Nothing but Coincidences: The Point-Coincidence Argument and Einstein's Struggle with the Meaning of Coordinates in Physics

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In his 1916 review paper on general relativity, Einstein made the often-quoted oracular remark that all physical measurements amount to a determination of coincidences, like the coincidence of a pointer with a mark on a scale. This argument, which was meant to express the requirement of general covariance, immediately gained great resonance. Philosophers like Schlick found that it expressed the novelty of general relativity, but the mathematician Kretschmann deemed it as trivial and valid in all spacetime theories. With the relevant exception of the physicists of Leiden (Ehrenfest, Lorentz, de Sitter, and Nordström), who were in epistolary contact with Einstein, the motivations behind the point-coincidence remark were not fully understood. Only at the turn of the 1960s did Bergmann (Einstein's former assistant in Princeton) start to use the term 'coincidence' in a way that was much closer to Einstein's intentions. In the 1980s, Stachel, projecting Bergmann's analysis onto his historical work on Einstein's correspondence, was able to show that what he started to call 'the point-coincidence argument' was nothing but Einstein's answer to the infamous 'hole argument.' The latter has enjoyed enormous popularity in the following decades, reshaping the philosophical debate on spacetime theories. The point-coincidence argument did not receive comparable attention. By reconstructing the history of the argument and its reception, this paper argues that this disparity of treatment is not justified. The paper will also show that the notion that only coincidences are observable in physics marks every critical step of Einstein's struggle with the meaning of coordinates in physics.

Keywords: General Relativity • Point-Coincidence Argument • Hole Argument • Observables • Operationalism

> I was too much a prisoner of the idea that our equations must fully reproduce [...] the relations between the phenomena and the chosen coordinate system, whereas we can be happy if they duly reproduce the mutual relations between the phenomena
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> Lorentz to Ehrenfest, Jan. 11, 1916

Introduction

In one of the opening sections of his 1916 review paper on general relativity, Einstein (1916a, 776–777) made the often-quoted remark that all our measurements amount to the determination of spacetime coincidences, like the coincidence of a pointer with a mark on a scale. Such statements did not entail any reference to coordinates, lending support to Einstein's stance that there is no reason to privilege a coordinate system over another. The sudden appearance of the 'coincidences'-parlance in Einstein's writings gained immediate resonance. Philosophers like Moritz Schlick (1917) found that Einstein's coincidence remark aptly expressed the conceptual novelty of general relativity with respect to previous spacetime theories. On the contrary, the young mathematician Erich Kretschmann (1918) deemed the argument as trivial and actually applicable in all spacetime theories. Kretschmann's objection was quite serious but was initially neglected; on the contrary Schlick's interpretation was essentially off-track but enjoyed a significant success (Giovanelli, 2013). However, in hindsight, despite the fact that both Schlick (Engler and Renn, 2013) and Kretschmann (Howard and Norton, 1993) have been credited for suggesting this argument to Einstein, both readings could not fully grasp the problem that Einstein's remark was trying to address, a problem that remained hidden in Einstein's private correspondence.

Nevertheless, led astray by Einstein's elliptic formulation, philosophers of the most disparate schools, a case of intellectual pareidolia, believed to see a confirmation of their views in Einstein's reduction of all measurements for the assessment of coincidences (Cassirer, 1921, 83; Petzoldt, 1921, 64; cf. Howard, 1991). Not surprisingly, the

argument continued to enjoy certain popularity among philosophers for several decades (e.g., Reichenbach, 1924, 1928; Popper, 1929). On the contrary, although physicists continued to struggle with the physical meaning of coordinates in general relativity (Darrigol, 2015), there is no evidence that Einstein appealed to the observability of coincidences again besides a cursory remark in his 1919 Berlin lectures on general relativity.[1] While he was moving toward the unified field theory-project, the positivist undertones of the language of 'coincidences' had probably become a liability rather than an asset (Giovanelli, 2016, 2018). As far as I can see, it was not until the turn of the 1960s that Peter G. Bergmann (1961), former assistant of Einstein at Princeton, used the language of 'coincidences' in order to address the problem of observables in general relativity.

In the early 1980s, John Stachel (1980), in a famous talk at the University of Jena, East Germany, applied the gist of Bergmann's 'measurability analysis' (Bergmann and Smith, 1982) to his archival work on Einstein's correspondence (cf. Stachel, 1993). Stachel realized that what he started to call the 'point-coincidence argument' was nothing but an attempt to overcome the infamous 'hole argument,' Einstein's refutation of the predictability of a set of field equations holding in any coordinate system. The argument was first mentioned in print by Roberto (Torretti, 1983), but a detailed historical reconstruction was laid down by John Norton (1984, 1987) in his pioneering investigations on Einstein's path to the 1915 field equations. Starting from a classical paper that Norton wrote soon thereafter with John Earman (1987), the 'hole argument' has started to enjoy enormous popularity in philosophical (cf. Rickles, 2007, for an overview) and, to a lesser extent, physical (Stachel, 2014) literature. In particular, it was pivotal for the revival of the substantivalism/relationalism debate that dominated the philosophical discussions in the ensuing decades (Pooley, 2013). By contrast, the point-coincidence argument seemed to have been relegated to the rank of antiquarian curiosity. The literature on this topic has usually focused on philosophers' misunderstanding of Einstein's remark (Ryckman, 1992; Howard, 1999) rather than on the actual impact of the point-coincidence argument on the history of physics. Over a century has now passed since the argument was last used by Einstein. Thus, this might be a good opportunity to try and reassess the meaning of Einstein's declaration that only coincidences are observable in physics, also in light of additional textual evidence that has recently emerged.

The goal of the present paper is to provide a reasonably self-contained and historically accurate overview of the history of the 'point-coincidence argument.' For this purpose, the paper attempts **(a)** to insert some sparse material with which Einstein scholars might already be familiar into a broader and unitary narrative arch. In particular, it will be shown that the appearance of the point-coincidences parlance in Einstein's writings marks all the main stages of Einstein's struggle with the meaning of coordinates in physics: from Einstein's reliance on the idea of a coordinate scaffolding (section 1.1) to its *Aufhebung* within the framework of special relativity (section 1.3); from his recognition of the physical insignificance of coordinates (section 1.3) to his concerns about its consequences for physics' predictability (section 2.1); and from his apparent full acceptance of the immateriality of the coordinate systems (section 4) to the reemergence of never-fading nostalgia for the idea of a physically preferred class of coordinates (section 5.1). Moreover, **(b)** the paper will integrate this material with a recently published correspondence with (section 3.1) and within (section 3.2) the Leiden physics community (Hendrik A. Lorentz, Paul Ehrenfest, Willem de Sitter, Johannes Droste, and Gunnar Nordström)—besides Berlin, the most important center of relativistic studies in Europe at that time.

These documents demonstrate that Einstein's confusion about the meaning of coordinates was not simply an idiosyncratic blunder, but the result of widespread and deep-seated prejudices with which early relativists could not easily dispense. Through the point-coincidence argument, Einstein successfully convinced the Leiden group to recognize that the choice of coordinates was completely arbitrary in general relativity. However, it was the Leiden group that pressured Einstein to resist the sirens of the privileged coordinate system in his search for gravitational wave solutions. It has seldom been noticed that, it was using point-coincidence argument once again in correspondence with Gustav Mie that Einstein could entirely eliminate the last remnant of the materiality of the coordinate system (section 5.2). Soon after that, **(c)** countering Kretschmann's 'triviality objection,' Einstein raised the point-coincidence argument to the status of a fundamental selection principle (section 6). While pre-general-relativistic theories can be always formulated in a way that *more than* spacetime coincidences were observable, general-relativistic theories are singled out by the fact that such formulations are impossible. *Nothing but* coincidences are observable in general relativity.

Thus, this paper will conclude that the point-coincidence argument, far from being a rhetorical trick to elude the hole argument, can be regarded as the starting point and at the same time the mature result of Einstein's 10-year-long struggle with the meaning of coordinates in physics. In particular, I hope that this historical reconstruction will convince the readers that the difficulties that Einstein and his interlocutors encountered should be classified under the somewhat worn-out philosophical category of 'operationalism' (Westman and Sonego, 2007, 2008). Although this terminology might appear as anachronistic to historians and old-fashioned to the philosophers of science, I think that it ultimately grasps the conceptual hurdles of Einstein and his interlocutors better than

[1] This is at least the last instance I was able to find.

the alternatives used in today's philosophical debate, in particular the substantivalism/relationalism opposition. Physicists struggled to understand what it means to express the positions of material points and the values of field quantities as functions of the coordinates when the latter has no direct operational meaning as readings on some physical scaffolding. The point-coincidence argument was nothing but an operationalist response to what was perceived as a failure of operationalism. Classical mechanics has redefined the operational meaning of at 'same place in different times,' special relativity that of 'same time at different places.' One could say that general relativity has forced physics to redefine the operational definition of 'at the same time at the same place.'

Modern relativists are already used to thinking of coordinates as meaningless parameters and find early 'reification' of the coordinate network hard to fathom. In this sense, the idea of 'coordinate scaffolding' can be probably regarded as an instance of what Gaston Bachelard (1938) has called an '*epistemological obstacle*'.[2] These obstacles are not of technical (either mathematical or experimental) nature, but rather of 'conceptual' one. Once they are overcome, it appears surprising that they might have constituted a hindrance in the first place. To capture this discrepancy, this paper makes large use of original texts and private correspondences in which these prejudices are recorded more or less immediately, when the information obtained *post facto*, after the 'obstacle' was removed, was not available. Breaking an epistemological obstacle is ultimately a philosophical act, despite being entangled in the technical details of a theory as complex as general relativity, an act that, in Bachelard's nomenclature, creates an '*epistemological rupture*.' Indeed, Einstein insisted until the end of his life that, minor new predictions aside, the greatest achievement of general relativity was the 'overcoming' of the concept of inertial frame (Einstein to Besso, Aug. 10, 1954; Speziali, 1972, Doc. 210). By showing that physics was still possible without a material reference frame, the point-coincidence argument had a third function, which, moving beyond Bachelard's terminology, might be called an '*epistemological reconciliation*,' the reestablishment of continuity after the rupture. Previous theories aimed to predict the position of point particles with respect to material scaffolding. However, at closer inspection, what these theories were actually predicting has always been the reciprocal coincidences of two physical systems. Ultimately, had the theory been sufficiently powerful, it would have been able to describe the dynamics of both systems conjointly (Giovanelli, 2014).

Modern presentations of relativity theory start at the end of this process when nothing but a structureless manifold of points is left. They attempt to grasp the nature of actual physical spacetime by adding further geometrical structure (topological, differential, affine, metric, etc.) without ever making reference to coordinates (Earman, 1989; Friedman, 1983). From the perspective of this coordinate-free approach, many of the problems discussed in this paper might appear technically trivial (Weatherall, 2016). However, for the historically minded philosopher of science, it is within the very process of 'becoming trivial' that most relevant conceptual issues are hidden. As a matter of fact, the history of physics has actually moved in the opposite direction (Norton, 2002). It started by introducing coordinates as reading on a material rods-and-clocks network, and it was subsequently pressured to progressively deprive it from any unnecessary structure. These two opposite strategies, the additive and subtractive strategies as they might be called (Norton, 1999), complement each other. Often in the history of physics, one needs to first enlarge the space of possibilities (the discovery of non-Euclidean geometries, non-Riemannian geometries, etc.) in order to be able to restrict it once again. As is seldom noticed, the point-coincidence argument played a central role in every relevant step of the process of liberation of physics from the prison of the coordinate grid. At every step of the process, it was thinking of measurements in terms of 'coincidences' that allowed Einstein to chip away yet another "rest of the materiality (*physikalischer Gegenständlichkeit*)" of space and time (Einstein, 1916a, 776).

1 The Materiality of the Coordinate System: A Proto-Version of the Point-Coincidence Argument

1.1 Coordinates as Readings on Rods and Clocks

In January 1911, when Einstein received a call to Prague (Illy, 1979) and was on the verge of leaving the University of Zurich, he was invited to give a farewell lecture on relativity at the local *Naturforschende Gesellschaft* (Einstein, 1911b). This lecture, in which Einstein used the term 'Relativity Theory' in a title for the first time, did not contain anything significantly new as far as physics was concerned. Thus, Einstein was uncomfortable with allowing its publication (Einstein to Schröter, Dec. 11, 1910; CPAE, Vol. 5, Doc. 237). However, this turned out to be a good opportunity to make some simple epistemological considerations. In particular, Einstein offered a pedagogical account of the role of the coordinate system in physics, presenting it as an extension of practices common in everyday experience. In a world with the electromagnetic field but without gravitational field, objects that carry no electrical charge move in straight lines, the strength of various forces in terms of how much they cause charged objects (objects subject to the forces) to deviate from uniform, straight-line motion. Of course, this claim is meaningless if the positions of the points are referred to an arbitrarily moving coordinate system (e.g., one

[2]Here, I use Bachelard's terminology with some freedom.

undergoing arbitrary rotation). When does it mean, however, to describe the position of a material point relative to a coordinate system?

> In mathematical physics, it is customary to relate things to coordinate systems [...] What is essential in this relating-to-something is the following: when we state anything whatsoever about the location of a point, we always indicate the *coincidence* of this point with some point of a specific other physical system. If, for example, I choose myself as this material point, and say, 'I am at this location in this hall,' then I have brought myself into spatial coincidence with a certain point of this hall, or rather, I have asserted this *coincidence*. This is done in mathematical physics by using three numbers, the so-called coordinates, to indicate with which points of the rigid system, called the coordinate system, the point whose location is to be described *coincides* (Einstein, 1911b, 2; my emphasis).

At this point, the use of the term 'coincidence' is still an isolated instance in Einstein's writings. Einstein seems to have introduced it merely to illustrate that, in his view, 'space' did not play for the physicist the role of an independent metaphysical entity but is embodied in a concrete, material scaffolding of some sort.

Ultimately, Einstein pointed out, to establish *where* an object is in space means to specify of the 'coincidence' of a point of the object with the point of another physical system, a '*body of reference*' (*Bezugkörper*); it might be the surface of the Earth, the city of Zurich, or the walls of the hall in which Einstein was giving his lecture. To communicate which point of the hall one is referring to, one might place there a material body of sufficiently small dimensions, say the podium of the speaker, as a 'label' that makes the point recognizable. However, if one wants to locate a point in the empty space over the podium, no such marks are available. Then, one might erect a bar perpendicular to the podium, one end of which coincides with the podium and the other with the point in question. The length of the bar, measured with a unit rod, allows marking the point in question so as to communicate its position to others. Proceeding in this way, one can progressively discard the specific characteristics and even the presence of a particular rigid body (the Earth, the hall, the podium, etc.).

As a 'body of reference,' one might choose three mutually perpendicular rigid material lines issuing from one point, the origin of the coordinate system, which define three planes. We construct perpendiculars to the coordinate planes (fig. 1) and count how many times a given unit measuring rod can be laid along these perpendiculars and mark off the values of such counting on the scaffolding. Wherever a point may be located, one can always think of a rigid rectangular system of unit rods that eventually coincide with the point under consideration. The position of a point can then univocally identified by means of three numbers, the so-called 'coordinates' x, y, z, that are the results of measurements made by rigid measuring rods, that is rods having a sequence of marks at regular intervals. The numbering system thus obtained is called a 'Cartesian coordinate system.' We will designate coordinate systems with the letter K.

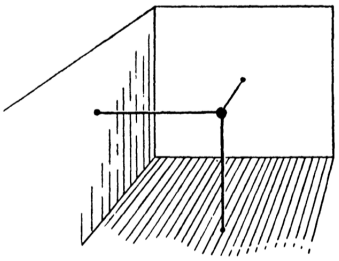

Figure 1 – From Einstein and Infeld, 1938, 212

If one wants to describe the motion of the material point, one needs to give the values of x, y, z as three functions of time, that is one needs to introduce a fourth parameter t. A point is at rest relatively to our reference body if these three functions are constant. To define the physical meaning of t, one might place clocks at each rods' intersections on the coordinate system K. A clock is a system which undergoes a physical process passing periodically through identical phases, as well as some kind of counter, e.g., a fixed-numbered dial. The numerical values of t correspond to the number of such oscillations marked-off by the displacement of the clock hands on the dial. We can predict the position, at any time, of a body falling parallel to the y-axis ($x = \text{const}, x = \text{const}$), and confirm our prediction by observation. If a graduated rod is placed beside y-axis, we can establish with what mark y on the rod the falling body will coincide at any given moment when the hand of a clock will coincide with the mark t. One might, e.g., predict that the coincidence of the particle with the earth is characterized $t = 4$ and by the $y = 0$.

As one can see, Einstein appears to have conceived physical space and time that corresponds to the mathematical space and time as fully 'arithmetized.' A point in space is *nothing but* a triple of numbers x, y, z, and instant of time is nothing but a number t. It becomes possible to treat spatial and temporal relations in a purely algebraic manner. The choice of the parameters x, y, z, t is, in principle, arbitrary. If the geometry of space is Euclidean, although one can resort to other numbering systems (polar, cylindrical coordinates, etc.),[3] Cartesian coordinates x, y, z are singled out by the fact that, given a unit of measure, coordinate numbers directly mean distances from the origin. Formally it means that it is advisable to chose the coordinate numbers so that the distance ds between any two arbitrarily close points x, y, z and $x + dx, y + dy, z + dz$ satisfy the condition

[3]Cf. below footnote 6.

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{1}$$

for every orientation of the interval ds. If, for instance, one chooses a different point of origin, or orients the coordinate axis in different directions, the same point will be every point is assigned a different set of coordinates numbers x, y, z. However, the distance ds between any to close points will be expressed by the same eq. (1). Such substitution of variables are called 'coordinate transformations,' and the equations relating them might be called 'transformation equations.' Euclidean *Geometry* is concerned with the substitutions of space coordinates x, y, z; *kinematics* considers coordinate transformations involving the time variable t as well.

What Einstein used the call the *geometric configuration* of a three-dimensional body with respect to K (Einstein, 1908, 439, 1910a, 28, 1911e, 510) is the totality of distances s between any pair of its points x, y, z as calculated using eq. (1). The *geometry of the rigid body* is the set of substitutions of x, y, z that lets the geometric configuration of a body unchanged.[4] All Cartesian coordinate systems K, K', K'' related by such coordinate transformations are geometrically equivalent. All one can say about them is that each Cartesian coordinate system has a different position or orientation with respect to the other systems and vice-versa. In an analogous way, Einstein labeled the *kinematic configuration* of a three-dimensional body with respect to K (Einstein, 1908, 439, 1910a, 28, 1911a, 510) the totality of the distances between any two of its points x, y, z at *the same time* t as calculated using eq. (1). The *kinematics of the rigid body* is the set of substitutions of the coordinates x, y, z, t that let the kinematics configuration of a body unchanged. All Cartesian coordinate systems K, K', K'' related by such transformations are kinematically equivalent. All one can say about them is that each Cartesian coordinate system is moving with respect to the other systems and vice-versa. Since it was tacitly assumed that $t = t' = t'', \ldots$, the geometric and kinematic configurations of a three-dimensional body are the same in all kinematically equivalent coordinate systems.[5]

However, the kinematic equivalence of the Cartesian coordinate systems in arbitrary relative motion does not necessarily hold from a dynamical point of view. The special role that the variables x, y, z, t play in physics is the consequence of the fact that all other dynamical variables (e.g., the electric and magnetic field vectors **E** and **B**) appearing in the laws of nature (Newton's laws, Maxwell equations, etc.) are expressed mathematically as single-valued functions of the coordinates. The coordinates x, y, z, t should be so chosen so that the functional relations between the kinematic and the dynamic variables can be presented in the simplest form. If one expresses, mathematically a dynamical law relating the dynamical variables in such form with respect to K, one can obtain the mathematical expression of such law in a different kinematically equivalent Cartesian coordinate system $K', K'', K''', \ldots$ by substituting the variables x, y, z, t measured from K with x', y', z', t' measured from K'. A rule which expresses the components of a certain quantity at a point in terms of the components of the same quantity at the same point with respect to another coordinate system is called a 'transformation law.' If the two expressions for the laws of nature in K and K' are identical—that is if the dynamical variables are the same functions of the new coordinates—K and K' can be considered equally 'good' from the dynamical point of view. There are 'good' and 'bad' Cartesian coordinate systems in relative motion from a dynamical point of view.

In particular, it could be shown that, if Newton's laws of motion for point particles can be written in the most simple form with respect to a coordinate system K, they can be written in the same form only in the coordinate systems $K', K'', K''', \ldots$ that are in uniform translational motion with respect to K. Such coordinate systems are single out by the fact that, if zero net forces act on a moving particle, its space coordinates x, y, z K change as a linear function of t. *True forces* cause charged particles to deviate from this 'standard' path. Although other coordinate systems can be used, only in such coordinate systems, coordinates x, y, z, t can be considered as the result of measurements made with 'good' rods and clocks. If one describes the motion of the same force-free particles with respect to a Cartesian coordinate system K' coinciding with K at $t = 0$, but rotating around the z-axis, the particle's coordinates x, y, z appear to change as nonlinear functions of t and extra terms depending on ω had to be added to Newton's laws. Multiplied for the mass of particles can be interpreted as *fictitious forces* deflecting *all* free particles from their uniform motion. Rods and clocks at rest in these systems do not reliably measure coordinates differences. The set of transformation equations relating mechanically equivalent coordinate systems were named by Philipp Frank (1908, 192) 'Galilean transformations.' The Galilean transformations constitute the algebraic expression of the Newtonian *kinematics of the rigid body in uniform parallel translation.*

As is well known, Maxwell electrodynamics is not compatible with this kinematics. It seemed to be possible to write Maxwell equations equations in their simplest form only with respect to *one* Cartesian coordinate system K, the coordinate system with respect to which the velocity of light is c, but not with respect to all other coordinate systems moving differently. A coordinate scaffolding is necessary not only for the description of locations but also for the representation of vectors, like the electric and magnetic vector fields $\mathbf{E}(x, y, z)$ and $\mathbf{B}(x, y, z)$. At each the

[4] E.g. the geometric configuration of a sphere of radius r is give by the equation $x^2 + y^2 + x^2 = r^2$. A sphere with respect to K is a sphere with respect all other Cartesian coordinate systems however positioned or oriented.

[5] A sphere $x^2 + y^2 + x^2 = r^2$ with respect to K will be a sphere with respect to all coordinate systems moving with respect to K.

point x, y, z at the time t one can decompose each field into in its three components $\mathbf{E}(E_x, E_y, E_z)$ and $\mathbf{B}(B_x, B_y, B_z)$ along the three axis of K. Once one has established where x, y, z is physically located, one can bring a suitable measuring device in coincidence with that point. It might be a spring balance, so graduated to measure force in an unaccelerated system, provided with an electric charge. The acceleration experienced by the charge in the x-direction will bring a pointer on a dial to coincide with a mark; the number one read is the value component of E_x of the electric field at x, y, z at the time t. A similar procedure can be used to measure **B**. One can then check whether the values of **E** and **B** predicted by Maxwell equations at x, y, z, t correspond to the measured ones. If one carries out the transformation laws of the field vectors **E** and **B** with respect to the Galilean transformations, one can calculate the components of **E** and **B** with respect to a moving system K'. The transformed quantities do not obey Maxwell equations in the same form, with respect to the new coordinate system K', that is **E** and **B** are the same functions of the new coordinates. The fields $\mathbf{E}(E_x, E_y, E_z)$ and $\mathbf{B}(B_x, B_y, B_z)$ measured with respect to the rest coordinate system K represented the 'real' states of the ether; the transformed field values $\mathbf{E}'(E'_x, E'_y, E'_z)$ and $\mathbf{B}'(B'_x, B'_y, B'_z)$ respect to a coordinate system K' moving in uniform translations with respect to K were deemed as calculation tools.

1.2 Questioning the Physical Meaning of Coordinates

However, this theoretical asymmetry between the rigid ether system K and the system K' moving in parallel translation with respect to it had no physical counterpart. Einstein (1905) realized that the apparent inconsistency between mechanics and electrodynamics could be removed by challenging an unanalyzed prejudice implicit in the Newtonian kinematics of the rigid body in parallel uniform translation. As is well known, by reflecting upon the way distant clocks are synchronized using light signals, Einstein realized that there is no *a priori* reason to assume that clocks at rest with respect to the rest system K measure the same numerical values of t as clocks at rest with respect to moving system K'. The result of rods-and-clocks measurements in two different coordinate system will assign not only different numbers to x, y, z, but also different numerical values to the time coordinate t. This result reshaped the traditional relations between the geometry and kinematics of the rigid body. "We obtain the shape of a body moving relative to the system K with respect to K' by finding the points of K' with which the material points of the moving body *coincide at a certain time t* of K'" (Einstein, 1911e, 510). Thus the assertion about the shape of a moving body acquire quite a complicated meaning since this shape can be ascertained only with the aid of determinations of time. The *geometric configuration* of a rigid body with respect to rest system K is not necessarily identical to its *kinematic configuration* with respect to moving K' as it was in it was tacitly assume in classical kinematics (Einstein, 1910a, 28)

A rigid body is defined thereby with respect to a coordinate system K, in turn the coordinate system K itself must be regarded as rigid body. As we have seen, the laws of mechanics, as well as those of electrodynamics, need a rigid body of reference for their formulation. Therefore, Einstein's introduction of new kinematics of the rigid body implies at the same time a redefinition of counts as a rigid coordinate system K. This redefinition had a sort of trickle-down effect. Einstein arrived at a set of linear transformation equations, the famous Lorentz transformations, relating the numerical values of the coordinates x, y, z, t of a point in K and the values x', y', z', t' that the same point in K' satisfying the condition that the velocity of light in vacuum is expressed by the same numerical value c in the two systems. Once we interpret coordinates x, y, z, t as measurable with identical rods and clocks (which have the same length and run at the same rate when compared with each other at relative rest), the introduction of the Lorentz transformations are not simply a conventional step, but implies predictions concerning the actual behavior of existing physical systems, predictions that can be experimentally verified or falsified, independently of any dynamical laws. As a consequence, they could be elevated to a *criterion* to judge well-establish dynamical laws (Einstein, 1910a, 136), and *heuristic* tool to find new ones (Einstein, 1911c, 13). As it turned out, Maxwell equations happen to already satisfy this condition. The vectors $\mathbf{E}(E_x, E_y, E_z)$ and $\mathbf{B}(E_x, E_y, E_z)$ play the same role in Maxwell equations with respect to K as the vectors $\mathbf{E}'(E'_x, E'_y, E'_z)$ and $\mathbf{B}'(E'_x, E'_y, E'_z)$ play in Maxwell equations referred to K'. Newton's law of motion had to be modified.

The impact of relativity theory on our conception of space and time was indeed deep. Classical mechanics revealed that 'the same place in different times' makes no sense unless a rigid 'body of reference' is pointed out; relativity theory forced us to recognize that that 'at the same time in different places' also depends on the 'body of reference.' Nevertheless, 'where' and 'when' something occur were always meant as a physical 'coincidences' with some material, non-accelerated 'body of reference.' Relativity theory was ultimately only changed our definition of what counts as as 'good' 'rigid coordinate scaffolding' K. As is well known, Einstein (1908) felt that the theoretical asymmetry between a 'good' acceleration-free $K(x, y, z, t)$ and the 'bad' coordinate system $K'(x', y', z', t')$ accelerating with respect to the former was at least questionable from an epistemological point of view. Thus, he sought a more encompassing set of transformation equations that would include Cartesian coordinate systems that are accelerating (and rotating) relative to each other. However, Einstein soon started to realize that the usual definition of coordinates in terms of cubical scaffolding of *rigidly connected* rods and clocks

had become problematic in uniformly accelerating and rotating coordinate systems.

As we have seen, relativity theory was ultimately conceived by Einstein as a new kinematics of the rigid body *in uniform parallel translation.* However, Einstein immediately started to suspect that the development of full relativistic rigid body kinematics, that would include accelerations and rotations, might have led to unexpected consequences (Einstein, 1907a,b). Indeed, Max Born (1909) soon showed that in relativity the kinematic definition of a three-dimensional rigid body, even in the simple case of linear uniform acceleration was a delicate matter. Rigidity, which was a kinematic 'property' of a three-dimensional body in the classical theory, turned to be a dynamic 'instruction' of how to apply forces to different points of a body so that the reciprocal distances between the body's points remain constant in the comoving frame. Paul Ehrenfest (1909) immediately pointed out that a 'Born rigid body' could not be put into rotation (Einstein, 1911e). Einstein sensed that the question of the rigid body in special relativity might have significant implications for the very definition of the coordinates in accelerated frames (Maltese and Orlando, 1995; Stachel, 1989).

After some years of work on the quantum problem (Einstein, 1909a,b, 1910b,c), in June 1911, while in Prague, Einstein (1911b) got back to the problem of gravitation. Einstein (1908) had singled out the well-known empirical fact that the ratio between gravitational 'charge' and inertial mass is constant, differently from, say, the ratio of electric charge and inertial mass. Making a particle more massive does not make it more resistant to gravitational attraction since it also increases its gravitational charge. Thus it become impossible to single out a 'good' unaccelerated coordinate system in which force-free particles move uniformly on straight lines. The same reference system can be interpreted, with equal justice, as a 'good' coordinate system K at rest in which there is a (homogeneous) gravitational field directed toward the $-z$-axis (with gravitational acceleration γ) or as a noninertial frame K' accelerating in the z-direction (with acceleration $-\gamma$). "If we accept this assumption," Einstein wrote, "we obtain a principle that possesses great heuristic significance" (Einstein, 1911d, 900).

It becomes possible to transfer the results obtained in an accelerated system to a system with a gravitational field. If a clock in an inertial system K reads time t, by a coordinate transformation one can infer that the same clock would read time $t' = t(1 + \gamma\xi/c^2)$ in a coordinate system K' in uniform acceleration with respect to K. However, such a system is indistinguishable from a system at rest in a gravitational field. Thus, one can predict that the rate of an identical clock t' would be $1 + \Phi/c^2$ (where $\Phi = \xi\gamma$ is the gravitational potential) slower if it is placed in an inertial system K at rest in a gravitational field. A clock at rest in the gravitational t field. Respect, to K' not only all free particles, but also light rays no longer travels along straight lines, because its x'-coordinate is no longer a linear function of t, etc.; thus the theory predicts that one should observe the same in a gravitational field. In general, if one knows the laws of nature (electrodynamics, thermodynamics, etc.) with respect to a gravitation-free system K, then one can, by a mere coordinate transformation, derive the laws relative to K' and, thus, the behavior of such system in a gravitational field.

In February and March 1912, Einstein (1912a, 1912c) published two further papers attempting to find the gravitational field equations for a theory of the static gravitational field with a variable velocity of light. Einstein expressed for the first time in a published paper his concerns regarding the physical significance that one can attribute to coordinates in an accelerating system. He considered an unaccelerated rigid framework of rods K, together with a set of suitably synchronized clocks at rest at each point of the framework. He then introduced a system $K'(x', y', z', t')$ accelerating relative to inertial systems $K(x, y, z, t)$. Einstein specified that K' is linearly accelerated with respect to K in "Born's sense" (Einstein, 1912a, 356), that is, in such a way that undergoes a rigid motion, in spite of not being a rigid body (Born, 1909). In such a system, differences of spatial coordinates x, y, z can still be measured by Euclidean rods at relative rest with respect to K'; however, one has to abandon the assumption that clocks directly measure the time coordinate t.

However, a Born rigid body cannot be put into rotation without stresses, that is will experience deformations which will be different for different materials. Therefore the rigid scaffolding used classical mechanics and relativity theory cannot be used as models for an accelerated frame of reference in the theory of gravitation. Euclidean geometry "most probably does not hold in a uniformly rotating system" (Einstein, 1912a, 356; cf. Kaluza, 1910). If the coordinate system $K'(x', y', z')$ rotates around the axis $z = z'$ respect to $K(x, y, z)$, a rigid unit rod with the same length in every position and every orientation would not directly measure spatial coordinate differences x', y', z'. Nevertheless, Einstein provisionally considered the interpretation of the coordinate system as a rigid material scaffolding as "permitted despite the fact that, according to the theory of relativity, the rigid body cannot possess real existence" (Einstein, 1912a, 356). In general, stresses always emerge if the motion is non-Bornian (Herglotz, 1911). However, at least in the case of linear acceleration, following Born's instructions, one can imagine that "the rigid measuring body being replaced by a great number of nonrigid bodies arranged in a row in such a manner that they do not exert any pressure on each other in that each is supported separately" (Einstein, 1912a, 356). This way, since distances do not change for the comoving observer, one can still resort to the direct definition of spatial coordinates. In a subsequent paper that was received in March (Einstein, 1912c), Einstein investigated the influence of the static gravitational field on the electromagnetic field.

He started from Maxwell equations that hold relative to a nonaccelerated system $K(x, y, z, t)$. "The physical meaning of the quantities appearing in these equations is a perfectly determinate one" (Einstein, 1912c, 150), he wrote. In such a system, x, y, z are measured by measuring rods laid along the rigid system K and t is measured by identically constituted clocks arranged at rest at the points of the system K. The vector fields **E** and **B** might be measured by spring balance and so on. One can obtain the form Maxwell equations in a uniformly accelerated system $K'(x, y, z, t)$ (in Born's sense), by applying the transformation laws for **E** and **B** corresponding to the coordinate transformations to a system accelerating along the x-axis, so that $x' = x + \mathbf{a}c/2t^2$. According to the equivalence hypothesis, these transformed equations also determined how the electromagnetic processes occur in a gravitational field. In principle, if this theory must have an empirical content, the transformed quantities should still have a defined physical meaning (Einstein, 1912b, 446). For example, one can measure the components of $\mathbf{E}'$ with respect to K' with a 'pocket' spring balance that can be transported at the point x', y', z' in K' with different gravitational potentials and check whether the predictions of the theory concerning the components of **E** and **B** are correct (Einstein, 1912b, 447).

This provisional compromise did not last long. Einstein soon realized that, in the general case of nonuniform fields, the statement of the principle of equivalence "can be valid *only for infinitely small fields*"; thus, a "Born's accel. *finite* system cannot be considered a static gravitational field" (Einstein to Ehrenfest, Jun. 20, 1912; CPAE, Vol. 5, Doc. 409), as Einstein initially thought. In a further paper (Einstein, 1912b), by addressing some objections of Max Abraham (1912a), Einstein started to realize that the role of coordinates in physics as readings on rods and clocks has to be completely abandoned. Following Abraham (1912a, 1912b), Einstein implemented for the first time Minkowski's (1909) formalism, in which the three-dimensional kinematics was translated into four-dimensional geometry. The space coordinates x, y, z and the time coordinate t were treated on equal footing. In analogy with eq. (1), the distance ds between two spacetime points can be expressed as functions of the spacetime coordinates as:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \,, \tag{2}$$

where c is variable $c(x, y, z)$ and plays the role of a scalar potential of the gravitational field. The equivalence principle opens up the possibility that the equations of the relativity theory that would also include gravitation may also be invariant with respect to acceleration and rotation transformations, that is, to nonlinear transformations. Einstein might have realized that, just like the coefficient c of the time coordinate is not constant in a linearly accelerating system, transforming the line element eq. (2) to rotating Cartesian spatial coordinates depends on coefficients that involve the angular frequency ω. This dependency has the consequence that also spatial coordinates do not directly correspond to rods-and-clocks readings. Already in a static gravitational field, Einstein commented that "*the spacetime coordinates will lose their simple physical meaning*" (Einstein, 1912b, 1064; my emphasis).

Einstein, as most physicists, considered the very notion of a massive rigid 'coordinate scaffolding' an indefeasible condition for the possibility of physics. When one claims 'a particle will be at x, y, z at t,' the latter numbers must be translated into something like 'the particle coincides with the southeast corner of the lab at 5 o'clock.' If an interpretation of this sort is not possible, the prediction does not seem to have any verifiable meaning. However, the 'materiality' of the coordinate scaffolding was put in serious danger by his theory of gravitation. This tension is clearly recognizable in a little known remark appended to a manuscript on special relativity on which Einstein worked in Prague and Zurich starting from 1912 (Rowe, 2008):

> One might ask whether such fastidious physical definitions for the time and space coordinates are really necessary, that is whether it is really necessary to burden the gentle and airy concepts of space and time with heavy rigid bodies and clocks. In my opinion, it is not *necessary*, but it is *advantageous* to do so. *It is also possible to regard the x, y, z, t as mere mathematical variables (parameters), which have only a meaning in facilitating the formulation of physical laws.* The laws formulated with the help of such parameters have then only in so far content as these parameters can be eliminated from several of them. This method of treatment does not alter the law by introducing arbitrary functions of these variables in the same way as x, y, z, t. *Our definitions for these coordinates can be regarded as a convenient method of elimination.* However, I believe that the considerations and definitions given here are sufficient for space and time only *as long as one forgoes the introduction of gravitation into the system of relativity* (Einstein, 1912–1914, ; fn. 60; my emphasis).

Already in this early passage, Einstein stated clearly that there are two possible ways to consider coordinates: (a) coordinates have physical significance. Coordinate numbers also inform us of the measurable space and time intervals; (b) coordinates are just parameters, catalog numbers that serve only to distinguish a point from another (Westman and Sonego, 2007). As Einstein conceded (cf. also Einstein, 1910a, 25-26; fn. 1), all physical theories, including mechanics and electrodynamics, in principle allow for both interpretations of the coordinates. It is possible to express the equations of physics using polar or cylindrical coordinates or other

arbitrary parametrizations. However, dynamical laws cannot be expressed in the most simple form in such systems. As it turns out, only those coordinate systems for which these laws assume their simplest form happen to be directly related to space and time measurements made by rods and clocks (a). The coordinate system can be replaced by the material coordinate scaffolding. Using a has a clear advantage over (b). If one follows (a), expressions like $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$ are not just a mathematical functions, but physically significant quantities.

The readings of rods and clocks at rest in a good coordinate system K give us x, y, z, t. At that point, one can place a detector, say a 'pocket' spring balance, which gives us the components of $\mathbf{E}$ and $\mathbf{E}$ with respect to a nonaccelerated coordinate system K. Using a different, say, accelerating, coordinate system K' will lead to different sets of components $\mathbf{E}'(x', y', z', t')$ and $\mathbf{B}'(x', y', z', t')$ which might be useful in performing some calculations. One must always get back to the 'good' nonaccelerated system K once the problem is solved. Only with respect to K the electromagnetic field can be split into $\mathbf{E}$ and $\mathbf{B}$, and those fields fields in their components in physically meaningful way that is accessible to measurement. The coordinate scaffolding can in fact be shielded from the effect of the electromagnetic field (e.g., by making it more massive). Thus, one can always recognize K as the 'good' non-accelerating frame with a sufficient degree of approximation, whereas K' is not. In this context, the use of rods and clocks could be regarded as an effective way to 'deparametrize' the theory. As Einstein seemed to allude at the end of the passage just quoted, the universal nature of gravitation made this last step impossible. A nonaccelerated reference system K behaves in the presence of a homogeneous gravitational field in the same way as if it were a suitably chosen accelerated reference frame K'. It becomes impossible to establish whether K is a 'good' coordinate in which nonaccelerated rods and clocks at rest reliably read coordinate differences or a 'bad' one K' accelerating in the opposite direction, in which coordinate numbers do not directly mean distances. Thus, one lacks a direct procedure to 'eliminate the parameters' and substitute them with meaningful numbers.

1.3 Coordinates as Physically Meaningless Parameters

By 1912, Einstein had reached the worrying conclusion that the meaning of coordinates as directly measurable with rods and clocks had to be abandoned in a theory of gravitation. The further steps from a scalar to a tensor theory of gravity are usually reconstructed relying on Einstein's later recollections (Einstein, 1923, 1933) but remain little documented by direct textual evidence. Einstein, at some point, while still in Prague, must have seen the analogy between Minkowski's line element $ds^2 = dx_1 + dx_2 + dx_3 + dx_4$ (setting $x_4 = -ict$) in an accelerated or rotational system and the line element in Gauss's theory of surfaces, the quadratic differential form:

$$ds^2 = Edp^2 + Fdpq + Gdq^2 \,. \tag{3}$$

Einstein might have learned about it in Carl Friedrich's Geiser's (1897–1898) lectures that he had attended at the ETH (Reich, 1992, 163ff.). In Gauss's approach, one can cover a curved surface using two families of nonintersecting curves so that, within each family, each curve is distinguished by the parameters p ($p = 1$, $p = 2$, etc.) and q ($q = 1$, $q = 2$, etc.). If these two families are thought to be infinitely dense, one can in principle label any point on the surface through their intersections $p = \text{const}$ and $q = \text{const}$. For example, the point (5, 3) is where the coordinate lines $p = 5, q = 3$ intersect. If one is dealing with a plane surface, one can always introduce a regular network in which, given a unit of measure, these numbers directly represent actual lengths from the origin $p = 0, q = 0$ using eq. (1). However, in the general case, one needs to know the three coefficients E, F, and G at every point (p, q) in order to convert small coordinate differences dq and dp into actual distances ds on the surface.[6] Different surfaces are represented by different quadratic forms; however, the opposite does not apply; the same surface can be covered with different grids of lines. In order to decide whether two quadratic forms determine the same surface, one needs to calculate the Gaussian curvature from E, F, and G and its first and second derivatives. Nevertheless, Einstein did not seem to be interested in eq. (3) inasmuch as it defines the intrinsic geometrical structure of the

[6] In Cartesian coordinates, once the origin and a unit of measure have been fixed, these numbers also mean distances. Thus, the small distance ds between any two neighboring points can be directly calculated from the difference between their coordinates $ds^2 = dq^2 + dp^2 (E = 1, F = 0, G = 1)$. This is not the case in other coordinate systems, like polar coordinates $p = r$, $q = \vartheta$ on a flat surface, so that line element takes the form $ds^2 = dr^2 + r^2 d\vartheta^2$ ($E = 1$, $F = 0$, $G = r^2 \sin^2$). Equal increments in radial distance dr correspond to real distances $ds^2 = dr^2$ from the center; however, small increments in angular coordinates $d\vartheta$ correspond to $1/\sqrt{G}$ real distances. On a flat surface, one can always switch back to Cartesian coordinates, in which the parameters p, q have a direct metrical meaning. However, this is not true in the general case. The fact that a sphere cannot be projected onto a plane without distortion can be expressed analytically by the fact that it is impossible to convert the quadratic differential form that holds on a sphere into the one holding on a place through a mere transformation of the independent variables. On a sphere, it is suitable to introduce coordinates $p = \vartheta$ and $q = \varphi$, which are the customary latitude (counted from the equator) and longitude (counted from the meridian of Greenwich) measured in radians. Constant angular increments $d\vartheta$ do not mean constant distance increments ds. Increments ds are related to coordinate increments by $ds^2 = R^2 d\vartheta^2 + R^2 \cos^2 \vartheta d\varphi^2$, where R is the radius of the globe that can be set equal to 1 ($E = 1$, $G = \cos^2 \vartheta$, $F = 0$). While differences in longitude $d\vartheta$ correspond to actual distances ds, differences in latitude $d\varphi$ correspond to $1/\sqrt{G}$ real differences ds. The distance between any two arbitrarily distant points can be measured by the minimal number of links of a chain of ds joining the two points.

surface, but because it offers a sort of 'algorithm' to convert meaningless parameters into measurable distances.[7]

Getting back to Zurich in August 1912, with the help of Marcel Grossmann (Sauer, 2013), Einstein became familiar with Ricci-Curbastro and Levi-Civita's 'absolute differential calculus' (Levi-Civita and Ricci-Curbastro, 1900; cf. Reich, 1994), which had set up a general method to deal with quadratic differential forms, like Gauss's line element. At the same time, he realized the fundamental importance of the 'world tensor' $T_{\mu\nu}$ to represent the source of the gravitational field (Norton, 1994). Just like someone who, knowing only electrostatic and magnetostatic phenomena, attempts to develop a field theory encompassing all electromagnetic phenomena, Einstein aimed to develop a field theory of gravitation starting only from Poisson's equations valid for static fields and slow-moving particles. The Zurich Notebook (CPAE, Vol. 4, Doc. 10) documents how Einstein was able to move from Newton's scalar theory of gravitation to one based on a quadratic differential form:

$$ds^2 = \sum G_{\mu\nu} dx_\mu dx_\nu \,, \tag{4}$$

where ds is a scalar and $G_{\mu\nu}$ (which was later written as $g_{\mu\nu}$) is a tensor that can be written as a matrix with four rows and four columns, or 16 components in total, where μ and ν range from 1 to 4. The number of independent components reduce to 10 if $g_{\mu\nu} = g_{\nu\mu}$. Einstein's struggles (Renn and Sauer, 2007) finally led to a two-part 'Outline' of a theory of relativity and gravitation coauthored with Grossmann, which was published in June 1913 (Einstein and Grossmann, 1913).

In the physics part of the *Entwurf* paper, Einstein formulated the equivalence principle in the following way. Let us introduce a system of reference $K(x, y, z, t)$ in a sufficiently small region of spacetime. With respect to this frame, a body moves uniformly in a straight line according to the equation $\delta(\int ds) = 0$, whereas

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx^2 \,,$$

where the coefficient c in $x_4 = -ict$ is the constant velocity of light. Einstein conceived a transformation from inertial K to accelerated frame K' as a coordinate transformation. Thus, if one describes the motion of a body from the perspective of a system $K'(x', y', z', t')$ moving with acceleration a in the direction along x, one has to introduce new coordinates x', y', z', t', which are nonlinear functions of x, y, z, t. The simplest transformationleads to the line element

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + x_1^2 a^2 x_4^2 \,.$$

In this system, all free objects and light rays appear to be uniformly accelerated in a direction opposite that of the acceleration of K' so that their free motion $\delta(\int ds) = 0$ will appear as curvilinear with respect to K.

In Minkowski's formalism, the difference between K and K' is easy to spot. In K, given an appropriate choice of the coordinates, the matrix $g_{\mu\nu}$ has constant values, with $-1, 1, 1$ and c^2 as diagonal terms, and all other terms are equal to 0. In K', $g_{\mu\nu}$ are functions of the coordinates; in particular, $g_{44} = a^2 x_1^2$ is a function of x_1. In pre-general-relativistic physics, the Cartesian coordinate system K was regarded as a 'good' coordinate system, with respect to which light rays and free particles move on straight lines; the non-Cartesian system K' was regarded as a 'bad' coordinate system (as the appearance of fictitious forces reveals), and the curvilinear paths of light rays and free particles with respect to K' had no physical meaning. The equivalence principle

[7] Cartography provides an excellent analogy (Janssen, 1998) to which we will return throughout the paper. Indeed, most of the confusions about coordinates analyzed in this paper make sense if we make the hypothesis that physicists thought in terms of map projections, rather than in terms of the intrinsic structure of surfaces. Let us say that one wants to draw a flat map of the Earth. If the difference in scale is reduced to the difference of the choice of the unit of measure, the problem of a perfect map is expressed by the demand that one should be able to establish a one-to-one correspondence between the two surfaces such that ds^2 of a sphere and ds^2 of the map coincide in corresponding points. One can map a sufficiently small portion of the globe, say a city, without distortions on a flat paper (or onto a surface developable into a plane, like a cylinder or a cone) covered by a Cartesian coordinate grid x, y. If one measures the coordinate difference dx between two streets on the map on the vertical x-axis ($dy = 0$) with a ruler, the distance measured will correspond to the actual distance between the two cities on the Earth (up to a difference in units). The same procedure is not possible for maps of larger portions of the globe in which distortions necessarily occur. In translating latitude and longitude into Cartesian coordinates by some projection function $x = f(\vartheta, \varphi)$, $y = f(\vartheta, \varphi)$, one can see that equal coordinate differences on the map do not correspond to equal distances on the surface. In order to recover real distances ds between two close cities on the Earth from their coordinates on the map x, y and $x + dx, y + dy$, one needs to know the three functions E, F, G at every point of the map. Let us consider a cylindrical projection. It can be shown that, in such projection, $E = \left(1 - \frac{y^2}{R^2}\right)$, $G = \left(1 - \frac{y^2}{R^2}\right)^{-1}$, $F = 0$. The functions E, F, G are the 'legend' of the map. If one measures equal distances $dx = 1$ along the x-axis on the map, one will get real distances on the globe $ds = \sqrt{E}dp$, which are $\neq 1$. The results become more distorted toward the poles. In a stereographic projection, the map legend is different: $E' = G' = \left(1 + \frac{x^2}{R^2} + \frac{y^2}{R^2}\right)^{-2} F'$. As one can see, the same coordinate difference $dx = 1$ measured on the projection plane is multiplied by a different factor $ds = \sqrt{E'}dx$. Therefore, the same coordinate differences dx and dy might correspond to vastly different real differences on the globe, depending on the projection used. Thus, without knowing the 'map legend'—that is, the functions E, F, G, it is impossible to predict the actual distance ds between two close cities from the small coordinate differences dx and dy of their coordinates on the map (cf. footnote 9). Needless to say, the relation is actually reversed the same ds corresponds to different dx, dy of its endpoints in different maps projections; dx, dy are meaningless.

eliminates the asymmetry between 'good' and 'bad' coordinate systems. A linearly accelerated coordinate system K' is indistinguishable from an inertial system K with a gravitational field. Thus, Einstein drew the following conclusion. The variability of $g_{\mu\nu}$ with respect to the coordinates x_1, x_2, x_3, x_4, which mathematically tells us that we are using non-Cartesian coordinates, could be identified physically in the presence of a homogeneous gravitational field.

This suggested that the components $g_{\mu\nu}$ could take the place of the single Newtonian gravitational potential φ also in the general case, in which non-constant $g_{\mu\nu}$ cannot be transformed away by a simple coordinate transformation. Einstein arrived at the view that the gravitational field is characterized by 10 spacetime functions $g_{\mu\nu}$, which acquire a double meaning. On the one hand, $g_{\mu\nu}$ are the potential of a physical field, comparable to the electromagnetic field, which assumes particular values at certain points identified by their coordinates x_ν. At the same time, $g_{\mu\nu}$ are geometrical quantities, conversion factors that relate coordinate differences dx_ν and real distances ds according to the formula eq. (4). Consequently, the direct relationships between the coordinate differences x_1, x_2, x_3, x_4 and rods-and-clocks measurements is lost:

> From the foregoing, one can already infer that there cannot exist relationships between the space-time coordinates x_1, x_2, x_3, x_4 and the results of measurements obtainable by means of measuring rods and clocks that would be as simple as those in the old relativity theory. With regard to time, this has already found to be true in the case of the static gravitational field. The question therefore arises, what is the physical meaning (measurability in principle) of the coordinates x_1, x_2, x_3, x_4 [...] From this one sees that, for given dx_1, dx_2, dx_3, dx_4, the natural distance that corresponds to these differentials can be determined only if one knows the quantities $g_{\mu\nu}$ that determine the gravitational field. This can also be expressed in the following way: the gravitational field influences the measuring bodies and clocks in a determinate manner (Einstein and Grossmann, 1913, 8).

In previous theories, there was a privileged class of rectangular coordinate systems in which, knowing the difference dx_1, dx_2, dx_3, dx_4 between the coordinates of any two points, one could predict the distance ds between them as it would be measured by nonaccelerated rods and clocks, and vice versa. In the *Entwurf*-Theory, by contrast, once one knows the coordinates of two spacetime points x_1, x_2, x_3, x_4 and $x_1 + dx_1, x_2 + dx_2, x_3 + dx_3, x_4 + dx_4$, without knowing the functions $g_{\mu\nu}$, one cannot predict the rods-and-clocks distance ds. However, in order to determine $g_{\mu\nu}$, one must already know the measured distance ds^2 between two points of coordinates x_1, x_2, x_3, x_4 and $x_1 + dx_1, x_2 + dx_2, x_3 + dx_3, x_4 + dx_4$. There seems to be a circle. Einstein famously broke the circle by relying on the fact that, in a sufficiently small region of spacetime, one can always introduce a coordinate system in which $g_{\mu\nu}$ can be taken to be constant in the first approximation, that is, in which special relativity holds.[8]

One can switch from the given arbitrary coordinate system to a rectangular coordinate system and determine the length ds^2 using rods and clocks at relative rest. This is called the 'natural' four-dimensional interval, as opposed to the 'coordinate' interval. This natural length is, by definition, a scalar and can be set equal to ± 1, up to an arbitrary choice of unit, and directly measured with rods and clocks. One can set, say, n_1 spacings between the atoms of a rock-salt crystal as $= -1$ and n_2 wave crests emitted by a cadmium atom as $= 1$. One can then switch back to the originally given coordinate system. Since the numerical value of ds^2 per definition does not change, from eq. (4), one can read off $g_{\mu\nu}$ for known ds^2 and dx_ν. Over larger regions of spacetime, it might be impossible to introduce a coordinate system in which coordinate differences mean actual distances. In the general case, $g_{\mu\nu}$ can be measured as the numbers by which coordinate differences have to be multiplied so that $ds^2 = 1$ in every position and in every orientation. For example, a unit rod laid along the x_1-axis ($dx_2 = 0, dx_3 = 0, dx_4 = 0$) will measure $-1 = \sqrt{g_{11}}dx_1$; a unit clock at rest will measure $1 = \sqrt{g_{44}}dx_4$. This measurement procedure is meaningful under the condition that the ratio n_1/n_2 is *not* influenced by the gravitational field so that equally constructed unit rods and unit clocks always measure the same $\pm ds = 1$. If this is the case, we can determine $g_{11} = ds^2/dx_1^2$, $g_{44} = ds^2/dx_4^2$ and in general all values of the $g_{\mu\nu}$.

The key to Einstein's success in the electrodynamics of moving bodies was his careful analysis of the procedure to which physics, using rods and clocks, gives physical meaning to kinematic variables *before* any dynamics. However, the introduction of gravitation forced Einstein to 'unlearn' precisely the strategy which had led to him striking success a decade earlier (Stachel, 1993). Rods and clocks, the same instruments that serve to measure distances and time intervals, also serve to measure the gravitational field $g_{\mu\nu}$. Since the quantities $g_{\mu\nu}$ enter into this relationship, the coordinates in themselves have no independent physical meaning and are reduced to mere numbers. The fact that coordinate parameters beside serving as labels also measure distances turned out to be an accident. As Einstein soon came to realize, the kinematic variables x_1, x_2, x_3, x_4 and the dynamical variables $g_{\mu\nu}$ became entangled into an inextricable knot. As any other field theory, general relativity aims to predict the components of the gravitational field $g_{\mu\nu}$ at a point x_1, x_2, x_3, x_4. However, $g_{\mu\nu}$ are also the 'measurement' field that allows calculating the distance of x_1, x_2, x_3, x_4 with respect to any other point. Einstein struggled for several

[8] This is usually taken as a stronger version of the equivalence principle. For the different meanings of the equivalence principle, cf. Lehmkuhl, 2019.

years to make peace with the fact that one does not know where x_1, x_2, x_3, x_4 is *before* the $g_{\mu\nu}$ field is introduced. However, as we shall see, Einstein would need at least three more years to realize that the location of x_1, x_2, x_3, x_4 is lost again *after* the $g_{\mu\nu}$ field has been removed.[9]

2 The (Nearly) Last Remnant of Materiality of Coordinate System

2.1 From the Besso-Einstein Argument to §12 Argument

In the *Entwurf* paper, Einstein and Grossmann (1913) were able to show that the equation of motion of a mass point in a gravitational field could be written in a coordinate-independent way, without introducing an otherwise nonaccessible privileged class of coordinate systems. However, they were not able to put forward a set of field equations with these covariance properties. In the Zurich notebook, we can follow in great detail Einstein's tentative search for of a set of differential equations connecting the source $T_{\mu\nu}$ and $\Gamma_{\mu\nu}$ with a second-rank tensor of the second-order quantities $g_{\mu\nu}$:

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu} \,.$$

The static limit of these equations was supposed to yield Poisson's equation with a single potential, as the static limit of Maxwell's theory yields the Coulomb law. In other terms, the field equations must be reduced to the case in which only the component g_{44} of the gravitational field is variable, that is, a function of spatial Euclidean coordinates. $\Gamma_{44} = -\Delta\gamma_{44}$. To obtain this result, Einstein started from an expression in which $\Gamma_{\mu\nu}$ is a two-index contraction of the Riemann tensor.[10] These equations hold in all coordinate systems, whereas Poisson's equation does not. Thus, Einstein needed to impose some coordinate requirement to eliminate unwanted terms and let only a single potential. In particular, as a modern relativist would do (Norton, 1984), he considered the requirement [11]

$$g^{\mu\nu} \left\{ \begin{array}{c} \tau \\ \mu\nu \end{array} \right\} = 0 \,. \tag{5}$$

These kinds of coordinate requirements, besides ensuring that the field equations reduce to their Newtonian limit, were also supposed to guarantee energy-momentum conservation, at the same time leaving enough freedom to include accelerating systems. Einstein seemed to consider coordinate requirements of this sort as a general *restriction* (Janssen and Renn, 2007) on class of allowable coordinate system introduced once and for all and not simply as a condition imposed provisionally to solve a particular problem.[12] Therefore, all attempts to proceed in this way lead to considerable difficulties. Switching from the mathematical to the physical strategy (Janssen and Renn, 2007), Einstein decided to settle with a set of field equations valid for arbitrary but linear transformations. Thus, ultimately, Einstein and Grossmann were not able to cast the 'Entwurf'-field equations in a form that would allow for arbitrary substitutions of the independent variables. Nevertheless, Einstein was convinced that he was on the right track. "The conviction to which I have slowly struggled through is *that there are no preferred coordinate systems of any kind*", Einstein wrote to Ehrenfest. "However, I have only partially succeeded, even formally, in reaching this standpoint" (Einstein to Ehrenfest, May 28, 1913; CPAE, Vol. 5, Doc. 441).

In June 1913, just around the time the *Entwurf*-Theory appeared in print, Einstein, with the help of Besso, had already started applying the new theory to calculate the gravitational field of the Sun using a weak field approximation, as documented in a manuscript found among Besso's papers after his death (CPAE, Vol. 4, Doc. 14). Einstein and Besso aimed to calculate deviations $\gamma_{\mu\nu}$ from Minkowski's values $\delta_{\mu\nu}$ so that $g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu}$. Thus, Einstein believed that it is possible to obtain a good approximation by taking these deviations into consideration, along with their derivatives, only where they appear linearly. Imposing the condition 5, the field equations take the following form:

$$\Box\gamma_{\mu\nu} = \frac{\partial^2\gamma_{\mu\nu}}{\partial x^2} + \frac{\partial^2\gamma_{\mu\nu}}{\partial y^2} + \frac{\partial^2\gamma_{\mu\nu}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\gamma_{\mu\nu}}{\partial t^2} = \kappa T_{\mu\nu} \,, \tag{6}$$

[9] As a comparison, one can think of the coordinates x, y on a map, the legend of which has become unreadable. If one knows that, according to a map, a treasure is buried at $x = 45$, $y = 85$ in the middle of the ocean (where no other 'marks' are available if not coordinate numbers), there would be no way to find the latitude and longitude p, q of the point on the Earth. The numbers x, y are meaningless without the map legend. Thus, independently of the coefficients E, F, and G, the point x, y on the projection plane is, so to speak, nowhere (cf. footnote 7). In general relativity, difficulties emerge from the fact that $g_{\mu\nu}$ are both the map legend (the metric tensor) and the treasure to be found (the gravitational field). Cf. footnote 13.

[10] The so-called Ricci tensor; cf. e.g. Janssen and Renn, 2015a for a recent account of Einstein's path toward field equations.

[11] This is called the harmonic coordinate condition.

[12] The editors of Renn et al., 2007 have introduced the illuminating distinction between 'coordinate conditions' and 'coordinate restrictions.' I will use 'coordinate requirements' to indicate the general categories, including both coordinate conditions and restrictions (cf. footnote 16). For a different interpretation, cf. Norton, 2005.

where $\Box$ is the d'Alembertian operator and $T_{\mu\nu}$ is the matter tensor for an incoherent mass flow; the field is treated as a static field, with slow particle motion, and $g_{\mu\nu}$ are assumed to vanish at infinity. $g_{\mu\nu}$ with μ and ν ranging from 1 to 3 were expected to be constant $= -1$, so at a certain instant in time $dx_4 = 0$, a rod would directly measure spatial coordinates and will not be distorted by the Newtonian gravitational field. By contrast, a clock at rest in such coordinate system $dx_1 = dx_2 = dx_3 = 0$ measures $d\tau^2 = g_{44}dx_4^2$; that is, the greater the masses arrayed in its vicinity are, the slower the clock runs. The value of the component g_{44} was calculated as an approximate solution of the *Entwurf* equations in first- and second-order approximations. Once one knows g_{44}, one can calculate the path of light rays and the planets' orbits. However, Einstein and Besso were not able to account for Mercury's perihelion anomaly. Thus, these calculations were never published (Janssen, 2003).

For our limited purposes, this manuscript is interesting for the following considerations. On [p. 1], Besso writes the matrix of the covariant values of $g_{\mu\nu}$ in first-order approximation:

$$g_{\mu\nu} = -1 \quad (\mu, \nu = 1 - 3) \qquad g_{44} = \frac{1}{c_0^2}\left(1 + \frac{A}{r}\right) \quad \text{where} \quad A = \frac{\kappa M}{4\pi}\,. \tag{7}$$

A remark in Besso's hand seems to reveal that Einstein and Besso might have become aware of the problem of the uniqueness of the solution they had obtained, as a remark in Besso's hand reveals: "Is the static gravitational field in [eq. (7)] $g_{\mu\nu} = 1$ to 3 $g_{44} = f(x, y, z)$ a particular one? Or is it the general one expressed in spec. coordinates?" (CPAE, Vol. 4, Doc. 14[p. 16]). Thus, Besso seems to have been somehow puzzled by the fact that $g_{\mu\nu}$ are determined only up to a coordinate transformation, that is, that one introduces different values of the components of the gravitational field by switching to a different coordinate system. For a weak field on a background Cartesian coordinate system, one can introduce an infinitesimal coordinate transformation $x_\nu + \xi_\nu$ (where ξ_ν is similar in size to $\gamma_{\mu\nu}$) and obtain different values for $g'_{\mu\nu} = \delta_{\mu\nu} + \gamma'_{\mu\nu}$. On the contrary, Einstein and Besso probably expected that, given initial and boundary conditions, the field equations determine $g_{44}(x, y, z)$ unambiguously (i.e., as a single-valued function), just like Poisson's equation determine the gravitational potential $\phi(x, y, z)$ in a Cartesian coordinate system.

Besso left Zurich for Gorizia and got back to Zurich around August 1913. Probably during this second visit, he jotted down some notes (the so-called Besso memo) about a discussion that he had with Einstein (Janssen, 2007). These notes seem to contain an objection against the very possibility of general covariance of such equations. Besso argued that, in the case of a central mass, such as the Sun, surrounded by empty space, due to the arbitrary choice of the coordinate system, the field equations (together with boundary conditions imposed at infinity) do not guarantee a unique determinable of $g_{\mu\nu}$ in the empty region. Besso's notes seem to reproduce a dialog between him and Einstein:

> [Besso:] The requirement of [general] covariance of the gravitational equations under arbitrary transformations cannot be imposed: if all matter [is given] were contained in one part of space and for this part of space a coordinate system [is given], then outside of it the coordinate system could still [essentially] except for boundary conditions be chosen arbitrarily, [through which the arbitrarily] so that a unique determinability of the g's cannot be obtained
>
> [Besso:] It is, however, not necessary that the themselves are determined uniquely, only the observable phenomena in the gravitation space, e.g., the motion of a material point, must be.
>
> [Einstein:] Of no use, since with ⟨the⟩ a solution a motion is also fully given. If in coordinate system 1, there is a solution K_1, then this same construct is also a solution in 2, K_2; K_2 however, also a solution in 1 (Besso Memo p. 2; quoted and translated in Janssen, 2007).

As Besso pointed out, once one calculates a solution $g_{\mu\nu} = K_1$, one could obtain a different solution $g'_{\mu\nu} = K_2$ by simply introducing a new coordinate system 2. In a generally covariant theory, this solution is just as good as the first since a coordinate system 2 is just as good as 1. Besso suggested that this might not be a problem. It is not necessary that the field equations determine $g_{\mu\nu}$ in a unique way; it is only necessary that the theory correctly predicts the observable phenomena, for example, that tomorrow morning, when the Sun is five degrees over the horizon, Venus will be visible at 12 degrees over the horizon (Rovelli, 2008). In the second part of the quote, Besso probably wrote down Einstein's critique of Besso's counterargument: the new solution K_2 in the coordinate system 2 'is also a solution in 1,' that is, the components of the solution K_2 can be calculated as functions of the 1-coordinate numbers. Thus, under the given initial and boundary conditions, the *Entwurf* field equations do not determine the gravitational field univocally with respect to the same coordinate system 1.

From June to early July, Ehrenfest and his wife came to Zurich from Leiden, where they discussed Einstein's new theory (Yavelov, 2002) together with the Finnish physicist Gunnar Nordström (1912, 1913a, 1913b), who had presented an alternative theory of gravity, which Einstein considered as a serious alternative to the *Entwurf*-Theory. As a letter to Lorentz in August 1913 revealed, at that time, Einstein was still unsatisfied with the field equations of

limited covariance. It seemed questionable that all equations of physics could be formulated without reference to a specific coordinate system, except the field equations that regulate the behavior of $g_{\mu\nu}$. Thus, the theory was like "hanging in the air" (Einstein to Lorentz, Aug. 14, 1913; CPAE, Vol. 5, Doc. 467). At some point, Einstein might have realized that his reply to Besso's argument could turn out to be useful. The discovery of the Besso memo (Janssen, 2007) revealed that the argument, far from being a refined reflection of the nature of spacetime, was more of a 'the Fox and the Grapes' kind of argument. The generally covariant field equations that he was not able to find were after all not desirable in the first place. Field equations that are valid in all coordinate systems do not uniquely determine the gravitational field (Einstein to Lorentz, Aug. 16, 1913; CPAE, Vol. 5, Doc. 470).

In the printed summary of a lecture delivered in September of 1913 at the 96th annual meeting of the *Schweizerische Naturforschende Gesellschaft* in Frauenfeld, Einstein pointed out for the first time that it is logically impossible to introduce generally covariant field equations (Einstein, 1913a, 8). In a footnote of the printed version of a lecture delivered at the *Gesellschaft Deutscher Naturforscher und Ärzte* in Vienna a few weeks later, Einstein already hinted that "in the last days" he had found "a proof that such a generally covariant solution to the problem cannot exist at all" (Einstein, 1913b, 1257; fn. 2). Since the lecture was published in December, it is hard to say whether this remark was added later. Only in November did Einstein explain to Ludwig Hopf (Einstein to Hopf, Nov. 2, 1913; CPAE, Vol. 5, Doc. 480) that he was now satisfied with his field equations of limited covariance, since generally covariant field equations were actually impossible. As Einstein wrote more explicitly to Ehrenfest in the ensuing weeks, "a unique determination [*eindeutige Bestimmung*] of the $g_{\mu\nu}$ out of the $T_{\mu\nu}$" was not possible without a special choice of the coordinate system. According to Einstein, this was "rigorously provable" (Einstein to Ehrenfest, Nov. 15, 1913; CPAE, Vol. 5, Doc. 484). Thus, as he wrote to Ehrenfest, the spacetime variables were completely arbitrary, but one can use the conservation laws for energy and momentum to restrict the admissible coordinate transformations to the linear ones (Einstein to Ehrenfest, Nov. 15, 1913; CPAE, Vol. 5, Doc. 484). The 'good' coordinate systems are, so to speak, tailored to the physical world instead of being given *a priori* (Einstein to Mach, Dec. 15, 1913; CPAE, Vol. 5, Doc. 497). At that time, the Leiden community started to become one of Einstein's most important 'sounding boards.' Einstein was even thinking about hiring Adriaan Fokker or Johannes Droste as assistants since they both showed "Lorentz's excellent training" (Einstein to Ehrenfest, Nov. 15, 1913; CPAE, Vol. 5, Doc. 484).

As is well known, Einstein would reproduce the argument against general covariance in published writings on four occasions (Einstein and Grossmann, 1914a; Einstein, 1914b; Einstein and Grossmann, 1914b; Einstein, 1914a). Starting from the first published on January 30, 1914, in some 'Comments' added to the reprint of the *Entwurf* paper (Einstein and Grossmann, 1914a), to avoid imposing arbitrary additional boundary conditions at infinity, Einstein might have inverted Besso's argument (Janssen, 2007). Instead of an insular matter distribution surrounded by an empty space, Einstein introduced an empty region L (which might stand for *Loch*, hole) surrounded by matter. In this region, no material process occurs (no electromagnetic field, no particles, etc.); that is, $T_{\mu\nu}$ vanishes. In general, points in space are identifiable using some material element that one can use to 'mark' the place in question. However, in empty space, points can be identified only by their coordinate numbers .[13] Given a solution $g_{\mu\nu}$ of the field equations within L, the general covariance of the equations allows us to introduce a new coordinate system x'_μ so that $x_\nu = x'_\mu$ at the boundary of L, but it is $x_\nu \neq x'_\mu$ inside of it (Einstein and Grossmann, 1914a, 260). One now relates everything to this new primed system, in which matter outside of L is represented by $T'_{\mu\nu} = T_{\mu\nu}$ [14] and the gravitational field inside of L by $g'_{\mu\nu} \neq g_{\mu\nu}$. Thus,

$$T'_{\mu\nu} = T_{\mu\nu} \qquad \text{outside of } L$$

does not imply

$$g'_{\mu\nu} = g_{\mu\nu} \qquad \text{inside of } L.$$

In other terms, it is possible that "*more than one* system of $[g_{\mu\nu}]$ pertains to the system $[T_{\mu\nu}]$" (Einstein and Grossmann, 1914a, 260; my emphasis). In order to achieve a *unique* determination of $g_{\mu\nu}$ (gravitational field) by $T_{\mu\nu}$ (matter), Einstein concluded that one has to restrict the choice of the coordinate system.

Einstein's argument against general covariance of the field equations appeared for quite a long time based on a trivial misunderstanding. Of course, it is not surprising that the components $g_{\mu\nu}$ change in a definite manner as

[13] As we have seen, relativists might be compared with someone who wants to find a treasure in the open ocean, where no marks (e.g., church towers, mountain peaks, or other prominent points) other than coordinate numbers are available (cf. footnote 9). Before solving the field equations, physicists are in possession of a map with the treasure's coordinates $x = 45$, $x = 85$ but do not know the map's legend and cannot calculate its latitude p and longitude p on Earth. The peculiarity of Einstein's theory of gravitation is that the legend of the map (the metric tensor) is, so to speak, hidden in the treasure chest (it is a solution of the field equations). The theory aims to find the values of the coefficients E, F, G at x, y in the open ocean (the treasure); however, knowing E, F, G (the map legend) as functions of x, y is the precondition for physically locating y, x (i.e., to calculate its actual p and q). This is the root of all sorts of confusions if one thinks in terms of the map and not in terms of the intrinsic geometrical structure of the Earth.

[14] The coordinate system has remained unchanged outside of L as well as the components of the matter tensor.

the coordinates x_ν are changed to x'_ν, for example, by switching from Cartesian to polar coordinates and from polar to cylindrical coordinates and so on (Hoffmann, 1982). However, the point that Einstein wanted to make appears to have been more subtle (Stachel, 1980). This is revealed by a remark added to a footnote. Einstein pointed out that "the independent variables x_ν" must be attributed to "*the same numerical values of* x'_ν" (Einstein, 1914b, 178; fn. ; my emphasis). This amounts to the same maneuver introduced in the Besso memo: the new solution $g'_{\mu\nu}$, obtained by switching to the coordinate system x'_ν, is evaluated with respect to the old coordinate system x_ν, that is, $g'_{\mu\nu}(x_\nu)$. It is important to emphasize that Einstein talked about the *numerical values* of variables. As we have seen, in Einstein's and most physicists' view at that time, the numbers x_ν do not simply serve to label points; the coordinate values x_ν *are* the points (cf., e.g., Study, 1914, 82,91). Thus the same numerical value x_ν with respect to the same coordinate system K is the *same point*. Switching from a coordinate system to another is nothing but switching from a set of numbers x_ν to another set of numbers x'_ν so that x'_ν is some smooth function of x_ν. In the generally covariant theory, all substitutions are allowed.

As we have seen, these numbers are meaningless. One needs to know $g_{\mu\nu}$ to be able to compute the relative distances of any point x_ν within the hole with respect to each material point x_ν on the boundary of the hole. Letting x_ν outside of the hole remain unchanged, one can introduce a new coordinate system x'_ν within the hole and thus a new corresponding set of $g'_{\mu\nu}(x'_\nu)$. One might also calculate the new numerical values of the coefficients $g'_{\mu\nu}$ as functions of the old coordinate values x_ν. If the field equations are generally covariant, this will be an equally admissible solution. Thus, the field equations seem to predict two different sets of $g_{\mu\nu}$ components at the *same* point x_ν, that is, the point at the same relative distances from the material points x_ν on the border of the hole.[15] In principle, one could apply the same reasoning to a non-generally covariant theory of gravitation, but the predictability issue would not emerge.

In fact, Einstein did not raise the Besso-Einstein argument against Nordström's scalar theory of gravitation (cf. Stachel, 1987), which, at that time, he considered as a good alternative to the *Entwurf*-Theory. In a paper coauthored by Fokker, Einstein presented Nordström's theory in a generally covariant form (Einstein and Fokker, 1914). However, the theory presupposed the existence of a 'good' coordinate system in which the velocity of light is constant. In such coordinate system, the diagonal $g_{\mu\nu}$ have the constant value $-1/\Phi^2, 1/\Phi^2$. Thus, $\pm 1/\Phi$ is the factor by which coordinate times or coordinate lengths must be multiplied to obtain the naturally measured times and lengths. Consider the solution of Nordström's theory $g_{\mu\nu}(x_\nu) = \mathrm{diag}(-1/\Phi^2, 1/\Phi^2)$ in empty space, where x_ν is a such a good coordinate system; mathematically, we are free to introduce a new coordinate system $g'_{\mu\nu}(x'_\nu)$, where $g_{\mu\nu} \neq \mathrm{diag}(-1/\Phi^2, 1/\Phi^2)$ and the velocity of light is not constant. However, physically, this is not a 'good' coordinate system. Thus, *a fortiori*, the new $g'_{\mu\nu}$ and the old x_ν are also not an allowable solution.

When Einstein left Zurich for Berlin in March 1914, he had fully convinced himself that a generally covariant theory was not desirable in the first place, since it violates the "condition that the fundamental tensor $g_{\mu\nu}$ should be completely determined" by the gravitation equations (Einstein and Grossmann, 1914b, 216). In a paper with Grossmann, submitted in May, Einstein derived the field equations from a variational principle (Einstein and Grossmann, 1914b) and convinced himself that the form Lagrangian was uniquely fixed by the energy and momentum conservations. Einstein results were summarized in the first systematic review of the *Entwurf*-Theory that was presented in October of 1914 before the Prussian Academy of Sciences (Einstein, 1914a) in Berlin. In §12 of this paper, Einstein reformulated more clearly the 'proof' of a 'necessary restriction' in the choice of coordinates. Given the matter distribution $T_{\mu\nu} \neq 0$ outside of an empty region Σ within which $T_{\mu\nu} = 0$, the field equations determine the quantities $g_{\mu\nu}$ as functions of x_ν relative to the coordinate system K. Even if $g_{\mu\nu}$ and their first partial derivatives $\partial g_{\mu\nu}/\partial x_\nu$ are given on the boundary of Σ, we can still change the coordinates inside the region and thus obtain a different solution. Einstein used the nonstandard notation $G(x)$ to indicate these functions. His argument runs as follows:

> Let us introduce a new coordinate system K' which coincides with K outside of Σ, but deviates from K inside of Σ. such, however, that the $g_{\mu\nu}$' relative to K' as well as the $g_{\mu\nu}$ (including their derivatives) are everywhere continuous. The totality of the $g'_{\mu\nu}$ is symbolically denoted by $G'(x')$, $G'(x')$ and $G(x)$ describe the same gravitational field. When we replace the coordinates x'_ν by the coordinates x_ν in the functions $g'_{\mu\nu}$, i.e., when we form $G'(x)$, then this $G'(x)$ also represents a gravitational field relative to K, which however, is not the same field as the factual (that is, the originally given) gravitational field. If we assume the differential equations of

[15] A simplified version of the argument in terms of geographical maps might be the following. We want to calculate the values of the components $E(x, y)$, $F(x, y)$, $G(x, y)$ at the point $x = 45, y = 85$ in the middle of the ocean where no other marks are present (cf. footnote 9). If we know that the map is, say, a stereographic projection (E, F, G as in footnote 7), we can calculate the values of the components $E(45, 85)$, $F(45, 85)$, $G(45, 85)$. Let us switch to a cylindrical projection (E', F', G' as in footnote 7); the correspondent calculations $E'(45, 85)$, $F'(45, 85)$, $G'(45, 85)$ would deliver three different numbers. Thus, cartography attributes two different sets of components of E, F, G and E', F', G' at the same point $x = 45$, $y = 85$. Cartography cannot predict the values of E, F, G univocally. The predictability of cartography can be saved once one realizes that the Cartesian coordinate numbers $x = 45$, $y = 85$ on the projection plane correspond to very different physical points p, q on the Earth depending on whether the calculations are made using the stereographic E, F, G or the cylindrical E', F', G'.

> the gravitational field to be everywhere covariant, then they are satisfied for $G'(x')$ relative to K' whenever they are satisfied for $G(x)$ relative to K. Therefore, they are also satisfied for $G'(x)$ relative to K. There are then two different solutions $G(x)$ and $G'(x)$ relative to K, even though the solutions coincide on the boundary of the domain Σ (Einstein, 1914a, 1067).

I have reported here this very well-known formulation of the argument since Einstein and his interlocutors repeatedly referred to 'the §12-argument.' Thus, it is useful to have it at hand. The two-step scheme $G(x) \rightarrow G'(x') \rightarrow G'(x)$ described in this passage would indeed become from now on the standard formulation of Einstein's argument against general covariance. It shows that generally covariant field equations seem to attribute different values $G(x)$ and $G'(x)$ of the gravitational potentials to the same x_ν within Σ with respect to the same coordinate system K. As a matter of fact, if one considers the four numbers x_ν as being physically the same point of K, throughout a three-step calculation, the theory cannot predict the components of the gravitational potential at that point unequivocally. Thus, in Einstein's view, the argument provided a valid reason to introduce specialization in the choice of coordinates. Einstein imposed four non-covariant coordinate requirements written compactly as $B_\mu = 0$ that guarantee the validity of the conservation laws (cf. Janssen and Renn, 2015b, for more details). These relations were to hold in all those coordinate systems that Einstein called 'adapted' to a given gravitational field. In this way, Einstein was able to restrict the range of allowable $g_{\mu\nu}$-system inside of Σ and at the same time to leave it sufficiently large to comply with requirement of the equivalence principle (cf. Abraham, 1914, 514).

2.2 Embracing the Mathematical Overdetermination of the Field Equations

Nevertheless, the *Entwurf*-Theory started to spark considerable interest. Lorentz insinuated that Einstein's use of 'adapted' coordinates was nothing but the introduction of privileged coordinate systems from the backdoor (Lorentz to Einstein, Jan. 1, 2015; CPAE, Vol. 8, Doc. 43).[16] Nevertheless, he derived the *Entwurf* field equations from Hamilton's principle (Lorentz, 1915). Moreover, he instructed his student Droste to calculate the gravitational field of a single mass point (Droste, 1914–1915) and two spherical fixed centers from the *Entwurf* equations (Droste, 1915). After his Wolfskehl lectures (CPAE, Vol. 6, appendix B) in Göttingen, which started to attract David Hilbert's interest in the theory, Einstein exchanged few letters with the Göttingen physicist Paul Hertz and tried to convince him that his argument against general covariance was analogous to the case of two surfaces that are 'developable' into each other and thus geometrically identical (cf. Howard and Norton, 1993). However, Einstein still considered Hertz's requirement "'that the world be able to developable upon itself'" atrocious (Einstein to Hertz, Aug. 14, 1915; CPAE, Vol. 8, Doc. 108).[17] A few months later, his confidence in the *Entwurf*-Theory started to deteriorate (CPAE, Vol. 8, Doc. 123), (CPAE, Vol. 8, Doc. 129). Einstein was lost again in the "chaos of possibilities" and started to reexamine the problem from the ground up (Einstein to Hertz, Aug. 22, 1915; CPAE, Vol. 8, Doc. 111).

On November 11, 1915, Einstein returned to a set of generally vacuum field equations that he had introduced in 1913 (Einstein, 1915c):

$$R_{\mu\nu} = \kappa T_{\mu\nu}\,,$$

where $R_{\mu\nu}$ is what we now call the Ricci tensor. He replaced the restriction imposed by the conservation laws with the requirement that the determinant of the metric satisfied the condition $\sqrt{-g} = 1$ (Einstein, 1915d). Covariance with respect to transformations that preserve this condition is sufficiently broad to include transformations between inertial and accelerated coordinate systems. However, g is a constant only if the scalar (the sum of the diagonal terms[18]) of the energy tensor of matter $T_{\mu\nu}$ vanishes, $\Sigma T^\mu_\nu = T = 0$, as in the case of the electromagnetic stress-energy tensor (Einstein, 1915e). This way, Einstein arrived at field equations that are equivalent to generally covariant ones but specialized by the requirement $\sqrt{-g} = 1$. If the determinant g of the metric tensor is -1, these

[16] Einstein tried to convince Lorentz that this was not the case (Einstein to Lorentz, Jan. 23, 2015; CPAE, Vol. 8, Doc. 47). The situation, he claimed, was similar to the theory of surfaces. If one says that the line element is only allowed to take the form $ds^2 = dp^2 + dq^2$, then one will indeed say that the surface is flat. However, if one imposes the restriction $ds^2 = E(p, q)dp^2 + G(p, q)dq^2$, then one will say nothing about the nature of the surface. $F = 0$ only imposes the choice of orthogonal coordinates. Spherical polar coordinates on a sphere and Cartesian coordinates on a plane are both orthogonal. Thus, Einstein did not think in terms of the intrinsic geometry of the surface, but in terms of the 'good' coordinate system in which the field equations are valid. The requirement $F = 0$ does not fix the coordinate system uniquely as Cartesian inertial coordinates in pre-general-relativistic theories. However, the fact that the restriction $F = 0$ is imposed once and for all is another instance of Einstein's confusion between 'coordinate restrictions' and modern 'coordinate conditions.' cf. footnote 12. This confusion will persist for at least two years after the final version of general relativity; cf. section 5.1 and section 5.2.

[17] This shows, again, that Einstein did not think in terms of the intrinsic geometry of the surface, but in terms of the relations between meaningless coordinate numbers and real distances, that is, in terms of distortions performed by different map projections. Using different map projections, one can obviously calculate the values of E', F', G' and E, F, G as functions of the same numbers $x = 35$, $y = 45$; however, $x = 35$, $y = 85$ is not the *same* physical point in both cases, the point with the same p and q on Earth. What Hertz tried in vain to explain to Einstein was that all this is completely irrelevant. All these 'maps' represent the same intrinsic properties of the surface. All intrinsic information, like what are the great circles, where they intersect, and so forth, is the same in all mappings, and it is preserved when the surface (or part of it) is bent without stretching.

[18] What we would now call the 'trace'.

field equations (with the Ricci tensor as a gravitational tensor) can be written in terms of the Christoffel symbols $\Gamma^{\tau}_{\mu\nu}$, which are now interpreted as the components of the gravitational field. For the vacuum field equations,

$$\sum_{a} \frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}} + \sum_{\alpha\beta} \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\upsilon a} = 0\,, \tag{8}$$

where the determinant of the metric is

$$\left|g_{\mu\nu}\right| = -1\,. \tag{9}$$

On November 18, Einstein reported to the Prussian Academy of Sciences an approximate static spherically symmetric solution to these truncated field equations, which explained the anomalous advance of the perihelion of Mercury and the slight rotation of its elliptic orbit in addition to its elliptic motion. Before proceeding to solve the equations, he introduced the following remarks:

> A point mass, the sun, is located at the origin of the coordinate system. The gravitational field this point mass produces can be calculated from these equations by means of successive approximations. Nevertheless, we should consider that the $g_{\mu\nu}$ are still not completely determined mathematically by the equations [eq. (8)] and [eq. (9)], because these equations are covariant with respect to arbitrary transformations of determinant 1. Yet we are justified in assuming that all these solutions can be reduced to one another by such transformations that they are distinguished (by the given boundary conditions) formally but not, however, physically, from one another. Consequently, I am satisfied for the time being with deriving here a solution, without discussing the question whether the solution might be unique (Einstein, 1915a, 832).

The case described is identical to that of the Besso memo, and indeed Einstein could rely on the calculations he had made with Besso Janssen, 2007. However, Einstein was now ready to accept that the field equations do not fix the solutions $g_{\mu\nu}$ uniquely, but only up to a coordinate transformation that respects the condition that the determinant of the metric is 1. All such intertransformable solutions are only formally but not physically different. A genuine overdetermination would arise only if the solution remained overdetermined with respect to a definite coordinate system. However, Einstein must have now realized that this was not the case. For the given initial conditions, the worldline of the particles is geodesics of $g_{\mu\nu}$. Since both field equations and equations of motion are generally covariant, if one switches to another coordinate system, the particles will describe the same paths with respect to the new coordinate system.

Einstein's calculations revealed that the theory was able to account for the residual advance of 43″ of arc per century that was unexplained in the Newtonian theory. This was a path-breaking result and Fokker reported that Einstein suffered heart palpitations following the discovery (Pais, 1982, 253). A few days later, on November 25, 1915, Einstein (1915b) arrived at the field equations that today we can read in textbooks (albeit in slightly different notation). Imposing the condition $\sqrt{-g} = 1$ again, Einstein could write the equations in terms of the Christoffel symbols and showed how they satisfied the conservation laws. Einstein's "boldest dreams have now been fulfilled" (Einstein to Besso, Dec. 10, 1915; CPAE, Vol. 8, Doc. 162). On December 14, he wrote to Schlick his "newly found result": an empirically sound theory, "whose equations are covariant with arbitrary transformations in the spacetime variables" (Einstein to Schlick, Dec. 14, 1915; CPAE, Vol. 8, Doc. 165). As a consequence, space and time are now deprived "*last rest of physical reality [Realität]*" (Einstein to Schlick, Dec. 14, 1915; CPAE, Vol. 8, Doc. 165; my emphasis).

The fact that the theory's equations could be simplified *a posteriori* by selecting and imposing the condition $\sqrt{-g} = 1$ was deemed "of no epistemological significance" (Einstein to Schlick, Dec. 14, 1915; CPAE, Vol. 8, Doc. 165). Somehow crowning Einstein's triumph, on December 22, Karl Schwarzschild sent Einstein a letter from the eastern front communicating that he had already managed to find an exact solution for a 'mass point' (Einstein to Schwarzschild, Dec. 22, 1915; CPAE, Vol. 8, Doc. 169). Schwarzschild kept the condition $\sqrt{-g} = 1$. After introducing the most general line element in rectangular coordinates, he went over to polar coordinates according to φ, ϑ, r, which however do not satisfy the condition $\sqrt{-g} = 1$ (even in flat spacetime in spherical coordinates $g = -r^4 \sin^2\vartheta$). He found a trick to circumvent this problem by defining a new radial coordinate $x = r^3/3$ and a new angular coordinate $\varphi = -\cos\vartheta$. He then obtained a new set of variables, 'polar coordinates with the determinant 1,' that might be called the Schwarzschild-coordinate system.[19] Presenting his solutions in these coordinates, he found that the equations for planet orbits remain exactly the one that Einstein had obtained in first-order approximation.

[19]This nomenclature is not the usual one since what we call Schwarzschild-coordinates were actually introduced by Droste. cf. section 5.1.

3 The Return of the Point-Coincidence Argument in Einstein's Private Correspondence

3.1 The Einstein-Ehrenfest Correspondence

Some of Einstein's closest interlocutors, like Ehrenfest, were puzzled: "The day before yesterday," he wrote to Lorentz, "I received from Einstein the offprints of his latest papers on the theory of gravitation" (Ehrenfest to Lorentz, Dec. 23, 1915; SCHAL, Vol. 2, Doc. 247). Ehrenfest had always found it hard to make sense of Einstein's 'adapted coordinates' (cf. above section 2.1). "But if I have properly understood what Einstein *now* intends, then his theory of November 25, 1915, claims to be entirely free of 'adaption'-restrictions of the coordinates. *Is that so?!*" (Ehrenfest to Lorentz, Dec. 23, 1905; SCHAL, Vol. 2, Doc. 247). In particular, Ehrenfest noticed that "[i]n 1914 Einstein developed an almost philosophical proof for the necessity of the 'adaptedness' of the coordinate system (I mean, e.g., §12 of his paper of 19.XI.1914. Is this proof correct?" (Ehrenfest to Lorentz, Dec. 23, 1905; SCHAL, Vol. 2, Doc. 247). Einstein must have sensed that he owed an explanation to his correspondents (Ehrenfest to Lorentz, Dec. 23, 1905; SCHAL, Vol. 2, Doc. 247). In a letter to Ehrenfest, he described himself jokingly as the guy who is used to "retract what he wrote in the preceding year" (Einstein to Ehrenfest, Dec. 26, 1915; CPAE, Vol. 8, Doc. 173). Einstein was in a somewhat embarrassing situation of having to justify yet another retraction. Possibly at around that time, Einstein may have found an apt 'rhetorical device' to escape from his own argument against general covariance.[20] This argument, the point-coincidence argument, made its first explicit appearance in a letter to Ehrenfest. This letter is well known in Einstein's scholarship:

> In §12 of my paper of last year, everything is correct (in the first 3 paragraphs) [...] A contradiction to the uniqueness of the event does not follow at all from the fact that both systems $G(x)$ and $G'(x)$, related to the same frame of reference, satisfy the conditions of the grav. field. [...]
>
> 1. *the reference system has no real meaning*
> 2. that the (simultaneous) materialization of two different g systems (more aptly put, two different grav. fields) within the same area of the continuum is [...] impossible.
>
> In place of §12 the following consideration must appear. *Whatever is physically real in events in the universe (as opposed to that which is dependent on the choice of a reference system) consist in spatio-temporal coincidences** For ex., the intersection points of two worldlines are real, or the statement that they do not intersect each other. [...] When two systems in the $g_{\mu\nu}$'s (or gen., the variables used to describe the world) constituted in such a way that the second can be obtained from the first by mere spacetime transformation, then they are entirely equivalent. This is because they have in common all the spatial-temporal point coincidences, that is, all the observables (Einstein to Ehrenfest, Dec. 26, 1915; CPAE, Vol. 8, Doc. 173; my emphasis)
>
> * and in nothing else!

The apparent nonuniqueness of generally covariant equations would be objectionable only if more than one $g_{\mu\nu}$ system would be possible in the *same coordinate system K*. However, this is in principle impossible. Not only are the $g_{\mu\nu}$ a physical field on a coordinate system, but also they tell us which coordinate system we are using. Therefore, the theory cannot predict the $g_{\mu\nu}$ at a point identified by its four coordinate numbers x. Changing coordinates, one obtains not only a new set of $G'(x')$, but also a different physical meaning of the same quadruple of numbers $x = x'$. Two worldlines that intersect at $x = x'$ of K according to $G(x)$ will indeed not intersect at the same point x of K according to $G'(x)$. However, Einstein came to realize that this not a problem: the same point should be defined physically as where the same worldlines meet. Coordinate numbers x serve only as a bookkeeping device for such coincidences. Thus, any choice of coordinate numbers is just good as any other (Einstein to Ehrenfest, Dec. 26, 1915; CPAE, Vol. 8, Doc. 173). Ultimately, according to Einstein, the point-coincidence argument showed "how natural the requirement of general covariance is" (Einstein to Ehrenfest, Dec. 26, 1915; CPAE, Vol. 8, Doc. 173). The specialization of $\sqrt{-g} = 1$, Einstein insisted, does not change the essence of the matter.

Thus, Einstein was convinced that he had sufficiently refuted his own "'philosophical' consideration" (Einstein to Ehrenfest, Dec. 29, 1915; CPAE, Vol. 8, Doc. 174). He wrote to Lorentz that he was "conducting a discussion with Ehrenfest " (Einstein to Lorentz, Jan. 1, 1916; CPAE, Vol. 8, Doc. 177), who was rather skeptical. As a matter of fact, Ehrenfest confessed to Einstein that Lorentz was not convinced either that his field equations with the condition $\sqrt{-g} = 1$ were sufficient to determine *das Geschehen*, the events (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[12], Doc. 177a). However, Lorentz, differently from Ehrenfest, did not know about Einstein's letter with the point-coincidence argument, yet. Nevertheless, Ehrenfest, even after having discussed Einstein's counter-§12-argument with his wife Tatjana and with Fokker, was still not convinced: "I will defend the philosophy of §12 against your refutation" (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[12], Doc. 177a). After having repeated nearly literally

[20] One can speculate that a paper by Kretschmann (Kretschmann, 1915; cf. Howard and Norton, 1993) that was published on December 21 or discussions with Schlick (see Engler and Renn, 2013) might have suggested to Einstein a suitable 'turn of phrase.' As far as I can see, it has never been noticed that Einstein used the same turn of phrase as early as 1911. cf. section 1.1.

some of the lines from Einstein's letter, he tried to explain his point of view with a thought experiment, concerning the curvature of light rays passing in the vicinity of the Sun. It was sort of an astronomical version of Young's double-slit experiment.

To make the rather abstract matter intuitive, Ehrenfest drew a picture of infinitely weak (of negligible self-gravitational energy) light waves coming from a star passing through the empty space around the Sun and reaching two telescope apertures that combine light waves to form a fringe pattern on a photographic plate (fig. 2). The only "object of observation" is the "blackening or not blackening of a photographic plate behind both telescopes" (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a). The physical situation is parametrized by the coordinate system $x_1, \ldots, x_4, T_{\mu\nu}, g_{\mu\nu}$. Let us assume that the field equations predict a destructive interference on part of the plate: "Bravo the photrogr. plate is darkened (a true 'coincidence (explanation') claim" (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a). However, one can proceed further and "see the philosophy of §12 at work", that in Ehrenfest's notation $G(x) \to G'(x') \to G'(x)$:

> In the cross-hatched region occupied by matter, we kept fix the description of the world-happening, whereas in the non-cross-hatched empty space (where there is only ∞ weak light!) the latter is strongly changed You look at me laughing quietly and you say 'go ahead, young friend, and describe if you like, the empty space
>
> *with old coordinates x*
> *and brand new $G'(x)$*
>
> nothing observable, no 'coincidence' would change!? Now I'm astonished and *angry* over your laugh and
>
> *I claim with a clenched fist:*
>
> '*If with the old x and the old $G(x)$ and the new $G'(x)$ one calculates the darkening of the plate, then one should calculate the non-darkening of the plate with old x and new $G'(x)$*' [The star, the sun and the telescope and the plate are not in empty but in matter space] (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a).

The position of the star, the telescope aperture, and the plate in the matter-filled regions are fixed in a provisionally chosen coordinate system K with x. One calculates a solution to the field equations and obtained $G(x)$; once one knows such coefficients, one can extract from the coordinate numbers x the reciprocal distances of the Sun, the stars, the aperture, and the photographic plates and, in general, of any points x respect to K. Subsequently, the coordinates in the matter-free region are changed to the coordinate system K' so that $x' = f(x)$. Following the instructions of Einstein's §12, one can produce a series of different solutions $G(x)$, $G'(x)$, $G''(x)$, and so forth on the same coordinate system x "Symbols are used precisely in the sense of your §12!—Thus $G'(x) \neq G(x)$" (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a). The trajectories of light rays appear to pass through different values of the same coordinate system x in each case. "Do you concede this point or do you dispute it?", Ehrenfest asked Einstein rhetorically (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a). If Einstein accepts Ehrenfest's reconstruction, then he has to admit that if the positions of the Sun, stars, aperture, and photographic plates are given, the theory cannot predict the interference patterns on the plate, which is precisely the 'coincidences' Einstein was talking about. Einstein should concede "with an icy polite smile" the failure of his "solemn formula [*Beschwörungsformel*]" in his Mercury-perihelion paper, that is, the claim that intertransformable solutions are formally different, but physically identical (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[13], Doc. 177a).[21]

However, by that time, Einstein had become fully confident that the *Lochbetrachtung* was flawed (Einstein to Besso, Jan. 3, 1916; CPAE, Vol. 8, Doc. 174). As he explained to Besso, the meeting of coordinate lines of K is not real; real are only the "encounters [*Begegnungen*]" of worldlines. In other terms: "nothing more" that point-coincidences (Einstein to Besso, Jan. 3, 1916; CPAE, Vol. 8, Doc. 174). This was the root of Ehrenfest's difficulties as well. Ehrenfest "instinctively treat[ed] the reference system as something 'real'" (Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180). Einstein considered this prejudice fully natural: "I myself needed so long to arrive at total clarity on this point" (Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180). In order to convince Ehrenfest, Einstein invited him to depict a simplified version of his thought experiment (a star, an aperture, and a plate) on a tracing paper superimposed onto a flat letter paper. On the flat tracing paper, one can trace a regular graph in which coordinates directly represent actual lengths (the time coordinate is not taken into account). A coordinate transformation,

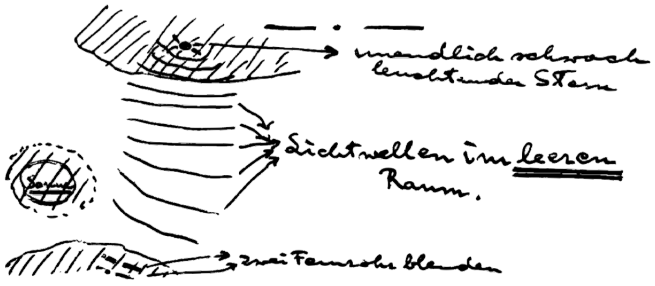

Figure 2 – Diagram in Ehrenfest's hand (Ehrenfest to Einstein, Jan. 1, 1916; CPAE, Vol. 8[12], Doc. 177a)

[21] The reference is to the passage cited above sec. 2.2.

which introduces curvilinear coordinates and a new set of $g_{\mu\nu}$, is represented as a deformation of the tracing paper. The boundary conditions at infinity, the coordinates of the star, the material point at the aperture, and the plate remain unchanged. The latter is compared again with the original diagram on the letter paper. The result is summarized in an often-quoted passage:

> In the above special case you obtain all the solutions that are a consequence of general covariance in the following way. Trace [the star, the aperture and the plate] onto completely deformable tracing paper. Then deform the tracing paper arbitrarily in the plane of the paper. Then make another tracing on the letter paper [...] When you relate the figure once again to the orthogonal writing paper coordinates, *the solution is mathematically different from the original*, and naturally also with respect to the $g_{\mu\nu}$. *But physically it is exactly the same, since the writing paper coordinate system is only something imaginary.* Always the same points are illuminated on the plate [...] The essential thing is: as long as the drawing paper, i.e. 'space,' has no reality, then there is no difference whatever between the two figures. *It all depends on coincidences, e.g., whether the plate points are hit by the light or not.* Thus, the distinction between your solutions *A* and *B* is merely a difference in presentation [*Darstellung*] with physical concordance [*Übereinstimmung*] (Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180; my emphasis).

In pre-general-relativistic theories, we know in advance that the orthogonal tracing paper is flat and that rectangular coordinates (flat tracing paper) are a 'good' coordinate system, which is physically realizable in the form of a material scaffolding. The goal of physics was ultimately to correctly predict the paths of particles and light rays with respect to this coordinate system. We could, of course, use a different curvilinear coordinate system (the tracing paper is distorted). Free particles would still follow time-like geodesics, and light rays would still follow null geodesics. However, they would impress different paths onto the letter paper. Nevertheless, before general relativity, we knew how to go back to the 'good' regular graph coordinate system, with respect to which the 'real' path of particles and light rays is defined. Since the deformed tracing paper was regarded as a 'bad' coordinate system, the theory predictability was safe. By contrast, by abolishing the distinction between 'good' and 'bad' coordinate systems, at first sight, general relativity seems to be incapable of determining uniquely the paths of light rays *with respect to the writing paper coordinate system.* The illusion disappears once one realizes that the regular graph paper has no independent physical reality. Thus, there is no reason to complain that the same diagram's lines do not intersect at the 'same point' of the latter paper. The 'same point' is there where the same diagram worldlines intersect [22]

3.2 The Lorentz-Ehrenfest Correspondence

Einstein's and Ehrenfest's conceptual difficulties might surprise the modern reader who is already accustomed to considering coordinates as essentially arbitrary systems of parameters. However, these difficulties were clearly not idiosyncratic. Also, a physicist of the status of Lorentz struggled to overcome the very same 'epistemological obstacle' (see Kox, 1985). On January 9, 1915, just a few days after Einstein's answer to Ehrenfest (Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180), Lorentz sent a long letter expressing his concerns regarding general relativity and presenting his own argument against general covariance. Lorentz considered the truncated vacuum field equations eq. (8). He argued that, given one solution I, $g_{\mu\nu} = F(x_\alpha)$, one can obtain a new one II, $g'_{\mu\nu} = F'(x'_\alpha)$, where $g'_{\mu\nu} = g_{\mu\nu} + \delta_{\mu\nu}$. This is done by introducing a small change of the spatial coordinates x_1, x_2, x_3 in the form $x'_\nu = x_\nu + \zeta_\nu$ (where ζ_ν is a small quantity of the 'first' order), whereas $x'_4 = x_4$. He imposed a condition on ζ_ν that lets $|g| = -1$ remain unchanged. The result was the following:

> If, e.g., I have the solution $g_{\mu\nu} = F(x_\alpha)$ (symbolically expressed), and replace x_α by x'_α, then by the transformation formulas, I can provide the values of $g'_{\mu\nu}$. I can express them in x'_α; suppose $g'_{\mu\nu} = F'(x'_\alpha)$. Then $g_{\mu\upsilon} = F'(x_\alpha)$ will also satisfy equations [8]. This is a new solution, differing from the first. [...] (I) and (II) now, in fact, differ physically, since in field (I) a material point moves uniformly along a straight line, whereas one can easily see that this is not the same in field (II). [...] One can also, taking (II) as a starting point [...] [and] derive a third solution; one can continue in this way [...] From the above it follows, it seems to me, that in the case we are considering of the matter-free field, equations ([8]) are, with continuity and the conditions at infinity, insufficient to determine the field; in contrast with Laplace's equation $\Delta\varphi = 0$, which in connection with the additional conditions requires that $\varphi = 0$ (Lorentz to Ehrenfest, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230).

Lorentz insisted that one would find a similar overdetermination by considering the Sun's gravitational field calculated by Einstein. The coordinate transformation $x'_\nu = x_\nu + \zeta_\nu$ can be applied to it as well. One can also make

[22] The deformation of the tracing paper described by Einstein is equivalent to the introduction of different maps on the same projection plain x, y, which correspond to the letter paper. On the map, the Greenwich meridian intersects the equator (both are 'great circles,' the shortest lines on the globe) at different points x, y and at different angles in different mappings, in stereographic, cylindrical, or other projections. However, this is not a problem for cartographers since they define the 'same point' as where the Greenwich meridian intersects the equator. This information is the same in all mappings.

the case that ζ_ν differs from 0 also in the space occupied by the Sun, then field (II) will pertain to a 'changed' Sun. . However, one can also set $\zeta_\nu = 0$ within the Sun and change only the external field so that the same distribution of matter would produce different fields. To escape this overdetermination, one could try to introduce additional conditions (e.g., the field is Minkowskian at infinity, symmetric, etc.). However, in Lorentz's view, this would not limit our freedom of choosing ζ_ν in the general case.

Ehrenfest replied immediately on the very same day (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230), with a letter written in German and a brief introduction and conclusion written in Dutch, "under the pressure of orthographical-grammatical boundary conditions" (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230). The day before, he had just received Einstein's answer to his question about the curvature of light beams (Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180). Ehrenfest enclosed Einstein's letter within the letter he sent to Lorentz. Ehrenfest did not find Einstein's reply very clear; however, thanks to a long discussion with his wife, he came to realize that Einstein was "entirely right" (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230). Ehrenfest relied again on the example of the curvature of light beams, which he meant to be "Ganz im Sinn des §12" (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230).

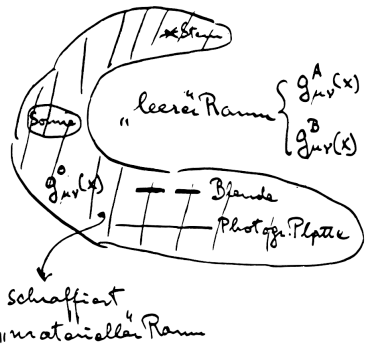

Figure 3 – Diagram in Ehrenfest's hand (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230)

He considered a material space made by a star, the Sun, an aperture, and a photographic plate on Earth (fig. 3). Exactly like in Einstein's §12, in the matter-free region around this material system, two observers A and B introduced two tensor fields $g^A_{\mu\nu}(x)$ and $g^B_{\mu\nu}(x)$ on the same coordinate system x_1, x_2, x_3, x_4, whereas the $g^0_{\mu\nu}(x)$ field in the matter region remained the same. When light rays $ds^2 = 0$ coming from a star enter into the matter-free region, they go through different coordinate values x_1, x_2, x_3, x_4 according to $g^A_{\mu\nu}(x)$ and $g^B_{\mu\nu}(x)$. In fact, the path of light rays is nothing but the shortest chain of ds whose links are null spacetime length so that small increments dx have natural spacetime distance $ds^2 = 0$. Which small increments dx correspond to such natural distance depends on the $g_{\mu\nu}$ systems, $g^A_{\mu\nu}(x)$ or $g^B_{\mu\nu}(x)$, that one uses to make the calculations. Thus, ultimately, given the coordinate system x, light rays will go through different values of x, since the same coordinate differences dx correspond to different real distances ds. However, in the meantime, Einstein had managed to convince Ehrenfest that the paths of light rays, calculated according to $g^A_{\mu\nu}(x)$ and $g^B_{\mu\nu}(x)$, will meet again at the same point x_1, x_2, x_3, x_4 on the matter region in which the coordinate system has remained unchanged and that they will do so with the same angle in both cases. Thus, one obtains the same darkening of the photographic plate.

Ehrenfest realized a key point. Many conceptual confusions emerged because physicists were not thinking 'geometrically' in terms of the intrinsic structure of spacetime, but rather algebraically in terms of transformations of variables.[23] Thus, he suggested to represent the 'world' as a four-dimensional E surface embedded in a five-dimensional Z space. The transformation considered by Einstein was nothing but bending *without stretching* of the parts of the surface between the stars and the aperture. The remaining parts of the surface were kept unchanged. This bending produced a different Θ surface: "This is, geometrically speaking, the result of §12" (Ehrenfest to Lorentz, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230). The two surfaces look differently in the embedding space (like a plane, a cone, and a cylinder look different). However, since they are developable onto each other, they share all intrinsic geometrical properties (like a plane, a cone, and a cylinder have the same flat geometry). The distance between two points, measured along the surface (i.e., along the shortest line joining them), is unchanged, and so is the angle between two lines that meet at a point. The paths of light rays are nothing but some of such lines. Thus, although observers A and B bend the central part of the surface differently in the matter-free region, inasmuch as that this does not stretch the surface, the light rays will meet again at the same angle with the telescope.

Ehrenfest was eager to know whether Lorentz considered his two solutions I and II as equivalent in this sense. Lorentz replied immediately; he "had read only a part of [Einstein's letter][24]" and he had realized that "he was entirely right". "I wrote to him straight away to withdraw my objections of yesterday", he concluded Lorentz to Ehrenfest, Jan. 10, 1916; SCHAL, Vol. 2, Doc. 231. Lorentz described to Ehrenfest with great clarity the 'epistemological obstacle' he had found difficult to overcome:

[23] In other terms, they reasoned in terms of different map distortions and not of the unique shape of the globe.

[24] Einstein to Ehrenfest, Jan. 5, 1916; CPAE, Vol. 8, Doc. 180.

> In connection with this I now realize that in the two fields I and II that I spoke of in my last letter,[25] it is true that the separate phenomena do not take the same course *in relation to the coordinate system*, but that in both of them the coincidences do occur in the same way [...] *I was too much a prisoner of the idea that our equations must fully reproduce* [...] *the relations between the phenomena and the chosen coordinate system, whereas we can be happy if they duly reproduce the mutual relations between the phenomena.* In physics, one has up to the present (though it appears to be unnecessary) aimed at drawing up equations that fully determine *the phenomena in relation to a chosen coordinate system* [...] But it now turns out that the 'field equations' are of a different nature. We cannot claim that they *determine* the $g_{\mu\nu}$. Instead, we must say that when for the $g_{\mu\nu}$ we take *arbitrary* functions that satisfy the equations, and then calculate all phenomena with these $g_{\mu\nu}$, we shall reproduce all coincidences well (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231; my emphasis).

For practical purposes, one should perhaps restrict the choice of $g_{\mu\nu}$ through suitable additional conditions. This restriction "would, however, now *have no sort of deeper meaning* and could be brought about in an arbitrary way" (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231; my emphasis). Thus, Lorentz realized that also the condition $\sqrt{-g} = 1$ has nothing special. We chose "the coordinate system such that $g = -1$", only for the reason that this would give us "the simplest possible solution (i.e., the values of $g_{\mu\nu}$) of the field equations" (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231). However, a less simple solution would be just as legitimate. "As far as the reproducing of the coincidences is concerned, the choice of the coordinate system must be entirely indifferent" (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231). Worldlines do not take the same course in relation to the coordinate system, but in both of them, coincidences do occur in the same way. Indeed, at closer inspection, a worldline is not a succession of coordinate values, but a succession of (possible) coincidences. "Theoretical physics," Lorentz concluded, "now begins to resemble the *geometria situs*" (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231).

Lorentz "congratulated Einstein on his brilliant results" (Lorentz to Ehrenfest, Jan. 11, 1916; SCHAL, Vol. 2, Doc. 231; see also Lorentz to Ehrenfest, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 232). However, as Ehrenfest pointed out in his reply, Lorentz's comparison of relativity theory to the *geometria situs* was too hasty (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233). "I know very well that you mean only to indicate a *direction*, not a *point*: the development from a lesser to a greater 'deformability'" (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233). According to Ehrenfest, general relativity was indeed "more 'topological' than ordinary theories, yet still not fully topological" (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233). The two solutions of the field equations that can be 'deformed' into one another in the sense of §12, that is, through a mere coordinate transformation, are equivalent (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233). This sort of deformation "causes the Einsteinian theory to *resemble analysis situs*, it is true, but to remain very certainly *infinitely* more limited" (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233). The correct analogy, according to Ehrenfest, was with Gauss's theory of surfaces. The four-dimensional world cannot be arbitrarily deformed like the rubber band of the *geometria situs*; it can only be "deformed 'without being stretched'" (Ehrenfest to Lorentz, Jan. 12, 1916; SCHAL, Vol. 2, Doc. 233) like how a cylinder can be developed onto a flat surface. This was precisely what Einstein stubbornly refused to accept from Hertz in August 1915: the world is developable onto itself (cf. section 2.2). Einstein thought of E, F, G as a calculation device that, given dq, dp, spits out ds, and at the same time E, F, G is a field with certain values at p, q. However, this way, he seems to have missed the key point: the entire class of inter-transformable E, F, G represents the same curved surface, with the same Gaussian curvature.

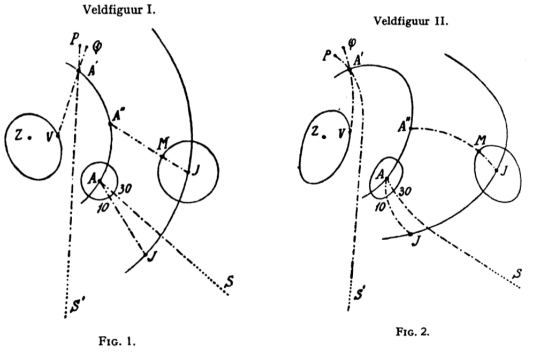

Figure 4 – Two equivalent field figures, Lorentz, 1917a

Einstein agreed with the "truly Ehrenfestian description of the telescope affair" (Einstein to Ehrenfest, Jan. 17, 1916; CPAE, Vol. 8, Doc. 182). In general, he was pleased with the reception that the theory had within the Leiden community, "a brilliant nook on this barren planet" (Einstein to Ehrenfest, Jan. 17, 1916; CPAE, Vol. 8, Doc. 182). Einstein wrote to Lorentz a few days later: "I see that you have thought over the theory entirely and have familiarized yourself with the idea that all of our experiences in physics refer to coincidences" (Einstein to Lorentz, Jan. 17, 1916; CPAE, Vol. 8, Doc. 245). This point of view quite consequently requires the formulation of generally covariant equations. Einstein read the Ehrenfest-Lorentz correspondence and confirmed that he himself "had hit upon this erroneous stance that corresponds to the standpoint held by you in the first of your letters" (Einstein to Lorentz, Jan. 17, 1916; CPAE, Vol. 8, Doc. 245). Einstein conceded that although he had presented the field equations

[25](Lorentz to Ehrenfest, Jan. 9, 1916; SCHAL, Vol. 2, Doc. 230).

in the $\sqrt{-g} = 1$ coordinates, it was important to present the field equations in a generally covariant form to avoid any arbitrariness. However, he also insisted that it was necessary to specialize the coordinate systems in some natural way, although he did not find a proper way to do so (Einstein to Lorentz, Jan. 17, 1916; CPAE, Vol. 8, Doc. 245). Einstein was particularly fond of how Lorentz had presented the point-coincidence-general covariance relations and he asked him to make these "considerations available to other physicists as well" (Einstein to Lorentz, Jan. 17, 1916; CPAE, Vol. 8, Doc. 245).

Lorentz agreed that a systematic presentation of the theory was a worthy endeavor (Lorentz to Ehrenfest, Jan. 22, 1916; SCHAL, Vol. 2, Doc. 235). As he suggested to Ehrenfest, he "could talk on this subject on Mondays at 10 o'clock for a few weeks, and use these lectures as the basis for a short paper or a little book" (Lorentz to Ehrenfest, Jan. 22, 1916; SCHAL, Vol. 2, Doc. 235). Indeed, it was Lorentz who used for the first time the point-coincidence argument in a public lecture in February 1916, explicitly referring to a correspondence that he had with Einstein (Lorentz, 1917b, 1342; fn. 3). Lorentz suggested that we could investigate the gravitational field of the Sun and we could perform a sort of 'scattering' experiment in a finite region surrounded by detectors. Given certain initial conditions, we would send a great number of material points and light rays, moving in all directions and with different velocities, and note all coincidences recorded by the detectors, that is, all "data by astronomical observations" that one could imagine to be infinitely detailed (Lorentz, 1917b, 1343).

One can represent these coincidences in a four-dimensional diagram, as a 'field figure,' a maze of worldlines and of light rays and their intersections, which can ultimately be everywhere-dense, a sort of continuously distributed matter without internal forces. This way, "each observed coincidence could be represented by an intersection of worldlines" (Lorentz, 1917b, 1343). The claim that Mars (M) is at a certain point at a certain instant with respect to the Sun (Z) means that an astronomical sight lines up the planet, a fixed star (S) and the telescope on A. This implies that the trajectory of a light ray joining the star and the orbit of the planet, passing through a telescope comoving with Earth (A), and leaving marks on a photographic plate is interrupted by the passage of Venus (V) when the handles of a clock are on a certain position on the dial (fig. 4). In previous theories, there was only one correct field figure; in general relativity, any deformation that allowed all intersections, and thus all coincidences and all observational data, to remain unchanged is just as good as any other (field-figure-I and field-figure II in fig. 4).[26] Mars is at the same point at the same time in all M cases.

4 The Point-Coincidence Argument in Einstein's 1916 Paper: Getting Rid of the Last Remnant of Materiality of the Coordinate System

As most readers might recall, Einstein used the point-coincidence argument for the first time in a published writing in his 1916 review paper (Einstein, 1916a; cf. Sauer, 2005; Janssen, 2005; Gutfreund and Renn, 2015), which was submitted in March. "I have worked very hard on a final formulation of general relativity that has now been fully developed" (Einstein to Zangger, Mar. 1, 1916; CPAE, Vol. 8[12], Doc. 196a). The argument appears in one of the first sections of the paper at the conclusion of what might appear as a sort of autobiographical, 'rational' reconstruction of the evolution of Einstein's attitude toward coordinates. "In classical mechanics, as well as in the special theory of relativity, the coordinates of space and time have a direct physical meaning" (Einstein, 1916a, 773). To say that a point event has coordinate x_1 means measuring off with a unit rod x_1 times from the origin of the coordinate system K along the axis of x_1. To say that an event has coordinate x_4 means that a unit clock at rest in K measured off x_4 periods at the occurrence of the event starting from x_0. Coordinates are labels that tell us *which* spacetime points we are referring to and at the same time they tell us *where* those points are, their relative spacetime distances from other points with respect to a 'good' coordinate system K. "This view of space and time has always been in the minds of physicists, even if, as a rule, they have been unconscious of it" (Einstein, 1916a, 774).

Working within the framework of special relativity, Einstein realized that this conception of coordinates could not be extended to accelerated systems. A coordinate system $K'(x', y', z', t')$ accelerating relative to a 'good' system $K(x, y, z, t)$ can be in principle maintained in a Born rigid motion, allowing for a direct measurement of x, y, z. However, one has to abandon the direct measurement of t with a clock (Einstein, 1916g). A coordinate system $K'(x', y', z', t')$ rotating around the $z = z'$-axis with respect to K, on the contrary, defies even this criterion of rigidity and does not allow for the measurement of x, y, z by means of rigid rods (Stachel, 1989). Since Einstein considered a uniformly rotating frame K' as equivalent to a system at rest K in a static gravitational field, he had to abandon the comfortable setting in which coordinate numbers meant also distances. One might ask what the physical content of a theory is in which the positions of material points and the values of field quantities are expressed as functions of the coordinates, but the latter are not measurable. In an enormously famous but quite elliptic passage, which does not betray the conceptual struggle needed to achieve the result, Einstein reminded

[26] Once again, Lorentz seems to conceive a field figure as a map projection on a flat projection plane footnote 7. A change of the coordinate system corresponds to a different map projection and thus to a deformation of the field figure as in fig. 4.

his readers of what is actually observable in a physical theory:

> That this requirement of general covariance, which takes away from space and time the last remnant of physical materiality [*Gegenständlichkeit*], is a natural one, will be seen from the following reflexion. All our spacetime assessments [*Konstatierungen*] invariably amount to a determination of spacetime coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings [*Begegnungen*] of two or more of these points. Moreover, the results of our measurings are nothing but assessments [*Konstatierungen*] of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time.
> The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences (Einstein, 1916a, 771; translation modified).

One might read this passage saying that the result of any measurement should ultimately ascertain whether a mark on one scale (a movable pointer or such) coincides with a certain mark on another scale. This statement does not entail any reference to coordinates. In a given coordinate system, two events are coincident if their coordinates are the same, but the relationships between space-time coincidences are independent of the coordinate system used (Ohanian, 1976, 253). Thus, coordinate numbers serve as individual 'names' for such coincidences (they tell *which* coincidence is referred to) do not carry any further 'metrical' information (they do not the us *where* this coincidence is).

The relationships between coordinate numbers and real distances can be extracted from the formula of the line element, which is now written so that the summation over repeated indices is assumed:

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu \,. \tag{10}$$

This formula tells us that when ds and dx_1, dx_2, dx_3, dx_4 are known numbers, the expression of the line element represents one equation for the determination of the unknowns $g_{\mu\nu}$. On the contrary, if $g_{\mu\nu}$ and dx_1, dx_2, dx_3, dx_4 are known, the equation spits out the number ds. The circular way in which these two operations are connected is at the origin of Einstein's difficulties of illustrating the relation between the predicted values of $g_{\mu\nu}$ and the observed ones, the ones that are measured with rods and clocks. This is revealed by the surprisingly misleading way in which Einstein presented the latter relations in the review paper.

After an introduction to the absolute differential calculus (Part *B*), Einstein derived the gravitational field equations (Part *C*) from a Lagrangian $\delta \int \mathrm{H} d\tau = 0$, and by imposing the condition $\sqrt{-g} = 1$. He then obtained the field equations in the following form:

$$\begin{gathered} \frac{\partial}{\partial x_\alpha} \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\mu\beta} \Gamma^\beta_{\nu\alpha} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \\ \sqrt{-g} = 1 \,. \end{gathered} \tag{11}$$

After a discussion of the form that $T_{\mu\nu}$ assumes in particular cases, in the last section, Einstein discussed the Newtonian approximation of weak fields, Minkowski's flat boundary conditions, and slow motion of particles. He applied this approximating procedure to obtain an approximate solution to his field equations for the exterior field of a static, spherically symmetric mass. In the first Newtonian approximation, Einstein obtained the following values for $g_{\mu\nu}$, written in arbitrary Cartesian coordinates, so that $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$, the radial distance between the Sun and a certain point. In this coordinate system, the approximate solution of the field equations takes the following form:

$$\begin{aligned} g_{\rho\sigma} &= -\delta_{\rho\sigma} - \alpha \frac{x_\rho x_\sigma}{r^3} (\rho, \sigma = 1, 2, 3) \\ g_{\rho 4} &= g_{4\rho} = 0 \\ g_{44} &= 1 - \frac{\alpha}{r} \,, \end{aligned} \tag{12}$$

where $\alpha = \kappa \mathrm{M}/4\pi$, with M being the mass of the Sun. It can be shown that this solution satisfies the condition $\sqrt{-g} = 1$. What remains now is to show how these predicted values can be compared with experience. Einstein's approach was somehow puzzling. As we have seen, it can be assumed that, in a small region of space,[27] we can introduce a Cartesian coordinate system in which $g_{11} = -1$, $g_{22} = -1$, and $g_{33} = -1$ and coordinates are measurable by unit rigid rods that have the same length in every position and every orientation. In the presence of the gravitational field, however, this construction would fail over a large region of space. Even in first approximation, the coefficients g_{11}, g_{22}, and g_{33} are not constant, as Einstein had initially believed, but just like the coefficient

[27] Over a single spacetime hypersurface.

g_{44},functions of the coordinates. If we want to check whether the calculated $g_{\mu\nu}$ values are 'true,' we will need to measure the observed $g_{\mu\nu}$ values using rods and clocks. Einstein described an idealized measurement procedure as follows. A unit rod laid along the radius x_1,

$$ds^2 = -1; \quad dx_2 = dx_3 = dx_4 = 0$$

,

does not measure directly the coordinate x_1, but

$$-1 = g_{11}dx_1^2 \qquad g_{11} = -\left(1 + \frac{\alpha}{r}\right).$$

On the contrary, a rod laid in the perpendicular direction reads directly the coordinates

$$ds^2 = -1; \quad dx_1 = dx_3 = dx_4 = 0$$

$$-1 = g_{22}dx_2^2 \qquad g_{22} = 1\,.$$

A clock placed at rest in the gravitational field

$$ds = 1; dx_1 = dx_2 = dx_3 = 0$$

measures

$$1 = g_{44}dx_4^2 \qquad g_{44} = -\left(1 + \frac{\alpha}{r}\right).$$

It is indeed unsettling that, in Einstein's view, measured in coordinates, a rod is *shortened* by the gravitational field by a factor of $\sqrt{g_{11}}$ and a clock is *slowed down* by a factor of $\sqrt{g_{44}}$ as if equal differences in coordinates x_1 and x_4 had a physical meaning of equal distances from the origin. This approach is visible in Einstein's 'backward derivation' of the red shift, in which equal differences in time coordinates give different proper time differences $dx_4 = 1/\sqrt{g}_{44}$ (Earman and Glymour, 1980b). A similar issue emerges, which follows Einstein's derivation of light deflection (Earman and Glymour, 1980a). Einstein introduced the velocity of light $\gamma = 1$ "in the sense of Euclidean geometry" (Einstein, 1916a, 806), and then he calculated the distorted values:

$$\gamma = \sqrt{\left(-\frac{g_{44}}{g_{22}}\right)} = 1 - \frac{a}{2r}\left(1 + \frac{x_2^{\,2}}{r^2}\right).$$

Thus, again, light rays follow paths in which small differences dx_ν of the successive values of x_ν are always $ds = 0$ in Cartesian coordinates. However, these paths are distorted by the gravitational field $g_{\mu\nu}$. However, this way, γ would depend on the choice of g_{44} and g_{22}.

In the approximation procedure adopted by Einstein, in which one starts from Cartesian coordinates, it seems natural to interpret coordinate differences as distances (e.g., r is the distance from the Sun) and $g_{\mu\nu}$ as a field that distorts such measurements. However, this approach has an obvious drawback. Since $g_{\mu\nu}$ are defined only up to a coordinate transformation, with a different choice of coordinates, rods and clocks would be shortened and slowed down differently by different factors $g_{\mu\nu}$. However, at this point, Einstein must have been aware that the relationships between coordinates and rods and clocks measurements are actually reversed, as he more aptly wrote a few lines later. Instead of measuring in terms of coordinates and considering the rod distorted, one should take "one and the same rod, *independently of its place and orientation*, as a realization of the *same interval* $[-ds]$" (Einstein, 1916a, 820; my emphasis). As it turned out, "Euclidean geometry does not hold even to a first approximation in the gravitational field" (Einstein, 1916a, 820). That is, near the Sun, it is impossible to cover a sufficiently large region of space with a coordinate system in which $g_{11} = g_{22} = g_{33} = -1$, and $ds = -1$ in every position and every orientation. This is like trying to wrap a flat piece of paper around a sphere. If we assume that identical clocks (say, identical atoms) always measure the same ds without being influenced by the gravitational field, in general, it is impossible to choose a coordinate system in which time coordinates are the result clock measurements, that is, where $g_{44} = 1$. Similarly, identical rods and identical clocks, wherever they are placed in the gravitational field, always measure the same velocity of light c.

Einstein's presentation of the practice of rods and clocks measurement in general relativity in terms of a field that distorts rods and clocks, and thus the paths of particles and light rays, is revealing of the fact that he seemed to vacillate between two different mental models: (a) a practically useful mental model in which $g_{\mu\nu}$ extract different 'natural' differences ds from equal differences in coordinates and (b) a theoretically more adequate mental model in which equal natural distances ds correspond to different coordinate distances according to different $g_{\mu\nu}$ systems.

Since Einstein did not think in geometrical terms (the intrinsic curvature of spacetime) but saw eq. (10) as a calculation tool to translate coordinate distances into real distances and vice versa, he seemed to switch from one mental model to the other depending on the circumstances. More precisely, although Einstein knew that (b) is, 'in theory,' the correct approach, he often reasoned 'in practice' in terms of (a). Given the coordinate differences of two points, what is the distance between them? The line element eq. (10) gives different answers to this question depending on the components of the $g_{\mu\nu}$. In my view, the way of posing this question is the source of most of the difficulties that we have encountered so far.

It is of course not that, using different intertransformable $g_{\mu\nu}$ as conversion factors, we calculate different ds from the *same coordinate increments* as in (a); rather, as in (b), using different $g_{\mu\nu}$, we can recover the same ds from *different coordinate increments*.[28] In spacetime, worldlines are parametrized by an infinitely large number of subsequent parameters x_1, x_2, x_3, x_4. Free-falling particles in an empty region of spacetime occupy the series of x_1, x_2, x_3, x_4, which satisfies the constraint that the chain of time-like ds connecting two successive coordinate increments x_ν and $x_\nu + dx_\nu$ is the longest among all alternatives. If one remains attached to (a), then the same series of coordinate numbers x_1, x_2, x_3, x_4 that represent the longest time-like worldline in the $g_{\mu\nu}$ system will not in $g'_{\mu\nu}$. Thus, the worldlines of two colliding free-falling particles A and B that intersect at the worldpoint x_1, x_2, x_3, x_4 according to $g_{\mu\nu}$, in general, will not according to $g'_{\mu\nu}$. However, according to (b), free-falling particles follow the longest chain of $ds = 1$ in all $g_{\mu\nu}$-systems; the fact that such world-lines can be parametrized by different series of x_1, x_2, x_3, x_4 is meaningless. In all $g_{\mu\nu}$ systems, the same worldline of A and B intersects at the same point, because the same point is defined operationally as where the same worldline intersects. "Every physical description resolves itself into a number of statements, each of which refers to the spacetime coincidence of two events A and B" (Einstein, 1916f, 65). Coordinates x_1, x_2, x_3, x_4 do not tell us *where* such coincidences happen with respect to a given coordinate system; the four numbers x_1, x_2, x_3, x_4 are only a bookkeeping system to keep track of *which* coincidences we are referring to in a given coordinate system.

The convincing power of the point-coincidence argument ultimately depended probably on the fact that it could present what was a radical redefinition of what is observable in physics as if, in a more careful analysis, it were already implicit in previous physics practices. This is probably a rational reconstruction of how Einstein himself stumbled on the point-coincidence argument. As we have seen, as early as 1911 (cf. section 1.1), Einstein had pointed out that, in physics and in everyday experience alike, "the position of an object in space is based on the specification of the point on a rigid body with which that event or object *coincides*" (Einstein, 1916f, 4; my emphasis). Einstein used this very same parlance again in his popular book on relativity (Einstein, 1916f) that he started to plan at the beginning of 1916 (Einstein to Besso, Jan. 3, 1916; CPAE, Vol. 8, Doc. 178) and concluded by the end of the year. As he pointed out further in the booklet, one needed only to take this conception to its extreme consequences.

Physicists used to think that the goal of a physical theory was to predict the successive positions x, y, z of, say, a material point with respect to a suitably chosen reference frame at time t measured by clocks placed at fixed positions on such scaffolding. At closer inspection, however, what they were actually able to predict was only the "the encounters [*Begegnungen*] of this [material] point with particular points" of a particular physical system that we have chosen as a "reference-body", at the moment of "the encounter of the hands of clocks with particular points on the dials" (Einstein, 1916f, 65). Thus, ultimately, even in pre-general-relativistic theories, spacetime predictions always referred to the encounters of at least two physical systems.

Pre-general-relativistic physics instinctively considered one system as somehow 'more important' than the other. One wants to know *where* the material point is on the scaffolding and not the other way around. Thus, it was natural to think of the scaffolding as a three-dimensional *rigid reference body* (endowed with clocks). *Where* is a material point could be expressed unambiguously in terms of relative distances from the origin. However, already in the case of uniform acceleration, the rigid reference body had to be substituted by a three-dimensional *deformable reference body* in Born rigid motion. Since this reference body could not be put into rotation, Einstein had to capitulate and drop the very notion of a reference scaffolding. He finally realized that physics could still get by with a more tenuous material medium, a "*reference-mollusk*" (Einstein, 1916f, 67), say, an infinitely dense cloud of particles carrying clocks. Physics can make meaningful predictions about the encounters of such particles: the free-falling clock such and such collides with a particle such and such, as the hand of the clock coincides with a certain mark on the dial. This redefinition of what is observable in physics, however, implied a more subtle

[28] Darrigol, 2015 labels (a) 'perverted geodesy' and (b) 'concrete geodesy.' In our cartographic analogy, model (a) corresponds to taking the Cartesian coordinate differences x, y on the projection plane as meaningful as in footnote 7 and distances in different maps as distortions; (b) means that every map represents the same latitude and longitude on the curved globe and that the projection distortions are meaningless as in footnote 6. According to Valente, 2018, Einstein embraced the perverted geodesy point of view. However, to me, this claim seems to be too strong and in conflict with too many passages in which Einstein declared coordinates as meaningless. In my view, Einstein's vacillation between (a) and (b) is the consequence of the fact that he did not think geometrically in terms of the intrinsic structure of spacetime (the globe). He rather regarded the $g_{\mu\nu}$, on the one hand, as conversion factors (map legend) between coordinate differences and real differences and, at the same time, as a physical field that has certain values at certain points with certain coordinates.

conceptual reorientation. The very question of *where* this encounter happens has become meaningless. "The last remnant of physical materiality [*Gegenständlichkeit*]" of the coordinate system, Einstein could triumphantly claim, has been dissolved (Einstein, 1916a, 776).

5 The (Very) Last Remnant of Materiality of the Coordinate System

5.1 A Galilean Coordinate System: The Physical Meaning of the Requirement $\sqrt{-g} = 1$

In spite of his efforts to convince his readers of the viability of the requirements of general covariance, it is somewhat surprising that, in his 1916 paper, Einstein did not give the field equations in a generally covariant form but rather in the form eq. (11), that is, by imposing the coordinate condition $\sqrt{-g} = 1$. As it was suggested by a manuscript for an unpublished appendix to the review article (CPAE, Vol. 6, Doc. 31), Einstein considered it possible to derive the field equations from a variational principle with the Ricci scalar chosen as a Lagrangian without the $\sqrt{-g} = 1$ condition. He also knew already that the conservation laws are satisfied without introducing such specialization. Thus, Einstein considered the field equations as generally covariant but only presented them in a special coordinate system in which the distinction between tensors and tensor densities disappears, simplifying the calculations. However, in May 1916, after sending to Ehrenfest the proofs of his review article (Einstein to Ehrenfest, Apr. 29, 1916; CPAE, Vol. 8, Doc. 218), Einstein wrote to him that his specialization of the coordinate system was "not *just* based on laziness", seemingly implying that the postulate $\sqrt{-g} = 1$ was more than a calculation trick (Einstein to Ehrenfest, May 24, 1916; CPAE, Vol. 8, Doc. 218). Even if the field equations should be presented in a fully generally covariant form, Einstein, nevertheless, considered that it is desirable to find a physical motivation for specializing the coordinate system *a posteriori*.

Exacts solutions were of course of particular interest. Schwarzschild's papers delivering exact internal and external solutions were issued in April (Schwarzschild, 1916b) and May (Schwarzschild, 1916a) in polar coordinates of determinant 1. Hans Reissner (1916) soon found a generalization of the Schwarzschild metric, including an electrical charge relying on Einstein's generally covariant formulation of Maxwell's equations (Einstein, 1916b). Nevertheless, approximation schemes played an equally important role in the early application of the theory. At that time, Einstein, like Schwarzschild himself, was searching for wave solutions of field equations, which were expected in a field theory of gravitation by analogy to electrodynamics. Einstein initially failed to find such a solution in $\sqrt{-g} = 1$ coordinates, raising doubts that it was even possible. Such coordinate systems were in fact in conflict with the coordinate condition eq. (5) that was suitable to write down a wave equation (Einstein to Schwarzschild, Feb. 19, 1916; CPAE, Vol. 8, Doc. 220).

Einstein found a workaround in June, as one can infer from a letter to Lorentz: "I myself worked on the integration of the field equations in first-order approximation and examined gravitational waves. The results are in part astonishing. There are three kinds of waves, though only *one* type transports energy" (Einstein to Lorentz, Jun. 17, 1916; CPAE, Vol. 8, Doc. 226). The reason for the breakthrough was again a contribution of the work of the Leiden community.[29] Lorentz's Monday morning lecture played an important role in making the Leiden physicists familiar with the formalism of general relativity (Kox, 1992). In May, Droste (1917), Lorentz's doctoral student, found independently of Schwarzschild an exact spherical symmetric static solution for the mass point, however, dropping the condition $\sqrt{-g} = 1$. In June, De Sitter (1917) started to calculate the motion of planets and the Moon using an approximate solution of the field equations. De Sitter used yet another coordinate system that was also used by Lorentz in his lectures (Lorentz, 1917c), a coordinate system in which the velocity of light depends on position but not on direction.[30] De Sitter must have managed to convince Einstein that gravitational wave solutions are easier to find in Lorentz-coordinates than in Einstein-coordinates. He communicated to him the approximate values of the diagonal elements of the gravitational field of a mass point in Lorentz-coordinates in a private correspondence.

On June 22, Einstein presented to the Prussian Academy of Sciences a paper on gravitational waves (Einstein, 1916d). He calculated the approximation

$$g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu} \qquad \delta_{\mu\nu} = \begin{cases} 0 & \text{if } \mu \neq \nu, \\ 1 & \text{if } \mu = \nu\,. \end{cases} \tag{13}$$

Einstein introduced the trace-reversed tensor $\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - 1/2\delta_{\mu\nu}\gamma$, which plays the role of a four-vector potential in electrodynamics. The specialization contained in eq. (13) is conserved if one performs an infinitesimal transformation on the coordinates $x'_\mu = x_\mu + \xi^\mu$. One can then impose the condition eq. (5),[31] which is a linearized approximation that allows rewriting the field equations for weak fields in a form familiar in electrodynamics:

[29] For the first extensive analysis of this episode, cf. Kennefick, 2007, 53ff. more technical details are discussed by Weinstein, 2016.

[30] This coordinate system is therefore called 'isotropic.' I will refer to it as or Lorentz-coordinates.

[31] The harmonic coordinate condition that corresponds to the electromagnetic Lorenz gauge.

$$\sum_{\nu} \frac{\partial^2}{\partial x_\nu^2} \bar{\gamma}_{\mu\nu} = 2\mathrm{K} T_{\mu\nu} \,. \tag{14}$$

For a given material system $T_{\mu\nu}$ at the origin of the coordinate system, $\bar{\gamma}_{\mu\nu}$ can then be determined "in a manner analogous to that of retarded potentials in electrodynamics" (Einstein, 1916d, 688). As the simplest example of application, Einstein calculated the gravitational field of a mass point of mass M, resting at the point of origin of the coordinates, and obtained the diagonal values of the $g_{\mu\nu}$: "Mr. De Sitter sent me these values by letter; they differ from those which I previously gave [in eq. (12)] only in the choice of the system of reference" (Einstein, 1916d, 692). Einstein further calculated the components of a gravitational field stress-energy pseudotensor t_ν^μ of gravitation in this coordinate system. De Sitter's coordinate choice did not satisfy the condition $\sqrt{-g} = 1$; however, Einstein realized that this choice of coordinates was advantageous in performing calculations. Indeed, in the rest of the paper, Einstein derived solutions of eq. (14) representing plane gravitational waves (whose wavefront is flat, rather than curved). Einstein classified the waves into three types: longitudinal, transverse, and a new type of waves. Using the component t_4^4 of the stress-energy pseudotensor that he calculated in the first part of the paper, he determined the energy transported by such waves. Einstein realized that such component vanishes for longitudinal and transverse waves.

Einstein wrote the following to de Sitter on June 22, the day he submitted his paper on gravitational waves (Einstein, 1916d) to the academy: "Your solution for the mass point [...] [o]bviously [...] differs from my old one in the choice of the coordinate system, but not intrinsically" (Einstein to de Sitter, Jun. 22, 1916; CPAE, Vol. 8, Doc. 227). However, Einstein realized that the two types of waves that do not transport energy in Lorentz-coordinates could be transformed away using Einstein-coordinates satisfying $\sqrt{-g} = 1$. Thus, in an appendix to the paper, Einstein suggested that the latter have a "deep-seated physical justification" (Einstein, 1916d, 696). Einstein's $\sqrt{-g} = 1$ coordinate system K should be regarded as a 'good' preferred coordinate system, whereas de Sitter's Lorentz-coordinate system K' is suitable for calculations but 'bad' from a physical point of view. Einstein went so far labeling these coordinate systems 'Galilean.' Indeed, to a certain extent, their role resembles that of an inertial system, a system with respect to which the real position of particles and components of fields are defined. As one can infer from a remark made on the margin of Einstein's letter, de Sitter was puzzled: "What is this 'Galileischer Raum'? Could one not just as well say the 'Aether'?" (de Sitter's margin note, in Einstein to de Sitter, Jun. 22, 1916; CPAE, Vol. 8, Doc. 227).

De Sitter returned to this issue in a letter written at the beginning of July.[32] He pointed out that "in order to determine the coordinate system one needs in general four conditions" (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). As Hilbert (1915) had explained in his 1915 paper, no definite solution of generally covariant field equations can be found, unless these four additional noncovariant equations are imposed (e.g., condition eq. (5)). However, once a solution is obtained, one must be able to transform the solution freely to other coordinate systems. On the contrary, if the four conditions fixing the coordinate system are chosen once and for all, "this would mean that one relinquishes relativity" and falls back to an absolute space and time that exists "independently of any physical observation and from the entire physics" (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). Therefore, de Sitter found it unsuitable that Einstein in his letter "called the coordinate system characterized through $-g = 1$ as a '*Galileischer Raum*' " (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). De Sitter pointed out that, imposing this requirement, one, like Schwarzschild, would buee forced to exclude polar coordinates; however, there is nothing 'ungalileian' in usual polar coordinates (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). As a matter of fact, Droste was able to find the same solution as Schwarzschild without imposing the condition $\sqrt{-g} = 1$.

According to de Sitter, imposing four conditions on the coordinates is indeed "a kind freezing or immobilization of the of space, so that the choice of coordinates in independent of the matter which might be present in space or of its motion and energy-changes" (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). In principle, one might also call "these frozen space in Galileo's name and deem the other spaces as non-real or apparent". However, in de Sitter's view, it must be emphasized that this distinction is "fully arbitrary" (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). "If other physicists had discovered in this or other spaces something advantageous, they would have the same right to call their space as the 'true' one and define it as 'galileian' or maybe even the 'aether'" (de Sitter to Einstein, Jul. 7, 1916; CPAE, Vol. 8[12], Doc. 227a). Hence, the role of the four-coordinate conditions is to help obtain a solution of a particular kind of problems. For example, the coordinate eq. (5) gives especially simple solutions in the case of linearized gravitational equations in empty space but might not be suitable in other cases. Thus, once a solution is obtained, they may be discarded, as one may throw away the proverbial ladder after climbing it up.

De Sitter interpreted Einstein's preferences for $\sqrt{-g} = 1$ coordinates in this way, but he was not sure whether he had misunderstood Einstein's stance. In his reply, written a week later, Einstein insisted on the importance "of

[32] In the manuscript, the letter is dated June 7. However, since it refers to Einstein's paper of June 26, the correct date is probably July 7.

further specializ[ing] the [coordinate] system in a natural way, even in the interests of better comparability of the found solutions" (Einstein to de Sitter, Jul. 15, 1916; CPAE, Vol. 8, Doc. 235). Indeed, without introducing a specialization of the coordinate system, no solution can be found. However, Einstein admitted that it was "not good if in my letter I called the system $\sqrt{-g} = 1$ 'Galilean'" (Einstein to de Sitter, Jul. 15, 1916; CPAE, Vol. 8, Doc. 235). Only a coordinate system in which all $g_{\mu\nu}$'s are constant should be called that. Einstein, in principle, agreed with de Sitter's interpretation of the distinction between 'real' and 'apparent.' If one finds a wave solution of the field equations, there are two possibilities. Either this wave-like process is independent of the choice of the coordinate system or it is just a coordinate effect. In the latter case, the wave motion can be transformed away by switching to another coordinate system and it is non-real. De Sitter's K' coordinates, that is, Lorentz-coordinates, were more advantageous for calculation. However, Einstein's K satisfying $\sqrt{-g} = 1$ had ultimately a special status from a physical point of view since it allows only waves that transport energy. Gravitational waves with respect to K are 'real,' whereas with respect to K' they are 'unreal' (Einstein to de Sitter, Jul. 15, 1916; CPAE, Vol. 8, Doc. 235).

However, Einstein's Leiden interlocutors were puzzled. As Lorentz wrote to de Sitter a day later, it was hard to avoid "the impression that E[instein] becomes somewhat unfaithful to his own doctrine now that he is going to privilege one particular kind of coordinate system and speak of a 'Galilei'schen Raum' " (Lorentz to de Sitter, Jul. 16, 1916). If the requirement $\sqrt{-g} = 1$ is used exclusively, this seems to violate the spirit of general relativity. Even if Einstein insisted that the field equations were still valid in all coordinates (Einstein to De Donder, Jul. 23, 16; CPAE, Vol. 8, Doc. 240), the distinction between a set of 'good' coordinate systems with respect to which phenomena look real and 'bad' one with respect to which they are 'apparent' is introduced again from the backdoor. The attitude of the Leiden group toward this issue seemed to have been quite different. De Sitter wrote a little paper in English in July, in which he put the problem of the choice of coordinates and notion of coordinate condition in a way that is closer to the modern view (De Sitter, 1916).

The 10 differential equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ do not determine $g_{\mu\nu}$ completely. If some set of $g_{\mu\nu}$ is a solution to Einstein's equations in one coordinate system x_1, x_2, x_3, x_4, it should also be a solution in any other coordinate system x_1, x_2, x_3, x_4. The coordinate system is specified by certain added coordinate conditions, four equations for $g_{\mu\nu}$ that must not be covariant. Different choices of these four conditions might suit different problems; however, choice among them is fully conventional In de Sitter's words,

> An important remark must still be made with respect to the equations $[G_{\mu\nu} = \kappa T_{\mu\nu}]$. Since $[G_{\mu\nu}]' = G'_{\mu\nu}$ and $\mathrm{T}_{\mu\nu} = \mathrm{T}_{\mu\nu}$ there are 10 of these equations, and there are also coefficients g_{ik} to be determined. However, the equations $[G_{\mu\nu} = \kappa T_{\mu\nu}]$ are not independent of each other, but four of them are consequences of the other six. They, therefore, are not sufficient to determine the g_ij. This is essential. For the $g_{\mu\nu}$ determine the character of the four-dimensional system of reference, and the principle of general relativity requires that this system can be arbitrarily chosen [...] In order to determine the $g_{\mu\nu}$ completely we must to the equations $[G_{\mu\nu} = \kappa T_{\mu\nu}]$ ad four additional conditions, which can be arbitrarily chosen, and which determine the choice of the system of reference. According to the form of these additional conditions the $g_{\mu\nu}$ will be different functions of the coordinates, the equations of motion will be different, the course of rays of light will be different, but *there will always be the same intersections of world-lines, and consequently all observable phenomena will be exactly the same*: they will only be described by a different system of space coordinates and a different time. One system may be more convenient than another-this is a matter of taste; but we cannot say that one system is true and another false (Sitter, 1916, 418–419).

Thus de Sitter resorted to the point-coincidence argument to show that the choice of the coordinate condition is arbitrary, and that none of such conditions, including Einstein's preferred condition $\sqrt{-g} = 1$, is privileged. The observable phenomena, that is the 'coincidences' are not affected by the coordinate system used, and this is all what counts from a physicist point of view.

After writing two fundamental papers on the emission and absorption of radiation in quantum theory (Einstein, 1916e,h), in the fall of 1916, Einstein stayed for a few weeks in the Netherlands. In the "unforgettable hours" (Einstein to Besso, Oct. 31, 2016; CPAE, Vol. 8, Doc. 270) he spent with Lorentz, Ehrenfest, Nordström, and younger physicists like Droste and Fokker, discussions regarding this issue must have inevitably ensued. Besides starting to address questions of cosmological nature with de Sitter (de Sitter to Einstein, Nov. 1, 1916; CPAE, Vol. 8, Doc. 272), in November, Einstein sent to Ehrenfest the galley proofs of his paper in which he derived the field equations from a Lagrangian without imposing the coordinate condition $\sqrt{-g} = 1$ and asked to show them to 'co-conspirators' (*X-Brüdern*) in Leiden (de Sitter to Einstein, Nov. 1, 1916; CPAE, Vol. 8, Doc. 272). The paper was nothing but the March 1916 appendix (CPAE, Vol. 6, Doc. 31). Einstein was now satisfied that the Hamiltonian function did not become heavily complicated even if one lets the coordinate system general, that is, without imposing such $\sqrt{-g} = 1$ condition. In particular, Einstein insisted that the derivation of the conservation that he obtained before imposing the condition $\sqrt{-g} = 1$ holds in all coordinate systems. As he wrote to Hermann Weyl, "one can see how the free eligibility [*Wählbarkeit*] of the reference system can be of advantage in the calculation" (Einstein to Weyl, Nov. 23, 1916; CPAE, Vol. 8, Doc. 278). Einstein's flirtation with the existence of privileged coordinate

systems seems to have come to an end.

5.2 *The Point-Coincidence Argument Again: The Correspondence with Gustav Mie*

At the beginning of December of 1916, Droste presented his spherical symmetric solution with additional results in the form of a doctoral dissertation in Dutch, which was published in the spring of 1917 (Droste, 1917). Opening the dissertation, Droste insisted that "he has not made use of the coordinate system often used by Einstein, where $\sqrt{-g} = 1$" (Droste, 1917, XII). According to Droste, Einstein's coordinate choice, on the one hand, did not significantly simplify the calculations and, on the other hand, diverted the attention from the particular characteristics of the equations to that of the coordinate system (Droste, 1917, XII). The choice of the coordinate system was recognized as fully arbitrary, "although one choice will be more effective than the other" (Droste, 1917, XII). As he wrote to Ehrenfest in the spring of 1917, Einstein found "Droste's dissertation [...] extraordinarily beautiful" (Einstein to Ehrenfest, May 25, 1917). Nevertheless, as late as the spring of 1917, he probably still believed that $\sqrt{-g} = 1$ coordinates somehow reflected the objective properties of spacetime. The condition is, for example, used in Einstein's cosmological work (Einstein, 1917a, 145). Einstein was again convinced to free himself of this last rest of materiality of the coordinate system through an exchange with the Leiden community (cf. Kennefick, 2007, 63ff. for more details; see also Weinstein, 2015, 254ff.).

In June, Ehrenfest wrote to Einstein that Nordström, newly engaged, was "preparing to write you" (Ehrenfest to Einstein, Jun. 14, 1917; CPAE, Vol. 8, Doc. 352). Nordström got back to Einstein only in August after his marriage (Nordström to Einstein, Oct. 23, 1917; CPAE, Vol. 8, Doc. 393). He had calculated the distribution of energy in a gravitational field with a spherical symmetry, which is the quantity t_4^4 (cf. also Lorentz, 1918, §48). Nordström relied on the notation introduced by Droste in his dissertation but used Schwarzschild's polar coordinates satisfying $\sqrt{-g} = 1$. He rewrote this equation in Cartesian coordinates so that $\sqrt{-g} = 1$ would be satisfied and calculated the components of the energy of the gravitational field using the formula of Einstein's 1916 general theory of relativity review paper (Einstein, 1916b, 806, eq. 50). Einstein had not performed this calculation in that paper since he made his calculations using Lorentz-coordinates. Nordström hence reached the unsettling conclusion that

$$t_4^4 = 0\,.$$

In other terms, "[t]he gravitational field, therefore, would have then no energy outside of the body" (Einstein to de Sitter, Sep. 22, 1917; CPAE, Vol. 5, Doc. 382), which was in disagreement with Einstein's result in his 1916 gravitational wave paper. Puzzled, Nordström checked his results and attached an addendum to the letter, in which he wondered whether "[t]he entire contradiction could lie in the fact that we are using different coordinate systems" (Einstein to de Sitter, Sep. 22, 1917; CPAE, Vol. 5, Doc. 382). In his 1916 gravitational wave paper, Einstein had used Lorentz-coordinates "characterized by the velocity of light being equal in all directions" (Einstein to de Sitter, Sep. 22, 1917; CPAE, Vol. 5, Doc. 382). As Nordström pointed out, an exact solution for the point mass in this coordinate system was also investigated by Droste in his dissertation. Relying on Droste's calculations, but keeping them at the same level of approximation that one can find Einstein's work, he found

$$t_4^4 = \frac{1}{4}\frac{\alpha^2}{r^4}\,.$$

Thus, the energy density of the gravitational field does not vanish in Lorentz-coordinates but does in Einstein's $\sqrt{-g} = 1$ system. As a consequence, the coordinate system that Einstein believed to be physically 'privileged' has a rather unphysical implication that the gravitational field of a point mass has no energy.

Einstein must have suspected some computational mistake. However, in October, Nordström confirmed his calculations using the approximate values and again obtained $t_4^4 = 0$ (Nordström to Einstein, Oct. 23, 1917; CPAE, Vol. 8, Doc. 393) when the system of coordinates is chosen so that the condition $\sqrt{-g} = 1$ is satisfied. Thus, Einstein preferred coordinates system had the counterintuitive implication that there appears to be no energy in the gravitational field of a point mass (Nordström, 1918a,b; cf. also Schrödinger, 1918). Using a different system of coordinates, this might not be the case, as in Lorentz-de Sitter's coordinates. After discussing the matter with Fokker, Nordström came to the conclusion that there was no particular reason why it was useful to impose such a condition. Indeed, $\sqrt{-g} = 1$ coordinates have a more direct physical interpretation, since they provide a fixed absolute measuring unit (Anderson and Finkelstein, 1971). The unit volume of a four-dimensional element is the same in coordinate units as in natural units[33] as in Galileian coordinates. However, it does not seem necessary for coordinate differences and natural distances to be expressed in the same units, and the latter might be treated as pure numbers (Nordström to Einstein, Oct. 23, 1917; CPAE, Vol. 8, Doc. 393).

At about the same time, the physicist Gustav Mie, who had critically followed the development of Einstein's theory since 1913, sent to Einstein at his request (Einstein to Mie, Dec. 14, 1917; CPAE, Vol. 8, Doc. 407) the published

[33]Hence, the label unimodular that we use for such coordinates, where 'modulus' is the volume element.

version of his Göttingen Wolfskehl lectures on relativity (Mie, 1917a,b,c). On December 22, Einstein repaid the courtesy by sending Mie his 1916 'Hamiltonian' paper (Einstein, 1916c) and his 1916 paper on gravitational waves (Einstein, 1916d). He admitted, however, that the latter contained a computational error. By that time, Einstein must have realized that he in that paper had used the trace-reversed tensor $\bar{\gamma}_{\mu\nu}$ instead of $\gamma_{\mu\nu}$ to calculate the energy density of gravitational waves t_4^4 (Einstein to Mie, Dec. 22, 1917; CPAE, Vol. 8, Doc. 416). Commenting on Mie's lectures, Einstein pointed out that he disagreed with Mie's "considerations about the necessity of the existence of *preferred coordinate systems*". "They can easily cause confusion", he added (Einstein to Mie, Dec. 22, 1917; CPAE, Vol. 8, Doc. 416). He revealed to Mie that he was preparing a new paper on gravitational waves, which he submitted by the end of December. It was presented by Planck at the end of January December (Einstein, 1918c).

In this classical paper, besides providing the so-called quadrupole formula (Kennefick, 2007, 65ff.), Einstein presented the role of coordinates and coordinate conditions in a way that appears closer to that of a modern textbook. Coordinate transformations involve four arbitrary functions, he wrote, so that four-coordinate conditions must be added to the six independent equations to permit unique solutions Einstein, 1918c, 155. This applies to linearized gravity as well. Given a solution $g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu}$, one might obtain a different solution by an infinitesimal coordinate transformation $x_\nu + \xi_\nu$ leading to $g'_{\mu\nu} = \delta_{\mu\nu} + \gamma'_{\mu\nu}$. However, this does not change the physical content. The freedom in the choice of the vector ξ_ν is merely an expression of the freedom of choosing coordinates. Gravitational waves that transport no energy can always be generated from a field-free system by a mere coordinate transformation Einstein, 1918c, 161. As a matter of fact, one can always introduce a sinusoidal behavior of $\gamma_{\mu\nu}$ by introducing the Minkowski metric in nonstandard coordinates. However, those waves are only apparent and should be distinguished from real waves, which cannot be introduced in this way.

Einstein's attitude toward the role of coordinates in physics at that time can be better appreciated by considering how he reacted to Mie's counter objection (Mie to Einstein, Feb. 5, 1918; CPAE, Vol. 8, Doc. 456). To make his case, Mie introduced a paradox that he had used in his Göttingen lectures (Mie, 1917a,b,c). He considered a rod at rest in a certain coordinate system. Using light rays as a standard, the rod appears to be straight. However, by introducing a fast-changing gravitational field and thus a withering coordinate system, the rod would appear to be curved and to move like a snake, without any physical cause (Illy, 1988). The two coordinate systems are indeed geometrically equivalent; however, physically, the first coordinate system is clearly better than the second one (Mie to Einstein, Feb. 5, 1918; CPAE, Vol. 8, Doc. 456). However, at this point, Einstein considered Mie's yearn for a 'good' coordinate system unacceptable. To make is stance clear, Einstein resorted again to the point-coincidence argument to reinstate his conviction that coordinates in general relativity are meaningless:

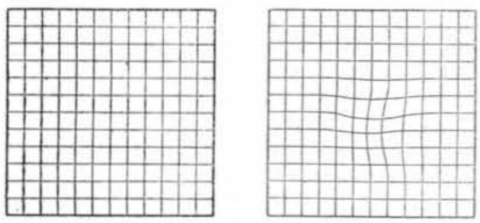

Figure 5 – Two equivalent coordinate systems, net I and net II (Mie, 1917c)

> I do not agree at all with your reflection about the bent (flapping) rod. *All physical descriptions that yield the same observable relations (coincidences) are equivalent in principle*, provided that both descriptions are also based on the same laws of nature. The choice of coordinates can have great practical importance from the point of view of clarity of description; in principle, though, it is entirely insignificant. It means nothing that 'arbitrary gravitational fields' occur, depending on the coordinate choice; it is not the fields themselves that lay claim to reality. They are merely analytical auxiliaries in the description of realities; in principle, one can actually only learn something about the latter by eliminating the coordinates. The ghost of absolute space haunts your rod example. The argument works *ad hominem* but does not hit the mark at all as I interpret it. It is not a question of a violation of logic. That a rod must either be straight or bent but cannot be both corresponds exactly to the objection advanced by philosophers against the special theory of rel. that the same body (at the same ⟨instant⟩ spacetime point) cannot be simultaneously at rest and in motion (Einstein to Mie, Feb. 8, 1918; CPAE, Vol. 8, Doc. 460; my emphasis).

Mie's mistake was after all a sort of a dynamical version of the same conceptual confusions that we have encountered in the previous pages. In his lectures, Mie imagined (fig. 5) that someone constructed a rectangular coordinate net (net I) using light rays (Mie, 1917c, 599). One can substitute this net with another curvilinear coordinate system (net II), which would correspond to the presence of a gravitational field. The light rays that appear optically straight in the first coordinate system would appear curved in the second coordinate system. According to the general relativity principle, net I is just as good as net II. However, this cannot be true. If someone introduces a gravitational field that changes periodically, light rays will appear as if they were moving like a vibrating string.[34] According to Mie, no one would take this result seriously. If all coordinate systems are on the

[34]In our preferred analogy, Mie's thought experiment corresponded to the case of a sort of a dynamical map projection of Earth, which

same footing mathematically, physically, net I will be a more reasonable coordinate system that represents the 'real' path of light rays. Given this premise, it is not surprising that Einstein relied once again on the point-coincidence argument to convince his interlocutor that this was not the case. Light rays will follow the same paths and intersect at the same angles in all coordinate nets, I and II, or whatever oscillation between them one can imagine. Thus, they will produce the same interference patterns on a photographic plate, that is, the same coincidences.

Mie had just discussed this issue with the young Königsberg mathematician, Kretschmann (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 465). Thus, in his reply, Mie alerted Einstein that the latter had shown that the point-coincidence argument was actually meaningless. In any theory, observations are reducible to point-coincidences, and thus any theory can be presented in an arbitrary coordinate system without changing its physical content (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 465). In a letter published in May, as Kretschmann's paper (Kretschmann, 1918) was already published, Mie tried to defend his point of view by pointing out that it was not dissimilar to the one Einstein had introduced in his 1916 gravitational wave paper. As we have seen, Einstein (1916d) had, in fact, insisted that there was 'a deep physical justification' for imposing the condition $\sqrt{-g} = 1$. As Mie pointed out, it was roughly the same attitude that he himself had taken in his Göttingen lectures. In particular, Einstein's 'deep physical justification' was "also almost exactly the same one I have explained with the slithering rod" (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 465). Searching for gravitational wave solutions, Einstein simply came upon a coordinate system in which such a vibrating rod would occur, and he rejected this "'undulatory oscillating coordinate system' as nonphysical" (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 532).

A mathematician, for sure, can consider all coordinate systems equivalent. After all, they are arbitrary numbering systems. However, a physicist, extracting quantities measurable with rods and clocks, light rays, and so forth from those numbers, is bound to prefer more reasonable coordinate systems over others (Mie, 1920a,b, 1921). In particular, one can never ascribe the same legitimacy to a coordinate system in which what Einstein called 'apparent' and Mie 'fictitious' gravitational phenomena exist—for example, gravitational waves that do not transport energy. All coordinate systems are equivalent "'in principle' "; by assuming the mathematician's standpoint, one can define everything "on sovereign authority" (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 465). However, from a physical point of view, the coordinate system in which the rod is not flapping is clearly better than the one in which it is not: "one can see clearly from this exactly why, as you say, 'there is profound physical justification for a coordinate choice' according to the condition $\sqrt{-g} = 1$" (Mie to Einstein, Feb. 17, 1918; CPAE, Vol. 8, Doc. 465). However, Einstein seemed to have now abandoned even the last trace of the materiality of the coordinate system. By changing the coordinates, one changes the form of the solution, but not its physical meaning. The distinction between a coordinate system 'good' only for calculations, and a coordinate system 'good' from a physical point of view could not be uphold. A nice plane wave, obtained by imposing the coordinate condition 5, becomes more complicated with a different choice of coordinates. Nevertheless, all gravitational waves that differ only by a coordinate transformation are physically the same. They produce the same elliptical oscillation in the distribution of suspended mirrored test masses, thus the same interference patterns in a gravitational wave detector. Ultimately, they produce the same 'coincidences.'

6 Conclusion: The Immateriality of the Coordinate System

On May 24, the *Annalen* published a brief programmatic paper in which Einstein, among other things, replied to Kretschmann's 'triviality-objection' against the point-coincidence argument. Einstein, so to speak, doubled down and elevated the point-coincidence argument to a fundamental physical principle in physics, the *relativity principle* (Gutfreund and Renn, 2015): "Nature's laws are merely statements about temporal-spatial coincidences" (Einstein, 1918b, 241). The requirement that only point-coincidences are observable, as the physical expression of the relativity principle, seems to have become independent of general covariance as a formal requirement (Lehner, 2005). It is indeed possible that Kretschmann's objection did not catch Einstein by surprise. As far as I can see, it was never noticed that Einstein was aware early on that any theory can be trivially presented in a way in which coordinates are meaningless parameters (cf. section 1.1 and sec. 1.2) without an operational interpretation.[35] The point of issue was a different one.

In previous theories, there was a way to eliminate the parameters; thus, it was simpler to formulate the laws of nature with respect to a class of privileged coordinate systems in which coordinate differences are physically meaningful numbers resulting from rods-and-clocks measurements. If one "demands this, then one confers to the magnitudes x, y, z, t physical meaning"; this is "*convenient (although not logically necessary)*" (Einstein to

changes cyclically and smoothly from, say, cylindrical to stereographic, and then back again to cylindrical, on the same flat projection plane. Three cities that lie on a vertical line do not do the same in a stereographic projection. Thus, in our dynamical map, the line connecting them would appear to move in a snake-like fashion on the projection plane. Again, the illusion emerges if one takes the coordinates x, y on the projection place as real distances. All these maps represent the same intrinsic geometrical structure of the globe. Most physicists seem to think in terms of the maps, not in terms of the globe.

[35]This corresponds to what Bergmann has called 'weak general covariance' (Bergmann, 1957, 11).

Adler, Aug. 4, 1918; CPAE, Vol. 8, Doc. 594; my emphasis). However, such 'deparameterization' is never possible in general relativity. Because of the equivalence principle, one has lost the ability to distinguish good, nonaccelerated rods and clocks that reliably measure coordinates from the bad accelerating ones whose measurement cannot be trusted. Einstein, with a creative leap of logic, concluded that there was no reason to prefer a coordinate system over another. The impossibility of formulating the theory with reference to a privileged class of coordinate systems gave to the "the relativity postulate [...] a heuristic meaning in spite of the apparent lack of real content" (Einstein, 1918b, deleted passage). While general-relativistic theories, as any other fundamental theory, cannot dispense with coordinates, they are singled out by the fact that they can only use coordinates as nonmeasurable variables (Einstein, 1918a, 699).[36] A solution of Maxwell equations $\varphi(x_\nu)$ is physically meaningful, only once one specifies how the parameter x_ν are measurable; on the contrary, the physical meaning of a solution of Einstein equations $g_{\mu\nu}(x_\nu)$ is what remains, once the dependence on the variables x_ν is eliminated.

The fact that Einstein's answer to Kretschmann appears to be nearly incomprehensible to the modern reader is possibly the consequence of our unfamiliarity with Einstein's algebraic, nongeometrical way of thinking. Einstein reasoned in terms of variables, kinematic, and dynamics variables, that enter into the equations of physics and of the procedures through which these variables can be measured. In general-relativistic theories "the connection between *quantities in equations* and *measurable quantities* is far more indirect than in the customary theories of old" (Einstein, 1918a, 700). In general relativity, it is impossible to devise an experiment that will measure whether $R_{\mu\nu} = 0$[37] at a point with the coordinates x_ν, because the values of the coordinates by themselves are not measurable. However, one can ask whether or not $R_{\mu\nu} = 0$ at the point where two particles meet. The point-coincidence argument was the operationalist answer to what was perceived as an operationalist conundrum, the impossibility to interpret the coordinates as readings on rods and clocks. The components $g_{\mu\nu}$ of the gravitational field at a certain location x, y, z, t are not measurable, they represent "nothing physically real but the gravitational field together with other data does" (Einstein, 1918a, 701).

In this sense, the point-coincidence argument, far from being a mere trick to escape from the hole argument, can be considered as Einstein's mature stance toward what is actually observable in physics. "Physical experiences [are] always assessments of point-coincidences (spacetime coincidences)" (Einstein, 1919, [p. 3]). As far as I can see, this was Einstein's last formulation of the point-coincidence argument. A nonrelativistic theory, one might say, contains not only statements about coincidences, but also statements about the coordinate systems that serve for their description. On the contrary, a general-relativistic theory entails only relationships between coincidences, the statements of which are independent of the choice of coordinates. Although confusions about the role of coordinates in general relativity seem to have been quite widespread among relativists for several years (Darrigol, 2015), the battle could be considered won (Einstein, 1920). Einstein, moving beyond general relativity toward the unified field theory program, seems to have lost interest in the 'operationalist' rhetoric of the observability of coincidences. The point-coincidence argument is not mentioned in his Princeton lectures (Einstein, 1922), Einstein's only presentation of the theory in textbook form.

Operationalism was for Einstein a 'throwaway philosophy' designed to be discarded once it played its iconoclastic role. As I plan to show elsewhere, it was Einstein's former assistant, Bergmann, who, in the 1950s, brought the language of 'coincidences' to the fore again. Einstein's 'operationalist' notion of 'coincidence' as the encounters of light rays and material particles was transformed into the 'realistic' notion of 'coincidence' as corealization of field values (Bergmann, 1961, 1962; Bergmann and Komar, 1962), ultimately of the gravitational field alone (Bergmann and Komar, 1960; Komar, 1958). A gravitational field has a certain value *where* four scalar functions of the some matter field have certain value. Where not matter is present, one can construct such scalars from the gravitational field itself. However, the gravitational field itself is, so to speak, *nowhere*. The "the world point", as Bergmann once wrote, "acquires reality only to the extent that it becomes the bearer of specified properties of the physical fields [...]. Perhaps this recognition will turn out to be of value in the years to come ..." (Bergmann, 1980, 176).

Abbreviations

CPAE Albert Einstein (1987–). *The Collected Papers of Albert Einstein*. Ed. by John Stachel et al. 15 vols. Princeton: Princeton University Press, 1987–.

EA *The Albert Einstein Archives at the Hebrew University of Jerusalem.*

MGA Hermann Minkowski (1911). *Gesammelte Abhandlungen*. Ed. by David Hilbert, Andreas Speiser, and Hermann Weyl. Leipzig: Teubner, 1911.

SCHAL Hendrik Antoon Lorentz (2008–). *The Scientific Correspondence of H.A. Lorenzt*. Ed. by A. J. Kox. New York: Springer, 2008–. I thank A.J. Kox for giving me access to the unpublished vol. 2.

[36] Strong general covariance in Bergmann's parlance (Bergmann, 1957, 11).

[37] The Ricci tensor.

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