What can bouncing oil droplets tell us about quantum mechanics?

Peter W. Evans[*1] **and Karim P. Y. Thébault**[†2]

[1] *School of Historical and Philosophical Inquiry*, University of Queensland
[2] *Department of Philosophy*, University of Bristol

October 28, 2019

Abstract

A recent series of experiments have demonstrated that a classical fluid mechanical system, constituted by an oil droplet bouncing on a vibrating fluid surface, can be induced to display a number of behaviours previously considered to be distinctly quantum. To explain this correspondence it has been suggested that the fluid mechanical system provides a single-particle classical model of de Broglie's idiosyncratic 'double solution' pilot wave theory of quantum mechanics. In this paper we assess the epistemic function of the bouncing oil droplet experiments in relation to quantum mechanics. Our analysis is framed by a two-way comparison between these experiments and recent philosophical discussions of confirmation via analogue black hole experiments (Dardashti *et al.*, 2015) and explanations via toy models (Reutlinger *et al.*, 2017). We find that the bouncing oil droplets are best conceived as *analogue representations* of quantum phenomena, rather than *analogue simulations*, and, furthermore, that their epistemic value should be understood in terms of how-possibly explanation, rather than confirmation. By pointing us towards the distinction between analogue simulation and analogue representation in current scientific practice the walker experiments evidence the wider philosophical moral that the relations of 'material surrogacy' and 'material representation' are distinct.

---

[*] email: p.evans@uq.edu.au
[†] email: karim.thebault@bristol.ac.uk

Contents

1 Introduction

In 2005, a team in Paris Diderot University led by Yves Couder and Emmanuel Fort discovered that an oil droplet bouncing on a vibrating fluid surface can be made to 'walk' horizontally across the surface. These 'walkers' display a kind of wave-particle duality: the bouncing droplet is self-propelled by interacting with the surface waves it creates. A series of subsequent experiments from both the team in Paris and an associated team led by John Bush at Massachusetts Institute of Technology (henceforth, the 'walker experiments') have since demonstrated that this fluid mechanical system displays behaviour that is typically considered to be quantum behaviour. This behaviour includes single and double slit diffraction and interference (Couder and Fort, 2006)[1] and quantised orbits of bound state pairs (Fort *et al.*, 2010), as well as phenomena that look analogous to quantum tunnelling (Eddi *et al.*, 2009), Schrödinger evolution of probabilities (Couder and Fort, 2012), and Zeeman splitting (Eddi *et al.*, 2012).

A string of strongly qualified suggestions have emanated from both the Paris and MIT teams that this fluid mechanical system provides a single-particle classical model of the pilot wave mechanism of de Broglie's idiosyncratic double solution pilot wave theory. For instance (emphasis added): "Such a system is *reminiscent* of the early de Broglie models for quantum systems" (Couder and Fort, 2012, p.2); "This *appears very close* to the hypothesis of a double solution put forward by de Broglie" (Couder and Fort, 2012, p.6); "our system *could be considered* as implementing at [a] macroscopic scale the idea of a pilot wave considered by de Broglie

---

[1] Although recent experiments contest the single and double slit diffraction and interference results: at best these phenomena are difficult to reproduce (Pucci *et al.*, 2018), and at worst cannot be reproduced at all (Andersen *et al.*, 2015; Batelaan *et al.*, 2016; Bohr *et al.*, 2016).

and Bohm for elementary particles at [a] quantum scale" (Fort *et al.*, 2010, p.17520); "in spite of the huge gap between the systems, *analogies exist with some aspects of* the particle-wave behaviour at [a] quantum scale" (Eddi *et al.*, 2011, p.459); "This hydrodynamic system *bears a remarkable similarity* to an early model of quantum dynamics, the pilot wave theory of Louis de Broglie" (Moláček and Bush, 2013, p.613); "The walker system *bears a notable resemblance* to an early conception of relativistic quantum dynamics, Louis de Broglie's double-solution pilot-wave theory" (Bush, 2015, p.170).

The basic idea behind this suggestive correspondence is that the motion of the oil droplet (the 'walker') is determined by the wave on the fluid surface in just the same way that the motion of a particle is determined by the phase of an associated wave in de Broglie's double solution formulation of the pilot wave approach to quantum theory. It is interesting to ask, however, whether and to what extent this classical fluid mechanical system can be considered a genuine *epistemic tool* to probe quantum behaviour. Our analysis is framed by a two-way comparison between these experiments and recent philosophical discussions of confirmation via analogue black hole experiments (Dardashti *et al.*, 2015) and explanations via toy models (Reutlinger *et al.*, 2017). We propose that despite a superficial similarity the epistemic function of the walker experiments is very different to that of analogue black hole experiments. The analogue black hole experiments, as reconstructed by Dardashti *et al.* (2015), exemplify the increasingly common scientific practice of analogue simulation, where a material system is manipulated and a formal relationship obtains between *empirical terms* (i.e. linguistic items that putatively correspond to physical phenomena) in the model of the system being manipulated (the source) and the model of the system about which we hope to learn (the target). In such circumstances it is possible that we can gain knowledge about *actual* phenomenal features of the target system. In contrast, in the case of the walker experiments, according to our rational reconstruction, the material system is manipulated with the epistemic goal of establishing a formal relationship between empirical terms in the source model (i.e. the model of the walker) and *extra*-empirical terms in the target model (i.e. terms corresponding to non-phenomenal aspects of the ontology of de Broglie's double solution theory). We categorise such activities as *analogue representation*, and argue that we should understand their primary epistemic function as the provision of *how-possibly explanations*. In this precise sense, our account of the epistemic function of the walker experiments has close parallels to the account of explanations via toy models provided by Reutlinger *et al.* (2017).

By pointing us towards the distinction between analogue simulation and analogue representation in current scientific practice the walker experiments evidence the wider philosophical moral that the relations of 'material surrogacy' and 'material representation' are distinct. The first, as illustrated by the analogue black holes example, is essentially a form of Ersatz experimentation, and is used by scientists to gain knowledge of the actual phenomena in a target system. The second, as illustrated by the walker experiments, is part of a much more general practice of scientific representation and gains its most epistemically important function in the context of how-possibly explanations provided by the material representation of extra-empirical terms describing non-phenomenal features of a target system. Whilst the two may often be performed together within one experiment, they are importantly different in both their logical form and inferential role. Furthermore, although relations of material surrogacy in general, and analogue simulation in particular, can always be interpreted as simultaneously functioning as material representations, the converse will not in general be true.

We proceed as follows. In §2 we describe in detail both de Broglie's double solution pilot wave theory and the walker experiments, with a view to identifying the relevant connections between the two. In §3 we discuss the various relevant ideas that can be drawn from the philosophical analysis of analogue experiments in general. First, we introduce the crucial distinction between analogue representation and analogue simulation. Second, we consider the subtle issue of the validation of representations and simulations, and the question of whether a single experiment can combine both material representational and material surrogacy functions. Finally, we employ these distinctions to examine the epistemic value of different forms of analogue experiments with a focus on confirmation and explanation. §4 then deploys this general framework to provide an answer to the question: What can bouncing oil droplets tell us about quantum mechanics? We consider in turn the surrogacy, representational and explanatory aspects of the experiments and offer our constructive, if rather deflationary, conclusion: we find that the bouncing oil droplets are best conceived as *analogue representations* of quantum phenomena, rather than *analogue simulations*, and that their epistemic value should be understood in terms of how-possibly explanation, rather than confirmation. In addition we warn that, due to the classicality of the walker experiments, their ability to be an analogue representation (let alone simulation) of any entanglement-based quantum phenomena that involve the violation of Bell-type inequalities is exceedingly constrained.

2 De Broglie's pilot wave theory and the walker experiments

Pilot wave theory was first proposed by de Broglie (1924, 1927a,b) before being independently redeveloped twenty five years later by Bohm (1952). Both formulations emphasise the existence of an actual configuration of particles that underpins the dynamics of a quantum system, where the actual positions and velocities of the constituent particles comprise a set of 'hidden variables' for the system. The claims concerning analogy emanating from the Paris and MIT teams mostly concern specifically de Broglie's pilot wave theory. It will prove worthwhile to briefly set out the basic distinction between de Broglie's original formulation of pilot wave theory and Bohm's subsequent formulation before we provide a more detailed account.

For de Broglie, quantum dynamics is characterised by the Schrödinger equation, which defines a 'pilot wave' that governs the evolution of the configuration of the system on configuration space, and a guiding equation, which determines the velocities of the particle trajectories prescribed by the evolution of the configuration as a function of a phase defined by the pilot wave. This velocity guiding equation is taken as the fundamental law of motion and so provides a first-order dynamics (Bacciagaluppi and Valentini, 2009, p.29). For Bohm, quantum dynamics is characterised as an extension of classical dynamics with the addition of a 'quantum potential' that is a function of the solution to the Schrödinger equation, and which contributes a quantum force on the particle configuration in addition to any classical Newtonian forces. The Newtonian relation between the quantum potential and the acceleration of the particle trajectories is taken as the fundamental law of motion and so provides a second-order dynamics, while de Broglie's guiding equation is taken in Bohm's formulation as a dynamical constraint (Bacciagaluppi and Valentini, 2009, p.29). Thus the basic difference between the two approaches is whether the quantum description is *fundamentally* Newtonian or not; Bohm adopting the former and de Broglie adopting the latter. This crucial difference between the two approaches thus amounts to a difference of *extra-empirical* structure. It will be worthwhile for the reader to keep this basic point of difference in mind as we dive into the detailed description that follows.

2.1 Double solution theory

Undoubtedly de Broglie's most famous contribution to quantum theory is the extension of Einstein's (1905) idea of wave-particle duality from the photon to massive particles. This idea is first articulated in de Broglie's doctoral thesis (de Broglie, 1924) but is not the central focus of that work. Rather, de Broglie's aim was to propose an equivalence between Fermat's principle of least time for describing rays of light and Maupertuis' principle of least action for describing moving bodies (de Broglie, 1924, p.56). In particular, the equivalence of Maupertuis' and Fermat's principles suggests that the role played by the 4-momentum, p_μ, in the motion of a body corresponds to the role played by the phase differential, $\mathrm{d}\phi$, in the propagation of a wave. Equating these two elements amounts essentially to a 4-vector generalisation of the Planck-Einstein relation, $E = h\nu$, to $w_\mu = \frac{1}{h}p_\mu$, with $w_\mu \propto \mathrm{d}\phi$ (Bacciagaluppi and Valentini, 2009, p.42).

This generalisation amounts to new law of motion for de Broglie – an early statement of his guiding equation – in which the momentum of a body (or particle velocity) is determined by the phase of an associated wave.[2] Significantly, such a law of motion is a departure from Newtonian mechanics: the law of inertia no longer applies to these quantum bodies, which would move along the rays of their associated wave, and so can deviate from a straight path without any applied forces (for instance, in the process of diffraction). De Broglie (1924, p.80) does presciently note, however, that one could recover the Newtonian picture by imagining a force to be active in such a process. This is precisely the step that Bohm was later to make.

The full presentation of de Broglie's theory – the one that we have come to know as pilot wave theory – is given in (de Broglie, 1927a). In contrast to Schrödinger, de Broglie explicitly hypothesises that particles be understood as solutions of the wave equations of motion, "the amplitude of which includes a peculiar singularity" (de Broglie, 1927a, p.225). Whereas Schrödinger develops a perspective in which 'particles' are an unnecessary part of the dynamics of the continuous phase wave Ψ (and Born goes one step further and explicitly attributes a statistical nature to this wave), de Broglie (1927a, p.225) is motivated by the possibility of a duality between Schrödinger's continuous solutions of the wave equations and his singular solutions representing the particles (hence his 'double solution' theory). He begins with the relativistic wave mechanical equation of motion (the Klein-Gordon equation) as a description of a particle in a constant potential, and considers solutions, $u(\mathbf{x}, t)$, with singular amplitude. From this he is able to derive a partial differential equation which, in the regime in which the

---

[2] For detailed exploration of the connection between this aspect of de Broglie's work and Schrödinger's derivation of the equation that bears his name see (Joas and Lehner, 2009).

behaviour of the amplitude singularity obeys the classical wave equation, approximates the relativistic equation of motion for a classical particle. Since in this classical regime the particle velocity aligns with the phase gradient, de Broglie (1927a, p.230) assumes that it does so in the non-classical regime also, reinforcing his strong commitment to the equivalence of the principles of Maupertuis and Fermat (particle and wave descriptions).

De Broglie then considers a continuous solution, $\Psi(\mathbf{x}, t)$, to the same wave equation (which is identical to the solutions to Schrödinger's wave equation). Whereas, in modern parlance, the $u(\mathbf{x}, t)$ are to be interpreted as ontic, since they represent real physical particles, the $\Psi(\mathbf{x}, t)$ are interpreted as epistemic, since they are taken to represent an *effective ensemble* of the singular particles. De Broglie derives a partial differential equation which, in the regime in which the continuous amplitude, a, is harmonic, i.e., $\nabla^2 a(\mathbf{x}) = 0$, approximates the wave equation for geometrical optics. He notes that the relativistic equation of motion for a classical particle and the wave equation for geometrical optics are the same equation of motion (the Hamilton-Jacobi equation) so long as the phase factors in each are identical. He then assumes that the phase factors are identical in the general (nonclassical) case, which ensures that the singular and continuous wave solutions are interdependent by virtue of sharing a common phase factor. It is this assumption that de Broglie refers to as the 'principle of the double solution', and this interdependence between the singular and continuous waves is the embodiment of the wave-particle duality at the core of de Broglie's double solution theory.

De Broglie (1927a, p.232) demonstrates that, from the identity of phase factors, one can understand Ψ as representing an effective ensemble particle density over space, where density is proportional to the square of the amplitude of Ψ. Moreover, de Broglie realises that interpreting Ψ as a representation of particle density has a novel consequence: equal probability over the initial positions of an ensemble of particles results in a set of possible particle trajectories and, since the trajectories are determined by surfaces of equal phase, the trajectories are *guided by* Ψ (de Broglie, 1927a, p.232; Bacciagaluppi and Valentini, 2009, p.59). For this reason, Ψ is called the guiding wave, or pilot wave, and the generalised quantum relation relating the phase of the guiding wave to the particle velocity is known as the guiding equation.

2.2 Ontology of the double solution theory

To understand the ontology of de Broglie's double solution theory it will prove highly instructive to contrast his interpretation with those of Schödinger and Bohm respectively.

Schrödinger was adamant that the continuous phase wave Ψ is not accompanied by an associated particle with a well-defined position or trajectory, rather 'particles' are to be identified with the spatially distributed wave packet (Bacciagaluppi and Valentini, 2009, p.116). However, de Broglie (1927a, p.238) notes, when one considers a many-body system of N particles, the solutions to the relevant wave equation exist in $3N$-dimensional configuration space. If this were the case, then it would be difficult to see the physical meaning of the coordinates used to construct the abstract configuration space in the first place. Moreover, it is unclear what physical meaning we should attach to the propagation of the continuous wave in an abstract configuration space. De Broglie points out that both of these difficulties disappear if we admit that particles always have well-defined positions and trajectories and that the physical picture of the system consists in N waves propagating in real 3-space rather than a single wave propagating in $3N$-dimensional configuration space.

This then is the foundation of de Broglie's interpretation of the pilot wave. By extending his framework for understanding $u(\mathbf{x}, t)$ and $\Psi(\mathbf{x}, t)$, and the phase identity between them, from the single-particle case to the many-body case (where there would now be many $u_i(\mathbf{x}, t)$), de Broglie is able to interpret Ψ as performing two distinct roles. Firstly, Ψ is a pilot wave: its phase, by being identical to the phase of the singular waves, determines the particle velocities of the system (and so the particle trajectories when the initial positions are given). Secondly, Ψ is a probability wave: the "fictitious" wave in configuration space in the many-body case plays the same role as the continuous wave in the single-particle case such that the square of its amplitude determines at each point the "probability of presence" of the particle configuration (and so the probability density of the particle trajectories when the initial positions are not given) (de Broglie, 1927a, p.240).

What we must not lose sight of here is that, for de Broglie, the guiding equation is the fundamental equation of motion for his new dynamics. That is, the insight at the heart of the equivalence between the principles of Maupertuis and Fermat – that there is phase harmony between the wave and particle aspects of the double solution to the dynamical equation – is the key motivation for de Broglie's pilot wave theory. So while Ψ plays the role of the pilot wave, it can only do so as a result of this phase harmony between it and the real u-waves in 3-space. The reason it is important to mention this here relates to a possibility that de Broglie (1927a, p.241) points out for understanding Ψ. De Broglie's velocity law of motion is derived by invoking his double solution principle. Recall that as a result of the principle, de Broglie simply assumes that the phase factors in the particle and wave motions are identical in the general, non-classical case on account of establishing that the particle velocity aligns with the phase gradient in the classical case. But one could just as easily assume the velocity law to hold as a *postulate* of the theory, rather than derive it from some underlying foundation. As a consequence, one could consider the pilot wave Ψ as physically real, and distinct from the reality of the material point, such that the motion of the material point is determined as a function of the phase of the pilot wave by the velocity law (Bacciagaluppi and Valentini, 2009, p.64), just as Bohm later suggested.

De Broglie does not advocate this position as the appropriate way to understand his theory. Indeed, de Broglie (1927a, p.241) is explicit about considering this understanding of Ψ as a real pilot wave in this sense with a "provisional attitude", preferring the above account of his double solution theory – and believing that the provisional account would ultimately lead one towards something analogous to the double solution theory. However, de Broglie (1927b) went on to present precisely this provisional theory at the fifth Solvay conference in 1927, with no use made of the principle of the double solution (Bacciagaluppi and Valentini, 2009, p.67–69). It was this provisional theory that became known as de Broglie's pilot wave theory, and it was this theory that Bohm rediscovered 25 years later, rectifying some of the outstanding problems identified with de Broglie's presentation of the theory at the fifth Solvay conference. Thus, in so far as Bohm's pilot wave theory is operationally equivalent to quantum mechanics (Bohm, 1952, p.166), so too is de Broglie's truncated 'provisional' theory operationally equivalent to quantum mechanics. But this equivalence belies the significant foundational differences between de Broglie's original double solution formulation and Bohm's later formulation.

The major divergence is that de Broglie takes his guiding equation to be a novel, non-Newtonian, fundamental law of motion. De Broglie is well aware of the possibility that the addition of a Newtonian force, underpinned by a new quantum potential, could plausibly account for the noninertial motion that results from this law, and he explicitly derives this classical formulation of the particle dynamics and, in the nonrelativistic limit, the additional potential energy term is precisely Bohm's 'quantum potential' (de Broglie, 1927a, p.237; Bacciagaluppi and Valentini, 2009, p.61). However, for de Broglie this is merely a demonstration of the relation between his new mechanics and classical mechanics. In contrast, the Newtonian relation between the quantum potential and the acceleration of the particle trajectories is taken by Bohm to be the fundamental law of motion, and this provides a second-order dynamics (while de Broglie's guiding equation is taken in Bohm's formulation as a dynamical constraint).

The divergence between de Broglie and Bohm is thus manifest at the *ontological* level. Whereas for Bohm (1952, p.170) the pilot wave Ψ is "a mathematical representation of an objectively real field", for de Broglie Ψ is a 'fictitious' probability wave and it is the u_i that are real waves in 3-space. Despite the fact that de Broglie never completed his double solution theory – notwithstanding later attempts (de Broglie, 1959, 1971)) – de Broglie had a clear idea of the ontology of his theory: an N-body system consists of N singular u-waves propagating in real 3-space, each defining particle motion according to the velocity equation, and whose phase is interrelated with the phase of the probability wave Ψ that guides and constrains system behaviour. Understanding the walker experiments as a concrete *representation* of this ontology will prove crucial to our analysis of the epistemic function of the experiments in the remainder of the paper.

2.3 The walker experiments

Consider a small, shallow rectangular bath oriented horizontally, filled with a layer of silicon oil, and parametrically driven by a low frequency generator to vibrate vertically. By piercing the fluid surface with a pin and then withdrawing quickly, a small oil droplet can be created which, due to the forced vibrations, bounces upon the fluid surface. There are two key features of the walker experiments. Strictly below the Faraday instability threshold,[3] beyond which forced standing waves appear on the surface of the fluid, the fluid surface is stable. When the bouncing droplet comes into contact with the surface it emits a travelling capillary wave (like a pebble dropped into a still pond) radially damped by the viscosity of the fluid. Thus, the first key feature is that each droplet is a local source of a standing Faraday wave that is sustained by the externally driven, vertically vibrating fluid (Protière *et al.*, 2005). During the time that the droplet is in flight above the fluid, this capillary wave evolves freely across the surface. When the droplet next bounces on the surface, small deviations in the flight of the droplet can cause the impact to occur to one side of the central crest of the capillary wave, and thus on an inclined surface. This deviation imparts a horizontal impetus on the droplet such that the

---

[3] The phenomenon of Faraday instability is closely related to the more familiar phenomenon of grains of sand on the surface of a beaten drum forming geometrical patterns (Faraday, 1831).

next surface impact again occurs on an inclined surface. As Protière *et al.* (2006, p. 92) put it:

> Each time the drop hits the surface a new dip forms, shifted from the trough that would have been formed by the evolution of the previous wave-packet. The resulting wave is thus the superposition of waves generated by a source that is slightly displaced at each jump.

Within the appropriate parameter regime, the Faraday wave spontaneously propels the droplet to 'walk' horizontally across the surface, coupling the motion of the droplet to the vertical displacement of the fluid surface (Protière *et al.*, 2006).

Significantly, the damping of the capillary waves that a droplet emits is inversely proportional to the distance to the Faraday instability threshold: the closer the driving frequency gets to the threshold, the longer the damping length becomes. This is then the second key feature of the walker experiments: due to the external impetus applied to vibrate the fluid, the standing Faraday waves created from each bounce of the droplet are sustained (again, in the appropriate parameter regime) for very many bounces of the droplet. The vertical displacement of the fluid surface at any point is thus the linear combination of very many distinct Faraday waves, and this provides the surface wave with a path memory of where the droplet has recently been (Eddi *et al.*, 2011). In other words, the increase in damping length gained by driving the vibration closer to the Faraday instability threshold enables a greater radius of interaction between the droplet and its environment.

The combination of the two key features of the walker experiments – that each droplet is a local source of a standing Faraday wave, and that the standing Faraday waves from each bounce of the droplet are sustained for very many bounces – yields a surface wave that is a superposition of many standing waves, each encoding information about the previous movement of the droplet. The subsequent motion of the droplet, bouncing off the fluid surface, is then a function of the information encoded in the surface wave:

> Here, each bounce of the droplet, by disturbing the interface, records information about the spatio-temporal localization of the collision. This information is stored because each bounce generates a sustained localized state of Faraday waves. The information being stored in waves, the data about the trajectory are cumulated in an interference pattern due to the waves' linear superposition. Later, as the drop collides again with the interface, it 'reads' this cumulated information and the local slope of the distorted surface determines the direction and amplitude of the next jump. The dual nature of the walker is contained in the path memory dynamics: the wave nature lies in the coding while the particle nature lies in the reading. Eddi *et al.* (2011, p.461)

This is how the oil droplet comes to be coupled to the surface wave, and so provide what we might call pilot wave dynamics: the "interplay between the droplet motion and its associated wave field makes it a macroscopic implementation of a pilot wave dynamics" (Couder and Fort, 2012, p.2).

Another feature of the walker experiments, related to path memory, has received far less attention (although Vervoort (2016) gestures towards it). Each time the droplet bounces on the fluid surface a distinct damped travelling capillary wave is emitted[4] that travels at a velocity

---

[4] "These waves are…the travelling equivalent of the standing Faraday waves usually observed" (Protière *et al.*, 2006, p. 91).

typically about 10 times the walking velocity of the particle (Protière *et al.*, 2006, p. 95). Given a boundary – such as a wall, a slit, a submerged barrier or a new particle with its own associated wave – within the damping length of the capillary wave, the surface wave that results from the superposition of successive capillary waves will contain a reflected component encoding information about this boundary. Since the particle is coupled to this wave, the motion of the particle is in part influenced by spatially remote boundaries (so long as they are within the damping length), producing what we might call 'nonlocal' effects. Significantly, some of these nonlocal boundaries lie in the 'future' path of the walking particle and it is this "echo-location" feature that explains, for instance, the diffraction of a particle walking through a single slit (Couder and Fort, 2012, p.2).

A combination of the path memory and these 'nonlocal' effects provides for some interesting properties of the droplet motion, the most significant being chaotic motion in the presence of obstacles. As Eddi *et al.* (2011, p.461) state:

> Other dramatic effects of the memory are observed whenever boundaries generate any kind of confinement of the walker. In these situations, the waves emitted in the past and reflected by the boundaries lead to a complex structure of the interference field and correspondingly to a disorder in the droplet motion.

In a more recent set of experiments performed by the team at MIT, in collaboration with the Paris team, a further significant phenomenon has been detected (Harris *et al.*, 2013). Given a defined bath geometry, a droplet will move chaotically under the guidance of the surface wave. After enough time, taking a record of the droplet's total path yields a location density map representing the statistical behaviour of the droplet. This map can be interpreted as the probability distribution for the droplet's location at some time. It turns out that this probability distribution is related to the Faraday modes of the bath geometry. There are then two wavelike modes of description for the droplet's behaviour: a surface wave (that is a superposition of capillary waves) guides (chaotically) the motion of the droplet, and a probability wave (that is a function of the geometry of the fluid boundary) constrains the distribution of the locations at which the droplet might be found at any time. Here is Harris *et al.* (2013, p.011001-4) on this point:

> We can thus understand the probability distribution as being a manifestation of the characteristics of the underlying trajectories. In the confined circular geometry, the pilot wave dynamics tends to drive the walker along circular orbits with radii corresponding to maxima of the cavity mode amplitude. Instead of being trapped on these orbits as in the low-path-memory limit, the walker wobbles around them and drifts between them; nevertheless, these unstable orbits leave their mark on the probability distribution.

This then completes the collection of phenomena that are the significant features of the walker experiments which suggest an analogy with de Broglie's double solution theory. Echoing our sentiment at the end of §2.2 above, Harris *et al.* (2013, p.011001-4) summarise nicely:

> Our study indicates that this hydrodynamic system is closely related to the physical picture of quantum dynamics envisaged by de Broglie, in which rapid oscillations originating in the particle give rise to a guiding wave field. The pilot wave theories of de Broglie and Bohm are often conflated; however, it is valuable to distinguish between them here for the sake of

> comparison with our system. According to Bohm, the particle is guided by its statistical wave, its velocity being equivalent to the quantum velocity of probability. According to de Broglie's double-wave solution, the particle is guided by a real wave (of unspecified origins) in such a way as to execute a dynamics whose statistics is described by standard quantum theory.

It should be clear that there are some striking analogies between the mechanism of the walker experiments and the ontology of de Broglie's double solution theory.[5] At the heart of these analogies is wave-particle duality. First and foremost, the interrelation between the surface wave of the fluid and the motion of the oil droplet appears analogous to the interrelation between each u-wave in 3-space and its associated singular amplitude. Through de Broglie's guiding equation, the particle-like behaviour of these singularities is closely aligned with the wave-like behaviour of their associated u-wave. Likewise, the particle-like behaviour of the walker is closely aligned with the wave-like behaviour of the surface wave.

Furthermore, the differentiation between the real u_i in 3-space determining the behaviour of quanta via the guiding equation and the pilot wave Ψ in configuration space that guides system behaviour through constraining probability current density (the two interrelated by the double solution principle) is analogous to the differentiation between the surface wave guiding the imminent trajectory of the oil droplet and the probability wave (related to the Faraday modes) that constrains the location density map representing the long-run statistical behaviour of the droplet. It is primarily for this reason that the fluid mechanical system more closely resembles the ontology of de Broglie's double solution theory than that of Bohm's pilot wave theory. Finally, we should not forget the lengthy list of typically quantum phenomena displayed by the fluid mechanical system: (possibly) single and double slit diffraction and interference, quantised orbits of bound state pairs, phenomena that look like quantum tunnelling, Schrödinger evolution of probabilities, and Zeeman splitting (but with the conspicuous absence of any entanglement-based quantum phenomena that involve the violation of Bell-type inequalities).

It remains to be seen whether this analogy implies that these experiments can tell us something about quantum mechanics. What value do these experiments have as epistemic tools to probe quantum theory? In order to answer this question, it will be helpful to consider a selection of issues relating to the philosophical analysis of analogue experiments in general terms.

3 Analogue experiments

3.1 Representation and simulation

Before we start our discussion it will be instructive to introduce some terminology relating to different parts of scientific theories and models and their putative correlates in the world. We will use the word *term* to denote any linguistic items within a theory or model. This includes terms in the logical sense, statements, equations, diagrams, or structures. Such 'terms' putatively stand in a relation of *representation* to elements of reality within the world. The precise nature of this representation relation is something regarding which we will endeavour to stay as neutral as possible – we take what we say below to be compatible with any of the various

---

[5] This point is also made by Bush (2015).

accounts in the contemporary literature.[6] We can distinguish three different classes of terms on the basis of the three different classes of elements of reality to which they can putatively correspond. First, we have *observable terms* that correspond to observable phenomena. In the context of physical science such observable phenomena will typically be physical quantities whose value can be directly measured or observed. For example, the angle between the angular diameter of the Moon seen from the Earth. It is important to distinguish between these terms and *empirical terms* that correspond to both observable and unobservable phenomena. That is, a larger set of elements of reality that (in the context of physical science) also typically includes physical quantities the value of which cannot be directly measured or observed, but rather only indirectly measured or inferred. The most vivid example of such a term is probably the mass of the Higgs boson, but various other examples can be found in both historical and contemporary physics.

As powerfully argued by Massimi (2007), building on the earlier ideas of Bogen and Woodward (1988), such unobservable phenomena are what is typically 'saved' by scientists in experimental particle physics, and should thus be taken as part of any adequate empiricist philosophy of science, *pace* Van Fraassen (1980). Further discussion of the observable versus unobservable phenomena distinction is found in Evans and Thébault (2019). For the purpose of the current paper, the more important distinction is between empirical terms and what we will call *extra-empirical terms*. These are linguistic items within a theory or model that putatively stand in a relation of representation to elements of reality within the world that are *non-phenomenal*. That is, elements of reality that are *not* physical quantities. The most powerful examples of such non-phenomenal elements of reality, to which we will refer back later, are absolute space in Newtonian mechanics and the wavefunction in quantum mechanics.

With our terminology in place, we can now introduce a distinction between two types of practice in contemporary physical science, both of which might be referred to as 'analogue experiments'.[7] The first, following the rational reconstruction of Dardashti *et al.* (2015), we call *analogue simulation*. This is when an experiment is conducted on a source system of one type of material constitution in order to learn about a target system, of another type of material constitution, on the basis of an isomorphism between terms in the respective modelling frameworks that provide adequate descriptions of the source and target in the relevant domain. A necessary feature of this notion of analogue simulation is that the isomorphism must connect at least some empirical terms defined in the two modelling frameworks. In fact, it is in virtue of the connection between such terms that the epistemic function of an analogue simulation can be fulfilled. Analogue simulations are designed for learning about empirical features of the

---

[6] An excellent recent discussion specifically relating to representation via material models is (Frigg and Nguyen, 2018). Further accounts, all of which we take to be compatible with our use of 'representation' below, are (Hughes, 1997; Giere, 1999; Suárez, 2004; Contessa, 2007; Bailer-Jones, 2009; Weisberg, 2012). A good overview of various connected issues is provided in (Gelfert, 2016, §2).

[7] We will not here consider the connection to the wide range of types of 'analogue experiments' found in the context of the life sciences. Whilst there are, for example, broad conceptual connections between our analysis below and the analysis of 'surrogate models' and 'model organisms', the differing degree of formalisation of the two sciences render the details of physical and biological analogue experiments importantly different. See, for example, (Bolker, 2009; Levy and Currie, 2014; Baetu, 2015). In interests of space, we will also neglect the subtle connection between analogue experiments and arguments by analogy. See (Bartha, 2019) for further discussion.

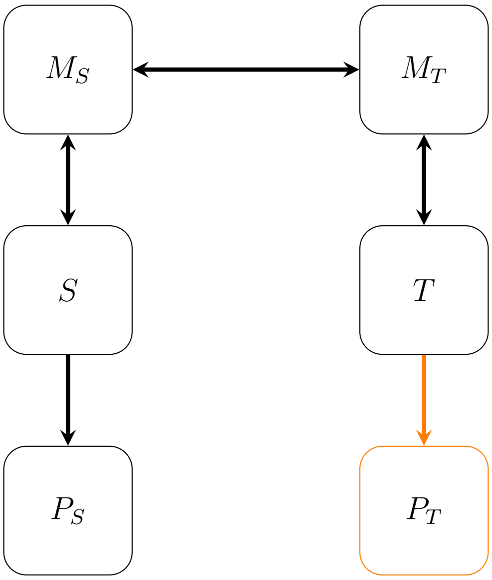

Figure 1: The structure of analogue simulation. A syntactic isomorphism between M_S and M_T is exploited to infer (inference in orange) from a phenomenon P_S of S the existence of an inaccessible phenomenon P_T of T.

target system via the manipulation of the source system. Dardashti *et al.* (2015, p.67) claim, then, that a system S provides an analogue simulation of a system T when: M_S is an adequate modelling framework for S within some domain of conditions D_S; M_T is an adequate modelling framework for T within some domain of conditions D_T; a syntactic isomorphism can be defined between M_S and M_T; under D_T, some property of T is observationally inaccessible; under D_S, S exhibits phenomenon P_S; thus, one can infer that under D_T, T will exhibit phenomenon P_T (Fig.1).

As indicated by the name, analogue simulation has a lot in common with the use of computer simulation since it involves 'programming' a physical source system such that there is a *quantitative correspondence* between terms in the mathematical model that describes the source system and the *counterpart terms* in the model that describes the target system.[8] Such activity has seen a growing number of applications in research in the contemporary context of quantum simulation (Cirac and Zoller, 2012; Georgescu *et al.*, 2014) and analogue gravity (Barceló *et al.*, 2011; Faccio *et al.*, 2013).

The most vivid modern example of an analogue simulation, and the inspiration for the analysis of Dardashti *et al.* (2015), is that of an analogue black hole or 'dumb hole'. Here the isomorphism is between the equations describing a black hole event horizon and those describing a sonic horizon in fluids, and the crucial empirical terms are those describing a thermalised photonic flux, Hawking radiation, and its thermal phononic counterpart (Unruh, 1981). The crucial characteristic of an analogue simulation is for our purposes that: i) the source-target isomorphism relation is a mathematical one at the level of models; and ii) the isomorphism is principally established in order to exploit structural similarities that relate to *empirical terms* in the respective models.

---

[8] This is unsurprising since core aspects of the Dardashti *et al.* (2015) conception of analogue simulation are drawn from earlier comparison of such practice with computer simulations due to Winsberg (2010, 2019).

The second type of experimental scientific practice, which can also be placed under the heading of analogue experimentation, we call *analogue representation*. This is a much broader category since it involves exploiting analogical relationships between source and target systems for essentially any scientific purpose. Most vividly, the use of plastic models of molecules by chemists is an example of analogue representation. In this case, the important relation is clearly *not* necessarily a formal relationship between mathematical models. Rather, the analogue representation gains its scientific value from the extent to which the source system itself (for instance, the plastic molecule) is appropriately *representative* of the abstract model of the target system (for instance, the theoretical model of the molecule).

Crucially here the relevant notion of representative correspondence need not be a precise mathematical one, and moreover need not be restricted to empirical terms within the abstract model of the target system. In fact, arguably one of the key virtues of analogue representations is that they allow us to produce *concrete representations of extra-empirical terms*. A particularly interesting form of analogue representation, which we will find to be of particular relevance to our case study, is when such representations are formalised via a syntactic isomorphism between modelling frameworks describing the source and target. In this context, analogue representation takes a structurally very similar form to analogue simulation, only with the isomorphism holding between empirical terms in the source model and extra-empirical terms in the target model. Just as analogue simulation via an experimental system is very similar in structural form to simulation via a computer, this kind of analogue representation via an experimental system has a natural correspondence to representations via a computer. Consider, for example, the representation of Newtonian space given by a 3D computer depiction of Euclidean geometry via a grid on a screen. This is a representation of a theoretical term within a modelling framework in virtue of an isometric mapping between the extra-empirical term $\mathbb{E}^3$ in the Newtonian model ('the target') and the grid lines on the screen ('the source').

These consideration point us towards a very general moral regarding the difference between two relations that might be called 'material surrogacy' and 'material representation'. The first is essentially a form of Ersatz experimentation, and is used by scientists to gain knowledge of the actual phenomena in a target system. The second is part of a much more general practice of scientific representation and relates to the representation via the source system of extra-empirical terms describing non-phenomenal features of a target system. Whilst the two may often be performed together within one experiment, they are importantly different in both their logical form and, as we shall see below, their inferential role. Furthermore, although relations of material surrogacy in general, and analogue simulation in particular, can always be interpreted as simultaneously functioning as material representations, the converse will not in general be true.

3.2 Validation

Conventional experiments only gain their value as epistemic tools via a process of validation (Franklin and Perovic, 2016). In conventional experiments in order to make inferences about a target system based on an experiment on a source a scientist requires arguments that they genuinely did learn about, first, the source system (the experiment is internally valid) and, second, that this knowledge is genuinely relevant to the features of a system or class of systems which they did not manipulate (the experiment is externally valid). In a conventional experiment, the experimenters will usually leverage the uniformity of particular material kinds as at least part of their justificatory story regarding why the source can stand in for the target in the appropriate sense.[9] Analogue simulations can be understood as subject to a structurally similar validation process. That is, their epistemic function requires that they must be supplemented with further arguments for internal and external validity. However, so far as external validation goes, such arguments are going to be of a very different form to those found in conventional experiment since the source and target are different kinds of material by construction. If analogue simulations are genuinely probative of empirical features of the target system it will typically only be due to these features being suitably independent of the material constitution. Building upon the work of Dardashti *et al.* (2015), Thèbault (2019) has argued that we can think of universality arguments as playing the role of external validation in the case of the analogue black hole example of analogue simulation. Hangleiter *et al.* (2017) give a parallel analysis for the case of analogue quantum 'emulations', and consider possible means of validation for an analogue experiment wherein a photonic source system is manipulated with the goal of learning about the phenomena of environment-assisted quantum transport in certain photosynthetic complexes.

This is where the case of analogue experiments understood as analogue representations looks remarkably different. When the source-target relation is between extra-empirical and empirical terms, considerations of material similarity are not of great relevance. Consider a plastic model of a plastic molecule, where the material of the plastic model is the same as the molecule which it represents. The analogue representation need not be established as a good representation of the extra-empirical terms that describe the molecule in virtue of the material similarity. Rather, it is a good representation in so far as it is a *useful* depiction of the theoretical model of the molecule for pedagogical purposes. How we should evaluate the success or failure of representation relationships is subtle and controversial.[10] It suffices for our purposes to note that given any of the various accounts of representation, the *style of argument* by which representation relations between source and target are justified is much broader and more flexible then the external validation arguments required in either conventional experiments or analogue simulations. In particular, neither material similarity nor material independence are necessary parts of such arguments. In the cases which are of interest to us, what is important is the isomorphism between extra-empirical terms in the target model that are difficult to conceptualise (say, the interrelation between the particle and pilot wave in the double solution ontology) and empirical terms in the source model that are visually identifiable (say, the interrelation between the walker and the surface wave). So long as that correspondence holds, the analogue experiment is usefully performing its representational function.

Just as the boundary between analogue representation and conventional experiment is not a strict one (consider analogue representations of bridges interacting with water or aeroplane wings in air tunnels), the boundary between analogue simulation and analogue representation

---

[9] The form of such inferences is explored in more detail in Evans and Thébault (2019).

[10] See the literature on representation in science to which we refer in fn.6.

is also fairly ambiguous. This point is made particularly clear by the observation that some analogue experiments function as a combination of both an analogue simulation (since there is an isomorphism between empirical terms in the source and target models) and as analogue representation (since the source system can also be considered a representation of extra-empirical terms in the target model). The similarity to computer simulation is again instructive. Consider a simulation of three particles moving according to the laws of Newtonian gravitation that is represented on a computer screen via three dots moving in the Euclidean grid lines described above. Clearly this is performing both surrogacy and representational functions. There is an isomorphism between the empirical terms (i.e. the relative positions and velocities of the particles) in the Newtonian model and the relative positions of the dots, and between an extra-empirical term (Euclidean space) and the grid lines. Similarly it is very plausible to think that an analogue experiment can perform both the surrogacy and representational function simultaneously.

Consider again then the case of an analogue black hole. In this case, as well as there being an isomorphism between observables in the fluid and black hole model (e.g. Hawking radiation) we can also think of the flow of the fluid as a representation of spacetime. Crucially, the question of whether the source system adequately plays the first, material surrogacy role is independent from the question of whether it plays the second, representational role.[11] Whilst a single analogue experiment may well be simultaneously interpretable as either an analogue simulation or analogue representation, for justificatory purposes we must be sensitive to important differences between these two functions.

3.3 Confirmation and explanation

With these distinctions and definitions in hand we can finally consider the question of what we can actually learn via analogue experiments. Again, it is worth making reference to the arguments of Dardashti *et al.* (2015) who claim that there can be 'confirmation via analogue simulation' in circumstances where the analogue experiment is externally validated by reference to universality arguments that show that the empirical phenomena being simulated are suitably independent of the differences in material constitution between source and target system. The idea is that the universality arguments provide a common empirically grounded reason for believing in the empirical validity of both the source and target modelling frameworks within their domain of application. In virtue of that connection, inductive support for the one is then taken to be also inductive support for the other. The claims of Dardashti *et al.* (2015) have drawn contrasting reactions. Subsequent analysis has included extensions in terms of formal frameworks for confirmation theory (Dardashti *et al.*, 2019; Feldbacher-Escamilla and Gebharter, forthcoming), further exploration of the connection to conventional experiments and computer simulations (Boge, 2018) and a (contentious) discussion of 'circularity' inherent in such patterns of inference (Crowther *et al.*, 2019; Bartha, 2019; Evans and Thébault, 2019).

---

[11] It is worth noting that, in this case, the experimenters are not, as it happens, particularly interested in the second function of their experiment since their main focus is on simulating empirical phenomena, and in justifying their inferences based upon universality arguments. Most likely this is because the mathematical models of a black hole in terms of Schwarzschild geometry already has a variety of simple visual representations (e.g. topographic diagram or Penrose diagram).

The last of these discussions is worth briefly summarising. Crowther *et al.* (2019) claim that confirmation via analogue simulation can only be given a non-circular justification in the case of accessible target systems. In response, Evans and Thébault (2019) first point out that, as an inductive form of inference, analogue simulation cannot be consistently classed as premise circular since, as ampliative inferences, inductive arguments cannot by definition be premise-circular. Then, a charitable reconstruction of the reasoning implicit behind the argument of Crowther *et al.* in terms of a problem of rule-circularity is shown also to fail, unless Crowther *et al.* are adopting the unreasonable (in this context) position of inductive sceptics. Finally, Evans and Thébault interpret Crowther *et al.* to be offering two reasonable grounds for worry based upon an appeal to inter-type uniformity or the confirmation of theories regarding inaccessible target systems. Evans and Thébault argue that to rule out gaining knowledge regarding astrophysical Hawking radiation based upon analogue experiments *in principle* is unreasonable. However, to fully meet a reasonable sceptical challenge, along the lines of their reconstruction of Crowther *et al.*, *in practice* a new generation of analogue experiments showing Hawking radiation in diverse media is needed.

Notwithstanding these debates it is abundantly clear that whilst the issue of confirmation in the context of analogue simulation is a live one – uncontestedly in the case of accessible target systems, contestedly for inaccessible target systems – it is simply not relevant to the case of analogue representations. A little consideration of the nature of inductive evidence and the fault lines of the contemporary debate regarding scientific realism demonstrates why. Consider the inferences that a canonical empiricist and realist seek to make based upon a valid experiment. Both empiricist and realist would agree that valid experiments will, at least in some cases, allow us to provide inductive evidence in favour of the *empirical adequacy* of a theoretical model; that is, the truth, within a certain domain, of empirical statements which can be derived from the model. This confirmation is built upon a correspondence between empirical terms in the model and observations drawn from the experiment.

We can set out the structure of the confirmation inference in simple Bayesian terms as follows. We denote by H the claim that a particular (plausible) theory is empirically adequate in a given domain, and E that a particular empirical prediction of the theory within that domain obtains. We thus have that $H \rightarrow E$, which means (almost trivially) that $P(H|E) > P(H)$, given that both $0 < P(H) < 1$ and $0 < P(E) < 1$, and so we have confirmation of H by E. It is instructive to break this very standard inference down a little. Bayes' theorem states:

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}, \tag{1}$$

where we take the marginals, $P(H)$ and $P(E)$, to be strictly in the interval $[0, 1]$, but otherwise rationally unconstrained. Given that we have assumed that $H \rightarrow E$, it follows necessarily that $P(E|H) = 1$. It then immediately follows that the confirmation measure $\triangle = P(H|E) - P(H)$ is greater than zero, and we have Bayesian confirmation of H by E.

So far this is relatively uncontroversial. What is a subject of enduring controversy is the further move that the realist seeks to make. That is, to argue that, in at least some cases, we are licensed to make a further meta-inference to the truth (or approximate truth) of extra-empirical terms based upon (for example) inference to the best explanation or the avoidance of 'miracles'. The details of this controversy are not important for our purposes. Rather, what is important is that in arguing that the extra-empirical terms can be confirmed by inductive evidence the realist should not attempt to proceed directly via induction based on the first-order evidence. This is for good reason: we have sufficient rationale to believe that theories with false extra-empirical terms can be empirically adequate in a variety of domains. The connection between the extra-empirical terms and the empirical observations is thus underdetermined to such a degree that it would be rather foolish to try to argue directly from empirical adequacy to truth in general terms. Moreover, we have, in fact, various general reasons, drawn from internal inconsistency (e.g. the well known problems with infinities in general relativity and quantum field theories), conflict between existing theories (such as quantum field theory and general relativity), and, most infamously, the history of science (Laudan, 1981; Lyons, 2002; Vickers, 2013), to expect that none of our current theories can be taken to be true *simpliciter*. Thus, strictly speaking, if we were going to put forward a Bayesian analysis where H is now the claim that a theory is true *simpliciter*, whilst we *would* have that $H \to E$, and thus that $P(E|H) = 1$, confirmation is blocked since we have good reason to set $P(H) = 0$. Furthermore, if we weaken the realist stance such that H is the claim that a theory is approximately true, not only is it no longer the case that $H \to E$, but we might plausibly set $P(E|H)$ close to zero (Lyons, 2003). We thus see that such direct arguments for confirmation of truth are on rather shaky ground. Realist arguments for confirmation must be founded, if they can be founded at all, on second-order evidence such as the continued success of science in general (Dawid and Hartmann, 2018).

The plausibility of making an argument for confirmation via analogue representation is even more tenuous. The correspondence between the extra-empirical terms in the target model and the empirical terms in the source model is obviously explained, without the truth of the former, since the source system has specifically been arranged to ensure the correspondence! Since it exists by our own arrangement, the correspondence between these terms carries no particular inductive weight, and so provides no license for taking analogue representations to be confirmatory. Again, a sketch of the relevant relationships in Bayesian terms will prove worthwhile. Take H to be the claim that a particular extra-empirical term in our target model is true (i.e. corresponds to an 'element of reality'), and take E to be the successful analogue representation of the empirical counterpart to that term in an experiment on a source system. The syntactic correspondence between extra-empirical terms in M_T and empirical terms in M_S is arguably independent of their respective empirical adequacy or truth. Thus, it seems difficult to avoid the conclusion that we should set $P(E|H) = P(E)$, and thus $P(H|E) = P(H)$, which implies $\triangle = 0$, and thus that no confirmation can obtain.

This brings us to the question of what analogue representations are for, if not confirmation. A tentative answer, that we will put to use below comes from the idea of *how-possibly explanation*. First introduced by Dray (1957, §VI), how-possibly explanation is a rival kind of explanation to deductive-nomological explanations.[12] Persson (2012) provides a productive

---

[12] For more discussion on how-possibly explanation see (Forber, 2010; Bokulich, 2014; Cuffaro, 2015). Hangleiter *et al.* (2017) argue that how-possibly understanding can be understood as a supplementary function

way, for our current purposes, to conceive of how-possibly explanation: how-possibly explanation offers a *potential* how-explanation that explores the space of possible explanantia for some explanandum. As part of formulating such how-possibly explanation, we must metaphysically and epistemically 'bracket', in the sense that the explanation is not about how the world actually is, nor do we know whether the explanation is true. Persson (2012, p.282) claims that the context of this type of how-possibly explanation

> is typically one of discovery, hypothesis generation, or the exploration of a range of possible explanations in a research environment where the explanandum phenomenon is accepted as a fact and now needs to be integrated with the system.

Reutlinger *et al.* (2017) rationally reconstruct the epistemic goal of toy modelling in science, claim that how-possibly explanation is one such goal, and go on to identify at least three epistemic functions of how-possibly explanation that endow it with value for the scientist. The first is a modal function: how-possibly explanation is valuable when "scientists want to understand whether and why some phenomenon is possibly or necessarily the case" (Reutlinger *et al.*, 2017, p.26). The second epistemic function is a heuristic function: how-possibly explanation is valuable when it stimulates further investigation into a more accurate model of the target system. The final epistemic function is a pedagogical function: how-possibly explanation is valuable when used for illustrative purposes to "enable students and researchers to quickly grasp the idea behind. . . the description of a phenomenon" (Reutlinger *et al.*, 2017, p.26).

We think it is plausible to take analogue representation to be a fruitful means to provide all three of these functions. The following section will articulate why, based upon the example of the bouncing oil droplet and pilot wave theory.

4 What can we learn from the walker experiments?

In this section we first introduce a reason to be optimistic about the possibility of the walker experiments being an analogue simulation of quantum phenomena (§4.1). We then provide a more significant reason to treat them as analogue representations (§4.2). In doing so, we argue that the walker experiments cannot provide inductive evidence for a pilot wave ontology for quantum phenomena, whether de Broglie's double solution theory or not. Finally, in §4.3, we consider the prospect that the walker experiments might still be valuable in terms of the how-possible explanation they provide.

4.1 Surrogacy aspect

An analogue simulation requires a syntactic isomorphism between two modelling frameworks that we take to be adequate models of the potential analogues. It is clear in the current context that one of those modelling frameworks is de Broglie's double solution theory, as a model of quantum phenomena. However, there does not seem to be an obvious candidate for a model that would stand in for the fluid mechanical walker system. Against this background, Borghesi

---

of certain forms of analogue simulation.

(2017) has developed a classical toy model in light of the fluid mechanical walker system and de Broglie's double solution theory.

Motivated by the desire to model the walker system in a relativistically covariant manner, Borghesi's model consists of a vibrating elastic medium carrying a transverse wave, ϕ,[13] and a point-like 'concretion' (a very concentrated heterogeneity of the medium itself). The mechanism of the model is comprised of an equation of motion for the concretion, in which the motion of the concretion is deflected by the gradient of ϕ at its location, and a wave equation that can be interpreted as an inhomogeneous Klein-Gordon-like equation in the presence of a wave source localised at the position of the concretion and dependent upon its vibration (Borghesi, 2017, p.938). A curious simplification of this toy model leads to interesting consequences.

To reflect better the interrelation in the walker experiments between the walker and the surface wave, Borghesi assumes that the concretion no longer acts as the wave source after "a kind of self-adaptive phenomenon between the transverse wave ϕ and the concretion" (Borghesi, 2017, p.939). This establishes an 'intimate harmony' between the wave and the concretion, which Borghesi labels 'symbiosis', such that there is no back-reaction of the concretion on ϕ (as is the case for the de Broglie guiding equation (Holland, 2005)). The regime in which symbiosis is possible requires that in the proper reference frame of the concretion, the concretion is located at a local extremum of ϕ. In this regime, the mass continuity equation leads to a concretion speed constant in time, and so reflects conservation of particle energy in the absence of an external potential, and it also leads to a relation that Borghesi calls the ϕ-guidance formula. In the low-velocity approximation, the ϕ-guidance formula reduces to be syntactically isomorphic to de Broglie's guiding equation.

Perhaps more remarkably, when the concretion is not the source of the wave, and assuming "that the time period of transverse oscillations is much shorter [than] the characteristic evolution time of the (perceivable) motion of the concretion", then the motion of the concretion (in symbiosis with the wave) is guided exclusively by the ϕ-guidance formula (Borghesi, 2017, p.941). That is, averaged over one transverse oscillation, the equation of motion for the concretion does not play any role in its motion. This 'cancellation' of the equation of motion in the symbiotic regime, due to the ϕ-guidance formula, ensures that the velocity of the concretion is directly proportional to the gradient of the wave, in contrast to an equation of motion which relates an external potential to particle acceleration.

In the symbiotic regime, where the concretion satisfies the ϕ-guidance formula, the following can also be derived from the model: a 'wave potential', syntactically isomorphic to the quantum potential derived by de Broglie (1927a) and Bohm (1952); an expression for the energy of the concretion in the low-velocity limit that contains a 'rest mass' term and an additional energy term, E_c, equal to a coefficient multiplied by an additional pulsation beyond the transverse vibrations of the wave, ω; a proportionality coefficient between wave and particle characteristics of the model that, when represented by the term $\hbar$, renders the additional energy term $E_c = \hbar\omega$, syntactically isomorphic to the Planck-Einstein relation; and a low-velocity approximation of the Klein-Gordon-like wave equation without source that is syntactically isomorphic to the free Schrödinger equation. Moreover, these expressions can be combined to produce an

---

[13] ϕ denotes a transverse displacement and not a phase.

expression for the energy and momentum of the concretion that are syntactically isomorphic to expressions that represent the energy and momentum of a quantum system. On this final isomorphism, Borghesi (2017, p.946) points out: “This point confirms in our system what de Broglie had suggested, here restricted to energy and momentum: the particle accounts for quantities commonly attributed to the wave-like nature of the system in quantum mechanics”.

Based on these claims, we can see that there is a syntactic isomorphism between at least some of the empirical terms in Borghesi’s fluid dynamical modelling framework for the walker system and de Broglie’s truncated provisional theory – that is, pilot wave theory. Moreover, and this is slightly more contentious, in so far as de Broglie toyed with some form of the Klein-Gordon equation as the wave equation for his u-waves, Borghesi derives a wave equation (from a principle of least action) that can be interpreted as an inhomogeneous Klein-Gordon-like equation in the presence of a wave source (although Borghesi himself does not go so far as to endorse any analogy on this point). It would be incorrect to claim that there is anything close to a syntactic isomorphism at play between these latter two wave equations, but the two wave equations have similar form (and, recall, de Broglie never found the precise wave equation for his u-waves).

Establishing a syntactic isomorphism is cause to be optimistic that the walker experiments might count as an analogue simulation of pilot wave theory. But there are good reasons to take this surrogacy aspect as a subsidiary function of the experiments and to understand the principal function of the experiments as representational.

4.2 Representational Aspect

Consider transcribing the relation between the walker experiments and pilot wave theory onto Fig.1. On the ‘source’ side of the diagram the transcription is straightforward. The source system, S, is the fluid mechanical walker system. The phenomena, P_S, are the range of typically quantum behaviours displayed by the walker system such as diffraction, interference, quantised orbits, tunnelling, *etc.* The modelling framework, M_S, is Borghesi’s classical toy model of the symbiotic concretion and elastic medium. On the ‘target’ side of the diagram the transcription is a little less clear. To begin with, in contrast to the analogue black hole case, the inaccessible piece of the puzzle that is the focus of this analogue experiment is not the phenomena P_T; in this case P_T is simply the quantum phenomena. The modelling framework, M_T, that we are taking to be syntactically isomorphic to Borghesi’s framework is de Broglie’s truncated provisional pilot wave theory. But what exactly is the target system, T?

In so far as P_T is the quantum phenomena, one might assume that the target system is simply any quantum system. But we do not need an analogue simulation to tell us about quantum systems; quantum systems are perfectly accessible and we have a perfectly good modelling framework that is highly empirically adequate: quantum mechanics. Thus it should be clear that the walker experiments are not attempting to provide evidence in favour of quantum mechanics *simpliciter*. If the walker experiments are showing us anything, they are providing us with a macroscopic representation of wave-particle duality, as portrayed by pilot wave theory. One might argue, then, that T must be a system of the sort described by pilot wave theory (that is, not simply *any* quantum mechanical system, but a pilot wave system). As such, the target seems to be extra-empirical in nature; T, the claim might go, is an ontological framework for quantum mechanics.

As we have argued above, any inference from the walker experiments to an ontological framework for quantum mechanics is not epistemically justifiable. Rather than playing the role of an analogue simulation, the walker experiments constitute an analogue representation. On account of the argument in §3.3, it is simply a category error to think that the sort of empirical support that experiments provide can be marshalled in favour of an extra-empirical claim as part of an analogue representation such as this – experiments cannot provide inductive support for extra-empirical statements. Quantum mechanics, as a modelling framework, notoriously underdetermines quantum ontology, and as striking as the typically quantum behaviour of the walker experiments is, they simply cannot provide epistemic warrant in favour of a pilot wave ontology for quantum mechanics.

This simple point makes it difficult to suggest that the walker experiments can be providing us with *inductive* evidence regarding de Broglie's interpretation of quantum mechanics, or quantum mechanics more generally. However, this need not render the walker experiments as epistemically worthless – after all, the reproduction of so many typically quantum phenomena, and the concrete representation of wave-particle duality via a pilot wave, particularly of the sort envisaged in de Broglie's double solution theory, is extraordinary. We contend that the epistemic value of the walker system should be understood in terms of how-possibly explanation.

4.3 Explanatory aspect

We can proffer here a rational reconstruction of the epistemic goals of the experimental teams undertaking the walker experiments and map it neatly over to the conception of how-possibly explanation from §3.3. Since any particular phenomenon only makes sense as an explanans in the context of a scientific model or theory, we take the explanans in the current case to be the walker experiments in the context of de Broglie's double solution theory. It then seems reasonable to suppose that the experimental teams undertaking the walker experiments are taking the explanandum to be the mechanism that underlies wave-particle duality in the double solution theory: by what physical mechanism, according to the double solution pilot wave theory, could the interrelation between particle and wave be established? The walker experiments, in combination with the double solution theory, demonstrate a possible (classical) mechanism through which that interrelation can be established. As an explanation of this explanandum, it does not seem that the Paris and MIT experimenters presume that the experiment is a representation of how the world actually is, nor that they know whether this mechanism for wave-particle duality is true of the double solution pilot wave theory. In other words, claims about the world have been metaphysically and epistemically bracketed. Furthermore, the walker experiments surely must be taken to be performed in the context of "discovery" and "hypothesis generation" in the face of acceptance that quantum mechanics renders wave-particle duality as a fact that is in need of integration, metaphysically speaking, with the rest of contemporary physics. Thus, given Persson's conception of how-possibly explanation, it seems a suitable characterisation of the utility of the walker experiments. But is there value in this utility?

Recall the three epistemic functions of how-possibly explanation that endow it with value according to Reutlinger *et al.* (2017): the modal function; the heuristic function; and the pedagogical function. All three can be seen to hold for the walker experiments: the first since the experiments are concerned with whether it is possible to represent coherently wave-particle duality by means of a pilot wave mechanism of the type envisaged by de Broglie's double solution theory; the second since the fluid mechanical model provides a concrete representation of the ontology of de Broglie's double solution theory, and therein a pilot wave mechanism for wave-particle duality, and this suggests that de Broglie's incomplete double solution theory might be worthy of renewed exploration; and the third since the walker experiments patently provide a clear representational tool for visualising a classical mechanism that underlies typically quantum behaviour, as well as more specifically a tool for visualising the pilot wave ontology of the double solution theory.[14]

On account of these epistemic functions of how-possibly explanation, the walker experiments have definite epistemic value as an analogue representation. This is despite the fact that there appears to be no epistemic justification for the walker system to play the role of an analogue simulation, or material surrogate, of quantum mechanics, and so tell us anything at all about quantum mechanics itself.

5 Final thoughts

There is evidently something remarkable about the fact that a series of experiments consisting of a vibrating bath of silicone oil sustaining bouncing droplets on its surface could provide a concrete representation of the ontology of a long forgotten pilot wave theory of quantum mechanics (one developed when quantum mechanics was in its infancy, to boot). As we have seen here, however, we must not be too hasty in considering the debate around the metaphysical consequences of quantum mechanics to be any closer to resolution as a result. The framework of analogue simulation is capable of providing epistemic licence to raise our probability that we will observe some phenomenon of a target system on account of observing some phenomenon in a source system. However, the relation between the walker experiments and de Broglie's double solution theory does not fit naturally into the analogue simulation framework. Rather, these experiments are better understood as an example of analogue representation. Significantly, we should not take analogue representation to establish anything beyond mere plausibility for the existence of elements of reality corresponding to extra-empirical terms, such as the ontology of quantum mechanics claimed by the double solution theory. Thus the walker experiments provide us with no justification to say anything more about quantum mechanics beyond the plausibility of some metaphysical framework – and pilot wave theory is already considered a plausible metaphysical framework for quantum mechanics. This notwithstanding, the walker experiments plausibly do provide us with a how-possibly explanation for the mechanism that underlies wave-particle duality in the double solution pilot wave theory. Not only does this provide a clear pedagogical tool for visualising both a classical mechanism underlying typically quantum behaviour and the pilot wave ontology of the double solution theory, but it also suggests that further investigation in quantum foundations into de Broglie's incomplete dou-

---

[14] One only need search YouTube for an array of examples of the pedagogical value of the walker experiments.

ble solution theory could plausibly prove fruitful. Furthermore, as already noted, by pointing us towards the distinction between analogue simulation and analogue representation in current scientific practice the walker experiments evidence the wider philosophical moral that the relations of 'material surrogacy' and 'material representation' are distinct.

There are many more points of interest concerning the relation between the walker experiments and quantum mechanics that have not been addressed in this paper. One such issue is the classicality of the fluid mechanical model. Attempts in recent years to describe typically quantum correlations in terms of classical causal modelling frameworks (Spirtes *et al.*, 2000; Pearl, 2009) have notoriously failed, and significant steps have been taken towards a quantum version of causal modelling (Leifer and Spekkens, 2013; Cavalcanti and Lal, 2014; Costa and Shrapnel, 2016; Allen *et al.*, 2017). Accordingly, one must attach a healthy level of scepticism to the idea that a classical fluid mechanical model could reproduce typically quantum correlations, as any classical mechanism looks likely to be too impoverished to do so (despite the arguments of Vervoort (2016)). And this is the rather sizeable elephant in the room in the context of asking what bouncing oil droplets can tells us about quantum mechanics: the walker experiments thus far performed evidently lack the capacity to act as either analogue simulations or analogue representations of quantum entanglement. As Bush (2015, p.287) puts it, since the typically quantum behaviour of the walker arises as a function of chaotic dynamics, "[t]he question remains open as to whether some combination of intrusive measurement and chaotic pilot wave dynamics might give rise to a hydrodynamic analog of entanglement." Progress on this issue could significantly change the nature of the relation between the walker experiments and quantum mechanics as we have presented it here but, as it stands, we do not expect the walker experiments to be an analogue representation or simulation of any entanglement-based quantum phenomena that involve the violation of Bell-type inequalities. However, we should certainly not dismiss the walker experiments as scientifically valueless on such a basis.[15] While one might portray quantum entanglement, or the measurement problem, as *the* defining feature of quantum theory, another legitimate candidate for this mantle is wave-particle duality. In so far as the walker experiments provide a concrete representation of wave-particle duality, and how-possibly explanation for the mechanism that underlies wave-particle duality in the double solution pilot wave theory, there remains in the walker experiments epistemic value for quantum foundations research.

Acknowledgments: This work grew out of a Research Group Fellowship in 2015 at the Munich Center for Mathematical Philosophy, with fellow group members Radin Dardashti, Matt Farr, and Alex Reutlinger. We are grateful to the hospitality of Ludwig-Maximilians-Universität and Stephan Hartmann for hosting us during the early stages of this research. For valuable discussion, comments, and feedback we are greatly appreciative to Guido Bacciagaluppi, Christian Borghesi, Eric Winsberg, and to audiences in Brisbane and Canberra. PWE's work on this paper was supported by the Templeton World Charity Foundation (TWCF 0064/AB38), the University of Queensland, and the Australian Government through the Australian Research Council (DE170100808). KT's work on this paper was supported by the Arts and Humanities Research Council, UK (AH/P004415/1).

---

[15] Indeed, it is worth noting the potential value of an incomplete analogy, or even failed analogy of the right sort, when it serves to reveal significant differences between the representation and the target.

References

Allen, J.-M. A., Barrett, J., Horsman, D. C., Lee, C. M. and Spekkens, R. W. (2017). Quantum Common Causes and Quantum Causal Models. *Physical Review X* **7**: 031021. doi:10.1103/PhysRevX.7.031021.

Andersen, A., Madsen, J., Reichelt, C., Rosenlund Ahl, S., Lautrup, B., Ellegaard, C., Levinsen, M. T. and Bohr, T. (2015). Double-slit experiment with single wave-driven particles and its relation to quantum mechanics. *Physical Review E* **92**: 013006. doi:10.1103/PhysRevE.92.013006.

Bacciagaluppi, G. and Valentini, A. (2009). *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*. Cambridge University Press, Cambridge. doi:10.1017/CBO9781139194983.

Baetu, T. M. (2015). The 'Big Picture': The Problem of Extrapolation in Basic Research. *The British Journal for the Philosophy of Science* **67**(4): 941–964. doi:10.1093/bjps/axv018.

Bailer-Jones, D. M. (2009). *Scientific Models in Philosophy of Science*. University of Pittsburgh Press.

Barceló, C., Liberati, S. and Visser, M. (2011). Analogue gravity. *Living Reviews in Relativity* **14**(1): 3. doi:10.12942/lrr-2011-3.

Bartha, P. (2019). Analogy and Analogical Reasoning. *In* E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Stanford University. Spring 2019 edition. plato.stanford.edu/archives/win2016/entries/reasoning-analogy/.

Batelaan, H., Jones, E., Huang, W. C.-W. and Bach, R. (2016). Momentum exchange in the electron double-slit experiment. *Journal of Physics* **701**: 012007. doi:10.1088/1742-6596/701/1/012007.

Boge, F. J. (2018). Why computer simulations are not inferences, and in what sense they are experiments. *European Journal for Philosophy of Science* **9**: 13. doi:10.1007/s13194-018-0239-z.

Bogen, J. and Woodward, J. (1988). Saving the Phenomena. *The Philosophical Review* **97**(3): 303–352. doi:10.2307/2185445.

Bohm, D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I. *Physical Review* **85**(2): 166–179. doi:10.1103/PhysRev.85.166.

Bohr, T., Andersen, A. and Lautrup, B. (2016). Bouncing Droplets, Pilot-Waves, and Quantum Mechanics. *In* J. Klapp, L. D. G. Sigalotti, A. Medina, A. López and G. Ruiz-Chavarría (eds.), *Recent Advances in Fluid Dynamics with Environmental Applications*. Springer International Publishing, Cham, pp. 335–349. doi:10.1007/978-3-319-27965-7_25.

Bokulich, A. (2014). How the Tiger Bush Got its Stripes: 'How Possibly' vs. 'How Actually' Model Explanations. *The Monist* **97**(3): 321–338. doi:10.5840/monist201497321.

Bolker, J. A. (2009). Exemplary and Surrogate Models: Two Modes of Representation in Biology. *Perspectives in Biology and Medicine* **52**(4): 485–499. doi:10.1353/pbm.0.0125.

Borghesi, C. (2017). Equivalent quantum equations in a system inspired by bouncing droplets experiments. *Foundations of Physics* **47**(7): 933–958. doi:10.1007/s10701-017-0103-6.

Bush, J. W. M. (2015). Pilot-Wave Hydrodynamics. *Annual Review of Fluid Mechanics* **47**(1): 269–292. doi:10.1146/annurev-fluid-010814-014506.

Cavalcanti, E. G. and Lal, R. (2014). On modifications of Reichenbach's principle of common cause in light of Bell's theorem. *Journal of Physics A: Mathematical and Theoretical* **47**(42): 424018. doi:10.1088/1751-8113/47/42/424018.

Cirac, J. I. and Zoller, P. (2012). Goals and opportunities in quantum simulation. *Nature Physics* **8**(4): 264–266. doi:10.1038/nphys2275.

Contessa, G. (2007). Scientific representation, interpretation, and surrogative reasoning. *Philosophy of science* **74**(1): 48–68. doi:10.1086/519478.

Costa, F. and Shrapnel, S. (2016). Quantum causal modelling. *New Journal of Physics* **18**(6): 063032. doi:10.1088/1367-2630/18/6/063032.

Couder, Y. and Fort, E. (2006). Single-particle diffraction and interference at a macroscopic scale. *Physical Review Letters.* **97**: 154101. doi:10.1103/PhysRevLett.97.154101.

——— (2012). Probabilities and trajectories in a classical wave-particle duality. *Journal of Physics: Conference Series* **361**(1): 012001. doi:10.1088/1742-6596/361/1/012001.

Crowther, K., Linnemann, N. and Wuthrich, C. (2019). What we cannot learn from analogue experiments. arXiv:1811.03859 [physics.hist-ph].

Cuffaro, M. E. (2015). How-Possibly Explanations in (Quantum) Computer Science. *Philosophy of Science* **82**(5): 737–748. doi:10.1086/683243.

Dardashti, R., Hartmann, S., Thébault, K. P. Y. and Winsberg, E. (2019). Hawking Radiation and Analogue Experiments: A Bayesian Analysis. *Studies In History and Philosophy of Modern Physics* **67**: 1–11. arXiv:1604.05932 [physics.hist-ph].

Dardashti, R., Thébault, K. P. Y. and Winsberg, E. (2015). Confirmation via Analogue Simulation: What Dumb Holes Could Tell us About Gravity. *The British Journal for the Philosophy of Science* **68**(1): 55–89. doi:10.1093/bjps/axv010.

Dawid, R. and Hartmann, S. (2018). The no miracles argument without the base rate fallacy. *Synthese* **195**(9): 4063–4079. doi:10.1007/s11229-017-1408-x.

de Broglie, L. (1924). *Recherches sur la théorie des quanta.* Ph.D. thesis, University of Paris.

——— (1927a). La méchanique ondulatoire et la structure atomique de la matière et du rayonnement. *Le Journal de Physique et le Radium* **8**: 225–241. doi:10.1051/jphysrad:0192700805022500.

——— (1927b). La nouvelle dynamique des quanta. *In The proceedings of the 1927 Solvay conference.* Translated and published in Bacciagaluppi and Valentini (2009), p.373–406.

——— (1959). L'interprétation de la mécanique ondulatoire. *Le Journal de Physique et le Radium* **20**(12): 963–979. doi:10.1051/jphysrad:019590020012096300.

——— (1971). L'interprétation de la Mécanique Ondulatoire par la Théorie de la Double Solution. *In* B. d'Espagnat (ed.), *Foundations of Quantum Mechanics*, Academic Press, New York, pp. 345–367.

Dray, W. H. (1957). *Laws and Explanation in History.* Oxford University Press, Oxford.

Eddi, A., Fort, E., Moisy, F. and Couder, Y. (2009). Unpredictable Tunneling of a Classical Wave-Particle Association. *Physical Review Letters* **102**: 240401. doi:10.1103/PhysRevLett.102.240401.

Eddi, A., Moukhtar, J., Perrard, S., Fort, E. and Couder, Y. (2012). Level Splitting at Macroscopic Scale. *Physical Review Letters* **108**: 264503. doi:10.1103/PhysRevLett.108.264503.

Eddi, A., Sultan, E., Moukhtar, J., Fort, E., Rossi, M. and Couder, Y. (2011). Information stored in faraday waves: the origin of a path memory. *Journal of Fluid Mechanics* **674**: 433–463. doi:10.1017/S0022112011000176.

Einstein, A. (1905). Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. *Annalen der Physik* **322**(6): 132–148. doi:10.1002/andp.19053220607.

Evans, P. W. and Thébault, K. P. Y. (2019). On the Limits of Experimental Knowledge. Manuscript in preparation.

Faccio, D., Belgiorno, F., Cacciatori, S., Gorini, V., Liberati, S. and Moschella, U. (2013). *Analogue Gravity Phenomenology: Analogue Spacetimes and Horizons, from Theory to Experiment*. Springer. doi:10.1007/978-3-319-00266-8.

Faraday, M. (1831). On a peculiar class of Acoustical Figures; and on certain Forms assumed by groups of particles upon vibrating elastic Surfaces. *Philosophical Transactions of the Royal Society of London* **121**: 299–340. doi:10.1098/rstl.1831.0018.

Feldbacher-Escamilla, C. J. and Gebharter, A. (forthcoming). Confirmation based on analogical inference: Bayes meets jeffrey. *Canadian Journal of Philosophy* .

Forber, P. (2010). Confirmation and explaining how possible. *Studies in History and Philosophy of Biological and Biomedical Sciences* **41**(1): 32–40. doi:10.1016/j.shpsc.2009.12.006.

Fort, E., Eddi, A., Boudaoud, A., Moukhtar, J. and Couder, Y. (2010). Path-memory induced quantization of classical orbits. *Proceedings of the National Academy of Sciences* **107**(41): 17515–17520. doi:10.1073/pnas.1007386107.

Franklin, A. and Perovic, S. (2016). Experiment in Physics. *In* E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Stanford University. Winter 2016 edition. plato.stanford.edu/archives/win2016/entries/physics-experiment/.

Frigg, R. and Nguyen, J. (2018). The turn of the valve: representing with material models. *European Journal for Philosophy of Science* **8**(2): 205–224. doi:10.1007/s13194-017-0182-4.

Gelfert, A. (2016). *How to Do Science with Models: A Philosophical Primer*. Springer, Switzerland. doi:10.1007/978-3-319-27954-1.

Georgescu, I. M., Ashhab, S. and Nori, F. (2014). Quantum simulation. *Reviews of Modern Physics* **86**: 153–185. doi:10.1103/RevModPhys.86.153.

Giere, R. N. (1999). Using Models to Represent Reality. *In* L. Magnani, N. J. Nersessian and P. Thagard (eds.), *Model-Based Reasoning in Scientific Discovery*, Springer, Boston, pp. 41–57. doi:10.1007/978-1-4615-4813-3_3.

Hangleiter, D., Carolan, J. and Thébault, K. (2017). Analogue quantum simulation: A philosophical prospectus. arXiv:1712.05809 [quant-ph].

Harris, D. M., Moukhtar, J., Fort, E., Couder, Y. and Bush, J. W. M. (2013). Wavelike statistics from pilot-wave dynamics in a circular corral. *Physical Review E* **88**: 011001. doi:10.1103/PhysRevE.88.011001.

Holland, P. (2005). Quantum back-reaction and the particle law of motion. *Journal of Physics A: Mathematical and General* **39**(3): 559–564. doi:10.1088/0305-4470/39/3/008.

Hughes, R. I. G. (1997). Models and Representation. *Philosophy of science* **64**: S325–S336. doi:10.1086/392611.

Joas, C. and Lehner, C. (2009). The classical roots of wave mechanics: Schrödinger's transformations of the optical-mechanical analogy. *Studies in History and Philosophy of Modern Physics* **40**(4): 338–351. doi:10.1016/j.shpsb.2009.06.007.

Laudan, L. (1981). A Confutation of Convergent Realism. *Philosophy of science* **48**(1): 19–49. doi:10.1086/288975.

Leifer, M. S. and Spekkens, R. W. (2013). Towards a formulation of quantum theory as a causally neutral theory of bayesian inference. *Physical Review A* **88**(5): 052130. doi:10.1103/PhysRevA.88.052130.

Levy, A. and Currie, A. (2014). Model Organisms are Not (Theoretical) Models. *The British Journal for the Philosophy of Science* **66**(2): 327–348. doi:10.1093/bjps/axt055.

Lyons, T. D. (2002). Scientific Realism and the Pessimistic Meta-Modus Tollens. *In* S. Clarke and T. D. Lyons (eds.), *Recent Themes in the Philosophy of Science: Scientific Realism and Commonsense*, Springer, Dordrecht, pp. 63–90. doi:10.1007/978-94-017-2862-1_4.

——— (2003). Explaining the Success of a Scientific Theory. *Philosophy of Science* **70**(5): 891–901. doi:10.1086/377375.

Massimi, M. (2007). Saving Unobservable Phenomena. *The British Journal for the Philosophy of Science* **58**(2): 235–262. doi:10.1093/bjps/axm013.

Moláček, J. and Bush, J. W. M. (2013). Drops walking on a vibrating bath: towards a hydrodynamic pilot-wave theory. *Journal of Fluid Mechanics* **727**: 612–647. doi:10.1017/jfm.2013.280.

Pearl, J. (2009). *Causality: Models, Reasoning, and Inference.* Cambridge University Press, New York, 2nd edition.

Persson, J. (2012). Three Conceptions of Explaining How Possibly—and One Reductive Account. *In* H. W. de Regt, S. Hartmann and S. Okasha (eds.), *EPSA Philosophy of Science: Amsterdam 2009.* Springer Netherlands, Dordrecht, pp. 275–286. doi:10.1007/978-94-007-2404-4_24.

Protière, S., Boudaoud, A. and Couder, Y. (2006). Particle–wave association on a fluid interface. *Journal of Fluid Mechanics* **554**: 85–108. doi:10.1017/S0022112006009190.

Protière, S., Couder, Y., Fort, E. and Boudaoud, A. (2005). The self-organization of capillary wave sources. *Journal of Physics: Condensed Matter* **17**(45): S3529. doi:10.1088/0953-8984/17/45/044.

Pucci, G., Harris, D. M., Faria, L. M. and Bush, J. W. M. (2018). Walking droplets interacting with single and double slits. *Journal of Fluid Mechanics* **835**: 1136–1156. doi:10.1017/jfm.2017.790.

Reutlinger, A., Hartmann, S. and Hangleiter, D. (2017). Understanding (with) Toy Models. *The British Journal for the Philosophy of Science* **69**(4): 1069–1099. doi:10.1093/bjps/axx005.

Spirtes, P., Glymour, C. N. and Scheines, R. (2000). *Causation, Prediction, and Search.* MIT Press, New York, 2nd edition.

Suárez, M. (2004). An Inferential Conception of Scientific Representation. *Philosophy of Science* **71**(5): 767–779. doi:10.1086/421415.

Thèbault, K. P. Y. (2019). What Can We Learn From Analogue Experiments? *In* R. Dardashti, R. Dawid and K. P. Y. Thèbault (eds.), *Why Trust a Theory?: Epistemology of Fundamental Physics*, Cambridge University Press, Cambridge, pp. 184–201. doi:10.1017/9781108671224.014.

Unruh, W. G. (1981). Experimental black-hole evaporation? *Physical Review Letters* **46**(21): 1351–53. doi:10.1103/PhysRevLett.46.1351.

Van Fraassen, B. C. (1980). *The Scientific Image.* Oxford University Press.

Vervoort, L. (2016). No-Go Theorems Face Background-Based Theories for Quantum Mechanics. *Foundations of Physics* **46**(4): 458–472. doi:10.1007/s10701-015-9973-7.

Vickers, P. (2013). A Confrontation of Convergent Realism. *Philosophy of Science* **80**(2): 189–211. doi:10.1086/670297.

Weisberg, M. (2012). *Simulation and similarity: Using models to understand the world.* Oxford University Press.

Winsberg, E. (2010). *Science in the Age of Computer Simulation.* University of Chicago Press, Chicago.

——— (2019). Computer Simulations in Science. *In* E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Stanford University. Spring 2019 edition. plato.stanford.edu/archives/spr2019/entries/simulations-science/.