Does the PBR theorem refute Bohmian mechanics?

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Abstract

The nature of the wave function has been a hot topic of debate since the early days of quantum mechanics. The recent PBR theorem proves that the wave function is ontic, representing the ontic state of a physical system or a concrete physical entity. On the other hand, Bohmian mechanics regards the wave function as not ontic but nomological, like a law of nature. This raises an interesting question: does the PBR theorem refute Bohmian mechanics? It seems widely thought that the ontic view and the nomological view are compatible, and thus the PBR theorem has no implications for Bohmian mechanics. In this paper, I argue that this is not the case. First, I point out that the nomological view and the ontic view are two different views of the wave function. This means that the result of the PBR theorem and Bohmian mechanics are incompatible. Next, I argue that the PBR theorem and Bohmian mechanics are nevertheless compatible, and the former does not refute the latter. The reason is that the PBR theorem and Bohmian mechanics are based on different fundamental assumptions, and in particular, Bohmian mechanics rejects one key assumption of the ontological models framework on which the PBR theorem is based. Finally, I argue that the rejection of this assumption will bring to our attention a potential important issue of Bohmian mechanics.

1 Introduction

It has been suggested that the wave function of the universe is not ontic, representing a concrete physical entity, but nomological, like a law of nature (Dürr, Goldstein and Zanghì, 1997; Allori et al, 2008; Goldstein, 2017). On this nomological view of the wave function, there are only particles in three-dimensional space in Bohmian mechanics. On the other hand, a general and rigorous approach called ontological models framework has been proposed to determine the relation between the wave function and the ontic state of a physical system (Spekkens, 2005; Harrigan and Spekkens, 2010), and several ψ-ontology theorems have been proved in the framework (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012, 2017; Hardy, 2013). In particular, the Pusey-Barrett-Rudolph theorem or the PBR theorem shows that in the ontological models framework, when assuming independently prepared systems have independent ontic states, the ontic state of a physical system uniquely determines its wave function, and the wave function of a physical system directly represents the ontic state of the system (Pusey, Barrett and Rudolph, 2012). An interesting question then arises: are the ontic view and the nomological view of wave function compatible? More specifically, is Bohmian mechanics consistent with the PBR theorem? Or does the PBR theorem already refute Bohmian mechanics?

This issue has not received much attention from researchers. Goldstein's (2017) authoritative review of Bohmian mechanics does not mention the PBR theorem. Esfeld et al's (2014) insightful paper about the ontology of Bohmian mechanics only refers to the theorem once without any discussion. Presumably it is thought that the PBR theorem and Bohmian mechanics are obviously compatible. They both say that the wave function is real for single physical systems after all. Furthermore, according to Esfeld et al (2014), although Bohmian mechanics says that the universal wave function is nomological, it regards the effective wave function of a subsystem in the universe as ontic, representing "an objective, physical degree of freedom belonging to the subsystem", and thus the theory is compatible with the PBR theorem. In this paper, I will argue that this view is not wholly correct, and a careful analysis of the compatibility between Bohmian mechanics and the PBR theorem will bring to our attention a potential important issue of the theory.

The rest of this paper is organized as follows. In Section 2, I first introduce Bohmian mechanics and the nomological view of the wave function. It is widely thought that this view can avoid the problems of interpreting the wave function as a physical entity over and above the particles in Bohm's theory. In Section 3, I then introduce the ontological models framework and the PBR theorem based on the framework. The PBR theorem proves that the wave function is ontic, representing the ontic state of a physical system or a concrete physical entity. In Section 4, I point out that the nomological view and the ontic view are two different views of the wave function. This means that the result of the PBR theorem and Bohmian mechanics are incompatible. In Section 5, I argue that the PBR theorem and Bohmian mechanics are nevertheless compatible, and the former does not refute the latter. The reason is that the PBR theorem and Bohmian mechanics are based on different fundamental assumptions, and in particular, Bohmian mechanics rejects one key assumption of the ontological models framework on which the PBR theorem is based. In Section 6, I argue that by rejecting this assumption to avoid the result of the PBR theorem, Bohmian mechanics denies the existence of a universal law, while this may prevent us from knowing the nature of the wave function in the theory. Conclusions are given in the last section.

2 Bohmian mechanics

Bohmian mechanics, being a modern formulation of Bohm's (1952) theory, is committed only to particles' positions and a law of motion that describes how the positions develop in time (Esfeld et al, 2014; Goldstein, 2017). The theory provides an ontology of quantum mechanics in terms of particles and their trajectories in physical space and time. The Bohmian law of motion is expressed by two equations, a guiding equation for the configuration of particles in three-dimensional space and the Schrödinger equation, describing the time evolution of the wave function that enters the guiding equation. The law can be formulated as follows:

$$\frac{dQ(t)}{dt} = v^{\Psi(t)}(Q(t)), \tag{1}$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $Q(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function of the particle configuration at time t, and v equals to the velocity of probability density in standard quantum mechanics. Moreover, it is postulated that at some initial instant t_0, the epistemic probability of the configuration, $\rho(t_0)$, is given by the Born rule: $\rho(t_0) = |\Psi(t_0)|^2$. This is called quantum equilibrium hypothesis, which, together with the law of motion, ensures the empirical equivalence between Bohmian mechanics and standard quantum mechanics.

The status of the above equations is different, depending on whether one considers the physical description of the universe as a whole or of a subsystem thereof. Bohmian mechanics starts from the concept of a universal wave function (i.e. the wave function of the universe), figuring in the fundamental law of motion for all the particles in the universe. That is, $Q(t)$ describes the configuration of all the particles in the universe at time t, and $\Psi(t)$ is the wave function of the universe at time t, guiding the motion of all particles taken together. To describe subsystems of the universe, the appropriate concept is the effective wave function in Bohmian mechanics.

The effective wave function is the Bohmian analogue of the usual wave function in standard quantum mechanics. It is not primitive, but derived from the universal wave function and the actual spatial configuration of all the particles ignored in the description of the respective subsystem (Dürr, Goldstein and Zanghì, 1992). The effective wave function of a subsystem can be defined as follows. Let A be a subsystem of the universe including N particles with position variables $x = (x_1, x_2, ..., x_N)$. Let $y = (y_1, y_2, ..., y_M)$ be the position variables of all other particles not belonging to A. Then the subsystem A's conditional wave function at time t is defined as the universal wave function $\Psi_t(x, y)$ evaluated at $y = Y(t)$:

$$\psi_t^A(x) = \Psi_t(x, y)|_{y=Y(t)}. \tag{3}$$

If the universal wave function can be decomposed in the following form:

$$\Psi_t(x, y) = \varphi_t(x)\phi_t(y) + \Theta_t(x, y), \tag{4}$$

where $\phi_t(y)$ and $\Theta_t(x, y)$ are functions with macroscopically disjoint supports, and $Y(t)$ lies within the support of $\phi_t(y)$, then $\psi_t^A(x) = \varphi_t(x)$ (up to a multiplicative constant) is A's effective wave function at t. It can be seen that the temporal evolution of A's particles is given in terms of A's conditional wave function in the usual Bohmian way, and when the conditional wave function is A's effective wave function, it also obeys a Schrödinger dynamics of its own. This means that the effective descriptions of subsystems are of the same form of the law of motion as given above.

Bohmian mechanics raises the question of the status of the wave function that figures in the law. The theory assumes the nomological interpretation of the wave function, according to which the relationship between the universal wave function and the motion of the particles should be conceived as a nomic one, instead of a causal one in terms of one physical entity acting on the other (Dürr, Goldstein and Zanghì, 1997; Goldstein and Teufel, 2001; Goldstein and Zanghì, 2013; Esfeld et al, 2014). In the words of Dürr, Goldstein and Zanghì (1997),

> The wave function of the universe is not an element of physical reality. We propose that the wave function belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material. We propose, in other words, that the wave function is a component of a physical law rather than of the reality described by the law. (p. 10)

The reasons to adopt this nomological view of the wave function come from the unusual kind of way in which Bohmian mechanics is formulated, and the unusual kind of behavior that the wave function undergoes in the theory. First of all, although the wave function affects the behavior of the configuration of the particles, which is expressed by the guiding equation (1), there is no back action of the configuration upon the wave function. The evolution of the wave function is governed by the Schrödinger equation

(2), in which the actual configuration $Q(t)$ does not appear. Since a physical entity is supposed to satisfy the action-reaction principle, the wave function cannot describe a physical entity in Bohmian mechanics.

Next, the wave function of a many-particle system, $\psi(q_1, ..., q_N)$, is defined not in our ordinary three-dimensional space, but in the $3N$-dimensional configuration space, the set of all hypothetical configurations of the system. Thus it seems untenable to view the wave function as directly describing a real physical field. In fact, the sort of physical field the wave function is supposed to describe is even more abstract. Since two wave functions such that one is a (nonzero) scalar multiple of the other are physically equivalent, what the wave function describes is not even a physical field at all, but an equivalence class of physical fields. Moreover, Bohmian mechanics regards identical particles such as electrons as unlabelled, so that the configuration space of N such particles is not the familiar high dimensional space, like R^{3N}, but is the unfamiliar high-dimensional space ${}^N R^3$ of N-point subsets of R^3. This space has a nontrivial topology, which may naturally lead to the possibilities of bosons and fermions. But it seems odd as a fundamental space in which a physical field exists.

Thirdly, the wave function in Bohmian mechanics plays a role that is analogous to that of the Hamiltonian in classical Hamiltonian mechanics (Goldstein and Zanghì, 2013). To begin with, both the classical Hamiltonian and the wave function live on a high dimensional space. The wave function is defined in configuration space, while the classical Hamiltonian is defined in phase space: a space that has twice as many dimensions as configuration space. Next, there is a striking analogy between the guiding equation in Bohmian mechanics and the Hamiltonian equations in classical mechanics. The guiding equation can be written as:

$$\frac{dQ}{dt} = der(log\psi), \tag{5}$$

where the symbol *der* denotes some sort of derivative. Similarly, the Hamiltonian equations can be written is a compact way as:

$$\frac{dX}{dt} = der(H), \tag{6}$$

where $der(H)$ is a suitable derivative of the Hamiltonian. Moreover, it is also true that both $log\psi$ and H are normally regarded as defined only up to an additive constant. Adding a constant to H doesn't change the equations of motion. Similarly, when multiplying the wave function by a scalar, which amounts to adding a constant to its *log*, the new wave function is physically equivalent to the original one, and they define the same velocity for the configuration in the equations of motion in Bohmian mechanics. Since the classical Hamiltonian is regarded not as a description of some physical entity, but as the generator of time evolution in classical mechanics, by the above analogy it seems natural to assume that the wave function is not a description of some physical entity either, but a similar generator of the equations of motion in Bohmian mechanics.

These analyses suggest that the wave function is nomological, describing a law and not describing some sort of concrete physical entity in Bohmian mechanics. A law of motion tells us what happens in space and time given the specification of initial conditions, but it is not itself a physical entity existing in space and time. The exact meaning of the wave function then depends on what exactly a law is. There are two main views about laws of nature in the literature, namely Humeanism and dispositionalism, and both of them can be drawn upon for developing the nomological interpretation of the wave function in Bohmian mechanics (Esfeld et al, 2014). By Humeanism about laws, there are only particles' positions in the ontology, while dispositionalism admits more in the ontology than particles' positions, namely the holistic disposition of all the particles in the universe.[1] My following analysis of Bohmian mechanics and the nomological view of the wave function is independent of how to understand laws of nature.

3 The PBR theorem

Although there are various reasons to adopt the nomological view of the wave function in Bohm's theory, there are also more rigorous arguments supporting the ontic view of the wave function. In this section, I will introduce the ontological models framework and an important ψ-ontology theorem, the PBR theorem.

Quantum mechanics, in its minimum formulation, is an algorithm for calculating probabilities of measurement results. The theory assigns a mathematical object, the wave function, to a physical system appropriately prepared at a given instant, and specifies how the wave function evolves with time. The time evolution of the wave function is governed by the Schrödinger equation, and the connection of the wave function with the results of measurements on the system is specified by the Born rule. At first sight, quantum mechanics as an algorithm says nothing about the actual state of a physical system. However, it has been known that this is not true due to the recent advances in the research of the foundations of quantum mechanics (see Leifer, 2014 for a helpful review).

First of all, a general and rigorous approach called ontological models framework has been proposed to determine the relation between the wave

[1]Note that Bohmian mechanics is also compatible with a primitivism about laws as suggested by Maudlin (2007). It has been argued that primitivism about laws faces a dilemma: "either it has to bite the bullet of conceiving the law as developing itself in time and as including differences that correspond to different initial wave-functions, or it has to conceive the universal wave-function as a physical entity." (Dorato and Esfeld, 2015)

function and the ontic state of a physical system (Spekkens 2005; Harrigan and Spekkens 2010). The framework has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that if a physical system is prepared such that the quantum algorithm assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. In general, for an ensemble of identically prepared systems to which the same wave function ψ is assigned, the ontic states of different systems in the ensemble may be different, and the wave function ψ corresponds to a probability distribution $p(\lambda|\psi)$ over all possible ontic states, where $\int d\lambda p(\lambda|\psi) = 1$.

There are two possible types of models in the ontological models framework, namely ψ-ontic models and ψ-epistemic models. In a ψ-ontic model, the ontic state of a physical system uniquely determines its wave function, and the probability distributions corresponding to two different wave functions do not overlap. In this case, the wave function directly represents the ontic state of the system.[2] While in a ψ-epistemic model, the probability distributions corresponding to two different wave functions may overlap, and there are at least two wave functions which are compatible with the same ontic state of a physical system. In this case, the wave function merely represents a state of incomplete knowledge - an epistemic state - about the actual ontic state of the system.

In order to investigate whether an ontological model is consistent with the quantum algorithm, we also need a rule of connecting the underlying ontic states with measurement results. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is determined by the ontic state of the system, along with the physical properties of the measuring device. Concretely speaking, for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the quantum algorithm then requires the following relation:

$$\int d\lambda p(k|\lambda, M)p(\lambda|\psi) = p(k|M, \psi), \tag{7}$$

where $p(k|M, \psi) = |\langle k|\psi\rangle|^2$ is the Born probability of k given M and the wave function ψ.

Second, several important ψ-ontology theorems have been proved in the ontological models framework (Pusey, Barrett and Rudolph, 2012; Colbeck and Renner, 2012, 2017; Hardy, 2013), the strongest one of which is the PBR theorem (Pusey, Barrett and Rudolph, 2012). The PBR theorem shows

[2]Note that the wave function is not necessarily complete, i.e. it does not necessarily represent the complete ontic state of a system.

that in the ontological models framework, when assuming independently prepared systems have independent ontic states, the ontic state of a physical system uniquely determines its wave function, or the wave function of a physical system directly represents the ontic state of the system. This auxiliary assumption is called preparation independence assumption.

The basic proof strategy of the PBR theorem is as follows. Assume there are N nonorthogonal quantum states ψ_i (i=1, ... , N), which are compatible with the same ontic state λ.[3] The ontic state λ determines the probability $p(k|\lambda, M)$ of different results k for the measurement M. Moreover, there is a normalization relation for any N result measurement: $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$. Now if an N result measurement satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, then there will be a relation $p(k_i|\lambda, M) = 0$ for any i, which leads to a contradiction.

The task is then to find whether there are such nonorthogonal states and the corresponding measurement. Obviously there is no such a measurement for two nonorthogonal states of a physical system, since this will permit them to be perfectly distinguished, which is prohibited by quantum mechanics. However, such a measurement does exist for four nonorthogonal states of two copies of a physical system. The four nonorthogonal states are the following product states: $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$,$|+\rangle \otimes |0\rangle$ and $|+\rangle \otimes |+\rangle$, where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The corresponding measurement is a joint measurement of the two systems, which projects onto the following four orthogonal states:

$$\begin{aligned}
\phi_1 &= \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle), \\
\phi_2 &= \frac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle), \\
\phi_3 &= \frac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle), \\
\phi_4 &= \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle), \qquad (8)
\end{aligned}$$

where $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. This proves that the four nonorthogonal states are ontologically distinct. In order to further prove the two nonorthogonal states $|0\rangle$ and $|+\rangle$ for one system are ontologically distinct, the preparation independence assumption is needed. Under this assumption, a similar proof for every pair of nonorthogonal states can also be found, which requires more than two copies of a physical system (see Pusey, Barrett and Rudolph, 2012 for the complete proof).

To sum up, the PBR theorem shows that quantum mechanics as an algorithm may also say something about the actual ontic state of a physical system. It is that under the preparation independence assumption, the

[3] It can be readily shown that different orthogonal states correspond to different ontic states. Thus the proof given here concerns only nonorthogonal states.

wave function assigned to a physical system, which is used for calculating probabilities of results of measurements on the system, is a mathematical representation of the ontic state of the system in the ontological models framework.

Here it may be worth noting that the PBR theorem also applies to the universe as a whole. First, the universe as a whole is a perfectly isolated system. Then, if the wave function of an isolated system is ontic, the wave function of the universe must be ontic too. One may object that since the second assumption of the ontological models framework concerns measurements, while nothing exists outside the universe and no measurements can be made on it, the results obtained based on the framework such as the PBR theorem may be not valid for the universe as a whole. However, this is a misunderstanding. The reason is that the measurements involved in the second assumption of the ontological models framework are not necessarily actual. The assumption is essentially about the connection between the ontic state of an isolated system and the Born rule. If only the Born rule is universally valid for any wave function and the universe as a whole has an ontic state, then this assumption will apply to the universe, and the results obtained based on the framework will be also valid for it.

Next, if the ontic state of every isolated subsystem of the universe is represented by a wave function, and in particular, when the subsystem is an entangled composite system, its ontic state is also represented by a wave function, then it is arguable that the ontic state of all these subsystems as a whole should be represented by an (entangled) wave function. In other words, the ontic state of the universe as a whole should be represented by a wave function. For example, suppose the wave function of each isolated system in the universe is a real physical field (Albert, 1996, 2013, 2015), then since the universe is composed of these fields, its ontic state must be also a field (in a space with the largest dimension).

There are three possible ways to avoid the result of the PBR theorem. The first is to deny the preparation independence assumption. Although this assumption seems very natural, it is rejected in some ψ-epistemic models (Lewis et al, 2012). The second is to deny the first assumption of the ontological models framework, i.e. denying that an isolated system has a state of reality, which is objective and independent of other systems including observers. Indeed, this assumption is rejected by Quantum Bayesianism or QBism (Fuchs et al, 2014) and other pragmatist approaches to quantum theory (Healey, 2017), where the wave function represents information about possible measurement results or it is only a calculational tool for making predictions concerning measurement results. The third way is to deny the second assumption of the ontological models framework. We will discuss this possibility later.

4 The wave function: ontic vs. nomological

It seems obvious that the ontic view and the nomological view are two different views of the wave function. According to the ontic view, the wave function, including the wave function of the universe, is ontic, representing the ontic state of a physical system or a concrete physical entity. While according to the nomological view, the universal wave function is not ontic but nomological, like a law of nature, and it does not represent a concrete physical entity.

This difference between the ontic view and the nomological view also exists for any isolated subsystems in the universe. For example, suppose the universe contains two isolated subsystems, and their wave function is a product state. Then, the ontic view regards the wave function of each subsystem as ontic, representing a concrete physical entity, while the nomological view still regards the wave function of each subsystem as nomological, representing no concrete physical entity.

What about the effective wave functions of the quasi-isolated subsystems of the universe? According to Esfeld et al (2014), the effective wave function of a subsystem encodes the non-local influences of other particles on the subsystem via the non-local law of Bohmian mechanics. For example, in the double-slit experiment with one particle at a time, the particle goes through exactly one of the two slits, and that is all there is in the physical world. There is no real physical field that guides the motion of the particle and propagates through both slits and undergoes interference. The development of the position of the particle (its velocity and thus its trajectory) is determined by the positions of other particles in the universe, including the particles composing the experimental setup, and the non-local law of Bohmian mechanics can account for the observed particle position on the screen (Esfeld et al, 2014). In this sense, one may say that the nomological view also regards the effective wave function as ontic, and thus it is consistent with the ontic view for the quasi-isolated subsystems of the universe.

However, it can be argued that the effective wave function of a subsystem of the universe does not encode the influences of other particles on the subsystem, and it cannot be wholly ontic according to the nomological view (see also Gao, 2017). First of all, consider the simplest case in which the universal wave function factorizes so that

$$\Psi_t(x,y) = \varphi_t(x)\phi_t(y). \tag{9}$$

where $x = (x_1, x_2, ..., x_N)$ is the position variables of N particles of a subsystem A of the universe, and $y = (y_1, y_2, ..., y_M)$ is the position variables of all other particles not belonging to A. Then $\psi_t^A(x) = \varphi_t(x)$ is subsystem A's effective wave function at t. In this case, it is uncontroversial that subsystem A and its environment, which are represented by $\varphi_t(x)$ and $\phi_t(y)$, respectively, are independent of each other. Thus, the effective wave function of subsystem A is independent of the particles in the environment, and it does not encode the non-local influences of these particles. As noted above, since the universal wave function factorizes in this case, the effective wave function of subsystem A is also nomological according to the nomological view.

Next, consider the general case in which there is an extra term in the factorization of the universal wave function:

$$\Psi_t(x,y) = \varphi_t(x)\phi_t(y) + \Theta_t(x,y), \tag{10}$$

In this case, the effective wave function of subsystem A is determined by both the universal wave function and the positions of the particles in its environment (via a measurement-like process). If $Y(t)$ lies within the support of $\phi_t(y)$, A's effective wave function at t will be $\varphi_t(x)$. If $Y(t)$ does not lie within the support of $\phi_t(y)$, A's effective wave function at t will be not $\varphi_t(x)$. For example, suppose $\Theta_t(x,y) = \sum_n f_n(x)g_n(y)$, where $g_i(y)$ and $g_j(y)$ are functions with macroscopically disjoint supports for any $i \neq j$, then if $Y(t)$ lies within the support of $g_i(y)$, A's effective wave function at t will be $f_i(x)$. It can be seen that the role played by the particles in the environment is only selecting which function the effective wave function of subsystem A is, while each selected function is independent of the particles in the environment and completely determined by the universal wave function.

Therefore, the effective wave function of a subsystem of the universe does not only encode the influences of other particles in the universe in general cases. When the effective wave function of a subsystem has been selected, the other particles in the universe will have no influences on the particles of the subsystem. For example, in the double-slit experiment with one particle at a time, the development of the position of the particle will not depend on the positions of other particles in the universe (if only the positions of these particles select the same effective wave function of the particle during the experiment, e.g. $Y(t)$ has been within the support of $\phi_t(y)$ during the experiment).

To sum up, the effective wave function of a subsystem of the universe is determined by both the universal wave function and the positions of the particles in its environment. As a result, the effective wave function cannot be wholly ontic, but must be partly nomological according to the nomological view. This means that the nomological view is not consistent with the ontic view for the quasi-isolated subsystems of the universe either.

5 Does the PBR theorem refute Bohmian mechanics?

The PBR theorem says that the wave function is ontic, representing a concrete physical entity, while Bohmian mechanics says that the wave function is nomological, and the ontology of the theory consists only in particles. Since the ontic view and the nomological view are different, Bohmian mechanics is obviously incompatible with the result of the PBR theorem. Then an interesting question arises: does the PBR theorem refute Bohmian mechanics? To answer this question, we need a more careful analysis.

As we have seen, the PBR theorem is proved based on three preconditions: (1) the quantum algorithm; (2) the ontological models framework; and (3) the preparation independence assumption. Bohmian mechanics admits the quantum algorithm, since it keeps the core of quantum mechanics. Moreover, Bohmian mechanics admits the preparation independence assumption, since two unentangled systems (whose wave function is a product state) have independent ontic states in the theory.[4] The crux is whether Bohmian mechanics also admits the ontological models framework.[5]

The ontological models framework has two fundamental assumptions. The first assumption says that if a physical system is prepared such that the quantum algorithm assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state. This assumption is accepted by Bohmian mechanics, which provides an ontology of quantum mechanics in terms of particles and their trajectories in physical space and time. According to the theory, an isolated system which can be assigned to a wave function is composed of particles, and the positions of these particles are the ontic state of this system. Note that the ontic state of the system also includes the disposition of these particles which determines their motion via the guiding equation according to the dispositionalist interpretation of Bohmian mechanics (Esfeld et al, 2014).

The second assumption of the ontological models framework says that when a measurement is performed, the behaviour of the measuring device is determined only by the ontic state of the system, along with the physical properties of the measuring device. For a projective measurement M, this means that the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. If Bohmian mechanics also admits this assumption, then it will be refuted by the PBR theorem. Fortunately, this is not true; Bohmian mechanics rejects the second assumption of the ontological models framework.

[4]Note that the result that different orthogonal states correspond to different ontic states can be derived in the ontological models framework without resorting to the preparation independence assumption or other auxiliary assumptions. Thus, even if Bohmian mechanics rejects the auxiliary assumptions such as the preparation independence assumption, there is still the question of whether it is consistent with the ontological models framework, as well as the issues that will be discussed later.

[5]The question of whether Bohm's theory is consistent with the ontological models framework has been discussed by several authors (Feintzeig, 2014; Leifer, 2014; Drezet, 2015). Here I will focus on the issue of whether Bohmian mechanics or the nomological view of the wave function is consistent with the ontological models framework.

In Bohmian mechanics, when a measurement is performed, the behaviour of the measuring device is determined not only by the ontic state of the system and the physical properties of the measuring device, but also by something else, the law of motion represented by the wave function. Concretely speaking, for a projective measurement M, the complete ontic state λ of a physical system and its wave function ψ both determine the probability of different results k for the measurement M on the system, which may be denoted by $p(k|\lambda, \psi, M)$. Note that when the wave function is not nomological but related to the state of reality, the second assumption of the ontological models framework should not be revised this way but keep unchanged, since the complete ontic state λ already includes all parts of the state of reality (see also Leifer, 2014; Drezet, 2015).

It can be seen that the PBR theorem cannot be proved based on this revised assumption. Let us remind the basic proof strategy of the PBR theorem. Assume there are N nonorthogonal quantum states ψ_i (i=1, ... , N), which are compatible with the same ontic state λ. According to the second assumption of the ontological models framework, the ontic state λ determines the probability $p(k|\lambda, M)$ of different results k for a measurement M. Moreover, there is a normalization relation for any N result measurement: $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$. Since there is an N result measurement that satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, there will be a relation $p(k_i|\lambda, M) = 0$ for any i, which contradicts the normalization relation.

Now if the second assumption of the ontological models framework is replaced by the revised assumption, namely that the probability of different results k for a measurement M on a physical system is determined not only by the ontic state λ of the system, but also by its wave function ψ, i.e. $p(k|\lambda, M)$ is replaced by $p(k|\lambda, \psi, M)$, then the above contradiction cannot be derived. The reason is as follows. Under the revised assumption, the original normalization relation for an N result measurement $\sum_{i=1}^{N} p(k_i|\lambda, M) = 1$ holds true only for systems with the same wave function, and for systems with different wave functions ψ_j ($j = 1, ..., N$), the normalization relation should be $\sum_{j=1}^{N} \sum_{i=1}^{N} p(k_i|\lambda, \psi_j, M) = 1$. Then, even if there is an N result measurement that satisfies the condition that the first state gives zero Born probability to the first result and the second state gives zero Born probability to the second result and so on, it will only lead to the relation $p(k_i|\lambda, \psi_i, M) = 0$ for any i. But this relation does not contradict the new normalization relation.

6 A deeper issue?

I have argued that the PBR theorem and Bohmian mechanics are compatible, and the former does not refute the latter. The reason is that the PBR theorem and Bohmian mechanics are based on different fundamental assumptions, and in particular, Bohmian mechanics rejects one key assumption of the ontological models framework on which the PBR theorem is based. As we will see, however, a further analysis of the rejection may bring to our attention a potential important issue of Bohmian mechanics.

The assumption that Bohmian mechanics rejects is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is determined only by the ontic state of the measured system, along with the physical properties of the measuring device. For a projective measurement M, this assumption means that the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. Bohmian mechanics replaces this assumption with the revised assumption that the probability of different results k for a measurement M on a physical system is determined not only by the ontic state λ of the system, but also by its wave function ψ, i.e. it replaces the response function $p(k|\lambda, M)$ with $p(k|\lambda, \psi, M)$.

First of all, this revised assumption already admits that the wave function is real for a single system. The wave function being nomological in Bohmian mechanics means that it is real for a single system. If the wave function is not real for a single system, then the response function for a single system should not explicitly depend on the wave function of the system. However, it is arguable that the wave function being real for a single system should not be assumed before our analysis; rather, it should be a possible result obtained at the end of our analysis. For it directly excludes the possibility that the wave function is not real for a single system (i.e. the ψ-epistemic view), and also leaves the question of why the wave function is real for a single system unanswered. This is unsatisfactory.

Next, by rejecting the second assumption of the ontological models framework, Bohmian mechanics denies the existence of a universal law that applies to different systems. This can be seen from the following simple example. Suppose in a universe there are only two independent systems, whose wave functions are different, being ψ_1 and ψ_2, respectively. In this case, the wave function of the universe is a product state $\Psi = \psi_1\psi_2$. If there is a universal law which applies to both systems, then the response functions for the two systems will be the same. In other words, the existence of a universal law supports the second assumption of the ontological models framework. Then the proof of the PBR theorem can go through, and by the theorem we can know that the wave function of each system represents the ontic state of the system, namely that the wave function is ontic.

On the other hand, in order to avoid the result of the PBR theorem, Bohmian mechanics assumes that the response function for each system depends not only on the complete ontic state of the system, but also explicitly on the wave function of the system. Since the wave functions of the two systems are different, the response functions for them are also different. This means that the law will be different for the two systems. Thus, Bohmian mechanics denies the existence of a universal law which applies to both systems.

As we know, it is a fundamental postulate of physics that the laws of motion are universal, applying to all systems and all circumstances. The second assumption of the ontological models framework is consistent with this postulate, and based on this assumption we can prove the PBR theorem and know that the wave function is ontic, representing the ontic state of a physical system or a concrete physical entity. By comparison, it seems that we can do little research on and know little about the physical world if rejecting this fundamental postulate.

Consider again the above example. If there does not exist a universal law which applies to both systems, then the response functions for the two systems will be not the same, and there will be no connection between these response functions either without further assumptions.[6] As noted before, we cannot directly assume that the response function is $p(k|\lambda, \psi, M)$, explicitly depending on the wave function, since it already admits that the wave function is real for a single system, being nomological. Then, given this little piece of information, we cannot prove the PBR theorem. Moreover, it is arguable that we cannot know the relationship between the wave function and the ontic state of each system, e.g. whether the wave function is ontic or epistemic or something else, without resorting to additional assumptions. The reason is that the consistency relation for each wave function, namely $\int d\lambda p(k|\lambda, M)p(\lambda|\psi) = |\langle k|\psi\rangle|^2$, cannot determine whether the probability distributions of the ontic state corresponding to two different wave functions such as $p(\lambda|\psi_1)$ and $p(\lambda|\psi_2)$ overlap when there is no connection between the response functions for different wave functions in the relation, such as $p_1(k|\lambda, M)$ for ψ_1 and $p_2(k|\lambda, M)$ for ψ_2. Since we cannot know whether the wave function is real for a single system, we cannot be sure whether the wave function is nomological and Bohmian mechanics is true either.

To sum up, Bohmian mechanics denies the existence of a universal law in order to avoid the result of the PBR theorem. But this will arguably prevent us from knowing the nature of the wave function in the theory.

[6]Note that if only the response functions are the same for the two systems with any different wave functions, then by the PBR theorem we can prove that the ontic states of the two systems are different in Bohm's theory. But this will reject Bohmian mechanics, according to which two systems with different wave functions may have the same ontic state (e.g. two particles have the same position).

7 Conclusions

The PBR theorem proves that the wave function is ontic, representing the ontic state of a physical system or a concrete physical entity. On the other hand, Bohmian mechanics regards the wave function as not ontic but nomological, like a law of nature. This raises interesting questions: are the ontic view and the nomological view of wave function compatible? If the answer is negative, then does the PBR theorem refute Bohmian mechanics? It seems widely thought that the ontic view and the nomological view are compatible, and the PBR theorem has no implications for Bohmian mechanics.

In this paper, I argue that this is not the case. First, I point out that the nomological view and the ontic view are two different views of the wave function. This means that the result of the PBR theorem and Bohmian mechanics are incompatible. Next, I argue that the PBR theorem and Bohmian mechanics are nevertheless compatible, and the former does not refute the latter. The reason is that the PBR theorem and Bohmian mechanics are based on different fundamental assumptions, and in particular, Bohmian mechanics rejects one key assumption of the ontological models framework on which the PBR theorem is based. Finally, I argue that by rejecting this assumption to avoid the result of the PBR theorem, Bohmian mechanics denies the existence of a universal law, while this will arguably prevent us from knowing the nature of the wave function in the theory.

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