Forthcoming in *The Routledge Companion to the Philosophy of Physics*

Newtonian Mechanics and its Philosophical Significance

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The theory that we now know as "Newtonian mechanics" is Newton's science of matter in motion. And its philosophical significance, in a sentence, is this: Newton gave us more than just an empirically successful theory of mechanics – he gave us an account of what *knowledge* of the physical world should look like, one that remains with us. But what is this account of physical knowledge? What is it that remains with us? Various answers to these questions have been given and they concern the *methodological character* of the laws of motion. What is methodologically rational about them? What is their distinctive feature? These are the questions on the table.

The structure of this article is as follows. I will begin by introducing the laws of motion, the relations among them, and the spatio-temporal framework that is implicit in them. Then I will turn to the question of their methodological character. This has been the focus of philosophical discussion from Newton's time to the present, and I will survey the views of some of the major contributors. A theme that runs through this section is that there is something in the spirit of Kant's analysis of Newtonian physics that is worth preserving, though distilling what that is is an open problem. I will conclude by showing that while Newtonian mechanics motivates a number of philosophical ideas about force, mass, motion, and causality – and through this, ideas about space and time – the laws are themselves the outcome of a philosophical or critical conceptual analysis. Therefore, taking some care to understand how the theory grew out of Newton's analysis of the conceptual frameworks of his predecessors and contemporaries is valuable for its insights into the nature of that activity.

A word about the scope of this article is in order. It is worth recalling that *Principia* contains two theories, the theory of mechanics and the theory of universal gravitation. The theory of mechanics is found in a few pages right at the start of *Principia* in "Axioms, or The Laws of

Motion," immediately following Newton's articulation of a few basic notions in "Definitions." The theory of universal gravitation is a derived theory within the mechanical theory, that is, once that theory has been extended, through a number of assumptions, to encompass planetary systems. My focus will be on the theory of mechanics.

What is this theory? It is a theory of causal interaction: it is about motion and the forces producing motion. Newton dealt with matter in resisting and non-resisting media. My focus will be the mechanics of point particles in non-resisting media – the most basic subject matter with which the theory deals. But it is worth mentioning, if only in passing, the formulations of Newtonian mechanics of Euler and Cauchy. (See Truesdell (1977) for details.) And there are still other formulations of Newtonian mechanics, notably those of Lagrange and Hamilton. These formulations are based on the principle of least action and they incorporate insights into the conservation of momentum and energy. They reveal a deep layer of structure exhibited by physical systems of many kinds and make them amenable to a similar treatment; in this way, these formulations extend Newtonian mechanics and greatly increase its computational power. They also provide a point of contact between classical and quantum theory. I will not discuss any of these formulations – my focus will be on "old-fashioned" Newtonian mechanics. (See Curiel (2014) and North (this volume) for a detailed account of the relation of the Lagrangian and Hamiltonian formulations to classical systems and to each other.)

It should also be noted that there is a reconstruction of Newtonian gravitation, patterned on Einstein's theory of gravitation, in which the basic account of motion is inseparable from the gravitational field: this is Cartan's reconstruction. On that reconstruction, gravitation is not a force causing acceleration but a manifestation of the curvature of space-time; as in Einstein's theory, the trajectories of free particles are geodesics (or "straight lines") of the (curved) space-time. Cartan's proposal is in several respects a natural and instructive way of thinking about Newtonian theory. (See Malament (2012) for details.) But I will deal only with Newtonian mechanics, independent of the gravitation theory.

Background: The Theory and the Spatio-temporal Framework Implicit in it

Newton's theory of causal interaction has three axioms, the laws of motion:

> *Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.*
>
> *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.*
>
> *To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and opposite in direction.* (Newton, 1726 [1999], pp. 416-7)

The laws, taken together, define and interpret the concept of inertial motion. This concept is the backbone of the theory, and, by examining how it is articulated, we will come naturally to all core aspects of the theory. The first law defines an ideal force-free trajectory, one from which a particle can be deflected by the action of some force, an objective cause. The first law alone, however, does not provide an account of inertial motion since we do not yet have a definition of force – it is a precondition for such a definition. It is the second law that defines and interprets the concepts of force and mass, tying them to acceleration. The acceleration of mass is the measure of the action of some force. The second law expresses a criterion for distinguishing free particles from particles acted upon by a force. Now, one might be tempted to suggest that the second law alone is enough for giving an account of inertia – one might suggest that the first law is a limiting case of the second, that is, when there is no force impressed. But the first law associates or coordinates a free particle with a particular kind of trajectory, a straight line. Hence, the first and second laws are interdependent.

The first and second laws provide a complete account of inertial motion, provided that one is interested only in ideal point particles. For actual bodies – bodies that are themselves composed of particles – the third law is a necessary condition for inertial motion. The forces among the constituent particles must be equal and opposite, failing which the body by its internal forces will accelerate of its own accord. (See the Scholium to the Laws of Motion, where Newton gives a proof of this; see also Samaroo (2018).) This is what the third law establishes and it is the basis for formulating a principle of conservation of momentum: in an isolated system the total momentum is conserved. Hence, the third law is also a criterion for distinguishing free particles from those acted upon by a force. What should be clear from this brief account is that inertia depends on all three laws for its articulation. It should also be clear that the laws of motion are mutually complementary. Only taken together do they determine Newton's theory of causal interaction.

Now, we can associate with any particle in inertial motion a reference frame. A reference frame is a "small" space, one in which we can describe the motions of bodies in the space among themselves, for example, using a coordinate system. But we can also perform mechanical experiments and calculate their outcomes using the laws of motion. In any such space, the outcomes will be the same. These "inertial frames" that are picked out by the laws of motion are the basis for empirical investigation in Newtonian mechanics.

With this brief account of Newtonian mechanics in hand, let us consider the spatio-temporal framework that is implicit in it. As we have seen, the laws of motion define inertial motion as that state in which a body unacted upon by forces, or on which the net force is zero, moves in uniform rectilinear motion. In other words, a body in inertial motion moves *equal distances in equal times*. In this way, the laws of motion define an ideal clock, which marks the "equable flow" of time. Furthermore, it is implicit in the theory that all inertial observers, and all ideal clocks, will measure proportional time intervals and agree on which events are simultaneous.

The concept of space that is implicit in Newtonian mechanics is tied to the inertial frame concept. As we have seen, an inertial frame is one in uniform rectilinear motion, one furthermore in which the outcomes of mechanical experiments, calculated using the laws of motion, are the same. And Newton noted that the same outcome would be obtained in any frame in uniform rectilinear motion relative to *it* – this is the Galilean principle of relativity. By way of this invariance property, we obtain an equivalence-class of inertial frames – the class of frames in which the outcomes of mechanical experiments are the same. The equivalence-class structure so determined *is* the structure of the space-time of Newtonian mechanics.

It is important to note that this structure admits of no distinction between rest and uniform motion – both are states of inertial motion. By contrast, Newton held that while inertial frames are empirically indistinguishable, they are not theoretically equivalent. They move with various velocities relative to what he called "absolute space," even if those velocities cannot be known. It was only in the nineteenth century, through the work of Neumann (1870), Thomson (1884), Lange (1885), and others, that absolute space was shown to be superfluous.

It is sometimes said that the concept of space-time has its origin in Einstein's special theory of relativity in 1905. But something that should be evident from the foregoing is that already in Newtonian mechanics there is a concept of space-time. The very concept of an inertial trajectory, which is the basis for Newton's theory of causal interaction, appeals not just to places and times but to *places connected at times*. And not only does an inertial trajectory connect places at times but it connects them in such a way that certain states of motion are well defined. (See Stein (1967), DiSalle (2006), and Malament (2012) for careful accounts of this; see also Earman (1989) and Weatherall (2016) for other accounts of the spatio-temporal framework that Newtonian mechanics is sometimes held to motivate.) To summarize, what we find in the laws of motion is not only an account of force, motion, and causality but also a spatio-temporal framework.

The Methodological Character of the Laws of Motion

The theory, now introduced in overview, is a paragon of empirical success. What is the rational justification for this success? This is the question with which methodological analysis is concerned. A methodological analysis asks two basic questions: what *kind* of principles are the laws of motion? What is their *role* in the conceptual framework of physics? For example, are the laws entirely determined by empirical evidence? Or do they reflect elements of choice, for example, considerations of simplicity? Or is their methodological character more complex? And, if so, what is their character?

One answer is that given by Hume (1777 [1975]), who took Newtonian science to be a revolutionary advance – he took it as the model for his "science of human nature." Hume regarded the laws of motion as empirical generalizations that are inductively derived from constant and regular experience, that is, from a set of empirical facts. Consider the following remark about the second law: "Geometry helps us apply this law …, but the law itself is something we know purely from experience, and no amount of abstract reasoning could lead us one step towards the knowledge of it" (*Enquiry,* IV). It is useful to recall Hume's claim that all objects of human reason are either "relations of ideas," of which his examples are the propositions of arithmetic and geometry, or "matter of fact," namely contingent empirical propositions such as “the Sun will rise tomorrow.” Evidently, Hume regards the laws of motion as matters of fact.

Hume’s view of the laws of motion singles out an obvious feature, namely that experience has a role to play in their formulation. But there are a number of criticisms one might raise against his view. One might say that we do not know the laws “purely from experience.” For example, there are no truly force-free bodies: inertial motion is an ideal state and we have no “impressions.” Hence, one could hardly say that the first law derives from mere induction. One might also take issue with Hume’s remark that geometry merely “helps us” apply the laws. The formulation of the laws *presupposes* a number of mathematical concepts, notably concepts belonging to Euclidean geometry and the calculus. In this way, one might argue that Hume’s division of the objects of reason into relations of ideas and matters of fact is inapt for the analysis of the laws of motion.

Hume’s failure to give a satisfactory characterization of the laws is naturally contrasted with Kant’s. Kant’s account is not without its own difficulties, but it captures a feature of the laws that has remained part of subsequent discussions. Much like Hume, Kant saw in Newton’s theory not only a revolutionary scientific discovery but a revolutionary philosophical advance. He saw a basis for criticizing the reigning Leibnizian tradition in which concepts of force and motion, space and time, substance and causality are applied to the “intelligible” world of monads.

In the First Critique (1787/[1998]), Kant asked, how has science achieved universal assent, while philosophy is the subject of endless dispute? What distinguishes scientific reasoning from philosophical reasoning, so that the former leads to principles that are necessary and universal, whereas the latter remains arbitrary and particular? How can philosophy start on “the secure path of a science”? Kant argued that nothing less than a Copernican Revolution in philosophy is needed. No longer should philosophy be done after the fashion of Leibniz and Wolff, or following an earlier empiricism: philosophy’s task is to reveal the structure of our faculty of understanding – the structure that the very possibility of knowledge implicitly presupposes. Kant’s theory of the constitution of experience provides an account of the *concepts* of this faculty and, of particular interest to us, the *principles* both “constitutive” and “regulative” by which they are applied to possible experience. The principles are rules that the understanding imposes on the appearances, in order to submit them rules. Without such rules, experience would be impossible. We would have nothing but a chaos of sensory appearances. The principles are about the world, and therefore "synthetic," but known through transcendental deduction, and therefore "a priori." Kant regarded the account of the constitution of experience as the true subject matter of metaphysics.

In the *Metaphysical Foundations* (1786 [2011]), Kant held the laws of motion to be just such principles. They have a constitutive function: they determine the concepts of the objects of enquiry; they make it possible for objects of knowledge to be objects of knowledge. The laws of motion are constitutive not only of a particular conception of force, mass, inertia, and causal interaction but of a spatio-temporal framework relative to which true motion can be understood.

Breaking with the Leibnizian tradition, Kant argued that our metaphysical concepts of force and motion, causal interaction, and space and time have no content at all except through their "sensible" counterparts, that is, through their articulation in the laws of motion. Kant's account is a landmark in the theory of knowledge, but it is problematic in at least one respect: Kant took Newton's laws to be the only ones that constitute the concepts of force, mass, and motion, and furthermore space, time, and causality. But though physics did not end with Newton, the idea that the laws of motion and certain other physical principles have a constitutive function was developed in the work of Kant's successors, notably in the work of Poincaré.

The idea that the laws of motion are definitions has a special place in the analysis of their methodological character. What is meant by "definition" is important here. For example, we might take a Russellian notion of definition as a starting point. For Russell (1897), the constituents of a sentence expressing a proposition must have independently grasped meanings. On such an account, the terms appearing in the laws – "force," "mass," "inertial motion" – would already have their meanings, independently of the theory determined by the laws.

But there is another way of thinking about the laws of motion: we might regard them as implicit definitions. We find this idea in the work of Poincaré and Duhem. Poincaré (1902 [1952]) argues against Russell's view that we can know the meanings of primitive terms directly, for example, by intuition or acquaintance. For Poincaré, the primitive terms are implicitly defined by the axioms in which they figure. The laws of motion, on his account, are definitions disguised as claims.

But there is further aspect to his view: Poincaré pointed out that a geometrical framework must be presupposed for the construction of a mechanical theory and he claimed that we can choose any one of the geometries of constant curvature, namely Euclidean, Bolyai-Lobatchevskian or Riemannian. For him, there is no fact of the matter about which of them is the actual space of experience, but, since the laws of mechanics will be simplest on a Euclidean background, he held that Euclidean geometry would always be preferred. He stressed that geometry, on its own, tells us nothing about the behaviour of physical objects, only geometry together with physical laws. He held that a geometrical framework and parts of the laws can be chosen arbitrarily – all that is required is that the remaining part of the laws be chosen such that the resulting theory is empirically adequate. For these reasons, he claimed that the laws of motion are conventions.

The presupposition of a Euclidean background is evident in the first law of motion, where "straight" is understood in the Euclidean sense. The first law defines the trajectory of a force-free body as a straight line – it establishes a correspondence between a physical object and a geometric notion, as part of a particular way of constructing a mechanical theory. But, for Poincaré, it is important that assuming a Euclidean background does not preclude the possibility that the completed theory or another theory that is in some sense more fundamental may lead us to revise our presuppositions about geometry.

What about the second law of motion? Taken on its own, we cannot speak of the truth or falsity of the relation expressed in the law because there is no experiment that settles the question. To speak of the truth or falsity of the law would be to assume that there is something prior to Newtonian mechanics that provides an independent definition of force – at least a definition that forms part of an empirically adequate theory of mechanics. For example, we cannot say that the relation between force and acceleration expressed in the law is imprecise, since any imprecision that we might notice while measuring some particular force only suggests to us that we need to look for the forces contributed by some yet-unnoticed bodies. But to say that there is no experiment that settles the question of the truth or falsity of the relation is not to suggest that the force law is not empirically constrained. The law can only be evaluated as part of the entire system of mechanics that it helps define.

What about the third law of motion? Poincaré notes that there are no perfectly isolated systems, only nearly isolated systems. When we observe such systems, we see that the constituent parts interact with one another such that they satisfy the third law and the centre of gravity of the system moves (nearly) uniformly in a straight line. Poincaré asks, could a more accurate experiment invalidate this? "What, in fact, would a more accurate experiment teach us? It would teach us that the law is only approximately true, and we know that already." (Poincaré, 1902 [1952], p. 105) The third law defines action and reaction to be equal and opposite – it expresses a criterion by which we can determine whether momentum is conserved in an isolated system.

Now, one might think that Poincaré's view that the laws of motion are conventions commits him to the view that they are arbitrary. But he is clear that the laws are not arbitrary:

> Are the laws of acceleration and of the composition of forces only arbitrary conventions? Conventions, yes; arbitrary, no – they would be so if we lost sight of the experiments which led the founders of the science to adopt them, and which, imperfect as they were, were sufficient to justify their adoption. (Poincaré, 1902 [1952], p. 110)

What we find in this passage is Poincaré's recognition that while the laws of motion reflect elements of choice, they are empirically constrained. The laws may of course be revealed to be bad definitions or to have only limited applicability, but they function nonetheless as implicit definitions of the basic concepts of mechanics.

Before pressing on, it is worth noting a (perhaps obvious) feature of laws of motion: they fail a condition of *observational non-creativity*, according to which, roughly speaking, a definition should have no observational consequences. This is a condition that definitions are required to satisfy if they are to be regarded as analytic. The laws of motion are definitions, but they are not analytic – they are empirically constrained.

The proposal that the laws of motion are correctly understood as definitions, and furthermore implicit definitions, takes us much of the way to later proposals. But the notion of implicit definition has its origin in nineteenth-century work in the foundations of geometry, where it is discussed without reference to physical theory. It is evident that the laws of motion have a feature that the axioms of geometry do not: not only do they implicitly define the concepts of mechanics but they interpret them. They coordinate theoretical concepts with empirically measurable correlates.

That certain principles have a defining and coordinating function was recognized by Reichenbach (1928 [1958]), even if he did not discuss the laws of motion explicitly. Reichenbach regarded relativity theory as well-established, but not well-understood. He sought to improve our understanding of it by revealing the physical presuppositions that underlie the application of relativistic geometry and chronometry. Specifically, he argued that their application depends on principles that he called "coordinative definitions." These definitions establish how the claims of a mathematical theory are transformed from mathematical truths into claims that can be revised on the basis of experience. To take a simple example, Euclidean geometry becomes a theory of applied or physical geometry by means of the principle of free mobility: practically rigid bodies undergo free motions without change of shape or dimension. This principle is a presupposition of our ability to perform the compass-and-straightedge constructions of Euclidean geometry, and in this way it controls the application of the theory.

Now, for Reichenbach, the main interest in identifying coordinative definitions resides in their capacity to isolate which among the assumptions that control the application of geometry and chronometry are *conventions* and which are *factual* claims. And it is central to his view that certain principles that control the application of geometry and chronometry are based on stipulations. For this reason, he held coordinative definitions to be arbitrary. We see this, for example, in his account of special relativity, in which he claims that the Einstein synchronization criterion rests on a stipulation about the to and fro velocities of light; hence the synchronization of distant clocks is a matter of convention.

The interpretive function of coordinative definitions led Reichenbach to regard them as constitutive principles. Coordinative definitions serve to apply an uninterpreted conceptual framework – the pure concepts of the understanding – to the world of experience. But while Kant held Newton's laws to be the unique set of principles that constitute the conceptual framework of physics, Reichenbach regarded them not as absolute but relative. He recognized that experience might lead us to mutually inconsistent coordinations that are relativized to particular contexts of enquiry, but have nonetheless a constitutive function.

Reichenbach's notion of coordination is incorporated into the recent work of Michael Friedman (e.g., 2001, 2010). Friedman's account of the laws of motion is found in his analysis of Newton's and Einstein's gravitation theories. According to Friedman, a satisfactory methodological analysis of these theories requires us to distinguish between three levels of enquiry. The *first level* is comprised of principles that are epistemologically distinguished by the fact that they define a space of intellectual and empirical possibilities, and so determine a framework of investigation. They articulate theoretical concepts and their physical interpretations. The *second level* is comprised of empirical hypotheses that are formulable within the framework. The *third level* is comprised of distinctly philosophical principles that motivate discussion of the framework-defining principles and the transition from one theory to another.

Friedman calls the first-level principles "constitutive principles." He includes in this category both mathematical principles or presuppositions and coordinating principles. The mathematical principles define a space of mathematical possibilities; they allow certain kinds of physical theories to be constructed. Among other examples, we find the calculus, linear algebra, and Riemann's theory of manifolds. The coordinating principles, which Friedman understands in Reichenbachian terms, interpret theoretical concepts. They express mathematically formulated criteria for the application of concepts such as force, mass, motion, electric field, magnetic field, and others.

On Friedman's analysis, Newtonian gravitation has as its *constitutive component* Euclidean geometry, the calculus, and the laws of motion. This component defines the space of intellectual and empirical possibilities that allows us to conceive of gravitation as a force, and that makes it possible to formulate the law of universal gravitation, an *empirical hypothesis*. Because constitutive principles articulate and interpret basic theoretical concepts, and so a framework of empirical investigation, Friedman regards them as relativized but nonetheless constitutive principles. They are not a priori, as Kant held them to be, but they are prior to the development of hypotheses about particular systems.

Friedman's approach to the analysis of physical theories is intended as a corrective to Quine's (1951) account of scientific knowledge. Quine took aim at the logical empiricists' account of theories with its distinction between the analytic and synthetic components of a theoretical framework. Quine represented scientific knowledge as a web of belief in which no satisfactory analytic-synthetic distinction can be drawn and in which all strands of the web meet the "tribunal of experience" on an equal footing. He claimed that in the case of a derivation where the conclusion conflicts with experience, there is nothing to prevent us from holding on to the conclusion by revising the logical and mathematical principles that were assumed in the derivation. It is Friedman's principal goal to show that there are distinctions between the components of our frameworks of physical knowledge, and that these components are stratified. Friedman argues, furthermore, that the components of our frameworks are not confirmed or infirmed as wholes, as Quine maintained. It makes little sense to speak of revising the constitutive component of a theory in the case of a conclusion that conflicts with experience since constitutive principles determine the framework of empirical investigation – the framework without which an empirical hypothesis could be neither formulated nor tested.

Friedman's proposal is significant for its restoration of the logical empiricists' idea that frameworks of physical knowledge are stratified. But I have argued (Samaroo, 2015) that Friedman's account of a constitutive principle is too broad: only coordinating principles should be regarded as constitutive. Friedman's inclusion of both mathematical and coordinating principles in the category of constitutive principles is intended to address Quine's contention that the mathematics involved in formulating a theory is just another element in the web of belief. Friedman argues that this view of the role of mathematics in physics fails to account for the way in which mathematics makes certain kinds of empirical theories intellectual possibilities; it fails to account for the way in which mathematics supplies some of the concepts required for formulating a theory and for deriving predictions. While I agree with Friedman about this, there are good reasons for taking constitutive principles to be only those principles that constitute or interpret theoretical concepts by expressing criteria for their application.

First, one might argue that including mathematical principles in a theory's constitutive component opens the notion of a constitutive principle to trivialization. One might argue that what is constitutive is relative to some particular formulation of a theory, and since what is constitutive in one is not constitutive in another, the very idea of a constitutive principle is undermined. For example, Newtonian mechanics admits of various formulations, some of which rest on radically different mathematical frameworks from others. Take, for example, the mathematical frameworks peculiar to analytic mechanics, which are very different from the one in which Newton worked. But, however the theory is formulated, Newtonian mechanics is the theory whose basic structure at least is constituted by the laws of motion.

Second, one might argue that including mathematical principles in a theory's constitutive component lends support to a main feature of Quine's account of theories. A Quinean might argue that if, e.g., the calculus and Euclidean geometry are constitutive components of Newtonian mechanics, then they are confirmed or infirmed along with the rest of the theory. Friedman argues against Quine that constitutive principles do not face the "tribunal of experience" on an equal footing with the empirical hypotheses whose formulation they permit – they are principles without which empirical hypotheses would make neither mathematical nor empirical sense, and without which no test would be possible. But the principles that establish Friedman's argument against Quine are not the mathematical principles, which, on their own, are subject to neither empirical confirmation nor disconfirmation, but the *coordinating principles* that interpret theoretical concepts and control and application of the mathematics. Hence, distinguishing the mathematical principles from the coordinating principles strengthens the case against Quine.

Most importantly, however, the inclusion of both mathematical principles and coordinating principles in a theory's constitutive component blurs the distinction between the theory's *factual* and *non-factual* components. By taking only coordinating principles to be constitutive, we can distinguish clearly between those components of our theories that are empirically constrained and those that are not; we can distinguish between those principles that define and articulate our epistemic relation with the world and those that are part of the formal background or language. The proposed limitation to the account of a constitutive principle is in no way intended to diminish the role of mathematical principles in the articulation and application of physical theories, but to clarify the fact that mathematical principles and coordinating principles have different criteria of truth. This proposal benefits the account of the stratification of our theoretical knowledge and allows a still stronger criticism of Quine's account to be given; it aims in this way to vindicate something close to the analytic-synthetic distinction that Quine rejected. The foregoing is only a brief overview; see Samaroo (2015) for a sustained critical analysis.

In light of these criticisms of Friedman's programme, what remains of the notion of a constitutive principle? And what of the methodological character of the laws of motion? I have argued that the notion of a constitutive principle – a principle that constitutes or interprets a theoretical concept by expressing a criterion of its application – has something to offer the account of the laws of motion. On this account, the laws of motion express criteria for the application of the concepts of force, mass, and inertial motion – and those that depend on these.

It is worth noting that this account of the laws of motion – that is, of the laws as *empirical criteria* – is essentially Einstein's. In a short but suggestive article, Einstein (1919 [2002]) sketched a distinction between theories that provide a general framework for physics ("principle theories" or "framework theories") and specific theories constructed within such a framework ("constructive theories"). Although Einstein's focus was relativity theory, Newtonian mechanics and Einstein's special and general theories are all framework theories. That is, they provide frameworks of constraints in which physical quantities can be constructed and whose evolution can be determined. As Einstein put it, these theories are based on "empirically discovered … general characteristics of natural processes" and they express "mathematically formulated criteria" that physical processes satisfy (Einstein, 1919 [2002], p. 213). These criteria enable us to articulate theoretical concepts such as force, mass, inertia, acceleration, rotation, and simultaneity; furthermore, they motivate spatio-temoporal frameworks. These theories must be presupposed for the construction of theories of special systems, for example, the theory of a point particle or that of a perfect fluid; in the Newtonian context, the theory of the gravitational field. So, while the frameworks articulated by Newtonian mechanics, special relativity, and general relativity are not a priori in any Kantian sense, they are prior in this particular sense. Nonetheless, the presupposition of these frameworks does not preclude the possibility that some new theory will motivate their replacement.

The laws of motion, in sum, are founded on experience and are in that sense synthetic, but they are not mere empirical generalizations, derived by induction. Nor are they synthetic a priori propositions, though they function as "constitutive a priori principles" in a particular sense of that term. They are certainly definitions, but they are not analytic in the sense of being true by mere stipulation or convention, because they are responsible to a body of observation and experiment, and to the pre-analytic concepts of which Newton gave an analysis. They are "analytic of" the concepts they determine. They implicitly define the basic concepts of mechanics that appear in them, but they do more than that: they interpret those concepts. They function as "coordinative definitions," but they are not arbitrary in the Reichenbachian sense of that term. Still, they have the constitutive function that Reichenbach sought to capture: they constitute or interpret theoretical concepts by expressing criteria for their application; furthermore, they control the application of a number of mathematical theories. More simply, perhaps, they are empirical criteria for the application of the basic concepts of mechanics. Newtonian mechanics, then, is a "framework theory" in Einstein's sense. It is a framework of investigation that is "prior" to the theories of special systems that we might pursue and evaluate, but it is evidently not a priori in the usual sense of the term.

The Laws of Motion as the Outcome of a Conceptual Analysis

I will close with a reflection on the nature of Newton's "activity" in constructing his theory of mechanics. The point I wish to make is this: while the laws of motion motivated philosophical ideas about motion, force, and causality, they come from Newton's analysis of what is presupposed in the dynamical reasoning of his predecessors and contemporaries. More specifically, the laws are formulations of the principles that Newton thinks are explicitly, as in the case of the first law, or implicitly, in the case of the second and third laws, presupposed in the reasoning of these figures – when they are reasoning properly, that is, solving problems successfully. In this way, the laws of motion are the outcome of a *philosophical* or *critical conceptual analysis* of the conceptual framework of the mechanical philosophers. I will give a brief account of a few of the main parts of that framework. This terrain has been covered by others and more carefully. I will introduce only as much as is needed to make my point.

The Early Modern current known as "the mechanical philosophy" represents the Universe as a mechanism, one subject to mechanical laws governing all matter and implying determinism. The central tenet of the mechanical philosophy is a principle of action or causality: all physical action is mechanical action, that is, action through pressure or impact. This is the basis for the reductionist view that all natural phenomena can be explained by mechanical processes. Some, though not all, of the mechanical philosophers held that all matter is composed of minute corpuscles and aggregates of them. Their configurations determine bodies' primary and secondary qualities. This, in overview, is the mechanical philosophy.

The mechanical philosophy was intended as a corrective to the hierarchical and teleological aspects of Aristotelian and scholastic-Aristotelian science that persisted. Its proponents took it to be a development of Galileo's programme of mechanical explanation. For this reason, one might begin with Galileo – after all, it was his arguments for the heliocentric hypothesis, and his proposals along the way, that undermined much of Aristotelian physics. From Galileo, we might move from figures such as Mersenne and Gassendi, through Hobbes and Boyle, to Descartes, Huyghens, and Leibniz. But I will focus on Descartes, whose *Principles of Philosophy* (1644 [1983]) was the standard work in seventeenth-century natural philosophy and whose physical principles Newton sought to refute.

Let us consider a few aspects of the mechanical philosophers' views on inertial motion, force, and the conservation of momentum. I will organize my discussion around their embryonic versions of these concepts – the concepts that Newton would subject to a critical analysis.

The first set of ideas is bound up with inertial motion. Two ideas are commonly held: that all motion is relative; a certain conception of inertial motion. Consider first the relativity of motion, an idea with which Descartes, Huyghens, and Leibniz are generally associated, though they understood "relativity of motion" differently. Descartes' criterion of "true motion" is in many respects singular and worth distinguishing from the others: a unique standard of motion that is also relative – relative, that is, to immediately contiguous bodies, which, however, provide a univocal reference. This is unlike the Leibnizian view, or "standard" relativism, according to which *any* two descriptions that agree on the relative distances, and so on changes of instantaneous "situation," are equivalent. This might aptly be called a "general principle of relativity." Now, on the surface, Huyghens shared with Leibniz the view that all motion is relative. But where Leibniz defended a "general relativity," Huyghens recognized that determining accelerations and rotations implicitly depends on a *privileged* state of uniform rectilinear motion relative to which they can be referred. This was also recognized by Newton – see, e.g., his criticism of Cartesian motion in *De grav* (c1660 [2004]). Both Newton and Huyghens saw clearly that such a state of motion is necessary for a satisfactory expression of the principles of mechanics. It is the recognition of this privileged state of motion – a state of "true motion" over and above the merely relative motions – that led Huyghens to formulate the first law of motion, which Newton embraced. For Newton, then, the first law expresses what was explicit in Huyghens' work and implicitly presupposed in other seventeenth-century accounts. (See Stein (1977) for references to the original sources and for translations of previously unpublished fragments of Huyghens.)

Consider also the idea of inertial motion. This is commonly held to have come from Descartes, though it was again Huygens who first stated it properly. For all that, it is worth considering the view of Descartes, who helped lay the foundations of mechanics by taking motion and rest to be primitive states of bodies that do not require further explanation. We find this in his first two laws of nature:

> The first law of nature: that each thing, as far as is in its power, always remains in the same state; and that consequently, when it is once moved, it always continues to move ... (*Principles*, II, §37)
>
> The second law of nature: that all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the centre of the circle which they are describing. (*Principles*, II, §39)

Many of the salient features of Newtonian inertia are there: a body at rest remains at rest; a body in motion perseveres in its state of motion; bodies move in straight lines. But the concept of motion that Descartes articulates in the first and second laws differs from Newtonian inertia in that motion and rest are different states. Descartes also fails to make clear the connection between motion and force: there is, for example, no recognition that in the cases of both rest and motion the net force on a body is zero. Nor is there any requirement that a body's state of motion be *uniform with respect to time*, that is, unaccelerated. The requirement of uniformity is essential: without it, there is no notion that a body moving inertially moves *equal distances in equal times*, and hence no notion of an ideal clock that marks time. On Descartes' account, there is no concept of inertial motion in any Newtonian sense, and hence no basis for articulating viable concepts of force and mass. Newton's formulation of the first law of motion reflects his understanding of the very features that the Cartesian account lacks; his formulation of the law is an explication of what a satisfactory account of force demands.

The second set of ideas I will consider is concerned with the concept of force or action. Here, too, Cartesian physics was a starting point for Newton's analysis. Descartes sought to free mechanics from the hierarchical and teleological aspects of scholastic science; he set out to rid physics of qualitative properties and to reduce everything to "certain dispositions of size, figure, and motion." For all that, Descartes discusses force in a way that is reminiscent of the scholastic-Aristotelian framework of impetus and resistance: he appeals to the power or tendency needed to maintain bodies in their state of rest or to keep them in rectilinear motion. There is a question whether a body's tendency to move or to remain at rest is an essential, God-given characteristic or whether it derives from extension and from the interactions of bodies among themselves.

Newton's critical analysis of Cartesian force follows naturally in several respects from his articulation of the concept of inertial motion. Newtonian inertia is a necessary presupposition for saying what an *objective cause* or *force* is: it is that which deflects a body from its uniform rectilinear trajectory. And through this account, we also gain the concept of inertial mass, understood as a body's resistance to changes to its state of motion. All three concepts – inertial motion, force, and inertial mass – are articulated at once. Only with Newton's laws, therefore, do we find an explication of force that is a full realization of the mechanical philosophers' ideal: a physical quantity known only through its *effects*. As is well known, however, Newton's account of force encompasses the action of fields of force on distant matter, something the mechanical philosophy cannot comprehend.

The third set of ideas is bound up with the conservation of a certain "quantity of motion" in collisions and interactions. Here, again, Cartesian physics is in the background. This is the subject of Descartes' third law:

> The third law: that a body, upon coming in contact with a stronger one, loses none of its motion; but that, upon coming in contact with a weaker one, it loses as much as it transfers to that weaker body (*Principles*, II, §40)

Descartes' account of the quantity of motion that is transferred from one body to another in a collision is clarified in §43. He takes that quantity to be the product of the size and speed of the body, where "size" appears to be understood as volume or bulk; there is no mention of the vector quantity velocity, only the scalar quantity "speed."

Descartes' account falls short of Newton's in several respects – see, e.g., the seven rules of impact (§46-52) where this is manifest – but the principle at issue is an embryonic version of the principle of conservation of momentum. Descartes' account is significant for being one of the first attempts at formulating the principle. But, for Descartes, the conservation of momentum has as much a theological as an empirical foundation: when God created the Universe, He gave to all bodies a certain quantity of motion, a quantity that He preserves at every successive moment, even when it is transferred (§62).

For a strictly empirical account of the conservation of momentum, we need to look elsewhere. The germ is already there in the work of Galileo and his contemporaries. But the first systematic accounts are found in the work of Wallis (1668), Wren (1668), and Huyghens (1669) on the laws of impact: Wallis dealt with inelastic collisions; Wren and Huyghens elastic ones. Their work is mentioned by Newton in the Scholium to the Laws and the third law of motion is based on it. Given two bodies A and B, the third law of motion defines their interaction as $F_{A\ on\ B} = - F_{B\ on\ A}$ and, in an isolated system, the corresponding expression for the conservation of momentum is $\frac{dp_A}{dt} = -\frac{dp_B}{dt}$. In this way, Newton not only incorporates the contributions of his contemporaries but explicates them in his theory of mechanics.

What should be clear from the foregoing is that the laws of motion, far from being radical, are implicitly and explicitly presupposed in the work of the mechanical philosophers. This was Newton's reason for taking them to be axioms. Newton's activity, then, is an eminently philosophical one: it is a critical conceptual analysis of confused concepts; it is also an analysis of successful practice that aims to discover the principles on which that practice depends. This culminates in the explication of the basic concepts of mechanics and the articulation of criteria for their application. These criteria are the basis for an empirically adequate theory of mechanics. We find in Newton's construction of his theory an exemplar of an approach to conceptual analysis that has been at the heart the foundations of physics, at least since Galileo, and at the heart of the analytic tradition, at least since Frege. Conceptual analysis, so understood, is the practice of identifying central features of a concept by revealing the assumptions on which use of the concept depends. (This way of expressing the basic idea of conceptual analysis is due to Demopoulos (2000, p. 220).) This practice proceeds by examining the use, misuse, and limitations of pre-existing concepts – in the case of interest to us, inertia, force, and mass – and revealing the assumptions on which their pre-analytic use depends. So, while Newton's theory of mechanics motivates philosophical ideas about matter and motion, space-time and causality, and philosophical debates about them, its philosophical significance goes deeper: the theory is a reminder of what conceptual analysis in the foundations of physics might aspire to. The laws of motion are the result of that analysis.

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