The hypothesis of “hidden variables” as a unifying principle in physics.

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03.01.2020

Abstract. In the debate whether ‘hidden variables’ could exist underneath quantum probabilities, the ‘no hidden-variables’ position is at present favored. In this article I attempt to provide a more equilibrated verdict, by pointing towards the heuristic and explanatory power of the hidden-variables hypothesis, in particular in its deterministic form. I argue that this hypothesis can answer three foundational questions, whereas the opposing thesis (‘no hidden variables’) remains entirely silent for them. These questions are: 1) How to interpret probabilistic correlation ? (a question considered by Kolmogorov “one of the most important problems in the philosophy of the natural sciences”, and first analysed by Reichenbach); 2) How to interpret the Central Limit Theorem ?; and 3) Are there degrees of freedom that could unify quantum field theories and general relativity, and if so, can we (at least qualitatively) specify them ? It appears that only the hidden-variables hypothesis can provide coherent answers to these problems; answers which can be mathematically justified in the deterministic case. This suggests that the hidden-variables hypothesis should be considered a legitimate candidate as a guiding, unifying principle in the foundations of physics.

1. Introduction.

The question whether the universe is ultimately deterministic or indeterministic (probabilistic) concerns one of the oldest debates in the philosophy of physics. The atomists Democritus and Leucippus were determinists, while Aristotle was arguably one of the first indeterminists, believing in irreducible chance or hazard. Modern quantum mechanics, a probabilistic theory, has convinced many that indeterminism wins; but a more careful analysis, based notably on the interpretation of the theorems of Gleason, Conway-Kochen and Bell, shows that the debate is actually undecided (cf. e.g. Wuethrich 2011). This may be a shared belief in the philosophy of physics community; but it surely is unpopular outside that community. A broader question can be condensed in following slogan: Can hidden variables exist underneath quantum probabilities? Can quantum probabilities be reduced to, or 'explained by', deeper-lying variables ? (A precise mathematical formulation of this question will be given in Section 2.) Assuming the existence of such hidden variables is a more general hypothesis than determinism: determinism corresponds to the extremal case where the variables are deterministic, a mathematically well-defined special case of more general probabilistic variables (cf. Section 2). Now, the generally received wisdom is again that such hidden variables can*not* exist, at least if one only considers *local* hidden variables – which is what I will do throughout this article, since nonlocal variables involve, by definition, superluminal interactions, so interactions that are not Lorentz-invariant and in overt contradiction with relativity theory. Thus, I will only inquire here about the possibility of local degrees of freedom (variables). Note that the hidden variables which could, under the assumption that the hidden-variables hypothesis (HV-hypothesis) is correct, explain quantum events, could be interpreted as the *causes* of these events[1]. Other philosophers, following notably Hume and Russell, believe one should jettison the notion of cause / causality. In order to avoid needless controversy I will keep the discussion essentially in terms of 'variables' rather than 'causes'.

The question of the possibility of hidden variables (HV) underlying quantum probabilities is likely most clearly investigated through Bell's theorem (Bell 1964). Indeed, Bell's theorem is the only standard chapter in physics that explicitly mentions the concepts of cause and determinism and investigates them in the light of quantum mechanics. Bell's seminal article of 1964 starts as follows: "The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality" (1964, p. 195). Now, according to Bell's theorem local HV theories contradict quantum mechanics in certain experiments. Since these Bell experiments have vindicated the quantum prediction countless times (just two of the latest examples are Handsteiner et al. (2017) and Rauch et al. (2018)), most scholars believe now that the prospects for local hidden-variable theories, and thus for the HV-hypothesis, are dim. In other words, according to the standard view quantum probabilities are irreducible in general; they cannot be understood as emerging from a deeper-lying HV level.

[1] In the literature, notably on Bell's theorem, the hidden variables λ are indeed often called 'causes', deterministic causes if x depends functionally on λ (as in (3b) below), probabilistic causes if the probability of x conditionally depends on λ (as in (1) and (2) below). Defining causes in this way has been done by several philosophers of science investigating causation (cf. Suppes 1970, Hausman and Woodward 2004, Hitchcock 2018).

My goal here is to argue that not only is the HV-hypothesis not refuted by Bell's theorem and the Bell experiments (in line, notably, with Wuethrich 2011), but this hypothesis, compared to its competitor ('no HV' or irreducibility), has the greater explicatory power. Specifically, I will show in Section 3 that the HV-hypothesis can provide coherent answers to three questions of the foundations of probability and of physics, whereas irreducibility remains entirely silent for them. These questions are: 1) How to interpret probabilistic correlation ? (a problem raised by Kolmogorov and first analyzed by Reichenbach); 2) How to interpret the Central Limit Theorem?; and 3) Are there variables that could unify quantum field theories and general relativity, and if so, can we say at least something about them ? It appears that in particular *deterministic* HV allow to mathematically prove answers to questions 1) and 3); and to conjecture a proof of an answer to question 2). To further illustrate the heuristic power of determinism[2], I will recall that it corresponds, at least in certain physical situations, to the 'best informed' epistemic position (Section 2).

This article is organized as follows. In Section 2 I will extract from the literature a straightforward definition of 'deterministic' and 'probabilistic / stochastic' variables, needed in the remainder of the article. This will allow to define 'reducibility' of (quantum) variables via (yet unknown) additional variables – the so-called hidden variables. I will need these concepts throughout the article. Next, it appears that some confusion exists regarding the concept of 'objective (and subjective) probability': this concept can have two quite different meanings. To illustrate my argument, it will be helpful to look in detail at a realistic case, namely the automated tossing of a large die: an experiment that can equally well be described as a deterministic or a probabilistic system, depending on the epistemic status of the experimenter (in line with e.g. Suppes 1993). Hence it will appear useful to introduce the notion of "relativity or subjectivity of (in)deterministic ascription". This simple thought experiment does not claim to provide new results, but will allow to illustrate and clarify the notions of reducibility and determinism, and thus to throw light on the three foundational questions mentioned above (related to probabilistic dependence, the Central Limit Theorem and the unification of QFT and GR). These questions are the subject of Section 3, containing the main results of this work. Section 4 will conclude.

[2] For a detailed treatment of physical determinism, see notably Earman (1986), (2008), (2009); and Hoefer (2016).

2. Definitions: reducibility, determinism, objectivity / subjectivity of probability.

Let us start by defining concepts. The mathematical representation of deterministic and probabilistic variables we will use here (definitions (1) - (3)) stems from the quantum foundations literature, specifically on Bell's theorem (see e.g. Bell (1964), Hall (2010), (2011), Friedman et al. (2019)); I will use the compact notations of these works. Before defining the HV-thesis or reducibility of (quantum) probability as a general principle, let us first define them *relative to some quantum probability*[3] $P(x|a)$, where 'x' is a quantum variable (say spin) and 'a' a measurement or analyzer variable (say the angle of a polarizer). By definition, $P(x|a)$ (or x) is *reducible* IFF $P(x|a)$ can be expanded (for all values of x) as a sum of conditional probabilities, where the sum runs over the values of some (hidden) variable λ:

$$\exists\ \lambda: \quad P(x|a) = \sum_\lambda P(x|a,\lambda).P(\lambda|a). \qquad \frac{\text{(reducibility of x)}}{\text{(HV}-\text{hypothesis for x)}} \tag{1}$$

In words, a variable λ exists that allows to expand $P(x|a)$ as given. In (1) all variables (x, λ and a) can be multi-component (n-tuples); in the following these variables all represent physical properties. Note that the expansion in (1) is *in principle* allowed by standard rules of probability theory; (1) therefore is physically meaningful if the λ represent physical properties. $P(x|a)$ is then *irreducible* IFF $P(x|a)$ cannot be expanded as a sum of conditional probabilities:

$$\nexists\ \lambda: \quad P(x|a) = \sum_\lambda P(x|a,\lambda).P(\lambda|a). \qquad \frac{\text{(irreducibility of x)}}{\text{("no HV for x")}} \tag{2}$$

So in (1) λ represents hypothetical variables on which the quantum property x would depend; but the experimental confirmation of Bell's theorem has convinced most researchers that such λ and probabilities as $P(x|a,\lambda)$ do not exist.

Determinism corresponds now simply to the extremal case of reducibility (1), when the probabilities $P(x|a,\lambda)$ have values 0 or 1 for all values of x and λ; i.e. when the values of x are

[3] A more precise notation for $P(x|a)$ would be: $P(X = x|A = a)$. But since no confusion is possible we will stick to our simpler notation and continue to use the symbol 'x' also for the variable, as is practical in a physics context. Instead of considering the conditional probability $P(x|a)$, we could equally well consider the unconditional $P(x)$, but introducing the variables 'a' now allows us to connect fluidly with *Problem 3* below. (Moreover, as many authors I believe that conditional probability is the natural probability here: the variables 'a' define the quantum experiment, in particular the analyzer variables, the specification of which is essential for a well-defined probability, cf. Vervoort (2010, 2012). But this is not essential here.) Also, as usual all variables in the following range over an interval defined by the system considered.

completely determined by the values of a and λ, so that a suitable function f exists for which x = f(a,λ). This leads to following definition (δ is the Kronecker-delta):

$$\exists\ (\lambda, \text{function } f):\ P(x|a) = \sum_\lambda P(x|a,\lambda).P(\lambda|a), \text{ where } P(x|a,\lambda) = \delta_{x,f(a,\lambda)}. \qquad \text{(3a)}$$

(determinism of x; x is a deterministic variable)

Equating $P(x|a,\lambda) = \delta_{x,f(a,\lambda)}$ in (3a) captures well our intuitions about determinism: if we know or specify enough variables ('information' if one prefers), we know whether x occurs or not with certainty. From (3a) follows a definition that is, in practice, equivalent:

$$\exists\ (\lambda, \text{function } f):\ x = f(a,\lambda). \qquad \text{(3b)}$$

(determinism of x; x is a deterministic variable)

Representation (3a,b) is widely used and uncontroversial for defining deterministic variables (compare e.g. Eq. (3b) to Eq. (1) in Bell's original paper (1964); and compare (3a,b) to Hall (2010) Eq. (6), Hall (2011) Eq. (3) and text above Eq. (22), or Friedman et al. (2019) Eq. (3)). I will further illustrate the utility of these definitions by examples.

We have thus defined reducibility (the HV-hypothesis) relative to some stochastic property / variable x. As a universal principle it can be defined as stating that *all* physical properties / variables / events are reducible in the sense (1). Determinism as a principle is then again a special case of the HV / reducibility-principle. In general, reducibility can loosely be read as stating that (quantum) variables can be "reduced to" or "explained by" additional variables, in a stochastic or deterministic way, depending on whether the $P(x|a,\lambda)$ are trivial or not. As said, in the Bell literature these additional variables λ are often called 'causes'; and it indeed seems that reducibility can be the key ingredient of, or even equated with, the hypothesis of causality (in line with the definitions of cause given by e.g. Suppes 1970, Hausman and Woodward 2004, Hitchcock 2018; see also footnote 1). But I will not explore this further here. Note also that determinism, according to which all (quantum) variables would be deterministic under the surface, is still compatible with a non-trivial probabilistic distribution $P(\lambda|a)$ over the hidden variables λ, as is best seen in (3a)[4]. So in a deterministic universe, in which (quantum) variables x are deterministic functions of hidden variables, there still could be a non-trivial distribution over the (initial values of the) hidden

[4] From (3a) follows: $P(x|a) = \sum_{\lambda_x} P(\lambda_x|a)$, where λ_x is defined by $f(a,\lambda_x) = x$.

variables. This agrees with the discussion in (Wuethrich 2011, p. 365), where the author distinguishes two roles probabilities can play in dynamical theories: 1) that of describing the dynamical evolution given a certain initial state of the physical system at stake; and 2) that of codifying a distribution over initial or boundary conditions ($P(\lambda|a)$ is an example of case 2)). Under the hardest form of determinism, this distribution $P(\lambda|a)$, taken over the 'initial values of the universe', would also be given by a δ-symbol (Kronecker or Dirac): $P(\lambda|a) = \delta(\lambda - \lambda_0(a))$. Then all variables in the n-tuple λ have one fixed value: Laplacianism in its purest form.

But as stated above, Bell's theorem, combined with the results of the Bell experiments, constitute a convincing physics-based argument for *ir*reducibility in the sense (2), so against the HV-assumption. Irreducibility could also be equated with the slogan "no hidden variables", neither deterministic nor probabilistic. (Of course, Bell's no-go argument only goes through if the universe is local. But we excluded the option of nonlocal HV from the start: they are in contradiction with relativity theory. Still, for completeness, it is useful to remember that the main argument for irreducibility, Bell's theorem, only holds if there really are no nonlocal interactions in this universe.)

The next conceptual clarification we need concerns the notions of 'objectivity' and 'subjectivity' of probability, which are used in the present context in two different ways. Some readers may be sympathetic to the idea that, in a sense, determinism is the intuitively most cogent form of reducibility. Notably, they make invoke an argument from homogeneity or simplicity: a universe with only one category (only deterministic events and systems) corresponds to the simplest, most homogeneous, most unified worldview – moreover the one that seems to take "the most information" or "the most adequate information" into account, at least if we can generalize from the example we will study below. This intuition lies at the basis of the historic "determinism versus indeterminism" debate. In the remainder of this Section I will comment qualitatively on this intuition (mathematical arguments are left to Section 3), while defining important concepts.

It is well known that probability has a remarkably complex epistemic nature; its interpretation is an intensely debated topic in philosophy of science (for reference works, see e.g. von Mises 1928/1981, 1964, Fine 1973, von Plato 1994, Gillies 2000; for recent work on the frequency interpretation, cf. Vervoort 2010, 2012). Among the many interpretations of probability on the market, such as the classical interpretation of Laplace and the propensity and frequency interpretations, there is also the *subjective* or Bayesian interpretation, which enjoys a relative popularity in quantum mechanics (Caves et al. 2002, 2007 and Bub 2007). On the subjective interpretation (of at least some schools), probability is a measure of the strength of belief, which an observer subjectively attributes to a random event (or hypothesis, depending on the specific interpretation). According to this view different observers may attribute different probabilities to the same event. *Objective* interpretations stipulate that the probability *of a well-defined event* is an 'absolute' measure, *the same for all observers* (von Mises 1928/1981, 1964, Fine 1973, von Plato 1994, Gillies 2000, Vervoort 2010, 2012) – an interpretation that resonates well with the objective character of physics. Note that in this broad philosophy of science context the question whether such objective interpretations of probability allow for a deterministic reduction is eluded – it is rarely if ever considered. So this is the first sense in which the objectivity of probability is discussed. In the philosophy of physics literature there is however a second, widely used sense of this notion: it associates subjective probability with the hypothesis of determinism, and objective probability with indeterminism (see e.g. Wuethrich 2011, p. 365). The underlying thought is that 'objective' (quantum) probability, as *irreducible (and non-trivial) probability in the sense defined above*, is the only possibility we are left with in a world that is at its most fundamental (say quantum) level indeterministic; while conceiving probability 'subjectively', as an expression of our ignorance of underlying deterministic processes, is appropriate in a deterministic world. But it is important to realize that even in the latter 'subjective' or rather 'reducible' interpretation of probability, probabilities can still be considered as objective measures in the sense used in the philosophy of probability – for instance as relative frequencies. As we will illustrate in a moment, even in a deterministic universe one can still define probabilities as frequencies that are measured and attributed the same value by agents with a radically different epistemic status. And even on an 'objective' or rather 'irreducible' reading of quantum probabilities, one might consider probability values as subjective degrees of belief, as in (Caves et al. 2002, 2007 and Bub 2007). As will become clear in the following, in the context of our present investigation it is more precise to consider not the probability *value* itself as subjective, but rather the "ascription of a deterministic or indeterministic character" *to the system or event under scrutiny*. It is this attribution that is subjective, in the sense that it depends on the observer's knowledge (cf. below). Hence also the added value of the notion of (ir)reducibility.

But sure enough, since Laplace, determinists typically understand probabilities as mathematical tools, capable of describing certain (ensembles of) objects in nature – tools that we only need when we do not have knowledge of the full set of variables and the full dynamics determining the behaviour of these objects, so when we cannot predict the full behaviour of these objects (as individual objects). But Laplace's demon knows everything so does not need probabilities: he predicts everything deterministically (he could still define and measure probabilities, though, see further). Hence on this view, the attribution of a deterministic or probabilistic character to an event / system depends on the knowledge of the subject, the observer. Let us call this the *relativity of (in)deterministic ascription*. If this Laplacian view is correct, determinism could be said to be more fundamental in that it corresponds to the enviable "truth before God's eye". In any case, according to Laplacian determinism probabilities would be reducible; or to put it still differently they would 'emerge' from a deterministic substratum – they would pop up and be useful in our human theories and experiments.

Now, there are countless examples where the relativity of (in)deterministic ascription is blatant. Consider following thought experiment, a mechanized version of a die toss, consisting of say N runs (tosses). The test involves a large (say 20x20x20 cm^3) cube made of soft plastic, positioned at the beginning of each toss with high precision in the middle of a drumhead. Underneath the drumhead a metallic pin moves quickly upward, imparting to the cube a vertical momentum so that it tosses around maybe once or twice; for each toss the mechanism can be started by a button. The outcome or event 'e' is the number $\in \{1,2,\ldots,6\}$ shown on the upper face of the cube, 'measured' after it lands on a table. Suppose that Alice wishes to ascertain whether this experiment is probabilistic and, if so, what are the values of the probabilities P(e); also suppose that Alice has no means to determine the position and force exerted by the pin (e.g., the pin-mechanism is screened off). The best she can do is to make a table and note, for a long series of tosses, the outcome e. In a trivial case she would find always the same result, say e = 6 (she is a valiant researcher and goes to N = 1000). In this case she would term the experiment or the system deterministic: she feels she can predict what would be the result of the (N+1)-th toss; so she assumes P(6) = 1 and P(1) = P(2) =… = 0. But consider now the case where each outcome looks perfectly random, unpredictable to her. Then she has to determine, based on her table, the frequencies $P_N(e) = \#_N(e) / N$, where $\#_N(e)$ is the number of times the outcome is e in N trials. Suppose she finds that for N = 1000, $P_N(e)$ comes close to 1/6 for all e, and moreover that, when comparing the $P_N(e)$ for N = 100, 200, 300,…1000, the $P_N(e)$ come (globally) closer and closer to 1/6. Realizing that this "frequency stabilization" is, besides the unpredictability of individual outcomes, the hallmark of a probabilistic system (von Mises 1928/1981, 1964; Vervoort 2010, 2012), she rather confidently concludes that the system is probabilistic and that the six probabilities P(e) are = 1/6 (to good approximation). She has done her job as a physicist.

Now, this experiment could of course be deterministic in disguise: an engineer, Bob, could have constructed the die and the system that commands the pin in such a manner that there is a perfect functional relation between the position coordinates of the pin impact and the outcome 'e'. E.g., the pin hits M = 10,000 positions on the lower surface of the die, say distributed over a square grid; once the first M grid positions have been probed the next runs will repeat the same sequence, so M is the periodicity of the impact points. If the system is well-calibrated and the die movement not too chaotic (the die is of soft material and rotates not more than once or twice), Bob can establish such a functional relation – a table – between impact position and outcome, while at the same time ensuring that the outcomes 1, 2, …, 6 arise in a random-looking sequence leading[5] to P(e) = 1/6 for all e, to excellent approximation. This example illustrates definitions (1) and especially (3): here λ is the pin position and x (= e) is the outcome of the die toss. P(x) (= P(e)) is well-defined and reduces to 0 or 1 if we specify λ ($P(x|\lambda) = 0$ or 1; x = *f*(λ) for some function *f*). (Note further that, in typical parlance, the pin position assuming a certain coordinate λ *causes* the die to show the result x. And if Bob uses enough engineering ingenuity he could even devise a system where the λ are *probabilistic* causes.)

So this is a system that the engineer Bob and any 'informed' experimenter will identify as a deterministic system – *in principle* each individual toss can be predicted in advance, more precisely there exists (before Bob's and God's eye) a functional relationship between pin positions and outcome. But Alice and any uninformed experimenter will deem the system probabilistic (recall that in this well-defined experiment the pin-mechanism is screened-off, Alice cannot peek behind the screens and do further experiments). So, this is an example of relativity or subjectivity of (in)deterministic ascription *regarding physical systems* – relativity with respect to the knowledge state of the agent inquiring about the *system*. Importantly, notice that this does not mean that probability *values* are subjective: both the informed Bob and the uninformed Alice will

[5] Of course, this assumes, among other things, that there are about M/6 grid positions that lead to each of the 6 outcomes e.

measure, when asked, the same probabilities P(e) *if they do the same experiment*[6]. Bob can put his table aside and measure and compute the same ratios $P_N(e) = \#_N(e) / N$ that Alice obtains; he should find the same results. Quite generally, in physics probabilities are measured as frequencies, even if the system is known or assumed to be deterministic deep down (see other examples below).

Note also that there is, *in principle but not always in practice*, a way for Alice to discover the deterministic nature of the system, by noting outcomes and by making N runs with N >> M, say N = n.M (n ≥ 2), with M the periodicity of the impact points. If she does so, she will notice that the outcomes repeat themselves with a periodicity M. She can then predict the individual outcomes with a likelihood that is proportional to n. In principle she can reach *quasi*-certainty about future outcomes, and that is all a physicist can ask for. But this procedure does not work if M, the number of hidden variable values, becomes too large to be practically accessed. In that case Alice, left to her devices, has no practical experimental means to discover the deterministic nature of the system. Interestingly, this variant of the experiment (M very large), is an example of experimental situations where the abundance of hidden information, the "too large number of (values of) hidden variables" prevents discovery of the deterministic nature of a system. This is essentially the same rationale used by proponents of determinism to address *Problem 3* below.

Other examples of probabilities emerging from an underlying deterministic dynamics can be found e.g. in fluid mechanical systems. Fluid mechanics is governed by deterministic equations, notably the Navier-Stokes equation; yet there exist an enormous variety of probabilistic features in fluids, gases, eddies, diffusive systems etc. Such properties seem to be all examples of variables of which the random nature is induced by the distribution of the initial values $P(\lambda|a)$, as discussed in Wuethrich (2011) (recall the above-mentioned second role probabilities can play). Another example of well-defined and non-trivial probabilities in deterministic systems is the following. A typical numerical method in statistical mechanics is the Monte-Carlo technique, quite faithfully reproducing probabilistic features of a wide variety of real-world stochastic systems. Now, this numerical simulation method always uses, in practice, a *pseudorandom* number generator, which allows to simulate physical randomness in some parameters of the system considered. Such a

[6] Hence the importance of realizing that (physical) probability values refer to *well-defined experiments,* as many philosophers have emphasised, notably von Mises, Popper and more recently van Fraassen (1980). Popper famously interprets probability as the propensity that a certain experimental set-up has to generate certain frequencies. Van Fraassen presents in his (1980, pp. 190 – 194) a logical analysis of how to link in a rigorously precise way experiments to probabilities. This idea is discussed in more detail in (Vervoort 2010, 2012).

generator is in reality deterministic, in that the generated numbers look randomly distributed but are actually generated by a complex function – deterministic by definition. Hence a dynamics that is fully deterministic can reproduce an enormous variety of systems of stochastic mechanics. Thus all these probabilistic systems can be understood to be deterministic under the surface[7].

3. Three questions.

The above examples indicate that the "better informed observer" perceives the deterministic nature of phenomena that his less informed counterpart deems probabilistic. Recall that determinism (of x) corresponds to the extremal case of reducibility, when all probabilities (P(x)) in (1) are reduced to 0 or 1 upon specification of enough λ. In view of such examples, it is only natural to inquire whether this deterministic reducibility as in (3) could hold for *all* probabilities; or at least, whether generalized reducibility as in (1) could hold for all probabilities. This is also the question that sparked Bell's inquiry (Bell was initially interested in deterministic reduction, but he and others soon realized that (1) is the more general case). I will now show that the HV-hypothesis or reducibility (1) allows to explain three fundamental problems for which irreducibility remains powerless. Certain mathematical results favor, moreover, deterministic reduction.

Problem 1. How to interpret statistical correlation ? Probability theory can be interpreted as a purely mathematical theory, but also as a physical theory – a theory describing random physical events (cf. von Mises 1928/1981, 1964, who develops such a physical interpretation in greatest detail). In a sense, it seems that probability theory could even be considered the most general of all physical theories, since it governs countless different types of physical systems, from almost all branches of physics. Quantum mechanics is a statistical discipline and complies with probability theory; but also general relativity is, as a deterministic theory, a special case of a probabilistic theory (with probabilities having values 0 or 1). One therefore suspects that the problems of the

[7] In this context, it would be interesting to look in detail at the question, launched notably by Suppes (1993), whether deterministic and indeterministic descriptions are observationally equivalent for micro-physical systems in general (it is clear that in the large-die experiment, they are not). However this would lead me too far: I refer to the discussion in (Wuethrich 2011, p. 367-368); I agree with its conclusion. In the last Section, I will conclude that there are good reasons to believe that any probabilistic theory presupposes a deterministic substratum. Whether this can be empirically tested is not so clear; but surely, if it turns out to be possible to construct deterministic theories for quantum mechanics, there seem no principled reasons to think there can be no new predictions. To make new predictions is surely the aim of a theory that would unify quantum mechanics and general relativity theory (cf. next Section).

foundations of probability theory are, or can be, also problems of the foundations of physics. Besides von Mises, several members of the prolific Russian school of probability (Kolmogorov, Chebyshev, Lyapunov, Khinchin, Markov, Gnedenko, Bernstein etc.) were interested in foundational aspects of probability theory (in this respect it is delightful to read e.g. Kolomogorov 1933/1956, Gnedenko 1967). For my present concerns the first relevant problem of the interpretation of probability is summarized in following remarkable quote by Kolmogorov in his reference work of 1933, *Foundations of the Theory of Probability* (1933/1956, p. 9):

> "In consequence, one of the most important problems in the philosophy of the natural sciences is – in addition to the well-known one regarding the essence of the concept of probability itself – to make precise the premises which would make it possible to regard any given events as independent. This question, however, is beyond the scope of this book."

Kolmogorov asks here whether there exist general conditions allowing one to know in advance (without doing a statistical test) whether two events or variables x and y are *independent* (de-correlated) or not, in symbols whether their joint probability satisfies following equation:

$$P(x,y) \; = P(x)P(y|x) \stackrel{?}{=} P(x)P(y). \tag{4}$$

An equality sign corresponds, by definition, to probabilistic independence. When dealing with a probabilistic question related, for instance, to the tossing of two dice, or in general to the joint occurrence of two events, one has to assume or guess whether these events / variables / systems are independent or not. Generally speaking, in real-world situations there is no rule; the only way to be sure is to experimentally verify Eq. (4). But even if no rule exists, one often 'intuits' correctly whether events and variables are dependent or not. (Upon reflection, the present author thinks this has a touch of magic to it[8]; in any case, he wholeheartedly agrees with Kolmogorov that the above question is essential.) E.g., two die throws by two different people under normal circumstances can safely be treated as independent. The interesting question is: *on what basis do we intuit this* ? Surely by our intuitive understanding, our feeling, that these events are not *causally connected*, i.e. that one event is not *determining* the other, and that there is no common cause determining the two events. My point is this: while probability theory remains entirely silent regarding the origin of stochastic (in)dependence – this is Kolmogorov's complaint –, determinism and more generally the HV-hypothesis offer an explanation. According to it, probabilistic dependence has to do with

[8] Or more prosaically, that causal intuitions are strongly developed in humans.

an underlying causal stratum, with causes connecting the dependent variables. Explicitly, two events are independent if one event is not causally determining the other one, and if there is no common cause determining the two events (in mathematical terms, the mentioned causes are variables). This analysis, usually attributed to Reichenbach (1956), is sometimes termed the 'qualitative part of Reichenbach's principle' (Cavalcanti and Lal 2014, Allen et al. 2017). The quantitative part of Reichenbach's principle can be formulated as follows: if the correlation between x and y is exclusively due to a common cause (there is no direct causation), and if z is a complete common cause for x and y, meaning that z is the set of all variables acting as common causes, then x and y must be conditionally independent given z (Allen et al. 2017). Mathematically, under the condition stated:

$$\text{If} \quad P(x,y) \neq P(x)P(y) \quad \text{then} \quad \exists\ z: \quad P(x,y|z) = P(x|z)\,P(y|z), \tag{5a}$$

where z is a complete common cause.

Reichenbach's principle has been verified countless times in classical macroscopic systems (cf. Spirtes et al. 2001, Pearl 2009). So Kolmogorov's problem of correlation can be explained by referring to hidden variables z, more precisely hidden *common* causes. What is most relevant for our argument, is that under the assumption that x and y are deterministic variables in the sense (3), one can *prove* that the qualitative part of Reichenbach's principle implies the quantitative part: this is done in (Allen et al. 2017). Moreover, it is straightforward to generalize the result by Allen et al. (2017) to the case of direct causation: if x would directly cause y, (5a) applies with z = x. So determinism is the only known assumption by which one can prove Reichenbach's principle (5a), in other words *that correlation occurs when (common) causes exist*. There is no explanation for correlation, neither the qualitative nor the quantitative part of Reichenbach's principle, if one assumes irreducibility.

I will prove here a related theorem ('Theorem 1'); a corollary of its proof will turn out to be useful for *Problem 3*: If x and y are deterministic variables (if P(x) and P(y) can be deterministically reduced as in (3)), then their joint probability satisfies:

$$\exists\ z: \quad P(x,y|z) = P(x|z)\,P(y|z), \tag{5b}$$

where z is the set (n-tuple) of variables that make P(x) and P(y) deterministic (note the formal similarity but also the difference with (5a)). In short, in a deterministic world, any joint probability can be factorized by the full set of causes. Indeed, if x and y are deterministic variables then also (x, y) is, and we have, according to (3a):

$\exists$ (λ_1, function f_1) : $P(x) = \sum_{\lambda_1} P(x|\lambda_1).P(\lambda_1)$, where $P(x|\lambda_1) = \delta_{x;f_1(\lambda_1)}$.

$\exists$ (λ_2, function f_2) : $P(y) = \sum_{\lambda_2} P(y|\lambda_2).P(\lambda_2)$, where $P(y|\lambda_2) = \delta_{y;f_2(\lambda_2)}$.

$\exists$ (λ_1,λ_2, functions f_1, f_2) : $P(x,y) = \sum_{\lambda_1\lambda_2} P(x,y|\lambda_1,\lambda_2).P(\lambda_1,\lambda_2)$, where

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))}.$$

If x is a deterministic function (f_1) of λ_1, then it is also trivially a function (f_1') of (λ_1, λ_2). Formally, we write: $f_1'(\lambda_1,\lambda_2) = f_1(\lambda_1)$. Then we have:

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))} = \delta_{x;f_1(\lambda_1)}\delta_{x;f_2(\lambda_2)} = P(x|\lambda_1)\ P(y|\lambda_2).$$

Further:

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))} = \delta_{(x,y);(f_1'(\lambda_1,\lambda_2),f_2'(\lambda_1,\lambda_2))} = \delta_{x;f_1'(\lambda_1,\lambda_2)}\delta_{y;f_2'(\lambda_1,\lambda_2)}$$
$$= P(x|\lambda_1,\lambda_2)\ P(y|\lambda_1,\lambda_2).$$

Thus we obtain (5b), with z = (λ_1,λ_2) the set of variables that deterministically reduces x and y. This also proves as a useful corollary that, if λ_1 is the minimal set of variables that deterministically reduces x, one has that $P(x|\lambda_1,\lambda_2) = P(x|\lambda_1)$, $\forall\ \lambda_2$: x is independent of all non-causally related variables.

There is a body of literature that corroborates the view that determinism explains correlation. Indeed, the assumption that determinism underlies probabilities, is an ubiquitous guidance principle in the statistical theory of causal modeling (Pearl (2009)); and see a recent treatment in physics (Allen et al. 2017). On this view, probabilities as $P(x|y)$ can ultimately be reduced to deterministic functions from (y, λ) to x, where λ are unobserved variables; and (y, λ) are termed the causes of x (Pearl 2009, p. 26; note the coherence with the conclusions we drew from the artificial die tossing experiment of the previous Section). Pearl states:

> "In this book, we shall express preference toward Laplace's quasi-deterministic conception of causality and will use it, often contrasted with the stochastic conception, to define and analyze most of the causal entities that we study. This preference is based on three considerations. First, the Laplacian conception is more general. Every stochastic model can be emulated by many functional relationships (with stochastic inputs), but not the other way around; functional relationships can only be approximated, as a limiting case, using stochastic models. Second, the Laplacian conception is more in tune with human intuition. […] Finally, certain concepts that are ubiquitous in human discourse can be defined only in the Laplacian framework. We shall see, for example, that such simple

> concepts as 'the probability that event B occurred because of event A' and 'the probability that event B would have been different if it were not for event A' cannot be defined in terms of purely stochastic models (Pearl 2009, p. 26)."

More generally (remaining in the more general stochastic case represented by (5a), without resorting to determinism), if a variable z exists complying with (5a), then the variables x and y are reducible according to the definition (1) (z takes the role of λ; if a variable z exists so that $P(x|z)$ is defined, then (1) holds by elementary rules of probability calculus). Again, even without a deterministic interpretation, we can causally interpret correlation and (5a): x and y are caused by z and z 'screens off' the correlation between them.

In sum, irreducibility or the no-HV-hypothesis, offers, it seems, no answer to the question what correlation is. But only determinism allows to prove what appears to be the essence of correlation, namely the quantitative part of Reichenbach's principle (5a). Of course, it is often believed that Reichenbach's principle and reducibility do not work for quantum correlations. But there is a way out of this conundrum, as shown under *Problem 3*.

Problem 2. How to interpret the Central Limit Theorem ? The history of the Central Limit Theorem (CLT), one of the corner pieces of probability theory, is extremely rich and many variants of it exist. Let us pay tribute here to only a few of the great names of mathematics having contributed to it: de Moivre, Laplace, Chebyshev, Lyapunov, Pòlya, Gnedenko, Kolmogorov (it seems that Lyapunov was the first to have given the exact proof of the common variant we consider here, cf. Tijms 2004, p. 169). As even a quick glance in probability books shows, there is an aura of venerability surrounding the CLT: statisticians are struck by the regularity this theorem brings in the randomness of probabilistic systems. Witness the eloquent words of Francis Galton[9] (1894, p. 66); Tijms calls the CLT the "unofficial sovereign of probability theory" (Tijms 2004, p. 169); others go as far as calling it "one of the most remarkable theorems in the whole of mathematics" (Mood and Graybill, 1963). This theorem states (cf. e.g. Tijms 2004, Gnedenko 1967):

[9] "I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the Law of Frequency of Error. The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshalled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along" (Galton 1894, p. 66)

> **CLT**. Any random variable that is the average, i.e. the weighted sum, of many independent random variables, has a normal distribution independently of the distribution of the contributing variables.

In pure mathematics this is treated, of course, as a purely mathematical theorem, but in textbooks for practitioners of probability theory the CLT is given a real-world *interpretation*. These textbooks typically present the theorem as an explanation of why the Gaussian distribution is so overwhelmingly present in the physical world, describing an unlimited variety of stochastic phenomena and properties (distribution of height; countless biological parameters; fabrication errors; diffusion phenomena; accident rates; meteorological data; etc.). The textbook rationale is straightforward enough: many phenomena are the average of many other independent phenomena (this is a direct interpretation of the mathematics of the CLT). But what does this really mean, in view of the fact that probability theory is a physical theory? In physics, it is hardly the case that a majority of properties are the average of many other properties; this would make physical properties rather trivial constructs as average sums. What one could say, more precisely, is that many physical properties are the *effect* of many (independent) *causes*. Now this again sneaks in causal terms in the interpretation of probability. A priori this may seem innocuous, but in the present context it is relevant. Again, key ingredients of probability theory are most naturally understood in terms of an underlying causal reality. As far as I know, irreducibility or indeterminism do not help to understand the CLT.

Can one prove the CLT starting from the above causal interpretation? I do not know, but in the deterministic case it is tempting to suggest following path for a proof in a hand-waving way. Suppose the CLT can indeed be understood as expressing that a property / variable y, *determined* by N variables ($x_1,\ldots,x_N$), is normally distributed under broad conditions. According to definition (3b) the way to express that y is deterministically caused by ($x_1,\ldots,x_N$) is: y = $f(x_1,\ldots,x_N)$ for some function *f*. The CLT considers a special case for the function *f*, namely y = $\frac{1}{N}\sum_i x_i$. Thus, one would have to prove the CLT from following generalized version of it:

> **CLT-gen**. Under broad conditions, a stochastic variable that is a *function* of many independent variables will be normally distributed, independently of the distribution of the contributing variables.

The proof would include, essentially, the specification of the mentioned 'broad conditions'. Here is a first hint. According to Taylor's theorem one can approximate a function $f(x_1,\ldots,x_N)$ as follows:

$$f(x_1,\ldots,x_N) = f(0,\ldots,0) + [\,\partial f / \partial x_1\,]_0 \,.\, x_1 + \ldots + [\,\partial f / \partial x_N\,]_0 \,.\, x_N + O[x_i^2]\,.$$

According to the CLT the dominant terms, linear in ($x_1,\ldots,x_N$), converge to a Gaussian[10].

Conversely, a deterministic interpretation of the CLT allows one to conjecture a generalized version of it, namely **CLT-gen** above.

Problem 3. Are there degrees of freedom that could unify quantum field theories (QFT) and general relativity (GR), and if so, can we specify them (even a little more, qualitatively) ? Of course, I will attempt here a qualitative answer, to a problem that is often considered as one of the greatest unresolved problems of physics. GR and QFT are generally believed to be mutually incompatible, at least in regions of extremely high mass and small dimensions, such as exist in black holes or at the Big-Bang. This incompatibility parallels the standard interpretation of Bell's theorem, which seems to constitute a further severe obstacle to the unification program. Indeed, on its standard interpretation, Bell's theorem shows that quantum mechanics cannot be derived from an underlying deterministic theory (as GR), in general not from any theory that introduces (local) hidden variables for quantum properties in the sense (1). In particular, deterministic and Lorentz-invariant degrees of freedom that could describe GR cannot describe[11] quantum mechanics as an irreducibly probabilistic theory in the sense (2). But it is also known that the no-go verdict of Bell's theorem is only valid if one adheres to *a particular form of determinism / reducibility*, *not* if one adheres to full determinism or full reducibility – often called 'superdeterminism' in the Bell literature. Indeed, Bell's theorem is based on an assumption of 'measurement independence', a condition of probabilistic independence or de-correlation stating that:

$$P(\lambda|a,b) = P(\lambda). \tag{6}$$

Here 'a' and 'b' are the left and right analyzer directions in a Bell experiment, and λ are the hidden variables (deterministic or stochastic) that would 'explain' and determine the quantum probabilities $P(x,y|a,b)$ measured in the Bell test (as in Eq. (7) below, which is an instance of (1)). Here 'x' and 'y' are the left and right polarizations or spins and $P(\lambda)$ is the probability distribution of the λ. As is well known, assumption (6) can only be justified by metaphysical arguments (notably based

[10] There are functions however that do not lead to a Gaussian distribution. For instance, one can show that a product of independent variables leads to a lognormal distribution, which is not Gaussian. As said, the main task would reside in the identification of the precise mathematical conditions under which functions lead to a Gaussian distribution.

[11] According to Bell's theorem only *non-local* HV variables could describe quantum mechanics, but such a theory would involve superluminal, non-Lorentz-invariant interactions. As stated, this possibility is not considered here.

on the notion of free will, subject to philosophical debate since millennia); for a detailed overview of these arguments, against or in favor of (6), see e.g. Lewis (2006)[12] and Wuethrich (2011). Most people accept (6) on the basis of the idea that freely chosen parameters as a and b cannot be stochastically determined by particle properties – such a 'superdeterministic' assumption would contradict free will or amount to a cosmic conspiracy. First note that reducibility applied *at face value* to the Bell correlation implies (cf. (1)):

$$P(x, y|a, b) = \sum_\lambda P(x, y|a, b, \lambda)\, P(\lambda|a, b). \qquad (7)$$

Inserting assumption (6) in (7) leads then immediately to the Bell inequality (if also the usual factorability or 'locality' condition (8) is assumed). I do not take strong sides here, but remark that (6) inserted in (7) amounts to a *partial* reducibility. Genuine reducibility implies (7) in full, so featuring the conditional probability, as follows by direct application of probability theory (a rule sometimes called "law of total probability"). Expression (7) leaves open the possibility that the correlation $P(\lambda|a, b)$ could itself be reduced through (common) causes between (λ, a, b) (see further, and the following footnote).

Some researchers, both philosophers and physicists, have contested the general validity of hypothesis (6) (cf. for instance Shimony et al. 1976, Brans 1988, Lewis 2006, Hall 2010, 2011, Nieuwenhuizen 2011, Vervoort 2013, 2018, 't Hooft 2014, 2016, 2017, Hossenfelder 2014); but surely superdeterminism remains a minority position. The gist of the argument, common to most of the mentioned authors, is that if one takes determinism seriously, it cannot be excluded that there exists, at least in principle, a correlation between the variables λ and (freely or randomly chosen) analyzer directions as (a, b). A straightforward way to read this rationale is that, in a fully deterministic universe, a and b are also deterministic parameters reducible to some causes (variables); these causes have their causes, etc. until the origin of the universe, the Big-Bang. Back to our problem: on this non-standard view the λ are interpreted *as degrees of freedom of a theory that describes the Big-Bang* (or the universe shortly after it); these degrees of freedom are common causes of x, y, a, b. So they remain correlated to 'everything', in particular (a,b), in contradiction with Eq. (6). Or, on a closely related reading, the λ could be 'particle' variables that share common

[12] Lewis proposes a new argument in favor of (6), but his main conclusion is that this assumption needs far more work before it can be accepted as uncontroversial.

causes with (a,b), these common causes (λ^*) being the ab-initio degrees of freedom just mentioned[13].

This view has, notably, been defended by Nobel laureate Gerard ‘t Hooft (2014, 2016, 2017). ‘t Hooft has laid the basis for developing a superdeterministic but local hidden-variable theory, even if he acknowledges, of course, that this program is not finalized. He terms his theory the “Cellular Automaton Interpretation” of quantum mechanics; it has the ambition to serve as the basis for a full theory of quantum gravity. In short, ‘t Hooft posits the existence of a basis of ultimate ‘ontological states’ (characterized by λ), with associated Hilbert space: a preferred basis of states of which all particles are composed and which deterministically evolve by permutations among themselves in discrete time intervals. On the nature of the hidden variables in his theory ‘t Hooft says: “This set consists of the states the universe can ‘really’ be in. At all times, the universe chooses one of these states to be in, with probability 1, while all others carry probability 0” (2016, p. 14). And:

> “Our theory is that there does exist a true, ontological state, for all atoms and all photons to be in. All ontological states form an orthonormal set, the elements of an ontological basis. The universe started out to be in such a state, and its evolution law is such that, at all times in the future, the universe will still be in an ontological state. Regardless which ontological initial state we start from, the state in the future will be an ontological one as well, that is, not a quantum superposition of different ontological states. What we have here, is a conservation law, the conservation of ontology. It selects out which quantum superpositions can be allowed and which not, just because, according to our model, the evolution law is ontological” (2017, p. 15).

And more specifically on the correlations violating Eq. (6):

> “How to explain this apparent ‘conspiracy’? A state considered in some experimental setup may either be a physical state, which we shall call ‘ontological’, or it is a superposition of ontological states. […] However, if an ‘ontological basis’ exists, which we believe to be the case, then there is a conservation law: the ontological nature of a state is conserved in time. If, at some late time, a photon is observed to be in a given polarization state, just because it passed through a filter, then that is its ontological state, and the photon has been in that ontological state from the moment it was emitted by its source. […] Indeed, the same conclusion can be reached by considering the black hole microstates, which quite possibly correspond to the ultimate, classical degrees of freedom of an

[13] Note that both these interpretations are in agreement with the interpretation of the correlation $P(\lambda|a,b)$ given under Problem 1 (qualitative part of Reichenbach’s principle), stating that correlation occurs through causes: either λ directly causes (a,b), in which case $P(a,b|\lambda)$ and therefore (by Bayes’ rule) $P(\lambda|a,b)$ are defined; or λ and (a,b) have common causes (λ^*). In the latter case, we have $P(\lambda|a,b) = \sum_{\lambda*} P(\lambda|a,b,\lambda^*)\, P(\lambda^*|a,b)$.

underlying theory, while they fundamentally arise at the Planck scale only" (2014, p. 13-14).

Even if much remains to be said about the concept of "conservation of ontology", there is a straightforward reading of 't Hooft's idea in plain physical terms, as mentioned above: conspiratorial-*looking* correlations that originate at the Big-Bang are conserved. In other words, $P(\lambda|a,b) \neq P(\lambda)$. It seems important to note that one does not need to ever be able to derive an *explicit* formula for these correlations *in practice*; for contesting the standard interpretation of Bell's theorem it suffices to assume that such correlations *exist*. And, deeming correlations 'conspiratorial' seems an utterly anthropocentric attitude; what looks conspiratorial to our limited minds, may still be perfectly physical (and obvious to a more developed intelligence !).

We may look at the problem from another angle, and note a deep connection between Bell's theorem and Reichenbach's principle (a connection which was noted from quite early on, see notably van Fraassen (1982)). In order to derive Bell inequalities from the expansion (7), one has to assume, besides (6), also following factorability condition:

$$P(x,y|a,b,\lambda) = P(x|a,\lambda)\ P(y|b,\lambda)\ . \tag{8}$$

This can be seen as an application of Reichenbach's assumption (5a) together with the assumption that $P(x|a,b,\lambda)$ does not depend on b (so $P(x|a,b,\lambda) = P(x|a,\lambda)$), and $P(y|a,b,\lambda)$ not on a. The condition (8) is often termed the 'locality' condition, and adopted as a consequence of relativity theory, since (x, b) as well as (y, a) are spacelike separated in advanced experiments. It is interesting to see how Bell derives (8) in his (1981) (see the discussion before and after Eq. (10) in this article): he does not mention Reichenbach but very well bases (8) on Reichenbach's idea that, if λ represents the common causes of x and y, (8) is 'reasonable' (if also locality is assumed).

Now, the analysis of correlation presented under *Problem 1* allows to see that (8) is an extremely mild assumption, with a plausibility that seems to caress certainty. Indeed, it was shown under *Problem 1* (see Theorem 1) that, in a deterministic world, and if one keeps $P(x|a,b,\lambda)$ and $P(y|a,b,\lambda)$, (8) holds necessarily true for λ = the set of all causes of (x, y); we do not need to invoke locality at this stage. That $P(x|a,b,\lambda) = P(x|a,\lambda)$ can also be given a causal reading: we saw in the corollary of Theorem 1 that x must be independent of b given (a,λ), since x cannot be a function of b; b cannot cause x (x and b are spacelike separated). This proves that in a deterministic scenario, (8) follows necessarily only from the condition $P(x|a,b,\lambda) = P(x|a,\lambda)$ (and a similar one for y), a condition which seems itself, especially in the light of Theorem 1 and its corollary, quite harmless. In sum, in a deterministic scenario (8) seems utterly plausible.

And yet, the fact that the Bell inequalities are violated in experiments, has convinced many that Reichenbach's principle does not work in quantum mechanics. But this is overlooking the possibility that the culprit for the discrepancy between the experimental results and Bell's assumptions may be assumption (6) – the neglect of (super)determinism, the neglect of full reducibility. Conversely, if one assumes (super)determinism, one saves Reichenbach's principle. Regrettably, this option is rarely if ever considered (cf. e.g. Cavalcanti and Lal 2014, a recent work investigating Reichenbach's principle in the light of Bell's theorem).

It is generally believed that the theory that can unify quantum mechanics and relativity theory should involve variables describing the Big-Bang. According to the rationale recalled above, Bell's problem, in a deterministic reading, points to precisely these degrees of freedom as a possible solution, since these are 'old' enough (lie far enough in the past light-cones) to be conceivable as common causes both of spin variables and 'freely chosen' analyzer angles. Once again, from the deterministic point of view, spins and analyzer angles *should* have common causes originating at the Big-Bang. Seen from this angle, it is somewhat surprising that superdeterminism is not a more popular assumption.

Therefore, the import of superdeterministic proposals as those of 't Hooft for my present argument is clear. Once more, it is the hypothesis of reducibility, in particular determinism, that suggests an answer to the question of Problem 3: it are precisely the degrees of freedom describing the beginning of the universe that can, on this interpretation, at once dissolve the no-go impediment of Bell's theorem *and* potentially unify GR and QFT (as they should do). Once more, irreducibility seems to offer no answer to *Problem 3*; rather, it seems to leave Bell's theorem as an obstacle to the unification of GR and QFT.

4. Conclusion.

In this article I have argued that the HV-hypothesis, or reducibility, has a greater explanatory power than its competitor, the no-HV hypothesis or irreducibility. Indeed, the HV-hypothesis can coherently answer three questions for which irreducibility remains entirely silent: 1) the interpretation of correlation and Reichenbach's principle; 2) the interpretation of the Central

Limit Theorem of probability theory; and 3) a question related to Bell's theorem and the unification of GR and QFT.

Even if it may well be that we will never know with certainty whether the universe is ultimately deterministic or probabilistic (cf. e.g. Suppes 1993, Wuthrich 2011) – *but what can we know with absolute certainty, in any natural science ?* –, it appeared that of the two possible forms of reducibility, deterministic reduction, in short the hypothesis of determinism, is the most successful one. To the best of my knowledge, only the assumption of deterministic HV allows to mathematically prove and explain the essential features of correlation, in the first place the quantitative part of Reichenbach's principle. This result is coherent with the largely deterministic understanding of probability as emerges e.g. from works as (Pearl 2009). Next, only the assumption of deterministic HV allows a (qualitative) interpretation of the Central Limit Theorem (and, for what it is worth, allows to envisage a proof of this theorem based on the latter interpretation). Last but not least, only the assumption of (super)deterministic[14] HV leads to a generally known solution to Bell's theorem. This solution is not very popular but should be reappraised, I believe, in view of its other heuristic merits just mentioned (note, finally, that determinism also allows to save locality and Reichenbach's principle). The usual practice in (philosophy of) physics is to adopt the principles *that can coherently explain the most facts*. And here it seems that reducibility/determinism wins. The burden of proof lies then with potential opponents to prove that irreducibility/indeterminism explains more. One might object that as long as quantum mechanics has not been shown equivalent to a (local) deterministic theory, indeterminism explains more. But that would be a category mistake, as it opposes a metaphysical principle (or isolated hypothesis, or heuristic principle, perhaps akin to a symmetry principle in physics) to a physical theory; the correct question is: which of the two principles, determinism or indeterminism, has – in principle – the greatest explanatory capacity? On the assumption of (super)determinism, there is no principled objection to the deterministic reduction of quantum mechanics.

The main goals of this article were: 1) to counterbalance the popular view that "hidden variables cannot exist", and 2) to propose the HV-hypothesis, in particular the hypothesis of determinism, as a strong explanatory and heuristic principle, notably one that can guide further research on topics touched upon here.

[14] It has been argued that probabilistic HV could equally well do the job (see e.g. Vervoort (2013)), but this solution is less well-known and less investigated.

Acknowledgements. I would like to thank Christian Wuethrich for discussion and decisive comments on an early draft. Further I thank, for helpful comments, participants to the QIRIF-2019 conference on quantum foundations in Vaxjo, Sweden, as well as Jean-Pierre Marquis, Yvon Gauthier, Mario Bunge, Vesselin Petkov, Yves Gingras, Henry E. Fischer.

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