and sometime rival of) the so-called *Adams Thesis* (Adams 1996, p. 3), according to which the "probability" of a conditional $A \to C$ should be the conditional probability $P(C|A)$. Kaufmann presents a scenario in which one is about to draw a ball from one of two bags. The likelihood of drawing from each bag, as well as the contents of each bag, are given in the following table.

$P(\text{Bag } X) = \frac{1}{4}$	$P(\text{Bag } Y) = \frac{3}{4}$
10 red balls,	10 red balls,
9 of them with a black spot	1 of them with a black spot
2 white balls	50 white balls

Kaufmann asks whether strength of belief in

(1) If I pick a red ball, it will have a black spot

ought to be 'high', 'fifty-fifty', or 'low'. Kaufmann writes:

> The judgment of nine out of ten informants to whom I posed this question in an informal survey, as well as my own intuition, is that the answer should be 'low'.

Accordingly, Kaufmann postulates a new, "local" method of calculating a probability for (1). This method may be motivated as follows. Consider an "expert" who knows which bag is in play. (Kaufmann doesn't resort to this device, but it's useful.) By Expert Reflection, one's strength of belief in (1) ought to be the expectation of the expert's strength of belief in (1). If it's Bag X, the expert's strength of belief in (1) will be $\frac{9}{10}$ (Kaufmann assumes that speakers do defer to conditional probability, conditional on the bag being fixed), and if it's Bag Y the expert's strength of belief in (1) will be $\frac{1}{10}$. So strength of belief in (1) (says Kaufmann) might plausibly be

(2) $P_l(R \to B) = P(R \to B|X)P(X) + P(R \to B|Y)P(Y)$

$$= P(B|RX)P(X) + P(B|RY)P(Y) = (\tfrac{9}{10})(\tfrac{1}{4}) + (\tfrac{1}{10})(\tfrac{3}{4}) = \tfrac{3}{10}.$$

(See Section 2 for more on the above display.) Here of course $B =$*Black Spot*, $R =$*Red*, etc. Kaufmann contrasts (2) with what he calls the "global" interpretation (i.e. the interpretation consistent with The Thesis):

(3) $P_g(R \to B) = P(B|RX)P(X|R) + P(B|RY)P(Y|R) = P(B|R) = \frac{6}{10}$.

He calls the move from $P(X)$ in (2) to $P(X|R)$ in (3) an "*abductive* inference to the best explanation for the (hypothetical) observation that the ball is red," observing that "This step is evidently not performed by those who give the conditional (1) a 'low' rating."

Michael Zhao (2015) and Justin Khoo (2016) have offered putative characterizations of the conditions under which a "local" reading exists. In Section 3 we present contrasting conditional pairs that cast doubt on the adequacy of these theories. One member of each pair is paradigmatically locally interpretable and the other looks to be so interpretable if the first is, yet giving *both* members the local interpretation feels incoherent. The lesson, we suggest, is that those who employ the local interpretation haven't thought carefully enough about how their resulting attitudes fit together with other attitudes they would adopt in similar situations according to similar rules. These troubles for the local interpretation are compounded in Section 4, where it is shown that Kaufmann's one attempt to demonstrate that the local interpretation can be employed as a basis for rational decision in some circumstances where local and global interpretations come apart founders on a mathematical error.[1]

A separate issue is that Kaufmann (2004) doesn't say what conditional sentences mean (e.g., whether they have truth conditions). Kaufmann (2009) addresses this gap, but for reasons we shall give in Section 5, the resulting semantics has at least one serious flaw. Taking cues from the exposition in Kaufmann (2004), we recover what we believe to have been Kaufmann's intention. To this end, we substitute for a model of van Fraassen (1976), from which the semantics of Kaufmann (2009) derives, an alterative model that turns out to be isomorphic to one developed by Andrew Bacon (2015). (We shall give a different, more notationally efficient, presentation.[2])

2. Triviality, Total Probability and the Resilient Equation

In the first line of (2) there appears to be an appeal to the law of total probability

$$P(A) = \sum_{i=1}^{n} P(B_i)P(A|B_i) \text{ for any partition } \{B_1, \ldots, B_n\} \qquad (TP)$$

[1]Kaufmann's rhetoric places a great deal of importance on this example–he essentially concedes that if there aren't scenarios where speakers are justified in basing decisions on locality, there is no reason to think that the local interpretation isn't a mere fallacy. Douven (in his footnote 4) misdiagnosed Kaufmann's example as a case in which true credence and rational action come apart.

[2]Briefly, we employ a countable set of ordinals as indexing set, whereas Bacon uses the naturals. So while there is an isomorphism between the models, Bacon must continually speak of an "equivalence" between "sequences" and "sequence of sequences", etc. Since any mapping that might establish this equivalence would encrypt the important orderings that are essential to the model, the approach involving ordinals (or some other sufficiently structured countable set) is more natural.

over the partition $\{X, Y\}$, with A the conditional $(R \to B)$. In the second line, meanwhile, the identity $P(R \to B|X) = P(B|RX)$ is used; Fitelson (2015) refers to this identity as the "Resilient Equation". The conjunction of these practices, taken as generally valid rules, has been shown to imply "triviality" by David Lewis (1976). Lewis employs (TP) for $A \to C$ over the partition $\{C, \neg C\}$. This appears to commit Thesis defenders who accept the Resilient Equation to the absurd conclusion $P(A \to C) = P(C)$, which shows in its turn that no Thesis-friendly probability space admits of a three cell measurable partition $\{E_1, E_2, E_3\}$ with positive measure cells: otherwise, one could set $A = E_1 \cup E_2$ and $C = E_1 \cup E_3$.

Thesis defenders therefore face a choice. Either they must disavow that (TP) applies to conditionals, or they must disavow the Resilient Equation. Though Kaufmann doesn't defend The Thesis in all contexts, he does believe that the global reading "generally accords well with pre-theoretical intuitions" and is typically available. Put to the same choice, then, he disavows (as generally valid; see Kaufmann 2004 p. 585 line 20) the Resilient Equation. For though he uses it in (2), he notes that it is 'not warranted by an "official" rule of the probability calculus', suggesting that its legitimacy there is exceptional.[3] Even so, as a sometime employer of this equation, he feels obligated to at least address the threat of "triviality", writing "I do not claim that conditionals have local interpretations with respect to just any variable."

Lewis (1976) discusses two types of Thesis proponents–those for whom conditionals are propositions, so that their "probabilities" are true probabilities, and those for whom conditionals are not propositions, so that their "probabilities" are "assertabilities", "felicities" or other "ersatz probabilities".[4] The former are obligated to deny the Resilient Equation; the latter are obligated to deny the Law of Total Probability (as applied to conditionals). Those who believe conditionals to be propositions believe them to be highly context-sensitive; those who believe they aren't employ a highly non-classical notion of (ersatz) conditional probability. Stalnaker and Van Fraassen are of the first type, Adams is of the second. Kaufmann's position, which is laid out in Kaufmann (2009), is ultimately something of a hybrid–he takes conditionals to have "valuations" that are sometimes truth values (i.e. 0 or 1) but can be intermediate (i.e. lie strictly between 0 and 1). On the other hand, he defines these valuations to be the expected values of true propositions lying in a more elaborately structured space. This, together with the fact that he unflinchingly employs the Law of Total Probability (and flags use of the Resilient Equation as exceptional) suggests to us that he essentially views conditionals as genuine, context sensitive propositions.

Kaufmann (2009) uses a model of van Fraassen (1976) as a "baseline" for his own semantics for conditionals. This model was engineered for the specific purpose of confirming The Thesis without lapsing into triviality, a property we believe Kaufmann

[3]Douven (2008) appears to agree, stating that the first line of (2), "being a mere application of the law of total probability, is unassailable."

[4]The latter proponents needn't commit on what these ersatz probabilities amount to. All that is necessary is that their assignment be descriptive of (appropriately vetted) speaker usage. So, they may regard "$P(C|A) = x$" as a formal paraphrase of an utterance such as "the probability of $A \to C$ is x". Such theorists wouldn't discuss "strengths of belief in" conditionals at all, if speakers didn't.

intended his baseline to exhibit.[5] However, there is a problem here. For whereas Kaufmann (2009) intends a semantics for arbitrarily nested conditionals, van Fraassen (1976) is explicit that he does not know whether his model confirms The Thesis for arbitrary nestings. (And in fact, it does not.) In an appendix we suggest that, since it does not, a similar model due to Andrew Bacon (2015) is the proper baseline for Kaufmann's semantics.

3. On the conditions under which the local interpretation is salient

Existing theories about when the local interpretation gains traction with speakers (see in particular Zhao 2015 and Khoo 2016) tend to focus on the existence of an appropriate "background variable" or "background partition". In this section, we'll give examples where $A \to B$ is predicted to have a salient local reading with respect to some background partition, but where the assumption that $\neg A \to B$ has such a reading with respect to the same background partition as well can lead to an intuitively incoherent combination of attitudes. This suggests, at the least, that the conditions under which such readings arise is more nuanced than has been acknowledged.

Assume for example that after the draw mentioned in (1) you'll continue to draw balls from the (same) bag, without replacement. We take the status of (1) to be the same in this revised scenario; in particular it still has local probability $\frac{3}{10}$ with respect to $\{X, Y\}$. Consider:

(4) If I pick a red ball first, the first red ball picked will have a black spot; and

(5) If I pick a white ball first, the first red ball picked will have a black spot

Here are three possible responses to (4)/(5) consistent with the view that the local reading is salient for (1).

First response: The local reading is salient for (1), but isn't for (4). This seems wrong, because (4) is equivalent to (1), in the sense that (4) and (1) have probabilistically equivalent antecedents, and their consequents are probabilistically equivalent conditional on their consequents. So the two propositions must have the same local probability, $\frac{3}{10}$, which (owing to the equivalence), ought to have similar "salience".

Second response: The local reading is salient for (4), but isn't for (5). This too seems wrong–there isn't any special relationship between "Red" (as opposed to "White") and the background partition. At any rate if it were right, it would be a phenomenon missed by every theory we are aware of concerning when the local reading is salient.

Third response: It's coherent to have equal strengths of belief in (4) and (5). But that's no good, because no speaker, if they thought they would be interpreted in this way, would utter (4) in the first place. Rather, they would utter the far more direct *the first red ball picked will have a black spot*, which has *true probability* equal to $\frac{3}{10}$. Local interpretation implausibly reduces strength of belief in this conditional (and similarly for a wide range of others) to strength of belief in the consequent.

[5]Kaufmann introduces several emendations to the model allowing Thesis-violating readings; we leave it to an enterprising reader to verify that one could, if one cared to, make similar emendations to the alternate baseline we recommend here.

Assuming there are no other possible responses to (4)/(5) consistent with the view that strength of belief in Kaufmann's original example can be coherently given by the local interpretation, the moral is clear: namely, that strength of belief in Kaufmann's original example cannot coherently be given by the local interpretation after all.

Other examples in the literature can be treated along similar lines. Consider for example the following scenario from Zhao (2015).

> **Subway.** ...waiting at the subway stop...you suddenly realize that you're not certain which day of the week it is. But you have an impression that it's the weekend, which causes you to be...$\frac{2}{3}$ sure that it is. ...on weekends, trains arrive...every half hour but are usually pretty empty; on weekdays, trains arrive once every five minutes and are generally crowded. How likely is it that *if a train arrives in the next five minutes, it will be crowded*?

Zhao answers "The intuitive answer seems to be: not very". But consider the following conditionals, which have equal local probabilities relative to the background partition {weekday, weekend}:

(6) If the next train does arrive in the next five minutes, it will be crowded; and

(7) If the next train does not arrive in the next five minutes, it will be crowded

The choices are as before. Either (6) doesn't have a salient local reading, which would be odd since it is equivalent to something that supposedly does, or (6) does and (7) doesn't, which violates a clear symmetry[6], or it's fine that both do, i.e. it's okay to have not lesser strength of belief in (7) than one has in (6)...indeed okay that strength of belief in both reduces to that in their common consequent, *the next train will be crowded.* (In which case, as before, why utter the antecedent at all?)

Yet another example from Kaufmann (2009):

(8) If you strike the match, it will light

The setup is that the match is probably wet, a struck wet match is less likely to light than a struck dry match, and you are more likely to strike a dry match than you are to strike a wet match. My strength of belief in (12) obviously won't change if I determine to strike the match if you don't. So:

(9) If you strike the match, it will light when struck

(10) If you don't strike the match, it will light when struck

This is a pair for which the local interpretivist faces the same bad alternatives.[7]

Kaufmann's (2009) treatment of more complex sentences does little to bolster the case for the local interpretation. Consider for example the sentence

(11) Harry will pass the test if he takes it, and he will win in the show if he is selected

[6]Of course, if one assumes that trains *necessarily* arrive every five minutes on weekdays, the antecedent of (7) entails *weekend*, potentially breaking symmetry. (We aren't this literal though.)

[7]A referee suggested that this example relies on a "somewhat awkward reading of a rightnested conditional". We don't agree that the reading is awkward, and in any event "it will light when struck" is not a conditional, whence (9) and (10) aren't rightnested conditionals.

Here Harry is either intelligent (probability .25) or not (probability .75). An intelligent person will pass the test with probability .9; respectively win in the show with probability .9 (conditional on taking the test; respectively being selected for the show). For a not-intelligent person these conditional probabilities are both .1. If we knew Harry to be intelligent, it seems we might assign (11) probability $\frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$; if we knew Harry to be not intelligent, it seems we might assign (11) probability $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$. Accordingly, Kaufmann assigns (11) probability $.25(.81)+.75(.01) = .21$.

Though (11) has the syntax of a conjunction, note the similarity to locality (with $\{\text{intelligent}, \text{not intelligent}\}$ playing the role of background partition). So if intuitions do support .21 for (11), does this create sympathy for the local interpretation?

We think not. For (11) has no "abductive inference" involved in its interpretation, and treating $\{\text{intelligent}, \text{not intelligent}\}$ as a background partition for locality leads to familiar troubles for "nearby" conditionals that do. Suppose for example that we learn that Harry will take the test and be selected for the show, and consider

(12) If Harry passes the test then he will win on the show

It seems clear that most speakers who would be inclined to report any strength of belief for (12) would report a high one. (Imagine approaching the limiting case in which the relevant conditional probabilities aren't .9 and .1, but 1 and 0.) Indeed, most would report high strength of belief in

(13) If Harry passes the test then he will win on the show, and if he fails the test he will lose on the show

as well. But if one treats $\{\text{intelligent}, \text{not intelligent}\}$ as a local partition, one assigns (13) probability .09 and (12) probability .3, which is implausibly equal to that of

(14) If Harry fails the test then he will win on the show

So...whence the intuitive appeal of .21 for (11)? We believe that this appeal (to those for whom it has any) derives from the fact that .21 is just the global probability of

(15) If Harry takes the test and is selected for the show, he will pass the test and win on the show

Another phenonemon speakers sympathetic to the local interpretation need to explain away is that it can violate the following desideratum for strength of belief measures.[8]

Weights: Suppose $\{A_1, \ldots, A_n\}$ is a partition of event space. For any event C, $\min\{P(A_i \to C) : i = 1, \ldots, n\} \leq P(C) \leq \max\{P(A_i \to C) : i = 1, \ldots, n\}$.

[8]A referee brashly rejected *Weights*, citing grounds that its violation in the example discussed here looks to be something like a typical case of the so-called "Simpson Reversal". That isn't right, though there is a relationship between Simpson Reversals and the issues under discussion. Recall that a "Simpson Reversal" is a case in which a weighted average $\sum_{i \in I} u_i a_i$ is larger than another weighted average $\sum_{i \in I} w_i b_i$ even though $a_i < b_i$ for every $i \in I$. A popular example: David Justice had a higher batting average than Derek Jeter in both 1995 and 1996, yet Jeter had the higher aggregate batting average over that two-year span. So if one were about to watch a randomly chosen major league at-bat from that two year period and chose to regard "year of at-bat" as a suitable background variable, only the global interpretation would assign *If Jeter is the batter, he will get a hit* higher probability than *If Justice is the batter, he will get a hit.*

Note that if *Weights* is satisfied, $P(C)$ is always a weighted average of the values $P(A_i \to C)$.[9] To motivate the independent plausibility of *Weights*, consider the pre-theoretical awkwardness of the following combination of attitudes:

(i) strength of belief in "if we watch the game then we'll order pizza" is $\frac{1}{2}$;
(ii) strength of belief in "if we don't watch the game then we'll order pizza" is $\frac{1}{2}$;
(iii) strength of belief in "we'll order pizza" is $\frac{1}{3}$.

Such combinations are realized under local interpretation. Consider the following:

$P(\text{Bag } X) = \frac{1}{2}$	$P(\text{Bag } Y) = \frac{1}{2}$
10 red balls,	20 red balls,
9 of them with a black spot	2 of them with a black spot
20 white balls	10 white balls
2 of them with a black slot	9 of them with a black spot

Suppose you are about to start picking a balls uniformly at random from whichever of the two bags is before you. According to the local interpretation:

(I) strength of belief in "if I pick a red ball first, the first ball picked will have a black spot" is $\frac{1}{2}$;

(II) strength of belief in "if I pick a white ball first, the first ball picked will have a black spot" is $\frac{1}{2}$;

(III) strength of belief in "the first ball picked will have a black spot" is $\frac{11}{30}$.

Academics having a theoretical story to tell about local interpretation may simply point to their models in order to "explain" such examples, but most rationally competent speakers will (pre-theoretically) reject these combinations of attitudes out of hand. If that's right, then the theorists need to explain why it isn't a strike against their theories–which were presumably developed in order to predict the attitudes of speakers–that they don't appear to do a very good job of prediction in such cases.

4. On a book of bets said to support the local interpretation

In Section 7 of his paper, Kaufmann writes:

> Assuming that the present proposal is descriptively correct, it raises a deeper question: Is it an account of a fallacy–one that is committed widely and systematically, but fallacious nonetheless–or is the departure from (The Thesis), at least in some circumstances, the "correct" interpretation of a conditional? (...) Are there situations in which it would be detrimental to base one's actions upon (The Thesis) and advantageous to follow the local interpretation? A negative answer would not imply that the local interpretation is not what speakers use, but only that it is not what they *should* use.

Kaufmann claims that the answer is "not negative". In support of this claim, he sets up the following scenario. At time 0, B pays a bookie $P(C|A)$. X's return on the wager is: $P(C|A)$ if $\neg A$, 1 if AC, 0 if $\neg CA$. B regards this wager as fair.

[9] This is trivial; every element of a closed bounded interval is a weighted average of its endpoints.

Between time 0 and time 1, both B and the bookie will learn whether A is true, but if in fact A is true the bookie will find out, in addition, "in what way" it is true, i.e. which of $AX_1, \ldots, AX_n$ is true, "where the X_i are the values of some variable $\mathbf{X}$ that we take to be causally relevant.... The conditional probability of C is not evenly distributed over all AX_i, and $\mathbf{X}$ does not causally depend on A." B will not have this additional information, but does know that the bookie will have it.

At time 1, B perceives her expected net payoff to be zero. (If $\neg A$ she knows that payoff to be exactly zero, and if A she knows it will be $1 - P(C|A)$ if C and $-P(C|A)$ otherwise, the former with probability $P(C|A)$.) The bookie's expected payoff, however (from her own perspective), can be positive or negative. If negative, she would like to make a new bet to exactly cancel the first.

Although this bet looked fair to B prior to the offer of it, the mere fact that the bookie wants to make it is evidence, for B, that she should not. Kaufmann claims (equation 30 in his paper) that the bookie's expected payoff at time 0 is now curiously influenced by B's refusal to do business with the bookie at time 1. Indeed, he claims that it is now "the weighted sum of these posterior payoffs for each X_i: $\sum_{X_i \in \mathbf{X}}(P(C|A) - P(C|AX_i))P(X_i)$."

That isn't right. Expected payoff for the bookie on AX_i is indeed $P(C|A) - P(C|AX_i)$, but on $\neg AX_i$ it is zero. The correct time zero expectation is therefore

$$\sum_{X_i \in \mathbf{X}} (P(C|A) - P(C|AX_i))P(AX_i)$$
$$= \sum_{X_i \in \mathbf{X}} \Big(\frac{P(CA)}{P(A)}\Big) P(AX_i) - \sum_{X_i \in \mathbf{X}} \Big(\frac{P(CAX_i)}{P(AX_i)}\Big) P(AX_i) = P(CA) - P(CA) = 0.$$

The fair price for the wager, then, is $P_g((A \to C) = P(C|A)$, not $P_l(A \to C)$ as Kaufmann (based on the fact that $\sum_{X_i \in \mathbf{X}}(P_l(A \to C) - P(C|AX_i))P(X_i) = 0)$ claims. It is implicit that Kaufmann thinks the fair price is the probability we should assign the conditional $A \to C$, so this is actually an argument *for* The Thesis.

5. Appendix: Baseline Semantics for the Local Interpretation

Thesis proponents face pitfalls. For an example, assume that $\{A, B, C, D\}$ partitions event space into equal measure events. A Thesis literalist might be tempted to write

$$P(\neg D \to A) = P(A|\neg D) = \frac{1}{3} \tag{5.1}$$
$$\neq \frac{1}{2} = P(A \vee D)P(A|A) + P(B \vee C)P(A|B \vee C) \tag{5.2}$$
$$= P(A \vee D)P(\neg D \to A|A \vee D) + P(B \vee C)P(\neg D \to A|B \vee C). \tag{5.3}$$

Passage from (5.2) to (5.3) goes, e.g., by the Resilient Equation which, recall, says that $P(X \to Y|Z) = P(Y|XZ)$ in general. A Thesis defender ought (so it might seem) to endorse the Resilient Equation, because if she were to learn Z, her new probability function would be $Q(\cdot) = P(\cdot|Z)$, and the resilient equation looks to follow in a line or two from $Q(X \to Y) = Q(Y|X)$. However, if we accept the Resilient Equation then (5.1)-(5.3) appears to violate the Law of Total Probability.

Thesis proponents have two options for explaining this away that are worth considering. On the first, indicative conditionals aren't propositions, and have neither truth conditions nor true probabilities. On this view, it would be better to write, say,

$$\begin{aligned} &P^*(\neg D \to A) = P(A|\neg D) = \frac{1}{3} \\ \neq &\frac{1}{2} = P(A \vee D)P^*(\neg D \to A|A \vee D) + P(B \vee C)P^*(\neg D \to A|B \vee C). \end{aligned}$$

Here $P^*(Z \to W|K) = P(W|ZK)$ (by definition) whenever Z and K are classical events (i.e. sentences containing no occurence of "$\to$"). P^* might be said to denote "assertability", "felicity", "strength of belief" or some other such notion, and would not be assumed to obey the probability axioms. It would have a few probability-like features (whatever is inherited from its definition), but lack others. Changing to P^* the instances of P evaluated at conditionals in (5.1)-(5.3) would thus eliminate the apparent violation of the Law of Total Probability there. Lewis (1976) writes:

> Adams himself seems to favor this hypothesis about the semantics of conditionals. ...I have no conclusive objection to the hypothesis that indicative conditionals are non-truth-valued sentences, governed by a special rule of assertability that does not involve their non-existent probabilities of truth. I have an inconclusive objection, however: ...(a) need to explain away all seeming examples of compound sentences with conditional constituents.

Such compound sentences appear to be well-formed and sometimes useful. ("That car is old, and if you honk its horn, a fuse will blow out.") Under the current proposal, either one should say that such sentences don't admit of ersatz probabilities (Adams' choice), or employ highly non-classical rules for their employment, e.g.[10]

$$\begin{aligned} &P^*((A \to C) \wedge B) = P(B|A)P^*(A \to C|B) \\ =&P(B|A)P(C|AB) = P(BC|A) = P^*(A \to BC). \end{aligned}$$

The proponent of such a rule will, in particular, deny the more familiar-looking identity $P^*((A \to C) \wedge B) = P(B)P^*(A \to C|B)$. On either account, the following "Total Law of Probability for Conditionals" replaces the usual Law:

$$P^*(A \to C) = \sum_{i=1}^{n} P(B_i|A)P^*(A \to C|B_i) \text{ for any partition } \{B_1, \ldots, B_n\}. \quad (TPC)$$

[10]This account of ersatz probabilities for conjunctions appears to avoid disaster (we believe it sidesteps triviality arguments such as that Fitelson 2015), but isn't particularly intuitive; note for example that it can happen that $P^*((A \to C) \wedge B) > P(B)$. Adams' course seems to us preferable.

Note the corresponding identity $P(C|A) = \sum_{i=1}^{n} P(B_i|A)P(C|AB_i)$. So this method of explaining away (5.1)-(5.3) is to hold onto the Resilient Equation and simply assert that the law corresponding to the Law of Total Probability for conditionals is (TPC).

The second method is to relinquish the Resilient Equation by giving conditionals truth conditions (and hence true probabilities) in accord with The Thesis. This is the approach of van Fraassen (1976). Van Fraassen's modestly Thesis-friendly model is relevant to our discussion because Kaufmann (2009) uses it as a "baseline" for his own semantics. The example we shall use to show that van Fraasen's model doesn't confirm The Thesis for arbitrary conditionals wreakes havoc with Kaufmann's model as well. The plan for the rest of this section, then, is to demonstrate where Thesis failure lies in van Fraassen's model, bring the correlative problems for Kaufmann's model into relief, and to finally propose a new baseline for the local interpretation.

Let $\mathbf{N}$ denote the set of natural numbers with zero. Let $(\mathcal{A}, P)$ be the agent's personal probability space (here $\mathcal{A}$ is a finite Boolean algebra of "classical events" and P a probability function having domain $\mathcal{A}$). For each $n \in \mathbf{N}$, independently sample x_n, an atomic event in $\mathcal{A}$, in accord with the probability function P. (One says that one is conducting "Bernoulli trials".) Let Q be product measure on the set of functions taking $\mathbf{N}$ to the set of atomic events in $\mathcal{A}$.

Given the complete sampling $\{x_n : n \in \mathbf{N}\}$, an event $A \in \mathcal{A}_0 = \mathcal{A}$ is true at n if and only if $x_n \subset A$. A conditional having the form $A \to C$, meanwhile, is true at n if and only if $x_{n+k} \subset C$, where k is the least natural number such that $x_{n+k} \subset A$. (This is of course an inductive definition.) Finally, for any sentence A, let $P(A) = Q($"A is true at 0"$)$.

Van Fraassen showed that this model elicits The Thesis for conditionals of the forms $A \to B$, $(A \to B) \to C$ and $A \to (B \to C)$. He further says (1976, p. 280) of such models that "although I do not know that the Thesis fails in them for more complicated conditionals, I expect it does". Our first task is to verify van Fraassen's "failure" intuition. In particular, we shall show that in a case where $\mathcal{A}$ is generated by three non-trivial atoms A, B and C, The Thesis fails for the conditional

$$Q = \Big((\neg B \to A) \to \neg B\Big) \to A.$$

Lemma 1. If $(\neg B \to A) \to \neg B$ is false then Q is true almost surely.

Proof. Let $k > 0$ be the minimum index at which $(\neg B \to A) \to \neg B$ is true. (Obviously there is such k with probability 1.) We need to show that A is true at k. Since $(\neg B \to A) \to \neg B$ is false at $k-1$ and true at k, it must be the case that $(\neg B \to A)$ is true at $k-1$ and $\neg B$ is false at $k-1$. (Otherwise, $(\neg B \to A) \to \neg B$ would come out true at $k-1$, since the $(\neg B \to A)$ world nearest to $k-1$ would be the $(\neg B \to A)$ world nearest k, which by hypothesis is a $\neg B$ world.)

Since $\neg B \to A$ is true at $k-1$ and $\neg B$ is false, $\neg B \to A$ must be true at k. (The $\neg B$ world closest to k is the $\neg B$ world closest to $k-1$, which by hypothesis is an A world.) But $(\neg B \to A) \to \neg B$ is also true at k, so its consequent $\neg B$ is true at k. Therefore (since $\neg B \to A$ is true at k) A is true at k, as desired.

Lemma 2. $P(A|(\neg B \to A) \to \neg B) < 1$.

Proof. Since A and C are non-trivial events and worlds 0 and 1 are independent on non-conditionals, there is strictly positive probability that C is true at 0 and A is true at 1. In such a case $\neg B \to A$ will be false at 0 and true at 1. So 1 will be the $\neg B \to A$ world closest to 0 and it is a $\neg B$ world, so $(\neg B \to A) \to \neg B$ is true at 0. Thus $P(\neg A \wedge ((\neg B \to A) \to \neg B)) > 0$, so that $P(\neg A|(\neg B \to A) \to \neg B) > 0$ also. Since $P(A|(\neg B \to A) \to \neg B) = 1 - P(\neg A|(\neg B \to A) \to \neg B)$, we are done.

Lemma 3. Suppose that $\neg E$ entails $E \to A$ almost surely, $P(\neg E) > 0$ and $P(A|E) < 1$. Then $P(E \to A) > P(A|E)$.

Proof. $P(E \to A) = P(\neg E) + P(AE) > P(\neg E)P(A|E) + P(E)P(A|E) = P(A|E)$.

With these lemmas in place we can now easily see why The Thesis fails for the conditional Q. To begin, let E be Q's antecedent, i.e. $(\neg B \to A) \to \neg B$. Then $\neg E$ entails $Q = E \to A$ almost surely by Lemma 1, and $P(A|E) < 1$ by Lemma 2. Finally, we have $P(\neg E) > 0$. For there is a strictly positive probability that B is true at 0 and A is true at 1, in which case the closest $\neg B$ world to 0 is 1, where A is true, meaning that $\neg B \to A$ is true at 0. But $\neg B$ is false, so $(\neg B \to A) \to \neg B$ is false. Therefore, by Lemma 3 we have $P(Q) = P(E \to A) > P(A|E)$, and we are done.

This example is a problem for Kaufmann's (2009) semantics. To illustrate the trouble, imagine a situation in which $P(A) = 10^{-20}$ and $P(B) = 1 - 10^{-10}$. Then Kaufmann assigns (see Kaufmann 2009, Theorem 5) $E = (\neg B \to A) \to \neg B$ probability equal to $P(\neg B)$, i.e. $P(E) = 10^{-10}$, and so by Lemma 1 must assign $Q = E \to A$ probability at least that of $\neg E$, i.e. $P(E \to A) \geq 1 - 10^{-10}$. But that $P(E \to A)$ should be so close to 1 seems intuitively unacceptable, given that A is vastly less likely than E.[11]

Fortunately, there's a way to tweak van Fraassen's model so as to bring it into conformity with The Thesis for arbitrary conditionals. Note however that the resulting model (cf. Bacon 2015) uses different "closeness" relations for conditionals of different complexities, and therefore does not employ Stalnaker's semantics precisely.

Again let $\mathbf{N}$ denote the set of natural numbers (with zero) and let $(\mathcal{A}, P)$ be the agent's personal probability space ($\mathcal{A}$ is a finite Boolean algebra). Denote by Ω the set of ordinal numbers $\{\sum_{i=0}^{k} n_i\omega^i : k \in \mathbf{N}, n_1, \ldots, n_k \in \mathbf{N}\}$. For each $\beta \in \Omega$, independently sample x_β, an atomic event in $\mathcal{A}$, in accord with P. Let Q be product measure on the set of functions taking Ω to the set of atomic events in $\mathcal{A}$.

Given the complete sampling $\{x_\beta : \beta \in \Omega\}$, an event $A \in \mathcal{A}_0 = \mathcal{A}$ is true at β if and only if $x_\beta \subset A$. A *degree 1 conditional*, i.e. a conditional having the form $A \to C$, where $A, C \in \mathcal{A}_0$, is true at β if and only if $x_{\beta+k} \subset C$, where k is the least natural number such that $x_{\beta+k} \subset A$. Let $\mathcal{A}_1$ be the Boolean algebra generated by $\mathcal{A}_0$ and the degree 1 conditionals. A *degree 2 conditional* is a conditional not having lesser degree and having the form $A \to C$, where each of $A, C \in \mathcal{A}_1$. Such a conditional is true at β if and only if $x_{\beta+k\omega} \subset C$, where k is the least natural number such that $x_{\beta+k\omega} \subset A$. Let $\mathcal{A}_2$ be the Boolean algebra generated by $\mathcal{A}_1$ and the degree 2 conditionals.

[11] A referee on the example: "the author has badly misunderstood...bizarre" etc. I nevertheless maintain (contra said referee) that it exhibits an unwelcome feature not anticipated by Kaufmann.

Continue in this fashion. Having constructed the finite Boolean algebra $\mathcal{A}_m$, define a *degree* $m+1$ *conditional* to be a conditional not having lesser degree and having the form $A \to C$, where each of $A, C \in \mathcal{A}_m$. Such a conditional is true at β if and only if $x_{\beta+k\omega^m} \subset C$, where k is the least natural number such that $x_{\beta+k\omega^m} \subset A$. Let $\mathcal{A}_{m+1}$ be the Boolean algebra generated by $\mathcal{A}_m$ and the degree 1 conditionals. Finally, for any sentence A, let $P(A) = Q($"A is true at 0"$)$.

The proof of Thesis confirmation is trivial. For sentences A and B, let m be the least natural number such that $\{A, B\} \subset \mathcal{A}_m$. (So that $A \to B$ is a degree $m+1$ conditional.) Let $x = P(A)$ and let $y = P(A \wedge B)$. The event "$A \to B$ is true at 0" is the disjoint union of the events "$A \wedge B$ is true at 0", "A is false at zero and $A \wedge B$ is true at ω^m", "A is false at zero and ω^m and $A \wedge B$ is true at $2\omega^m$", ... Thus $P(A \to B) = Q($"$A \to B$ true at 0"$) = y + (1-x)y + (1-x)^2 y + \cdots = \frac{y}{x} = P(B|A)$.

This proof highlights the role of the independence of the trials: the truth of A at 0 shouldn't correlate with the truth of A at any other $\beta = k \in \mathbf{N}$ (for classical events A); the truth of $A \to B$ at 0 shouldn't correlate with the truth of $A \to B$ at any other ordinal $\beta = k\omega$ (for classical events A, B), etc.

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