The Evidential Conditional

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This paper outlines an account of conditionals, the *evidential account*, which rests on the idea that a conditional is true just in case its antecedent supports its consequent. As we will show, the evidential account exhibits some distinctive logical features that deserve careful consideration. On the one hand, it departs from the material reading of 'if then' exactly in the way we would like it to depart from that reading. On the other, it significantly differs from the non-material interpretations of 'if then' which hinge on the Ramsey Test, advocated by Adams, Stalnaker, Lewis, and others.

1 OVERVIEW

Logicians have always been tempted by the thought that 'if then' expresses a relation of support. The meaning of 'support' can be articulated in various ways by using everyday words: one can say that the antecedent of a conditional must provide a *reason* to accept its consequent, or that the latter is justifiedly *inferred* from the former. Here is a telling quote drawn from Mill:

> When we say, If the Koran comes from God, Mohammed is the prophet of God, we do not intend to affirm either that the Koran does come from God, or that Mohammed is really his prophet. Neither of these simple propositions may be true, and yet the truth of the hypothetical proposition may be indisputable. What is asserted is not the truth of either of the propositions, but the inferribility of the one from the other.[1]

The thought that 'if then' expresses a relation of support owes its intuitive appeal to the fact that, in most cases, conditionals can be paraphrased by using words such as 'reason' or 'infer'. Consider the following examples:

(1) If it's pure cashmere, it will not shrink

(2) If you drink a beer, you'll feel better

(3) If it is snowing, then it is cold

It seems correct to say that the antecedent of (1) supports its consequent, as the following reformulations suggest:

(4) If it's pure cashmere, that is a reason for thinking that it will not shrink

(5) If it's pure cashmere, we can infer that it will not shrink

[1] Mill [26], p. 102.

Similar considerations hold for (2) and (3). What one wants to say when one utters (2) is that drinking a beer makes you feel better, so that if you drink it, you'll experience that effect. In the case of (3), again, the antecedent provides a reason to accept the consequent, although in this case the event described by the antecedent does not cause the event described by the consequent.

Of course, there are cases in which no paraphrase in terms of 'reason' or 'infer' is available. Typically, concessive conditionals do not admit reformulations along the lines suggested. Suppose that we intend to go out for a walk and we hope that it will be sunny. We can nevertheless assert what follows:

(6) If it will rain, we will go

In this case it would be inappropriate to say that the rain provides evidence to think that we will go. What we mean, instead, is that we will go anyway, that is, in spite of the rain. So the following seem correct reformulations of (6):

(7) Even if it will rain, we will go

(8) If it will rain, we will still go

More generally, concessive conditionals are suitably phrased by using 'even if' or 'still', and do not imply support in the sense considered. Nonetheless, the range of cases in which the notion of support seems pertinent is sufficiently large and representative to deserve separate study.

Despite the plain intelligibility of paraphrases such (4) and (5), the notion of support proves hard to capture at the formal level. This explains the multiplicity and the heterogeneity of the attempts that have been made so far to define a connective with the property desired. At least two main lines of thought have been explored, both of which significantly depart from the material interpretation of 'if then'. One option is to treat conditionals as strict conditionals by defining support in terms of necessitation: a conditional is true just in case its antecedent necessitates its consequent[2]. Another option is to provide a non-monotonic formal treatment of conditionals which aims to capture the idea that a conditional is true just in case its antecedent supports its consequent[3].

The account of conditionals outlined here belongs to the second category. The interpretation of 'if then' that we will explore is stronger than the material interpretation but weaker than the strict interpretation. We will call it *evidential interpretation*, as it rests on the idea that a conditional is true just in case its antecedent provides evidence for its consequent, where 'provides evidence for' is another way of saying 'supports'. The evidential interpretation may be regarded as one coherent reading of 'if then', although it is not necessarily the only admissible reading. We will not address the thorny question whether there is a unique correct analysis of 'if then', because the main points that we will make can be acknowledged without assuming that an affirmative answer can be given to that question. If different readings of 'if then' are equally admissible, the evidential interpretation is one of them.

[2] This option has been developed in different ways by Lycan [24], Gillies [13], Kratzer [20], Iacona [15], and others.

[3] Among the most recent attempts, Rott [30] contains a pioneering discussion of 'if' and 'because', relying on a variation of the belief revision formalism. The ranking-theoretic account offered in Spohn [33] explicitly involves the idea of the antecedent as providing a reason for the consequent. The approach to conditionals outlined in Douven [8] and Douven [9] employs the notion of evidential support from Bayesian epistemology. Krzyzanowska, Wenmackers, and Douven [17], van Rooij and Schulz [36], and Berto and Özgün [4] provide further examples.

Interestingly, the notion of support seems to apply equally well to indicative conditionals and to counterfactuals. Although we will focus on indicative conditionals, what we will say about this notion can easily be extended to counterfactuals. In particular, the distinction between evidential and concessive readings of 'if then' is orthogonal to the distinction between indicative and subjunctive conditionals. For example, the following sentences exhibit the same difference that obtains between (1) and (6):

(9) If it were cashmere, it would not shrink

(10) If it were raining, we would go

While (9) can be paraphrased by means of sentences that resemble (4) and (5), the most appropriate reformulations of (10) are sentences that resemble (7) and (8).

The structure of the paper is as follows. Section 2 provides a first informal sketch of the evidential account. Section 3 introduces a modal language that includes the symbol $\triangleright$, which represents our reading of 'if then'[4]. Sections 4-8 spell out some important logical properties of $\triangleright$. Section 9 explains how $\triangleright$ differs from the conditional as understood by Adams, Stalnaker, Lewis, and others. Finally, section 10 shows how the evidential interpretation can also be framed in terms of assertibility.

2 THE CORE IDEA

The evidential interpretation, in a way, stems from the same intuition that prompts the material interpretation and the strict interpretation. When one asserts a conditional 'If A, then C', one seems to imply that A and the negation of C do not go well together, that is, that there is something wrong with A being true and C being false. The following table provides a visual representation of this intuition:

1	1	
1	0	×
0	1	
0	0	

The four rows displays the four combinations of truth values that A and C can take, and the × in the second row indicates that 10 is the bad combination. The material interpretation and the strict interpretation develop this intuition in two different ways. According to the former, 'If A, then C' is true if and only if 10 does not actually occur, that is, it is not the case that A is true and C is false. According to the latter, 'If A, then C' is true if and only if 10 cannot occur, that is, it is impossible that A is true and C is false. Yet there is a third way of looking at the × in the second row, which is no less plausible than the other two. The reading of the table that we want suggest is that 'If A, then C' is true if and only if 10 cannot *easily* occur, that is, it is a remote possibility that A is true and B is false.

What does it mean that 10 cannot easily occur? It means that the worlds in which it occurs — the 10-*worlds* — are distant from the actual world if

[4] This symbol is borrowed from Spohn [33].

compared with those in which it does not occur. Consider (2). The following diagram describes a case in which (2) is true:

— — — 11
— — — — — 10
01
— — 00

Here the length of each dashed line indicates the distance from the actual world of the closest world in which the respective combination occurs. That is, any world in which you drink a beer without feeling better is more distant from the actual world than some world in which this does not happen. The following diagrams, instead, describe two cases in which (2) is false:

— — — — — 11
— — 10
01
— 00

— — 11
— — — 10
01
— — — — — 00

In the first case, the closest world in which you drink a beer without feeling better is less distant from the actual world than the closest world in which you drink a beer and feel better. In the second case, the closest world in which you drink a beer without feeling better is less distant from the actual world than the closest world in which you don't drink a beer and don't feel better.

This interpretation is modal rather than material, but without being strict. The contrast between materiality and modality is a fundamental point of dispute that emerged from the very beginning of the debate on conditionals. According to Sextus Empiricus, the Stoics disagreed with each other on at least two views of conditionals. Philo advocated the material view:

> a true conditional is one which does not have a true antecedent and a false consequent[5]

Chrysippus, instead, advocated a different view:

> a conditional holds whenever the denial of its consequent is incompatible with its antecedent[6]

Chrysippus' view can be construed in different ways, because incompatibility can be understood in different ways. The strict interpretation and the evidential interpretation provide two alternative specifications of the condition that the denial of the consequent is incompatible with the antecedent. On the understanding of Chrysippus that we will suggest, to say that the denial of C is incompatible with A is to say that the worlds in which A is true and C is false are distant from the actual world[7].

[5] Sextus Empiricus, *Outlines of Pyrrhonism*, II, 110-12.
[6] Sextus Empiricus, *Outlines of Pyrrhonism*, II, 110-12.
[7] For extensive discussions of the passage quoted, see Sanford [32], p. 25, Lenzen [21], pp. 15-19.

In order to provide a perspicuous representation of comparative measures of distance, we will employ the system of spheres adopted by Lewis in his semantics for counterfactuals. We will imagine non-actual worlds as ordered in a set of spheres around the actual world, depending on their degree of similarity to the actual world. This is a reasonably neutral formal tool which can be used without being committed to the rest of Lewis' view about conditionals and modal metaphysics.[8]

In the framework of the system of spheres, the condition that 10 cannot easily occur may be phrased as follows: there is a 10-free sphere — a sphere that contains no 10-worlds — in which A is true in some world and C is false in some world. These two conditions on the 10-free sphere guarantee the connection between A and C in the following sense. First, if there is a 10-free sphere in which A is true in some world, then some 11-worlds are closer to the actual world than any 10-world. This is essentially the Ramsey Test as understood by Stalnaker and Lewis: in the closest worlds in which A is true, C must be true as well[9]. Second, if there is a 10-free sphere in which C is false in some world, then some 00-worlds are closer to the actual world than any 10-world. This may be called *Reverse Ramsey Test*: in the closest worlds in which C is false, A must be false as well. The conjunction of the Ramsey Test and the Reverse Ramsey Test — call it *Chrysippus Test* — characterizes the evidential interpretation as we understand it[10].

The Chrysippus Test is stronger than the Ramsey Test exactly in the way that seems required in order to preserve the intuition that A must support C. Suppose that we are in a situation like that depicted in the third diagram above, that is, a situation in which there is a 10-free sphere and C is true throughout the sphere, while A is true in some world in the sphere but false in others. This situation represents a scenario in which C can easily hold regardless of A, and in which we would be inclined to say that C does not hold in virtue of A, so that it is not the case that a provides a reason to accept C. The Reverse Ramsey Test, unlike the Ramsey Test, is not satisfied in such a case: even if the closest 10-worlds may well be distant, they are no more distant than the closest 00-worlds. Thus, even if the falsity of C is kept away from the actual world, it is not because of the truth of A.

The following example illustrates the kind of situation described:

(11) If you drink a beer, the sun will rise tomorrow

(11) passes the Ramsey Test: the closest worlds in which you drink a beer are worlds in which the sun will rise tomorrow. But it does not pass the Reverse Ramsey Test: it is not the case that the closest worlds in which the sun will not rise tomorrow are worlds in which you don't drink a beer. Even if the absence of sunrise is a remote possibility, its distance from the actual world does not depend on your beer. Thus, an account of conditionals based on the Chrysippus Test will predict that (11) is false. The latter prediction is exactly what one should expect from the evidential interpretation: the antecedent of (11) does *not* provide a reason for accepting its consequent[11].

The hypothesis that emerges from these initial informal remarks is that 'If A, then C' is non-vacuously true just in case there is a 10-free sphere in

[8] Lewis [23], pp. 13-19.

[9] The Ramsey Test comes from Ramsey [29]. Stalnaker [34] and Lewis [23] adopt the modal interpretation suggested.

[10] Note that the two conditions on the 10-free sphere leave open the question whether the 10-free sphere contains 01-worlds. Requiring that it contains 01-worlds in addition to 11-worlds and 00-worlds would make the test unnecessarily stronger.

[11] Douven [8] discusses similar examples.

which A is true in some world and C is false in some world. This account of non-vacuous truth, as we shall see, can plausibly be combined with a standard characterization of vacuous truth. If A is impossible — that is, true in no world — or C is necessary — that is, true in every world — then there are no 10-worlds at all, so 10 is a maximally remote possibility. In other terms, the absence of 10-worlds entails necessitation, which may be regarded as the strongest form of support: if A necessitates C, then C follows from A.

3 DEFINITIONS

To phrase in formal terms what we have just said, we will define a modal language called **L**. The symbols of **L** are the letters $p, q, r, ...$, the connectives $\sim, \supset, \wedge, \vee, \triangleright, \Box, \Diamond$, and the brackets $(,)$. The formulas of **L** are defined by induction in the usual way: $p, q, r...$ are formulas; if α is a formula, $\sim\alpha, \Box\alpha, \Diamond\alpha$ are formulas; if α and β are formulas, $\alpha \supset \beta, \alpha \wedge \beta, \alpha \vee \beta, \alpha \triangleright \beta$ are formulas.

DEFINITION 1 Given a non-empty set W, a system of spheres O over W is an assignment to each $w \in W$ of a set O_w of non-empty sets of elements of W — a set of spheres around w — such that:

1. if $S \in O_w$ and $S' \in O_w$, then either $S \subseteq S'$ or $S' \subseteq S$;
2. $\{w\} \in O_w$;
3. if $S \neq \bigcup O_w$, then there is a S' such that $S \subset S'$ and $S' \subseteq S''$ for every S'' such that $S \subset S''$.

Clause 1 says that O_w is *nested*. This condition is essential, otherwise we would have two spheres S, S' and two worlds w', w'' such that $w' \in S$ but $w' \notin S'$, and $w'' \in S'$ but $w'' \notin S$. That is, w' would be more similar to w than w'' and w'' would be more similar to w than w'.

Clause 2 implies that O_w is *centered* on w. If $\{w\} \in O_w$, then by clause 1 we have that, for every $S \in O_w$, $\{w\} \subseteq S$, given that S is assumed to be non-empty. This means that w belongs to every sphere around w. The idea that underlies centering is that the innermost sphere is a singleton because no other world is as similar to w as w itself is.

Clause 3 states the *limit assumption*, according to which, for every sphere smaller than $\bigcup O_w$, there is a smallest sphere around S: getting closer and closer to S we eventually reach a limit. In the specific case in which $S = \{w\}$, this means that there is a sphere that contains the worlds closest to w. Although Lewis finds this assumption questionable for metaphysical reasons, we think that we can live with it[12].

A model for **L** is defined in terms of a system of spheres as follows:

DEFINITION 2 A model for **L** is an ordered triple $\langle W, O, V\rangle$, where W is a nonempty set, O is a system of spheres over W, and V is a valuation function such that, for each atomic formula α of **L** and each $w \in W$, $V(\alpha, w) \in \{1, 0\}$.

The truth of a formula of **L** in a world w in a model is defined as follows:

DEFINITION 3

[12] Further constraints on O might be added. One is *closure under union*: if $S \subseteq O_w$ and $\bigcup S$ is the set of all w' such that w' belongs to some member of S, then $\bigcup S \in O_w$. Another is *closure under intersection*: if $S \subseteq O_w$ and $\bigcap S$ is the set of all w' such that w' belongs to every member of S, $\bigcap S \in O_w$. A third constraint is *uniformity*: for every $w, w' \in W$, $\bigcup O_w = \bigcup O_{w'}$;. Each of these constraint is reasonable. However, we will not assume them as part of the definition of O because they are not strictly necessary for our purposes. See Lewis [23], pp. 14-15, 120-121.

1 If α is atomic, $[\alpha]_w = 1$ iff $V(\alpha, w) = 1$;

2 $[\sim\alpha]_w = 1$ iff $[\alpha]_w = 0$;

3 $[\alpha \wedge \beta]_w = 1$ iff $[\alpha]_w = 1$ and $[\beta]_w = 1$;

4 $[\alpha \vee \beta]_w = 1$ iff either $[\alpha]_w = 1$ or $[\beta]_w = 1$;

5 $[\alpha \supset \beta]_w = 1$ iff either $[\alpha]_w = 0$ or $[\beta]_w = 1$;

6 $[\alpha \triangleright \beta]_w = 1$ iff the following conditions hold:

   (a) for every $w' \in \bigcup O_w$, if $[\alpha]_{w'} = 1$ and there are no w'' and S such that $w'' \in S$, $w' \notin S$, and $[\alpha]_{w''} = 1$, then $[\beta]_{w'} = 1$;

   (b) for every $w' \in \bigcup O_w$, if $[\beta]_{w'} = 0$ and there are no w'' and S such that $w'' \in S$, $w' \notin S$, and $[\beta]_{w''} = 0$, then $[\alpha]_{w'} = 0$;

7 $[\Box\alpha]_w = 1$ iff, for every w' in every $S \in O_w$, $[\alpha]_{w'} = 1$;

8 $[\Diamond\alpha]_w = 1$ iff, for some w' in some $S \in O_w$, $[\alpha]_{w'} = 1$.

In clause 6, (a) expresses the Ramsey Test, or at least one widespread understanding of it: β must be true in the closest worlds in which α is true. (b) expresses the Reverse Ramsey Test: α must be false in the closest worlds in which β is false. Note that if α is impossible, the antecedent of (a) is false for every world, and the consequent of (b) is true for every world. Similarly, if β is necessary, the consequent of (a) is true for every world, and the antecedent of (b) is false for every world. This means that $\alpha \triangleright \beta$ is vacuously true when α is impossible or β is necessary. Instead, when α is true in some world and β is false is some world, (a) and (b) entail that there is a 10-free sphere where α is true in some world and β is false in some world.

Validity, indicated by the symbol $\vDash$, is defined in terms of truth in a world in a model:

DEFINITION 4 $\vDash \alpha$ iff α is true in every world in every model.

Logical consequence is defined accordingly for every finite set of formulas $\alpha_1, ..., \alpha_n$ and every formula β:

DEFINITION 5 $\alpha_1, ...\alpha_n \vDash \beta$ iff $\vDash (\alpha_1 \wedge ... \wedge \alpha_n) \supset \beta$.

In the following sections we will employ these definitions to elucidate the logical properties of the evidential interpretation. As we will show, $\triangleright$ differs from $\supset$ exactly in the way we would like it to differ from $\supset$, and exhibits distinctive logical features that deserve careful consideration. From now on, we will consider an arbitrary model $\langle W, O, V \rangle$, and use the symbol $\vDash_{PL}$ to indicate logical consequence in a classical propositional language.

4 SOME RELATIVELY UNCONTROVERSIAL PRINCIPLES

Let us start with some very basic and relatively uncontentious principles that hold for $\supset$. The first is *Modus Ponens*: 'If A, then C' and A entail C. This is the simplest and most fundamental rule of inference involving conditionals, and most theorists of conditionals agree on its centrality. The evidential account validates Modus Ponens:

FACT 1 $\alpha \triangleright \beta, \alpha \vDash \beta$ (*Modus Ponens* ✓)

Proof. Assume that $[\alpha \triangleright \beta]_w = 1$ and $[\alpha]_w = 1$. Since there are no w' and S such that $w' \in S$, $w \notin S$, and $[\alpha]_{w'} = 1$, by (a) we get that $[\beta]_w = 1$. □

The second principle, *Superclassicality*, says that 'If A, then C' is true whenever C logically follows from A. The evidential account validates this principle, as is plausible to expect:

FACT 2 If $\alpha \vDash_{PL} \beta$, then $\vDash \alpha \triangleright \beta$ (*Superclassicality* ✓)

Proof. Assume that $\alpha \vDash_{PL} \beta$. Then, for every w, there is no w' such that $[\alpha]_{w'} = 1$ and $[\beta]_{w'} = 0$. It follows that (a) and (b) are both satisfied, so that $[\alpha \triangleright \beta]_w = 1$. □

If β logically follows from α, then α provides a conclusive reason for accepting β. Note that a direct corollary of fact 2 is that $\vDash \alpha \triangleright \alpha$, given that $\alpha \vDash_{PL} \alpha$.

Two further principles, which involve the modal operator $\Box$, are *Necessary Consequent* and *Impossible Antecedent*: 'If A, then C' is true when C is necessary or A is impossible. The evidential account validates these two principles, given its treatment of vacuous truth:

FACT 3 $\Box\alpha \vDash \beta \triangleright \alpha$ (*Necessary Consequent* ✓)

Proof. Assume that $[\Box\alpha]_w = 1$. Then by definition $[\beta \triangleright \alpha]_w = 1$. □

FACT 4 $\Box{\sim}\alpha \vDash \alpha \triangleright \beta$ (*Impossible Antecedent* ✓)

Proof. Assume that $[\Box{\sim}\alpha]_w = 1$. Then by definition $[\alpha \triangleright \beta]_w = 1$. □

5 SOME HIGHLY CONTROVERSIAL PRINCIPLES

Now we will show that the evidential account invalidates some highly contentious principles that hold for $\supset$. In the material interpretation, the mere falsity of A or the mere truth of C suffices for the truth of 'If A, then C', that is, *False Antecedent* and *True Consequent* hold for $\supset$. This is commonly regarded as a reason to doubt the material interpretation. For example, it is quite implausible that the following sentences are true:

(12) If the Colisseum is in Paris, then I will win the lottery

(13) If the Colisseum is in Paris, then it is in Rome[13]

$\triangleright$ differs from $\supset$ in this respect. On the evidential account, (12) and (13) are false. More generally, the evidential account invalidates False Antecedent and True Consequent.

FACT 5 ${\sim}\alpha \nvDash \alpha \triangleright \beta$ (*False Antecedent* ×)

Proof. Suppose that $[\alpha]_w = 0$ and that, for some w', $[\alpha]_{w'} = 1$, $[\beta]_{w'} = 0$, and $w' \in S$ for every $S \neq \{w\}$. In this case $[{\sim}\alpha]_w = 1$. But $[\alpha \triangleright \beta]_w = 0$, for w' violates (a). □

FACT 6 $\beta \nvDash \alpha \triangleright \beta$ (*True Consequent* ×)

Proof. Suppose that $[\beta]_w = 1$ and that, for some w', $[\alpha]_{w'} = 1$, $[\beta]_{w'} = 0$, and $w' \in S$ for every $S \neq \{w\}$. In this case $[\alpha \triangleright \beta]_w = 0$, for w' violates (b). □

[13] Edgington [11], section 2.3, presents False Antecedent and True Consequent as "the best-known objection to the material account".

A closely related principle that holds for $\supset$ is *Linearity*: for every A and C, either 'If A, then C' or 'If C, then A' is true. For example, the following disjunction is true in the material interpretation:

(14) Either if it is snowing then I will win the lottery or if I will win the lottery then it is snowing

Again, $\rhd$ differs from $\supset$ in this respect. On the evidential account, (14) is false because in each disjunct the antecedent and the consequent are not related in the right way. More generally, the evidential account invalidates Linearity.

FACT 7 $\nvDash (\alpha \rhd \beta) \vee (\beta \rhd \alpha)$ (*Linearity* $\times$)

Proof. Suppose that $[\alpha]_w = 0$ and $[\beta]_w = 0$. Let w' and w'' be such that $[\alpha]_{w'} = 1$, $[\beta]_{w'} = 0$, $[\alpha]_{w''} = 0$, $[\beta]_{w''} = 1$, and that $w' \in S$ and $w'' \in S$ for every $S \neq \{w\}$. In this case $[\alpha \rhd \beta]_w = 0$ because w' violates (a). Moreover, $[\beta \rhd \alpha]_w = 0$ because w'' violates (a). □

Another principle that is closely related to False Antecedent and True Consequent is *Conditional Proof*: if A, together with a set of premises, entails C, then 'If A, then C' follows from those premises. If Conditional Proof holds, the same goes for False Antecedent and True Consequent. This is why the evidential account does not validate Conditional Proof.

FACT 8 Not: if $\Gamma, \alpha \vDash_{PL} \beta$, then $\Gamma \vDash \alpha \rhd \beta$ (*Conditional Proof* $\times$)

Proof. Conditional Proof fails because it entails False Antecedent and True Consequent. Suppose that if $\Gamma, \alpha \vDash_{PL} \beta$, then $\Gamma \vDash \alpha \rhd \beta$. Since $\sim\alpha, \alpha \vDash_{PL} \beta$ and $\beta, \alpha \vDash_{PL} \beta$, we get that $\sim\alpha \vDash \alpha \rhd \beta$ and $\beta \vDash \alpha \rhd \beta$, contrary to facts 5 and 6. □

Two further properties of the material interpretation are widely regarded as counterintuitive. One is *Monotonicity*: 'If A, then C' entails 'If A and B, then C'. The other is *Transitivity*: 'If A, then C' and 'If C, then B' entail 'If A, then B'. Examples such as the following, due to Adams, have been taken to show — and plausibly so, we think — that conditionals as they are used in ordinary language are neither monotonic nor transitive:

> If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life.
>
> If Brown wins the election, Smith will retire to private life. If Smith dies before the election, Brown will win it. Therefore, if Smith dies before the election, then he will retire to private life[14].

The evidential account can explain the apparent invalidity of these arguments. It is possible that the premise of the first argument is true but its conclusion is false, for only the former passes the Ramsey Test. Similarly, it is possible that the premises of the second argument are true but its conclusion is false, for only the former pass the Ramsey Test. More generally, the evidential account invalidates Monotonicity and Transitivity.

FACT 9 $\alpha \rhd \gamma \nvDash (\alpha \wedge \beta) \rhd \gamma$ (*Monotonicity* $\times$)

[14] Adams [1], p. 166.

Proof. Suppose that $[\alpha \triangleright \gamma]_w = 1$ and, for some S, there is no $w' \in S$ such that $[\beta]_{w'} = 1$. Suppose also that outside S there is a w'' such that $[\alpha]_{w''} = 1$, $[\beta]_{w''} = 1$, $[\gamma]_{w''} = 0$, and w'' belongs to every S' bigger than S. In this case $[(\alpha \wedge \beta) \triangleright \gamma]_w = 0$ because w'' violates (a). □

FACT 10 $\alpha \triangleright \beta, \beta \triangleright \gamma \nvDash \alpha \triangleright \gamma$ (*Transitivity* ×)

Proof. Transitivity fails because it entails Monotonicity, given Superclassicality. Suppose that $\alpha \triangleright \beta, \beta \triangleright \gamma \vDash \alpha \triangleright \gamma$, and assume that $[\alpha \triangleright \beta]_w = 1$. Since by fact 2 $[(\alpha \wedge \gamma) \triangleright \alpha]_w = 1$, from this assumption we get that $[(\alpha \wedge \gamma) \triangleright \beta]_w = 1$, contrary to fact 9. □

6 CONTRAPOSITION AND RIGHT WEAKENING

The facts outlined in sections 4 and 5 are results on which most non-material accounts of conditionals tend to converge: Modus Ponens, Superclassicality, Necessary Consequent, and Impossible Antecedent are widely accepted as sound, while False Antecedent, True Consequent, Linearity, Monotonicity, and Transitivity are widely rejected as counterintuitive. The facts outlined in this section and in the next two, instead, concern principles on which there is no such agreement. The evidential account crucially differs from other non-material accounts with respect to these principles.

One fact that deserves attention concerns *Contraposition*: 'If A, then C' entails 'If not-C, then not-A'. The evidential account validates Contraposition, so it agrees with the material interpretation in this respect. To illustrate, consider the inference from (1) to (15):

(15) If it will shrink, then it is not pure cashmere

This inference seems valid, and the same goes for similar inferences that involve (2) and (3) as premises. More generally, the evidential account validates Contraposition.

FACT 11 $\alpha \triangleright \beta \vDash \sim\beta \triangleright \sim\alpha$ (*Contraposition* ✓)

Proof. Assume that $[\alpha \triangleright \beta]_w = 1$. Then (a), for every w', if $[\alpha]_{w'} = 1$ and there are no w'' and S such that $w'' \in S$, $w' \notin S$, and $[\alpha]_{w''} = 1$, then $[\beta]_{w'} = 1$, and (b) for every w', if $[\beta]_{w'} = 0$ and there are no w'' and S such that $w'' \in S$, $w' \notin S$, and $[\beta]_{w''} = 0$, then $[\alpha]_{w'} = 0$. (a) and (b) are respectively (b) and (a) for $\sim\beta \triangleright \sim\alpha$. Therefore, $[\sim\beta \triangleright \sim\alpha]_w = 1$. □

This fact is a distinctive feature of the evidential account. Unlike the principles considered in the previous two sections, Contraposition is neither widely accepted nor widely rejected. Some theorists of conditionals regard it as counterintuitive. Here is a classical example due to Stalnaker:

> 'If the US halts the bombing, then North Vietnam will not agree to negotiate'. A person would believe that this statement is true if he thought that the North Vietnamese were determined to press for a complete withdrawal of US troops. But he would surely deny the contrapositive, 'If North Vietnam agrees to negotiate, then the US will not have halted the bombing'[15].

[15] Stalnaker [34], p. 39.

However, these examples can hardly prove that Contraposition fails in the evidential interpretation. As has been noted by Lycan, Bennett and others, the alleged counterexamples to Contraposition typically involve a concessive reading of the premise[16]. Therefore, they loose their grip on any interpretation which rules out such a reading. This is precisely the case of the evidential interpretation: a conditional is true in the evidential sense only if it is false in the concessive sense. Thus, a conditional that is true solely in the concessive sense, such as 'If the US halts the bombing, then North Vietnam will not agree to negotiate', is false in the evidential account. This conditional does not pass the Reverse Ramsey Test, for it is not the case that the closest worlds in which North Vietnam will agree to negotiate are worlds in which the US keep bombing. If North Vietnam will not agree to negotiate, it is not because the US halts the bombing, but rather in spite of that fact. More generally, insofar as the alleged counterexamples to Contraposition involve a concessive reading of the premise, they do not work in the evidential interpretation because their premise turns out to be false on that interpretation.

A closely related fact concerns *Right Weakening*, the principle according to which if B logically follows from C, 'If A, then C' entails 'If A, then B'. Right Weakening holds for $\supset$. However, it does not hold for $\triangleright$. To see why, consider the following example, taken from Rott:

> It makes perfect sense to say 'If you pay an extra fee, your letter will be delivered by express', because the fee will buy you a special service. But it sounds odd to say 'If you pay an extra fee, your letter will be delivered', because the letter would be delivered anyway, even if you did not pay the extra fee[17].

In the evidential interpretation, the first conditional is plausibly true. Very likely, if your letter will be delivered by express, it is because you paid the extra fee. The fulfilment of the Reverse Ramsey Test is key here: among the worlds in which your letter will not be delivered by express, those in which you did not pay the extra fee are closer than those in which you paid it. However, the second conditional may easily be false: it is not in virtue of the payment of the extra fee that your letter will be delivered. Arguably, this conditional does not pass the Reverse Ramsey Test: there is no reason to think that, among the worlds in which your letter is not delivered at all, those in which you did not pay the extra fee are closer than those in which you paid it. The closest worlds in which the letter is not delivered will rather have other kinds of features, like the occurrence of some accident, in virtue of which the delivery failed altogether, regardless of your payment of the extra fee. Therefore, the second conditional does not follow from the first. And since the consequent of the first — once naturally formalized in a propositional language — entails the consequent of the second, this shows that the evidential account invalidates Right Weakening.

The failure of Right Weakening can be proved in more general terms as follows:

FACT 12 Not: if $\beta \vDash_{PL} \gamma$, then $\alpha \triangleright \beta \vDash \alpha \triangleright \gamma$ (*Right Weakening* $\times$)

Proof. Right Weakening fails because it entails Monotonicity, given Contraposition. If it were the case that, if $\beta \vDash_{PL} \gamma$, then $\alpha \triangleright \beta \vDash \alpha \triangleright \gamma$, from the

[16] Lycan [24], p. 34, Bennett [3], pp. 32 and 143-144.
[17] Rott [31], p. 6.

assumption that $[\alpha > \gamma]_w = 1$ we would get that $[(\alpha \wedge \beta) \rhd \gamma]_w = 1$, contrary to fact 9. The reason is that $\alpha \rhd \gamma$ entails $\sim\gamma > \sim\alpha$ by fact 11, and $\sim\alpha \vDash_{PL} \sim\alpha \vee \sim\beta$. By fact 11 $\sim\gamma \rhd (\sim\alpha \vee \sim\beta)$ entails $\sim(\sim\alpha \vee \sim\beta) \rhd \sim\sim\gamma$, which is logically equivalent to $(\alpha \wedge \beta) > \gamma$. □

This proof shows the connection between Contraposition and Right Weakening: if Monotonicity fails, then either Contraposition or Right Weakening must fail as well. This is why facts 11 and 12 are closely related.

7 CONDITIONAL EXCLUDED MIDDLE AND CONJUNCTIVE SUFFICIENCY

One rather debated principle that holds for $\supset$ is *Conditional Excluded Middle*: for every A and C, either 'If A, then C' or 'If A, then not-C' is true. Some non-material accounts of conditionals preserve this principle, while others deny it. The key question is whether 'Not: if A, then C' entails 'If A, then not-C'. If it does, then Conditional Excluded Middle straightforwardly follows from Excluded Middle, according to which either 'If A, then C' or 'Not: if A, then C' is true, otherwise it does not follow.

The evidential account invalidates Conditional Ecluded Middle. Consider the following sentences:

(16) If planet nine exists, then the EU will collapse within 5 years

(17) If planet nine exists, then the EU will not collapse within 5 years

Since the existence of planet nine and the collapse of the EU are totally unrelated, (16) and (17) are both false, so the same goes for the disjunction of (16) and (17). More generally, $\sim(\alpha \rhd \beta)$ does not entail $\alpha \rhd \sim\beta$, so $(\alpha \rhd \beta) \vee (\alpha \rhd \sim\beta)$ does not follow from $(\alpha \rhd \beta) \vee \sim(\alpha \rhd \beta)$.

FACT 13 $\nvDash (\alpha \rhd \beta) \vee (\alpha \rhd \sim\beta)$ (*Conditional Excluded Middle* $\times$)

Proof. Suppose that $[\alpha]_w = 1$ and $[\beta]_w = 0$. Let w' be such that $[\alpha]_{w'} = 1$, $[\beta]_{w'} = 1$, and $w' \in S$ for every $S \neq \{w\}$. In this case $[\alpha \rhd \beta]_w = 0$ because w violates both (a) and (b). Moreover, $[\alpha \rhd \sim\beta]_w = 0$, for w' violates (b). □

A related principle that holds for $\supset$ but not for $\rhd$ is *Conjunctive Sufficiency*: 'A and C' entails 'If A, then C'. Even supposing that the antecedent and the consequent of (16) are both true, it does not follow that (16) is true. The same goes for (17). We take the failure of Conjunction Sufficiency to be a plausible result. If A and C are totally unrelated, it is definitely false that A provides a reason to accept C, or that if C holds, it holds in virtue of A.

FACT 14 $\alpha \wedge \beta \nvDash \alpha \rhd \beta$ (*Conjunctive Sufficiency* $\times$)

Proof. Suppose that $[\alpha]_w = 1$, $[\beta]_w = 1$, and for some w', $[\alpha]_{w'} = 1$, $[\beta]_{w'} = 0$, and $w' \in S$ for every $S \neq \{w\}$. In this case $[\alpha \wedge \beta]_w = 1$. But $[\alpha \rhd \beta]_w = 0$, for w' violates (b). □

Conjunctive Sufficiency is related to Conditional Excluded Middle in the following way. If 'Not: if A, then C' entails 'If A, then not-C', as required in order to derive Conditional Excluded Middle from Excluded Middle, then Conjunction Sufficiency holds. On the assumption that 'A and C' is true, the supposition that 'Not: if A, then C' is true leads to a contradiction if it entails 'If A, then not-C', so its negation follows by reductio, which is equivalent to 'If A, then C'. Therefore, since Conjunction Sufficiency fails for $\rhd$, the same must hold for Conditional Excluded Middle.

Leaving aside the relation between Conjunctive Sufficiency and Conditional Excluded Middle, fact 14 is particularly interesting because it shows that a principled distinction can be drawn between two claims that are usually conflated. One is centering, understood as a condition on the system of spheres based on a metaphysical assumption. The other is Conjunctive Sufficiency, the logical rule just discussed. In the semantic framework offered by Lewis, if one assumes centering, one gets Conjunctive Sufficiency. As Lewis himself suggests, one can avoid this result by replacing centering with a weaker condition, *weak centering*, that is, by replacing clause 2 of definition 1 with the conditiont that w belongs to every sphere around w, without requiring that the innermost sphere is a singleton[18]. This is why in the literature on conditionals it is quite common to talk about Conjunction Sufficiency and centering as if they were the same thing. However, this coincidence breaks down in our semantic framework: even if one assumes centering, as in definition 1, one does not get Conjunctive Sufficiency. This shows clearly that the question whether Conjunction Sufficiency holds does not reduce to the choice between centering and weak centering.

8 CONNEXIVITY

The last four principles that we will consider have been extensively discussed in relation to connexive logics. Connexive logics are characterized by two main theses which do not hold for $\supset$. One is *Aristotle's Thesis*: for every A, 'Not: if A, then not-A' is true. The other is *Boethius's Thesis*: 'If A, then C' entails 'Not: if A, then not-C'. Some connexivists have suggested that what is needed to validate Aristotle's Thesis and Boethius' Thesis is a suitable reading of Chrysippus' claim that a conditional is true when the negation of its consequent is incompatible with its antecedent[19].

Although we doubt that Aristotle's thesis and Boethius' thesis hold unrestrictedly, we believe that the idea of connexivity rests on a solid intuition, and that to some extent it is plausible that this intuition goes back to Chrysippus. Consider the following sentence:

(18) If it is snowing, then it is not snowing

It is quite natural to think that there is something wrong in (18). In the material interpretation, however, (18) is true when it is not snowing, so our negative reaction can be correct only if it is snowing. This is an odd thing to say. The impression of falsity that we get when we look at (18) has nothing to do with the weather. If we feel that there is something wrong in (18), it is not because we look out the window. It seems that (18) is false *no matter whether* it is snowing. So we find plausible to say that what makes (18) false is that the negation of its consequent is patently compatible with its antecedent.

This is not to say that every conditional of the form 'If A, then not-A' is intuitively false. For example, we have no clear intuitions about (19):

(19) If it is not the case that either it is snowing or it is not snowing, then either it is snowing or it is not snowing

[18] Lewis [23], p. 29

[19] McCall [25] and Wansing [38] suggest that the idea of connexivity go back to Chrysippus. The relation of inconsistency defined in Nelson [27] has been taken to provide such a reading.

More generally, when A is impossible, it is reasonable to think that 'If A, then not-A' is vacuously true. This is why we doubt that 'Not: if A, then not-A' is always true[20].

The same goes for Boethius' thesis. For example, it is plausible that if (3) is true, then (20) is false:

(20) If it is snowing, then it is not cold

But it is not obvious that the same holds for any two conditionals of the same form. For example, it is reasonable to think that (21) and (22) are both true:

(21) If it is snowing and it is not snowing, then it is snowing

(22) If it is snowing and it is not snowing, then it is not snowing

More generally, we think that Aristotle's thesis and Boethius' thesis are plausible only insofar as they entail two weaker claims which may be called *Restricted Aristotle's Thesis* and *Restricted Boethius' Thesis*: if A is possible, then 'Not: if A, then not-A' is true, and if A is possible, then 'If A, then C' entails 'Not: if A, then not-C'[21].

Similar considerations hold for a third connexivist thesis, *Abelard's Thesis*, according to which 'If A, then C' entails 'Not: if not-A, then C'[22]. Consider the following sentence:

(23) If it is not pure cashmere, it will not shrink

It makes perfect sense to think that if (1) is true, then (23) is false. However, there are cases in which two conditionals of this form may reasonably be regarded as true:

(24) If it is snowing, then either it is snowing or it is not snowing

(25) If it is not snowing, then either it is snowing or it is not snowing

More generally, we think that Abelard's thesis is plausible only insofar as it entails a weaker principle, *Restricted Abelard's Thesis*: if C is not necessary, then 'If A, then C' entails 'Not: if not-A, then C'.

The evidential interpretation behaves exactly as we would expect. First, (18) is false because it is trivially not the case that the worlds in which it is snowing are more distant from the actual world than those in which it is snowing and not snowing. Instead, (19) is vacuously true because its antecedent is impossible. Second, if (3) is true, then (20) is false: if it is cold in the closest worlds in which it is snowing, it cannot be warm in such worlds. Instead, (21) and (22) are vacuously true because its antecedent is impossible. Third, if (1) is true, then (23) is false, because it is impossible that both (1) and (23) pass the Chrysippus Test: if the closest worlds in which it will shrink are worlds in which it is not pure cashmere, it cannot be the case that the closest worlds in which it will shrink are worlds in which it is pure cashmere. Instead, (24) and (25) are vacuously true because their consequent is necessary.

Now we will prove that Restricted Aristotle's Thesis, Restricted Boethius' Thesis, and Restricted Abelard's Thesis hold for $\triangleright$. In order to do so, we will prove a stronger principle, *Restricted Selectivity*[23]:

[20] In this respect, a possible divergence from Chrysippus' original position must be acknowledged. In fact, it is a controverial matter whether Chrysippus regarded conditionals with impossible antecedents as true.

[21] This is essentially the point made in Iacona [16]. Restricted versions of connexive principles are also considered in Lenzen [22], Kapsner [19], and Unterhuber [35].

[22] See Estrada Gonzáles and Ramírez-Cámara [12], pp. 346-348.

[23] See Huber [14], p. 531.

FACT 15 If $\beta \vDash_{PL} \sim\gamma$, then $\Diamond\alpha, \alpha \triangleright \beta \vDash \sim(\alpha \triangleright \gamma)$ (*Restricted Selectivity* ✓)

Proof. Assume that $\beta \vDash_{PL} \sim\gamma$, $[\Diamond\alpha]_w = 1$, and $[\alpha \triangleright \beta]_w = 1$. Since $[\Diamond\alpha]_w = 1$, α is true in some worlds. Since $[\alpha \triangleright \beta]_w = 1$, for every w' such that $[\alpha]_{w'} = 1$ and there are no w'' and S such that $w'' \in S$, $w' \notin S$, and $[\alpha]_{w''} = 1$, then $[\beta]_{w'} = 1$. Since $\beta \vDash_{PL} \sim\gamma$, it follows that $[\sim\gamma]_{w'} = 1$. So $[\gamma]_{w'} = 0$. Therefore, $[\alpha \triangleright \gamma]_w = 0$, and consequently $[\sim(\alpha \triangleright \gamma)]_w = 1$. □

FACT 16 $\Diamond\alpha, \alpha \triangleright \beta \vDash \sim(\alpha \triangleright \sim\beta)$ (*Restricted Boethius' Thesis* ✓)

Proof. This follows directly from Restricted Selectivity. Assume that $[\Diamond\alpha]_w = 1$ and $[\alpha \triangleright \beta]_w = 1$. Since $\beta \vDash_{PL} \sim\sim\beta$, by fact 15 $[\sim(\alpha \triangleright \sim\beta)]_w = 1$. □

FACT 17 $\Diamond\alpha \vDash \sim(\alpha \triangleright \sim\alpha)$ (*Restricted Aristotle's Thesis* ✓)

Proof. This follows from Restricted Boethius' Thesis, given Superclassicality. Assume that $[\Diamond\alpha]_w = 1$. Since $[\alpha \triangleright \alpha]_w = 1$ by fact 2, it follows by fact 16 that $[\sim(\alpha \triangleright \sim\alpha)]_w = 1$. □

FACT 18 $\Diamond\sim\beta, \alpha \triangleright \beta \vDash \sim(\sim\alpha \triangleright \beta)$ (*Restricted Abelard's Thesis* ✓)

Proof. This follows from Resticted Selectivity, given Contraposition. Assume that $[\Diamond\sim\beta]_w = 1$ and $[\alpha \triangleright \beta]_w = 1$. By fact 11 the latter entails that $[\sim\beta \triangleright \sim\alpha]_w = 1$. So, by fact 15 $[\sim(\sim\beta \triangleright \alpha)]_w = 1$, given that $\sim\alpha \vDash_{PL} \sim\alpha$. This means that $[\sim\beta \triangleright \alpha]_w = 0$. But if so, $[\sim\alpha \triangleright \beta]_w = 0$ as well, for $\sim\beta \triangleright \alpha$ and $\sim\alpha \triangleright \beta$ have the same truth conditions (switch (a) and (b) as in the proof of fact 11). Therefore, $[\sim(\sim\alpha \triangleright \beta)]_w = 1$. □

9 COMPARISONS

The facts set out in sections 4-8 delineate the distinctive logical profile of the evidential interpretation. In terms of strength, the evidential interpretation lies between the strict interpretation and the material interpretation, because $\Box(\alpha \supset \beta)$ entails $\alpha \triangleright \beta$, and $\alpha \triangleright \beta$ entails $\alpha \supset \beta$:

FACT 19 $\Box(\alpha \supset \beta) \vDash \alpha \triangleright \beta$ (*Strict to Evidential* ✓)

Proof. Assume that $[\Box(\alpha \supset \beta)]_w = 1$. Then, for every w', if $[\alpha]_{w'} = 1$, then $[\beta]_{w'} = 1$, and for every w', if $[\beta]_{w'} = 0$, then $[\alpha]_{w'} = 0$. Therefore, $[\alpha \triangleright \beta]_w = 1$. □

FACT 20 $\alpha \triangleright \beta \vDash \alpha \supset \beta$ (*Evidential to Material* ✓)

Proof. Assume that $[\alpha \triangleright \beta]_w = 1$. Then, for some S, there is no $w' \in S$ such that $[\alpha]_{w'} = 1$ and $[\beta]_{w'} = 0$. Since $w \in S$, $[\alpha \supset \beta]_w = 1$. □

As we saw in section 5, the evidential interpretation differs from the strict interpretation in that it invalidates Monotonicity and Transitivity[24].

A second and more interesting point is that the evidential interpretation significantly differs from the accounts of conditionals advocated by Adams, Stalnaker, Lewis, and others. We will use the term 'suppositional interpretation' for any such account, regardless of the specific traits characterizing each of them. Broadly speaking, in the suppositional interpretation, a conditional means that its consequent is credible enough given its antecedent. That is,

[24] Influential analyses of non-material monotonic conditionals have been sometimes integrated in so-called dynamic semantics. A thorough comparison with these approaches would reveal even more divergences from ours. For instance, in Veltman's theory, presented in Veltman [37], True Consequent is valid while Modus Tollens is not.

on the supposition that its antecedent holds, there are good chances that its consequent holds. Just as the evidential interpretation, the suppositional interpretation lies between the strict interpretation and the material interpretation. But it is weaker than the evidential interpretation: if a conditional is true in the evidential sense, then it is true in the suppositional sense, but not the other way round. Truth in the suppositional sense is defined solely in terms of the Ramsey Test, so it holds no matter whether the Reverse Ramsey Test is satisfied.

The relation between the evidential interpretation and the suppositional interpretation can be expressed more precisely by adopting the symbol $\Rightarrow$ for the latter interpretation, that is, by assuming that, for any two formulas α, β and any world w, $[\alpha \Rightarrow \beta]_w = 1$ if and only if condition (a) of clause 6 of definition 3 is satisfied. On this assumption, we have the following equivalence:

FACT 21 $[\alpha \rhd \beta]_w = 1$ iff $[(\alpha \Rightarrow \beta) \wedge (\sim \beta \Rightarrow \sim \alpha)]_w = 1$

Proof. Assume that $[\alpha \rhd \beta]_w = 1$. Since (a) holds for α and β, $[\alpha \Rightarrow \beta]_w = 1$. Since (b) holds for α and β, (a) holds for $\sim \beta$ and $\sim \alpha$, hence $[\sim \beta \Rightarrow \sim \alpha]_w = 1$. Therefore, $[(\alpha \Rightarrow \beta) \wedge (\sim \beta \Rightarrow \sim \alpha)]_w = 1$. The proof of the right-to-left direction is similar. □

Fact 21 shows that $\rhd$ is definable in terms of $\Rightarrow$. The opposite is also true, although less trivial, that is, $\Rightarrow$ is definable in terms of $\rhd$.[25]

FACT 22 $[\alpha \Rightarrow \beta]_w = 1$ iff $[(\alpha \wedge \beta) \vee (\alpha \rhd (\alpha \wedge \beta))]_w = 1$

Proof. Assume that $[(\alpha \wedge \beta) \vee (\alpha \rhd (\alpha \wedge \beta))]_w = 1$. If $[\alpha \wedge \beta]_w = 1$, then w verifies both the antecedent and the consequent of (a), while every other world falsifies its antecedent. Therefore, $[\alpha \Rightarrow \beta]_w = 1$. If $[\alpha \rhd (\alpha \wedge \beta)]_w = 1$, then by definition $[\alpha \Rightarrow (\alpha \wedge \beta)]_w = 1$, which means that $\alpha \wedge \beta$ is true in the closest worlds in which α is true. It follows that β is true in the closest worlds in which α is true, that is, $[\alpha \Rightarrow \beta]_w = 1$.

Now assume that $[\alpha \Rightarrow \beta]_w = 1$. Then either $[\alpha]_w = 1$ or $[\alpha]_w = 0$. If $[\alpha]_w = 1$, then $[\beta]_w = 1$, for w verifies the antecedent of (a), so it must verify its consequent. It follows that $[\alpha \wedge \beta]_w = 1$, and consequently that $[(\alpha \wedge \beta) \vee (\alpha \rhd (\alpha \wedge \beta))]_w = 1$. If $[\alpha]_w = 0$, then $[\sim \alpha]_w = 1$, so $[\sim \alpha \vee \sim \beta]_w = 1$. This entails that $[(\sim \alpha \vee \sim \beta) \Rightarrow \sim \alpha]_w = 1$, for w verifies both the antecedent and the consequent of (a), while every other world falsifies its antecedent. It follows that $[\sim (\alpha \wedge \beta) \Rightarrow \sim \alpha]_w = 1$, given that $\sim \alpha \vee \sim \beta$ is logically equivalent to $\sim (\alpha \wedge \beta)$. Moreover, the assumption that $[\alpha \Rightarrow \beta]_w = 1$ entails that $[\alpha \Rightarrow (\alpha \wedge \beta)]_w = 1$: if β is true in the closest worlds in which α is true, then so is $\alpha \wedge \beta$. By fact 21 we get that $[\alpha \rhd (\alpha \wedge \beta)]_w = 1$, hence that $[(\alpha \wedge \beta) \vee (\alpha \rhd (\alpha \wedge \beta))]_w = 1$. □

The difference between the evidential interpretation and the suppositional interpretation emerges clearly if we consider the principles discussed in sections 6-8. As explained in section 6, Contraposition holds for $\rhd$. The examples that are usually taken to show that Contraposition fails, such as the inference about the US and North Vietnam, typically include concessive conditionals as premises, so they do not work if conditionals are understood evidentially. By contrast, Contraposition does not hold for $\Rightarrow$. The same examples work if conditionals are understood suppositionally, for their

[25] We owe this equivalence result to Eric Raidl, who provides an extensive analysis of the mutual definability of different kinds of conditional in Raidl [28].

premises turn out to be true. By and large, concessive conditionals may be described as conditionals that are true just in case they are true in the suppositional sense but not in the evidential sense[26].

Right Weakening produces opposite results: while the evidential interpretation invalidates it, the suppositional interpretation validates it. The examples that can rightfully be taken to show that Right Weakening fails, such as the inference about the letter, work only if conditionals are understood evidentially. In the evidential understanding, the conclusions of such inferences are false. Instead, the same examples do not work if conditionals are understood suppositionally, for their conclusions turn out to be true.

As explained in section 7, Conditional Excluded Middle and Conjunctive Sufficiency do not hold for $\triangleright$. The schema 'Either if A then C, or if A then not-C' has apparently false instances, and the same goes for the inference from 'A and C' to 'If A, then C'. Instead, both principles hold for $\Rightarrow$. Although the core idea of the suppositional interpretation — Ramsey's original idea — by itself does not entail Conditional Excluded Middle, and can be developed in the way suggested by Lewis, a natural reading of that idea accords perfectly well with Conditional Excluded Middle: to say that C does not hold on the supposition that A holds is to say that not-C holds on that supposition, so if 'If A, then C' is false, 'If A, then not-C' must be true. This is the reading adopted by Adams and Stalnaker. Conjunctive Sufficiency is valid as well: if A and C actually hold, then it is obviously the case that there are good chances that C holds on the supposition that A holds.

Finally, the evidential interpretation validates Restricted Aristotle's Thesis, Restricted Boethius' Thesis, and Restricted Abelard's Thesis. The suppositional interpretation agrees with it on the first two theses, but it crucially differs with respect to the third. Consider (11). Since (11) is acceptable in the suppositional sense, if we replace its antecedent with 'You don't drink a beer' we obtain a conditional which is also acceptable in the suppositional sense: there are good chances that its consequent holds on the supposition that its antecedent holds.

Not only the account outlined in this paper differs from the suppositional theories of conditionals in the way explained, but it also differs in important respects from some recent attempts to provide a non-monotonic theory of conditionals based on the notion of support. One is Rott's treatment of "difference-making" conditionals, which adopts a strengthened version of the Ramsey Test in the context of the classical theory of belief revision. Rott's account, like ours, invalidates Monotonicity and Right Weakening. Unlike ours, however, it does not retain Contraposition, even though Contraposition is consistent with the rejection of Monotonicity, provided that Right Weakening fails. This result has no obvious intuitive rationale. Once the concessive reading of 'if then' is ruled out, and the alleged counterexamples such as that considered in section 6 loose their grip, it is no longer clear what reason one may have for rejecting Contraposition[27].

The other example is Douven's epistemic analysis of conditionals, which relies on a notion of evidential support defined in terms of degrees of belief. Douven's account yields a considerably weak logic, in which several widely accepted principles, including Modus Ponens, turn out to be invalid. Therefore, it significantly differs from our account, which preserves Modus Ponens and other basic principles[28].

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[26] This is in line with the analysis of "even if" suggested in Douven [9], p. 119.
[27] Rott [31]. The account provided in Berto and Özgün [4] also entails failure of Contraposition.
[28] Douven [9], ch. 5.

In this paper we have pursued a truth-conditional approach to conditionals, that is, we have defined the evidential interpretation by specifying the conditions under which a conditional is true on that interpretation. More specifically, truth has been defined relative to worlds, as in any standard modal language. Accordingly, the notion of logical consequence adopted to illustrate the logical features of the evidential account is also standard.

This is not the only possibility, however. As is well known, an alternative route is available, whereby truth conditions are deliberately avoided, and logical principles are derived from the notion of assertibility. This is the route followed by Adams in his influential work on conditionals. According to Adams, 'If A, then C' is assertible to the extent that the probability of C conditional on A is high. In this analysis, the degree of assertibility of 'If A, then C' relative to a probability distribution P is thus $P(\mathrm{C}|\mathrm{A})$, and the corresponding degree of "uncertainty" is 1 minus the degree of assertibility. Apart from specific limitations in the expressive power of the underlying language, the logic of the suppositional interpretation is then preserved in the assertibility approach provided that a valid inference is defined as having the sum of the uncertainties of the premises as an upper bound for the uncertainty of the conclusion under any probability assignment[29].

Adams's account of the assertibility of a conditional offers a plausible interpretation of the Ramsey Test, but leaves no room for the Chrysippus test. In fact, a very high assertibility of 'If A, then C' is compatible with a comparably high probability of A given not-C, as illustrated by the case of (11). So it seems that a conditional can be highly assertible even if the negation of its consequent is not at odds with its antecedent in a most natural sense. All this is standard and well received in the literature, especially among authors who — unlike us — are skeptical either about possible worlds or about the very idea of truth as applied to conditionals[30]. More generally, the logic of the suppositional interpretation largely survives across the divide between accounts based on truth versus assertibility conditions, and this is quite rightly taken as a sign of the strength of that interpretation. It is then an interesting question whether something similar can be said with respect to the evidential intepretation.

Interestingly, this is indeed the case. The key point is to give an analogue representation of the assertibility of a conditional. For the limiting cases where $P(\mathrm{C}) = 1$ or $P(\mathrm{A}) = 0$, the default option is to follow Adams and posit the assertibility of 'If A, then C' to be (vacuously) maximal (i.e., 1). Besides, earlier work in the probabilistic analysis of evidential support supplies an effective solution for the more interesting cases where $P(\mathrm{C}) < 1$ and $P(\mathrm{A}) > 0$, namely, equating the degree of assertibility of 'If A, then C' given a certain probability distribution P with

$$\frac{P(C|A) - P(C)}{1 - P(C)}$$

if $P(\mathrm{C}|\mathrm{A}) \geq P(\mathrm{C}) > 0$, and 0 otherwise. Intuitively, this is a measure of the proportion of the initial uncertainty of C (that is, $1-P(\mathrm{C})$) that is cancelled by the upward jump (if any) of the probability of C due to A (that is, $P(\mathrm{C}|\mathrm{A}) - P(\mathrm{C})$)[31]. So 'If A, then C' turns out to be at least minimally

[29] Adams [1], Adams [2].
[30] For example Kahle [18], or Edgington [10].
[31] See Crupi and Tentori [6], and Crupi and Tentori [7].

assertible only if the supposition of A increases the probability of C. Crucially, this account of the assertibility of 'If A, then C' does combine the ideas of the Ramsey and the Reverse Ramsey test. Here is why. Suppose that the assertibility of 'If A, then C' relative to P is higher than a given threshold value, say, higher than 0.8. Then, on very mild background assumptions, one can prove both that the probability of C given A is also higher than 0.8 and that the probability of A given not-C is lower than 1-0.8=0.2. So a high degree of assertibility of 'If A, then C' as just defined implies both that C is highly probable given A and that not-C makes A improbable, thus being at odds with it.

Once the assertibility of an evidential conditional is characterized in probabilistic terms, one can apply Adams's idea of validity and check what logical principles are thus validated. In an extended investigation along this lines, we have shown that the resulting logic implies exactly the same pattern of validities and invalidities derived from our truth-conditional discussion above[32]. So the evidential interpretation is similar to the suppositional interpretation in this important respect: its specific logical behaviour is robust across alternative frameworks and can be motivated even without the modal apparatus employed here.

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REFERENCES

[1] E. W. Adams. The Logic of Conditionals. *Inquiry*, 8:166–197, 1965.

[2] E. W. Adams. *A Primer of Probability Logic*. CSLI Publications, 1998.

[3] J. Bennett. *A Philosophical Guide to Conditionals*. Clarendon Press, Oxford, 2003.

[4] F. Berto and A. Özgün. Indicative Conditionals: Probabilities and Relevance. *forthcoming*, 2020.

[5] V. Crupi and A. Iacona. Three ways of being non-material. *manuscript*, 2019.

[6] V. Crupi and K. Tentori. Confirmation as partial entailment: A representation theorem in inductive logic. *Journal of Applied Logic*, 11:364–372, 2013.

[7] V. Crupi and K. Tentori. Measuring information and confirmation. *Studies in the History and Philosophy of Science*, 47:81–90, 2014.

[8] I. Douven. The Evidential Support Theory of Conditionals. *Synthese*, 164:19–44, 2008.

[9] I. Douven. *The Epistemology of Conditionals*. Cambridge University Press, 2016.

[32] Crupi and Iacona [5].

[10] D. Edgington. Do Conditionals Have Truth Conditions? *Critica*, 18:3–30, 1986.

[11] D. Edgington. Indicative conditionals. In *Stanford Encyclopedia of Philosophy*. Stanford University, 2014.

[12] L. Estrada-Gonzáles and E. Ramírez-Cámara. A Comparison of Connexive Logics. *IFCOLog Journal of Logics and their Applications*, 3:341–355, 2016.

[13] A. S. Gillies. On Truth-Conditions for If (but Not Quite Only If). *Philosophical Review*, 118:325–349, 2009.

[14] F. Huber. The logic of theory assessment. *Journal of Philosophical Logic*, 36:511–538, 2007.

[15] A. Iacona. Indicative conditionals as strict conditionals. *Argumenta*, 4:177–192, 2018.

[16] A. Iacona. Strictness and Connexivity. *Inquiry*, forthcoming.

[17] S. Wenmackers K. Krzyżanowska and I. Douven. Inferential Conditionals and Evidentiality. *Journal of Logic, Language and Information*, 22:315–334, 2013.

[18] R. Kahle. Against possible worlds. In *Dialogues, Logics, and Other Strange Things: Essays in Honour of Shahid Rahman*, pages 235–253. College Publications, 2008.

[19] A. Kapsner. Humble connexivity. *Logic and Logical Philosophy*, 28:513–536, 2019.

[20] A. Kratzer. *Modals and Conditionals*. Oxford University Press, Oxford, 2012.

[21] W. Lenzen. A Fresh Look at the Stoic's Debate on the Nature of Conditionals. *manuscript*, 2019.

[22] W. Lenzen. Leibniz's laws of consistency and the philosophical foundations of connexive logic. *Logical and Logical Philosophy*, forthcoming.

[23] D. Lewis. *Counterfactuals*. Blackwell, 1973.

[24] W. G. Lycan. *Real conditionals*. Oxford University Press, Oxford, 2001.

[25] S. McCall. A history of connexivity. In D. M. Gabbay et al, editor, *Handbook of the History of Logic*, volume 11, pages 415–449. Elsevier, 2012.

[26] J. S. Mill. *A System of Logic, Ratiocinative and Inductive (1843)*. Harper and Brothers (Eighth Edition), 1882.

[27] E. Nelson. Intensional relations. *Mind*, 39:440–453, 1930.

[28] E. Raidl. Definable Conditionals. *Topoi*, forthcoming.

[29] F. Ramsey. General Propositions and Causality (1929). In D. H. Mellor, editor, *Philosophical Papers*, pages 145–163. Cambridge University Press, 1990.

[30] H. Rott. Ifs, though, and because. *Erkenntnis*, 25:345–370, 1986.

[31] H. Rott. Difference-making conditionals and the relevant ramsey test. *manuscript*, 2019.

[32] D. Sanford. *If P, then Q*. Routledge, 2003.

[33] W. Spohn. A ranking-theoretical approach to conditionals. *Cognitive Science*, 37:1074–1106, 2013.

[34] R. Stalnaker. A theory of conditionals. In F. Jackson, editor, *Conditionals*, pages 28–45. Oxford University Press, 1991.

[35] M. Unterhuber. Beyond System P. Hilbert-style convergence results for conditional logics with a connexive twist. *IFCOLog Journal of Logics and their Applications*, 3:377–412, 2016.

[36] R. van Rooij and K. Schulz. Conditionals, Causality and Conditional Probability. *Journal of Logic, Language and Information*, 28:55–71, 2019.

[37] F. Veltman. Data Semantics and the Pragmatics of Indicative Conditionals. In J. S. Reilly C. A. Ferguson E. C. Traugott, A. Ter Meulen, editor, *On Conditionals*, pages 147–167. Cambridge University Press, 1986.

[38] H. Wansing. Connexive logic. In E. Zalta, editor, *Stanford Encyclopedia of Philosophy*. Connexive Logic http://plato.stanford.edu/archives/spr2016/entries/logic-connexive/, 2016.