Semi-formally stated open problems on computable sets $\mathcal{X} = \{n \in \mathbb{N} : \varphi(n)\}$, where $\varphi(n)$ has the same intuitive meaning for every $n \in \mathbb{N}$ and the finiteness (infiniteness) of $\mathcal{X}$ remains conjectured

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Abstract

Let $b = ((24!)!)!$, and let $\mathcal{P}_{n^2+1}$ denote the set of all primes of the form $n^2 + 1$. Let $\mathcal{M}$ denote the set of all positive multiples of elements of the set $\mathcal{P}_{n^2+1} \cap (b, \infty)$. The set $\mathcal{X} = \{0, \ldots, b\} \cup \mathcal{M}$ satisfies the following conditions: `(1)` card($\mathcal{X}$) is greater than a huge positive integer and it is conjectured that $\mathcal{X}$ is infinite, `(2)` we do not know any algorithm deciding the finiteness of $\mathcal{X}$, `(3)` a known and short algorithm for every $n \in \mathbb{N}$ decides whether or not $n \in \mathcal{X}$, `(4)` a known and short algorithm returns an integer n such that $\mathcal{X}$ is infinite if and only if $\mathcal{X}$ contains an element greater than n. The following problem is open: *define a set $\mathcal{X} \subseteq \mathbb{N}$ such that $\mathcal{X}$ satisfies conditions* `(1)-(4)` *and a known and simple formula $\varphi(x)$ satisfies $\mathcal{X} = \{n \in \mathbb{N} : \varphi(n)\}$, where $\varphi(n)$ has the same intuitive meaning for every $n \in \mathbb{N}$* `(5)`. The statements $\varphi(n)$ in condition `(5)` have always the same intuitive meaning, if the predicate $\varphi(x)$ expresses a *natural property*, the term propounded by the philosopher David Lewis (1941-2001). Let $f(3) = 4$, and let $f(n+1) = f(n)!$ for every integer $n \geqslant 3$. For an integer $n \geqslant 3$, let Ψ_n denote the following statement: *if a system of equations $\mathcal{S} \subseteq \left\{x_i! = x_{i+1} : 1 \leqslant i \leqslant n-1\right\} \cup \left\{x_i \cdot x_j = x_{j+1} : 1 \leqslant i \leqslant j \leqslant n-1\right\}$ has only finitely many solutions in positive integers $x_1, \ldots, x_n$, then each such solution $(x_1, \ldots, x_n)$ satisfies $x_1, \ldots, x_n \leqslant f(n)$.* We prove that for every statement Ψ_n the bound $f(n)$ cannot be decreased. The author's guess is that the statements $\Psi_3, \ldots, \Psi_9$ are true. We prove that the statement Ψ_9 implies that the set $\mathcal{X}$ of all non-negative integers n whose number of digits belongs to $\mathcal{P}_{n^2+1}$ satisfies conditions `(1)-(5)`.

Key words and phrases: computable set $\mathcal{X} \subseteq \mathbb{N}$ whose finiteness remains conjectured, computable set $\mathcal{X} \subseteq \mathbb{N}$ whose infiniteness remains conjectured, David Lewis's notion of a natural property, intuitive meaning of a mathematical formula, Zenkin's super-induction.

1 Introduction and basic definitions and lemmas

In this article, we discuss semi-formally stated open problems on computable sets $\mathcal{X} = \{n \in \mathbb{N} : \varphi(n)\}$, where $\varphi(n)$ has the same intuitive meaning for every $n \in \mathbb{N}$ and the finiteness (infiniteness) of $\mathcal{X}$ remains conjectured.

Definition 1. *Let $b = ((24!)!)!$.*

Lemma 1. $b \approx 10^{10^{10^{25.16114896940657}}}$.

Proof. We ask Wolfram Alpha at `http://wolframalpha.com`. □

Definition 2. *We say that an integer $m \geqslant -1$ is a threshold number of a set $\mathcal{X} \subseteq \mathbb{N}$, if $\mathcal{X}$ is infinite if and only if $\mathcal{X}$ contains an element greater than m, cf. [10] and [11].*

If a set $\mathcal{X} \subseteq \mathbb{N}$ is empty or infinite, then any integer $m \geqslant -1$ is a threshold number of $\mathcal{X}$. If a set $\mathcal{X} \subseteq \mathbb{N}$ is non-empty and finite, then the all threshold numbers of $\mathcal{X}$ form the set $\{\max(\mathcal{X}), \max(\mathcal{X}) + 1, \max(\mathcal{X}) + 2, \ldots\}$.

Definition 3. *We say that a non-negative integer m is a weak threshold number of a set $\mathcal{X} \subseteq \mathbb{N}$, if $\mathcal{X}$ is infinite if and only if* $\mathrm{card}(\mathcal{X}) > m$.

Theorem 1. *For every $\mathcal{X} \subseteq \mathbb{N}$, if an integer $m \geqslant -1$ is a threshold number of $\mathcal{X}$, then $m+1$ is a weak threshold number of $\mathcal{X}$.*

Proof. For every $\mathcal{X} \subseteq \mathbb{N}$, if $m \in [-1,\infty) \cap \mathbb{Z}$ and $\mathrm{card}(\mathcal{X}) > m+1$, then $\mathcal{X} \cap [m+1,\infty) \neq \emptyset$. □

We do not know any weak threshold number of the set of all primes of the form n^2+1. The same is true for the sets

$$\left\{n \in \mathbb{N} : 2^{2^n} + 1 \text{ is composite}\right\}$$

and

$$\{n \in \mathbb{N} : n! + 1 \text{ is a square}\}$$

Lemma 2. *For every positive integers x and y, $x! \cdot y = y!$ if and only if*

$$(x+1=y) \vee (x=y=1)$$

Lemma 3. *(Wilson's theorem, [2, p. 89]). For every integer $x \geqslant 2$, x is prime if and only if x divides* $(x-1)!+1$.

2 Open Problems 1 and 2

The following two open problems cannot be stated formally as they refer to the intuitive meaning of a mathematical formula and the current mathematical knowledge.

Open Problem 1. *Define a set $\mathcal{X} \subseteq \mathbb{N}$ that satisfies the following conditions:*
(1) $\mathrm{card}(\mathcal{X})$ *is greater than a huge positive integer and it is conjectured that $\mathcal{X}$ is infinite,*
(2) *we do not know any algorithm deciding the finiteness of $\mathcal{X}$,*
(3) *a known and short algorithm for every $n \in \mathbb{N}$ decides whether or not $n \in \mathcal{X}$,*
(4•) *a known and short algorithm returns an integer n such that $\mathcal{X}$ is infinite if and only if* $\mathrm{card}(\mathcal{X}) > n$,
(5) *a known and simple formula $\varphi(x)$ satisfies $\mathcal{X} = \{n \in \mathbb{N} : \varphi(n)\}$, where $\varphi(n)$ has the same intuitive meaning for every $n \in \mathbb{N}$.*

Open Problem 2. *Define a set $\mathcal{X} \subseteq \mathbb{N}$ such that $\mathcal{X}$ satisfies conditions* (1)-(3), (5), *and a known and short algorithm returns an integer n such that $\mathcal{X}$ is infinite if and only if $\mathcal{X}$ contains an element greater than n* (4).

The statements $\varphi(n)$ in condition (5) have always the same intuitive meaning, if the predicate $\varphi(x)$ expresses David Lewis's *natural property*. For the meaning of this term, the reader is referred to [1].

Theorem 2. *Open Problem 2 claims more than Open Problem 1.*

Proof. By Theorem 1, condition (4) implies condition (4•). □

3 A partial solution to Open Problem 2

Edmund Landau's conjecture states that the set $\mathcal{P}_{n^2+1}$ of all primes of the form n^2+1 is infinite, see [5, pp. 37–38] and [7]. Let $\mathcal{M}$ denote the set of all positive multiples of elements of the set $\mathcal{P}_{n^2+1} \cap (b,\infty)$.

Theorem 3. *The set $\mathcal{X} = \{0,\ldots,b\} \cup \mathcal{M}$ satisfies conditions* (1)-(4).

Proof. Condition (1) holds as $\mathrm{card}(\mathcal{X}) > b$ and the set $\mathcal{P}_{n^2+1}$ is conjecturally infinite. By Lemma 1, due to known physics we are not able to confirm by a direct computation that some element of $\mathcal{P}_{n^2+1}$ is greater than b. Thus condition (2) holds. Condition (3) holds trivially. The set $\mathcal{M}$ is empty or $\mathcal{M}$ is an infinite subset of (b,∞). Therefore, b is a threshold number of $\mathcal{X}$. Thus condition (4) holds. □

4 The statements Ψ_n $(n = 3, 4, 5, \ldots)$, which seem to be true for every $n \in \{3, \ldots, 9\}$

Let $f(3) = 4$, and let $f(n+1) = f(n)!$ for every integer $n \geqslant 3$. For an integer $n \geqslant 3$, let $\mathcal{U}_n$ denote the following system of equations:

$$\begin{cases} \forall i \in \{1, \ldots, n-1\} \setminus \{2\} \; x_i! & = & x_{i+1} \\ x_1 \cdot x_2 & = & x_3 \\ x_2 \cdot x_2 & = & x_3 \end{cases}$$

The diagram in Figure 1 illustrates the construction of the system $\mathcal{U}_n$.

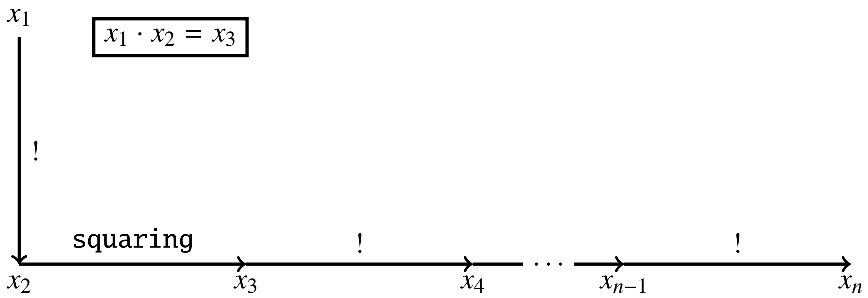

Fig. 1 Construction of the system $\mathcal{U}_n$

Lemma 4. *For every integer $n \geqslant 3$, the system $\mathcal{U}_n$ has exactly two solutions in positive integers, namely $(1, \ldots, 1)$ and $\big(2, 2, f(3), \ldots, f(n)\big)$.*

Let

$$B_n = \Big\{x_i! = x_{i+1} : \; 1 \leqslant i \leqslant n-1\Big\} \cup \Big\{x_i \cdot x_j = x_{j+1} : \; 1 \leqslant i \leqslant j \leqslant n-1\Big\}$$

For an integer $n \geqslant 3$, let Ψ_n denote the following statement: *if a system of equations $\mathcal{S} \subseteq B_n$ has only finitely many solutions in positive integers $x_1, \ldots, x_n$, then each such solution $(x_1, \ldots, x_n)$ satisfies $x_1, \ldots, x_n \leqslant f(n)$.* The statement Ψ_n says that for subsystems of B_n with a finite number of solutions, the largest known solution is indeed the largest possible. The author's guess is that the statements $\Psi_3, \ldots, \Psi_9$ are true.

Theorem 4. *Every statement Ψ_n is true with an unknown integer bound that depends on n.*

Proof. For every positive integer n, the system B_n has a finite number of subsystems. □

Theorem 5. *For every statement Ψ_n, the bound $f(n)$ cannot be decreased.*

Proof. It follows from Lemma 4 because $\mathcal{U}_n \subseteq B_n$. □

5 The statement Ψ_9 solves Open Problem 2

Let $\mathcal{A}$ denote the following system of equations:

$$\begin{cases} x_2! & = & x_3 \\ x_3! & = & x_4 \\ x_5! & = & x_6 \\ x_8! & = & x_9 \\ x_1 \cdot x_1 & = & x_2 \\ x_3 \cdot x_5 & = & x_6 \\ x_4 \cdot x_8 & = & x_9 \\ x_5 \cdot x_7 & = & x_8 \end{cases}$$

Lemma 2 and the diagram in Figure 2 explain the construction of the system $\mathcal{A}$.

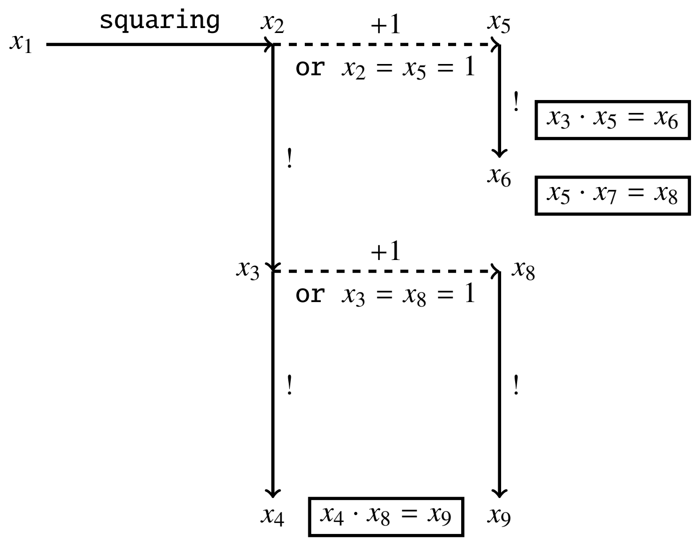

Fig. 2 Construction of the system $\mathcal{A}$

Lemma 5. *For every integer $x_1 \geqslant 2$, the system $\mathcal{A}$ is solvable in positive integers $x_2, \dots, x_9$ if and only if $x_1^2 + 1$ is prime. In this case, the integers $x_2, \dots, x_9$ are uniquely determined by the following equalities:*

$$\begin{aligned}
x_2 &= x_1^2 \\
x_3 &= (x_1^2)! \\
x_4 &= ((x_1^2)!)! \\
x_5 &= x_1^2 + 1 \\
x_6 &= (x_1^2 + 1)! \\
x_7 &= \frac{(x_1^2)! + 1}{x_1^2 + 1} \\
x_8 &= (x_1^2)! + 1 \\
x_9 &= ((x_1^2)! + 1)!
\end{aligned}$$

Proof. By Lemma 2, for every integer $x_1 \geqslant 2$, the system $\mathcal{A}$ is solvable in positive integers $x_2, \dots, x_9$ if and only if $x_1^2 + 1$ divides $(x_1^2)! + 1$. Hence, the claim of Lemma 5 follows from Lemma 3. □

Lemma 6. *There are only finitely many tuples $(x_1, \dots, x_9) \in (\mathbb{N} \setminus \{0\})^9$ which solve the system $\mathcal{A}$ and satisfy $x_1 = 1$.*

Proof. If a tuple $(x_1, \dots, x_9) \in (\mathbb{N} \setminus \{0\})^9$ solves the system $\mathcal{A}$ and $x_1 = 1$, then $x_1, \dots, x_9 \leqslant 2$. Indeed, $x_1 = 1$ implies that $x_2 = x_1^2 = 1$. Hence, for example, $x_3 = x_2! = 1$. Therefore, $x_8 = x_3 + 1 = 2$ or $x_8 = 1$. Consequently, $x_9 = x_8! \leqslant 2$. □

Theorem 6. *The statement Ψ_9 proves the following implication: if there exists an integer $x_1 \geqslant 2$ such that $x_1^2 + 1$ is prime and greater than $f(7)$, then the set $\mathcal{P}_{n^2+1}$ is infinite.*

Proof. Suppose that the antecedent holds. By Lemma 5, there exists a unique tuple $(x_2, \dots, x_9) \in (\mathbb{N} \setminus \{0\})^8$ such that the tuple $(x_1, x_2, \dots, x_9)$ solves the system $\mathcal{A}$. Since $x_1^2 + 1 > f(7)$, we obtain that $x_1^2 \geqslant f(7)$. Hence, $(x_1^2)! \geqslant f(7)! = f(8)$. Consequently,

$$x_9 = ((x_1^2)! + 1)! \geqslant (f(8) + 1)! > f(8)! = f(9)$$

Since $\mathcal{A} \subseteq B_9$, the statement Ψ_9 and the inequality $x_9 > f(9)$ imply that the system $\mathcal{A}$ has infinitely many solutions $(x_1, \dots, x_9) \in (\mathbb{N} \setminus \{0\})^9$. According to Lemmas 5 and 6 the set $\mathcal{P}_{n^2+1}$ is infinite. □

Let $\mathcal{F}$ denote the set of all non-negative integers k whose number of digits belongs to $\mathcal{P}_{n^2+1}$.

Lemma 7. card($\mathcal{F}$) *is greater than a huge positive integer.*

Proof. The number $(13!)^2 + 1 = 38775788043632640001$ is prime, see [8]. □

Theorem 7. *The statement* Ψ_9 *implies that* $\mathcal{X} = \mathcal{F}$ *satisfies conditions* (1)-(5).

Proof. Suppose that the antecedent holds. Since the set $\mathcal{P}_{n^2+1}$ is conjecturally infinite, Lemma 7 implies condition (1). Conditions (3) and (5) hold trivially. By Theorem 6, $\underbrace{9\ldots9}_{b \text{ digits}}$ is a threshold number of $\mathcal{X}$. Thus condition (4) holds. By Lemma 1, due to known physics we are not able to confirm by a direct computation that some element of $\mathcal{P}_{n^2+1}$ is greater than $f(7) = ((24!)!)! = b$. Thus condition (2) holds. □

6 Open Problems 3 and 4

Definition 4. *Let* (1⋄) *denote the following condition:* card($\mathcal{X}$) *is greater than a huge positive integer and it is conjectured that* $\mathcal{X} = \mathbb{N}$.

Definition 5. *Let* (2⋄) *denote the following condition: we do not know any algorithm deciding the equality* $\mathcal{X} = \mathbb{N}$.

The following two open problems cannot be stated formally as they refer to the intuitive meaning of a mathematical formula and the current mathematical knowledge.

Open Problem 3. *Define a set* $\mathcal{X} \subseteq \mathbb{N}$ *that satisfies conditions* (1⋄)-(2⋄), (2)-(3), (4•), *and* (5).

Open Problem 3 claims more than Open Problem 1 as condition (1⋄) implies condition (1).

Open Problem 4. *Define a set* $\mathcal{X} \subseteq \mathbb{N}$ *that satisfies conditions* (1⋄)-(2⋄) *and* (2)-(5).

Open Problem 4 claims more than Open Problem 2 as condition (1⋄) implies condition (1).

Theorem 8. *Open Problem 4 claims more than Open Problem 3.*

Proof. By Theorem 1, condition (4) implies condition (4•). □

7 A partial solution to Open Problem 4

Let $\mathcal{V}$ denote the set of all positive multiples of elements of the set

$$\{n \in \{b+1, b+2, b+3, \ldots\} : 2^{2^n} + 1 \text{ is composite}\}$$

Theorem 9. *The set* $\mathcal{X} = \{0, \ldots, b\} \cup \mathcal{V}$ *satisfies conditions* (1⋄)-(2⋄) *and* (2)-(4).

Proof. The inequality card$(\mathcal{X}) > b$ holds trivially. Most mathematicians believe that $2^{2^n} + 1$ is composite for every integer $n \geqslant 5$, see [3, p. 23]. These two facts imply conditions (1⋄) and (2⋄). Condition (3) holds trivially. Since the set $\mathcal{V}$ is empty or $\mathcal{V}$ is an infinite subset of (b, ∞), the integer b is a threshold number of $\mathcal{X}$. Thus condition (4) holds. The question of finiteness of the set $\{n \in \mathbb{N} : 2^{2^n} + 1 \text{ is composite}\}$ remains open, see [4, p. 159]. Hence, the question of emptiness of the set

$$\{n \in \{b+1, b+2, b+3, \ldots\} : 2^{2^n} + 1 \text{ is composite}\}$$

remains open. Therefore, the question of finiteness of the set $\mathcal{V}$ remains open. Consequently, the question of finiteness of the set $\mathcal{X}$ remains open and condition (2) holds. □

8 Open Problems 5 and 6

Definition 6. *Let* `(1*)` *denote the following condition:* $\mathrm{card}(\mathcal{X})$ *is greater than a huge positive integer and it is conjectured that* $\mathcal{X}$ *is finite.*

The following two open problems cannot be stated formally as they refer to the intuitive meaning of a mathematical formula and the current mathematical knowledge.

Open Problem 5. *Define a set* $\mathcal{X} \subseteq \mathbb{N}$ *that satisfies conditions* `(1*)`, `(2)-(3)`, `(4•)`, *and* `(5)`.

Open Problem 6. *Define a set* $\mathcal{X} \subseteq \mathbb{N}$ *that satisfies conditions* `(1*)` *and* `(2)-(5)`.

Theorem 10. *Open Problem 6 claims more than Open Problem 5.*

Proof. By Theorem 1, condition `(4)` implies condition `(4•)`. □

9 A partial solution to Open Problem 6

A weak form of Szpiro's conjecture implies that there are only finitely many solutions to the equation $x! + 1 = y^2$, see [6].

Lemma 8. *([9, p. 297]). It is conjectured that* $x! + 1$ *is a square only for* $x \in \{4, 5, 7\}$.

Let $\mathcal{W}$ denote the set of all integers x greater than b such that $x! + 1$ is a square.

Theorem 11. *The set*

$$\mathcal{X} = \{0, \dots, b\} \cup \{k \cdot x : (k \in \mathbb{N} \setminus \{0\}) \wedge (x \in \mathcal{W})\}$$

satisfies conditions `(1*)` *and* `(2)-(4)`.

Proof. Condition `(1*)` holds as $\mathrm{card}(\mathcal{X}) > b$ and the set $\mathcal{W}$ is conjecturally empty by Lemma 8. Condition `(3)` holds trivially. We do not know any algorithm that decides the emptiness of $\mathcal{W}$ and the set

$$\mathcal{Y} = \{k \cdot x : (k \in \mathbb{N} \setminus \{0\}) \wedge (x \in \mathcal{W})\}$$

is empty or infinite. Thus condition `(2)` holds. Since the set $\mathcal{Y}$ is empty or $\mathcal{Y}$ is an infinite subset of (b, ∞), the integer b is a threshold number of $\mathcal{X}$. Thus condition `(4)` holds. □

10 The statement Ψ_6 solves Open Problem 6

Let C denote the following system of equations:

$$\begin{cases} x_1! &= x_2 \\ x_2! &= x_3 \\ x_5! &= x_6 \\ x_4 \cdot x_4 &= x_5 \\ x_3 \cdot x_5 &= x_6 \end{cases}$$

Lemma 2 and the diagram in Figure 3 explain the construction of the system C.

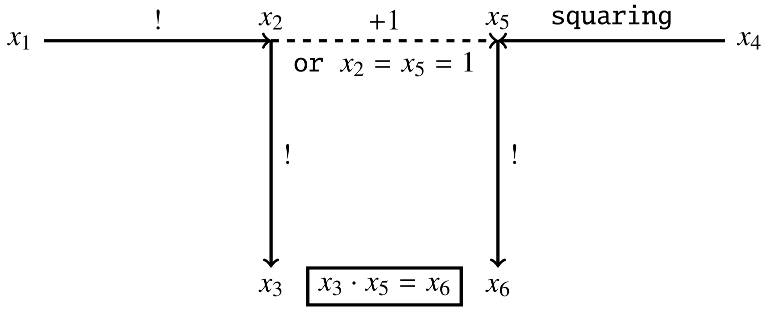

Fig. 3 Construction of the system C

Lemma 9. *For every $x_1, x_4 \in \mathbb{N} \setminus \{0, 1\}$, the system C is solvable in positive integers x_2, x_3, x_5, x_6 if and only if $x_1! + 1 = x_4^2$. In this case, the integers x_2, x_3, x_5, x_6 are uniquely determined by the following equalities:*

$$\begin{aligned} x_2 &= x_1! \\ x_3 &= (x_1!)! \\ x_5 &= x_1! + 1 \\ x_6 &= (x_1! + 1)! \end{aligned}$$

Proof. It follows from Lemma 2. □

Theorem 12. *If the equation $x_1! + 1 = x_4^2$ has only finitely many solutions in positive integers, then the statement Ψ_6 guarantees that each such solution (x_1, x_4) belongs to the set* $\{(4, 5), (5, 11), (7, 71)\}$.

Proof. Suppose that the antecedent holds. Let positive integers x_1 and x_4 satisfy $x_1! + 1 = x_4^2$. Then, $x_1, x_4 \in \mathbb{N} \setminus \{0, 1\}$. By Lemma 9, the system C is solvable in positive integers x_2, x_3, x_5, x_6. Since $C \subseteq B_6$, the statement Ψ_6 implies that $x_6 = (x_1! + 1)! \leqslant f(6) = f(5)!$. Hence, $x_1! + 1 \leqslant f(5) = f(4)!$. Consequently, $x_1 < f(4) = 24$. If $x_1 \in \{1, \dots, 23\}$, then $x_1! + 1$ is a square only for $x_1 \in \{4, 5, 7\}$. □

Theorem 13. *Let $\mathcal{X}$ denote the set of all non-negative integers n which have $(((k!)!)!)!$ digits for some* $k \in \{m \in \mathbb{N} : m! + 1 \text{ is a square}\}$. *We claim that the statement Ψ_6 implies that $\mathcal{X}$ satisfies conditions* `(1*)` *and* `(2)-(5)`.

Proof. Let $d = (((7!)!)!)!$. Since $7! + 1 = 71^2$, we obtain that $\{\underbrace{1\ldots1}_{d \text{ digits}}, \dots, \underbrace{9\ldots9}_{d \text{ digits}}\} \subseteq \mathcal{X}$. By this and Lemma 8, condition `(1*)` holds. Conditions `(2)-(3)` and `(5)` hold trivially. By Theorem 12, the statement Ψ_6 implies that $\underbrace{9\ldots9}_{d \text{ digits}}$ is a threshold number of $\mathcal{X}$. Thus condition `(4)` holds. □

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