On the Very Idea of Distant Correlations

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Abstract

Contemporary debate over laws of nature centers around Humean supervenience, the thesis that everything supervenes on the distribution of non-nomic facts. The key ingredient of this thesis is the idea that nomic-like concepts—law, chance, causation, etc.—are expressible in terms of the *regularities* of non-nomic facts. Inherent to this idea is the tacit conviction that regularities, "constant conjunctions" of non-nomic facts do supervene on the distribution of non-nomic facts. This paper raises a challenge for this conviction. It will be pointed out that the notion of regularity, understood as statistical correlation, has a necessary conceptual component not clearly identified before—I shall call this the "conjunctive relation" of the correlated events. On the other hand, it will be argued that there exists no unambiguous, non-circular way in which this relation could be determined. In this regard, the notion of correlation is similar to that of distant simultaneity where the necessary conceptual component is the one-way speed of light, whose value doesn't seem to be determined by matters of (non-nomic) facts.

Keywords: regularity, correlation, causal explanation, Common Cause Principle, EPR–Bell, distant simultaneity, Humean supervenience

1 Introduction

Suppose that two fair coins are tossed at the two ends of a corridor and the succession of outcomes is registered. Compare the four possible patterns of results depicted in Fig. 1. It is apparent that patterns (b), (c) and (d), by contrast with pattern (a), indicate strong *correlation* between the events at the two ends of the corridor. What seems to be distinctive about these three patterns is that one can find a suitable assignment of the outcomes on the two sides such that whenever Heads comes up on one side the corresponding outcome is Tails on the other side, and vice versa. Notice, however, that in fact this condition is also satisfied in pattern (a). As about half of the outcomes are Heads and Tails on both sides, one can always find such a pairing of them according to which, at least in a large portion of the tosses, Heads on one side will correspond to Tails on the other, and vice versa (see Fig. 2). Although such a correspondence can be established, no one would regard pattern (a) as being indicative of any kind of *real, meaningful* correlation whatsoever. There must be more to correlation than the mere existence of such an arbitrary correspondence. There must be a difference between a true regularity and a gratuitous association of events here and there. What makes the difference?

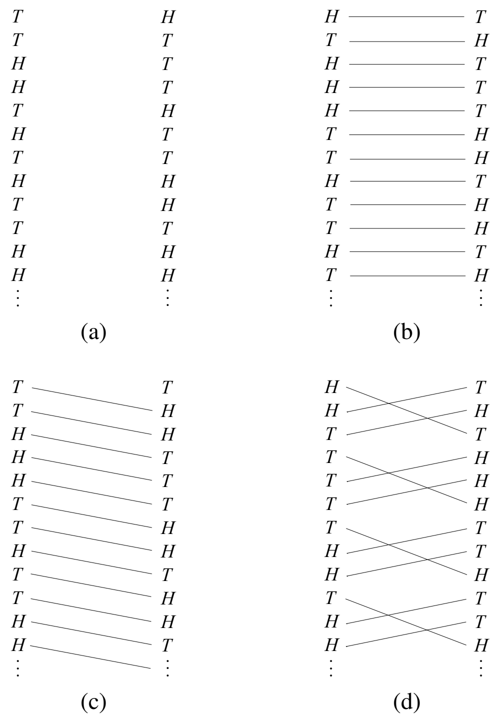

Figure 1: *Four possible patterns of results of coin tosses at the two ends of a corridor: (a) no correlation between the outcomes on the two sides; (b) whenever Heads comes up on one side, the simultaneous outcome is Tails on the other side, and vice versa; (c) whenever Heads/Tails comes up on the left hand side, the outcome of the succeeding toss is Tails/Heads on the right hand side; (d) a more complex regularity between the outcomes on the two sides*

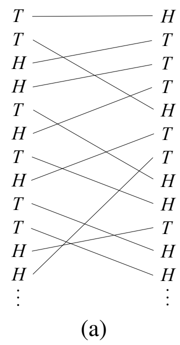

Figure 2: *Even if the pattern of results shows no regularity whatsoever, one can always find such a pairing of the outcomes according to which Heads on one side will correspond to Tails on the other, and vice versa*

This question is somewhat analogous to the problem of distant simultaneity in the philosophy of space and time (Einstein 1905, 38–40; Janis 2014). Under what conditions can one say that two events occur simultaneously (in a given frame of reference)? Perhaps there is an evident answer when the two events have the same or neighboring spatial location. But when the two events are far apart in space there no longer exists an obvious way to tell if they are simultaneous—"simultaneity" no longer has an obvious *meaning*—, and a non-trivial physical definition is required.

Correlation means that two events always or often occur in conjunction. The notion of "in conjunction" is much like "simultaneous." In some cases there is a natural way to understand this term. Suppose a coin is tossed repeatedly. We find that whenever it hits the table loudly the coin comes up Heads, and whenever it hits the table quietly it comes up Tails. That is to say, "loud" and "Heads," and "quiet" and "Tails" always happen *in conjunction*. Here, the relation "in conjunction" means that events "loud" and "Heads" occur in the *same* toss; that is, properties "loud" and "Heads" are properties of *one and the same* phenomenon, a particular toss of the coin. But when the correlating events belong to different phenomena, perhaps far apart in space and time, there no longer exists an obvious reading of when the events in question occur "jointly." In the two coin example above there is no obvious way to determine which pairs of tosses, one from the left and one from the right, happen "in conjunction." Much like in the case of simultaneity, what we need is a non-trivial physical definition of a conjunctive relation for distant/distinct events.

The paper is devoted to unpacking this problem. In Section 2 I shall give a more precise formulation of the question. Section 3 provides an instructive illustration in the context of the EPR–Bell problem in quantum mechanics. Section 4 will survey possible answers and at the same time argue that none of them is unproblematic. I conclude in Section 5.

2 An Essential Conceptual Component of Statistical Correlation

When I say "correlation" or "regularity" I mean the following notion of *statistical correlation*. Consider an experiment repeatedly performed for a large number of times. Each run of the experiment is a *particular* event; a little piece of the history of universe located at a definite spacetime region. Each of these particular events may instantiate certain event *types* $A,B,C,...$, together with their Boolean combinations, characterizing what happens in the given run. One can count how many times the different event types $A,B,C,...$ occurred in the ensemble of runs; let $p(A), p(B), p(C),...$ denote the relative frequencies. Event types A and B are said to be *independent* if

$$p(A \wedge B) = p(A)\, p(B) \tag{1}$$

otherwise they are *correlated*. We say that A and B are *maximally correlated* if

$$p(A \wedge B) = p(A) = p(B) \tag{2}$$

or

$$p(A \wedge \neg B) = p(A) = p(\neg B) \tag{3}$$

We shall use the term *perfect correlation* for the case when only one of the two equalities holds in (2) or (3); an example is

$$p(A \wedge B) = 0 \tag{4}$$

In the single coin example in Section 1 the ensemble of runs consists of the consecutive tosses. The event types we consider are "Heads," "Tails," "loud" and "quiet." There is maximal correlation between "Heads" and "loud," and "Tails" and "quiet," for example:

$$\frac{1}{2} = p(\text{Heads}) = p(\text{loud}) = p(\text{Heads} \wedge \text{loud}) \neq p(\text{Heads})\, p(\text{loud}) = \frac{1}{4} \tag{5}$$

In the two coin example in Section 1, take the collection of the simultaneous pairs of tosses as the ensemble of runs. Event types can be associated with the subsets of the four elementary outcomes $(H,H),(H,T),(T,H),(T,H)$ a double toss may possibly result in; for example $H_{\text{left}} = \{(H,H),(H,T)\}, T_{\text{right}} = \{(H,T),(T,T)\}$. Pattern (b) in Fig. 1 shows maximal correlation between event types "Heads" and "Tails" on the opposite sides; for example:

$$\frac{1}{2} = p(H_{\text{left}}) = p\left(T_{\text{right}}\right) = p\left(H_{\text{left}} \wedge T_{\text{right}}\right) \neq p(H_{\text{left}})\, p\left(T_{\text{right}}\right) = \frac{1}{4} \tag{6}$$

The notion of relative frequency and hence statistical correlation require three conceptual components: 1) a statistical ensemble $\mathscr{E}$ of particular events, 2) a set of outcomes X and a Boolean algebra Σ of its subsets corresponding to the event types possibly instantiated,[1] 3) a function $o : \mathscr{E} \to X$ that specifies which outcomes and thereby which event types are realized in which elements of the ensemble. Once they are given, relative frequencies and hence correlations are then determined. In what follows I shall refer to a collection $(\mathscr{E}, X, \Sigma, o)$ of these necessary components as a *frequency space*.

A frequency space provides the minimal conceptual framework in which one can meaningfully talk about frequencies and correlations; in which a phenomenon can be meaningfully described in terms of frequencies and correlations of events. In concrete examples of correlations, however, the frequency spaces in question often possess a specific extra structure, originating in the specific way in which these frequency spaces are constructed. To see this, consider three further examples of correlations.

Sea levels in Venice and bread prices in London The statistical ensemble consists of pairs of particular events that characterize the changes of sea level in Venice and the changes of bread price in London in each year, respectively. Allegedly (Sober 1988), there is a correlation between event types "increase in sea level" and "increase in bread price."

Goals on the football pitch and jubilations in the stand The statistical ensemble consists of pairs of particular events that characterize what happens on the football pitch in a given 10 seconds of the game and what happens in the stand in the subsequent 10 seconds, respectively. There is a correlation between event types "goal" and "jubilation."

Open umbrellas and moving windshield wipers The statistical ensemble consists of pairs of particular events that characterize how many open umbrellas are present on

[1] X is sometimes called "sample space." Note that the elements of the sample space should not be confused with the elements of the statistical ensemble. The elements of the sample space are event *types*, while the elements of the statistical ensemble are *token* events. In the two coin example, the sample space we consider is $\{(H,H),(H,T),(T,H),(T,H)\}$; while the statistical ensemble consists of particular instances of double tosses, different elements of which may realize one and the same element of the sample space.

the streets and how many moving windshield wipers are present on the roads under various weather conditions, respectively. There is a correlation between event types "many open umbrellas" and "many moving windshield wipers."

Notice the important feature these examples, together with the two coin example, share. In all these cases *two* distinct, well-defined phenomena are present, with distinct, well-defined spatiotemporal localizations: the things in Venice and in London, on the football pitch and in the stand, on the streets and on the roads, on the left and on the right side of the corridor. Each of the two phenomena *itself* gives rise to well-defined statistical data and admits a description in terms of a well-defined frequency space. In the above examples the frequency spaces we consider provide the *joint statistical descriptions* of the corresponding two phenomena. Correlations between the two phenomena arise as a particular feature of this joint description. The following is a simple characterization of what a joint description actually means.

Definition 1. Let $\mathscr{F}=(\mathscr{E},X,\Sigma,o)$, $\mathscr{F}'=(\mathscr{E}',X',\Sigma',o')$ and $\mathscr{F}''=(\mathscr{E}'',X'',\Sigma'',o'')$ be frequency spaces.

- The *product* of $\mathscr{F}$ and $\mathscr{F}'$ is the frequency space $(\mathscr{E}\times\mathscr{E}',X\times X',\Sigma\otimes\Sigma',o\times o')$, where $\Sigma\otimes\Sigma'$ is the standard product of subset algebras, and $o\times o':\mathscr{E}\times\mathscr{E}'\to X\times X',(e,e')\mapsto(o(e),o'(e'))$.
- $\mathscr{F}$ is a *restriction* of $\mathscr{F}'$ if $\mathscr{E}\subseteq\mathscr{E}',X=X',\Sigma=\Sigma'$ and $o=o'|_{\mathscr{E}}$.
- Let $\mathscr{F}''$ be a restriction of the product of $\mathscr{F}$ and $\mathscr{F}'$. $\mathscr{F}''$ is a *conjunctive product* of $\mathscr{F}$ and $\mathscr{F}'$ if the following holds:

  (i) for all $A\in\Sigma,A'\in\Sigma'$

  $$p''\left(A\times X'\right)=p\left(A\right) \tag{7}$$
  $$p''\left(X\times A'\right)=p'\left(A'\right) \tag{8}$$

  where p,p' and p'' denote relative frequencies associated with frequency spaces $\mathscr{F},\mathscr{F}'$ and $\mathscr{F}''$, respectively.

  (ii) every $e\in\mathscr{E},e'\in\mathscr{E}'$ occurs in at least one pair in $\mathscr{E}''$.

The conjunctive product $\mathscr{F}''$ of $\mathscr{F}$ and $\mathscr{F}'$ incorporates the minimal conditions under which a frequency space can be seen as a joint description of two statistical phenomena $\mathscr{F}$ and $\mathscr{F}'$: it represents all the components and retains all the facts of $\mathscr{F}$ and $\mathscr{F}'$ in the sense that (i) it preserves the frequencies of events in Σ and Σ', and (ii) it uses all the statistical information that is gathered in $\mathscr{E}$ and $\mathscr{E}'$. The simplest way to specify the $\mathscr{E}''\subseteq\mathscr{E}\times\mathscr{E}'$ that actually figures in a conjunctive product is to establish a *pairing* of the elements of the ensembles $\mathscr{E}$ and $\mathscr{E}'$, as the following trivial observation shows.

Proposition 2. *Let $\mathscr{F}''$ be a restriction of the product of $\mathscr{F}$ and $\mathscr{F}'$. Assume that there exists a bijection $c:\mathscr{E}\to\mathscr{E}'$ such that $\mathscr{E}''=\{(e,c(e))|e\in\mathscr{E}\}$. Then $\mathscr{F}''$ is a conjunctive product of $\mathscr{F}$ and $\mathscr{F}'$.*

In the two coin example let $\mathscr{E}$ and $\mathscr{E}'$ be the ensembles of tosses on the left and right hand sides, with particular outcomes as shown in Fig. 1 (c). The different kinds of pairing of tosses depicted in Fig. 3 correspond to different choices of bijection c, leading to different statistical ensembles $\mathscr{E}''$ of pairs of tosses. Different choices of c mean different specifications of those particular events, one from the left and one

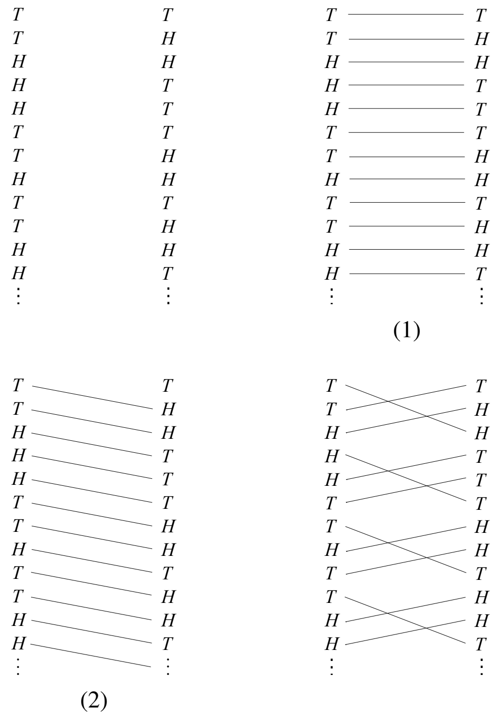

Figure 3: *The pattern of outcomes depicted in Fig. 1 (c). The two statistical ensembles of single-tosses on the left and right hand sides are merged together to a joint ensemble of double-tosses through different conjunctive relations. The outcomes are independent relative to the conjunctive relation of simultaneous tosses (1), while they are maximally correlated relative to the conjunctive relation of delayed tosses (2)*

from the right, whose *joint occurrence*, whose *conjunction* is seen as a "run" of the double-tosses.

Let A and B denote event types. When we say A and B are correlated, this is to express that the instances of A and B happen more frequently (or less frequently) in conjunction than as it follows from independence. However, when A and B belong to different phenomena, described by different frequency spaces, the meaning of when instances of A and B, particular events of two different ensembles, occur *in conjunction* is not a priori given. It is the joint description, the conjunctive product of the two frequency spaces that links together the otherwise unrelated phenomena by defining what is meant by "in conjunction." To emphasize this fact, I shall say that the pairs of particular events of $\mathscr{E} \times \mathscr{E}'$ which actually figures in the statistical ensemble $\mathscr{E}''$ of conjunctive product $\mathscr{F}''$, stand in *conjunctive relation*—relative to $\mathscr{F}''$ (cf. Fig. 3). Accordingly, those events that do stand in conjunctive relation will be seen as "jointly occurring"—relative the conjunctive product in question.

The notion of correlation is essentially relative to the specification of the conjunctive relation: in the sense that without specifying the conjunctive relation there is no joint description of distinct phenomena and therefore it doesn't make sense to talk about their correlations; and in the sense that different choices of the conjunctive relation lead to different statistical ensembles over which the values of relative frequencies and hence the facts of correlation will in general differ. Indeed, compare patterns (1) and (2) in Fig. 3: while the outcomes of tosses on the two sides are independent relative to the conjunctive relation of simultaneous tosses, they are maximally correlated relative to the conjunctive relation of delayed tosses.

There is an obvious way in which relative frequencies depend on the choice of a statistical ensemble. In patterns Fig. 1 (a)–(d) the frequencies of Heads and Tails are evaluated over different statistical ensembles. These ensembles differ in the sense that they correspond to different statistical data gathered in different experiments; the elements of the ensembles are different particular events pertaining to different pieces of the history of universe. However, what pattern (1) versus (2) in Fig. 3 shows is that even if one and the same experiment, one and the same set of statistical data, one and the same pattern of particular events is considered, the statistical ensemble and hence frequencies and correlations are still not unambiguously determined. For, to have that, the conjunctive relation has to be specified *additionally*.

As the coin example illustrates there may be various different ways in which the conjunctive relation can be set (Fig. 3), each of them leading to a different description of the phenomenon of $\mathscr{F}$ and $\mathscr{F}'$ taken together. In fact, Definition 1 gives no restriction on the conjunctive relation, as long as conditions (i) and (ii) in the definition remain satisfied. In this sense, an arbitrary pairing $c : \mathscr{E} \to \mathscr{E}'$ is admissible. On the other hand, the intuition about the coin case tells us that there must be some constraint on the freedom of how we put together the joint description of $\mathscr{F}$ and $\mathscr{F}'$. It is clear from the example in Fig. 2 that there are illegitimate cases of choosing the conjunctive relation—otherwise an arbitrary pattern of events ought to be seen as evidence of (maximal) correlation.[2] There must then exists a well-defined set $\mathscr{C}$ of subsets of

[2]Conditions (1)–(3), defining the notions of independence and correlation, are not expected to hold exactly in general, but only with some finite precision depending on the size of the statistical ensemble. In this sense there is no sharp boundary between cases of statistical independence and correlation, and the question of independence versus correlation should be seen as a matter of degree rather than a matter of kind. Note however that the choice of the conjunctive relation may alter whether a pattern of data is seen as evidence of independence or *maximal* correlation (with some finite precision), which are the two *extremes* on the correlation scale.

$\mathscr{E} \times \mathscr{E}'$ corresponding to those joint statistical ensembles/conjunctive relations that can be regarded as admissible, in the sense that they result in meaningful joint descriptions. Correlation of the phenomena $\mathscr{F}$ and $\mathscr{F}'$ means that one can find a suitable element of set $\mathscr{C}$ such that with respect to this joint ensemble some event types in $\Sigma \otimes \Sigma'$, characterizing the respective phenomena, are correlated. What is the condition that specifies $\mathscr{C}$?[3]

To see the significance of this question, an important remark is in order. One might think that the problem of setting the conjunctive relation is essentially related with, and thus dissolvable by rejecting, *frequentism*. Frequentism is a philosophical theory of probability that identifies the notion of probability with (finite or limiting) relativity frequency (Hájek 1997; 2009). It must be emphasized that the problem we are discussing does *not* presuppose such an identification; in fact, nowhere we made any reference to the notion of probability. Rather, our problem concerns the notion of regularity/regular connection which we indeed analyze in terms of (finite) relative frequency. The notion of regularity is not necessarily related to a "probabilistic" phenomenon (except on a frequentist understanding of "probabilistic"), and must be analyzable independently of any probability talk.

On the other hand, I believe that any talk of *probabilistic* correlation of distant events will eventually face the problem of conjunctive relation—no matter which analysis of probability one my favor. This is because probability statements, on any interpretation, are tested against (but not identified with!) finite relative frequencies. Consider a simple example. One asserts that the tosses of the two coins at the ends of the corridor are probabilistically independent, in the sense that the probability of obtaining each of the four possible outcome pairs is claimed to be 1/4. Here "probability" may mean propensity, credence, Laplacian probability, whatever interpretation of probability one may like. But when this assertion is confronted with evidence, what we will ultimately do is tossing the coins many times and seeing whether or not the outcomes are statistically correlated. Now, in doing so, which pairing of the outcomes should we consider?

To make a stronger case for this last point, it will be instructive to see a real world example. The EPR–Bell problem in quantum mechanics is certainly not merely a problem of the frequentist. Nevertheless, as we will see in the next section, the issue of the conjunctive relation provides a new idea for resolving it.

3 Are the EPR Correlations Real Correlations?

The EPR–Bell problem in quantum mechanics consists in the existence of a set of correlations produced by quantum phenomena which cannot be accommodated in the causal order of the world; in other words, which cannot be explained by a local hidden variable theory.

The basic setup of a 2×2 EPRB-type experiment is recalled in the spacetime diagram depicted in Fig. 4. We have eight different event types: a_1, a_2, b_1, b_2 are the choices of measurement directions $\mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2$, and A_1, A_2, B_1, B_2 are the events of the corresponding "up" detections, in the two wings of the experiment. The statistical ensemble on which the EPR statistics is based consists of the consecutive repetitions of the former spacetime diagram (Fig. 5). For the sake of simplicity, we give a typical example for relative frequencies observable in such an experimental scenario

[3] This condition may or may not be specifiable merely in terms of $\mathscr{F}$ and $\mathscr{F}'$.

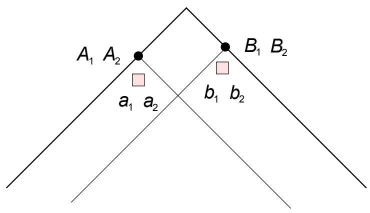

Figure 4: *The spacetime diagram of a single run of the EPRB experiment*

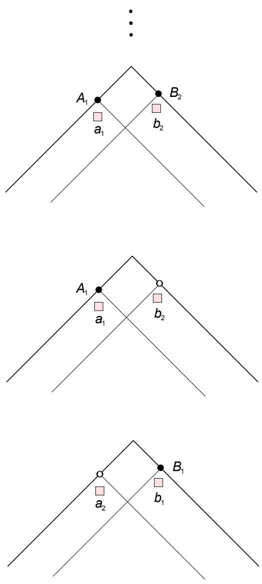

Figure 5: *The statistical ensemble consists of the consecutive repetitions of the space-time diagram in Fig. 4*

(Szabó 2001, 217–218):

$$p(a_i) = p(b_j) = \frac{1}{2} \tag{9}$$

$$p(A_i) = p(B_j) = \frac{1}{4} \tag{10}$$

$$p(a_1 \wedge a_2) = p(b_1 \wedge b_2) = 0 \tag{11}$$

$$p(A_i \wedge a_i) = p(A_i) \tag{12}$$

$$p(B_j \wedge b_j) = p(B_j) \tag{13}$$

$$p(a_i \wedge b_j) = p(a_i)\, p(b_j) \tag{14}$$

$$p(A_i \wedge b_j) = p(A_i)\, p(b_j) \tag{15}$$

$$p(a_i \wedge B_j) = p(a_i)\, p(B_j) \tag{16}$$

$$p(A_1 \wedge B_1) = p(A_1 \wedge B_2) = p(A_2 \wedge B_2) = \frac{3}{32} \tag{17}$$

$$p(A_2 \wedge B_1) = 0 \tag{18}$$

which correspond to relative measurement angles $\sphericalangle(\mathbf{a}_1, \mathbf{b}_1) = \sphericalangle(\mathbf{a}_1, \mathbf{b}_2) = \sphericalangle(\mathbf{a}_2, \mathbf{b}_2) = 120°$ and $\sphericalangle(\mathbf{a}_2, \mathbf{b}_1) = 0°$; and we set the arbitrary values of frequencies of measurement choices in (9) for the sake of convenience. As we can see, the outcomes of measurements in the two wings are correlated:

$$p(A_1 \wedge B_1) - p(A_1)\, p(B_1) = \frac{1}{32} \tag{19}$$

$$p(A_1 \wedge B_2) - p(A_1)\, p(B_2) = \frac{1}{32} \tag{20}$$

$$p(A_2 \wedge B_2) - p(A_2)\, p(B_2) = \frac{1}{32} \tag{21}$$

$$p(A_2 \wedge B_1) - p(A_2)\, p(B_1) = -\frac{1}{16} \tag{22}$$

On the other hand, statistics (9)–(18) violate the Clauser–Horne inequalities

$$\begin{aligned} -1 \leq\; & p(A_1 \wedge B_1 | a_1 \wedge b_1) + p(A_1 \wedge B_2 | a_1 \wedge b_2) \\ & + p(A_2 \wedge B_2 | a_2 \wedge b_2) - p(A_2 \wedge B_1 | a_2 \wedge b_1) \\ & - p(A_1 | a_1) - p(B_2 | b_2) \leq 0 \end{aligned} \tag{23}$$

$$\begin{aligned} -1 \leq\; & p(A_2 \wedge B_1 | a_2 \wedge b_1) + p(A_2 \wedge B_2 | a_2 \wedge b_2) \\ & + p(A_1 \wedge B_2 | a_1 \wedge b_2) - p(A_1 \wedge B_1 | a_1 \wedge b_1) \\ & - p(A_2 | a_2) - p(B_2 | b_2) \leq 0 \end{aligned} \tag{24}$$

$$\begin{aligned} -1 \leq\; & p(A_1 \wedge B_2 | a_1 \wedge b_2) + p(A_1 \wedge B_1 | a_1 \wedge b_1) \\ & + p(A_2 \wedge B_1 | a_2 \wedge b_1) - p(A_2 \wedge B_2 | a_2 \wedge b_2) \\ & - p(A_1 | a_1) - p(B_1 | b_1) \leq 0 \end{aligned} \tag{25}$$

$$\begin{aligned} -1 \leq\; & p(A_2 \wedge B_2 | a_2 \wedge b_2) + p(A_2 \wedge B_1 | a_2 \wedge b_1) \\ & + p(A_1 \wedge B_1 | a_1 \wedge b_1) - p(A_1 \wedge B_2 | a_1 \wedge b_2) \\ & - p(A_2 | a_2) - p(B_1 | b_1) \leq 0 \end{aligned} \tag{26}$$

where $p(\cdot|\cdot)$ denotes conditional relative frequency defined by Bayes's rule. As the appropriate Bell-type theorem shows (Hofer-Szabó et al. 2013, 149–152; Szabó 2008,

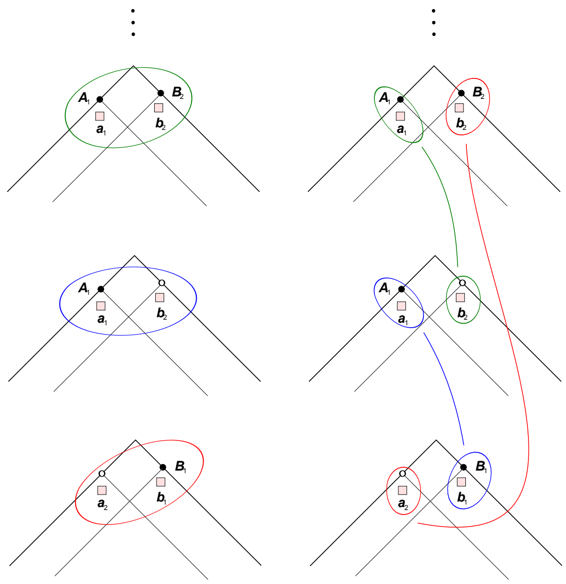

Figure 6: *Re-pairing measurements in the two wings changes the conjunctive relation and thereby the statistical ensemble*

15–20), the satisfaction of these inequalities is a necessary condition for the existence of a local hidden variable providing a causal explanation for correlations (19)–(22), under the assumptions of the finite speed of causal influence (that may propagate between the two wings) and the causal autonomy of the choices of measurements settings ("no-conspiracy"). The EPR–Bell problem thus consists in the fact that EPR correlations like (19)–(22) have no causal explanation.

On the basis of the observations of Section 2 I shall now outline a new idea for resolving the EPR–Bell problem.

Suppose that the coin tosses at the two ends of the corridor are performed in two cabins physically separated from the outside world, so that no causal signal whatsoever from outside can effect the outcomes of the tosses (not even from a "common cause"). Now, would the "jumpy" correspondence in Fig. 2 count as an unexplainable correlation which (together with the further physical constraints apply in this case) cannot be accommodated in the causal order of the world? Of course not, simply because there is no correlation at all. Only an inadmissible conjunctive relation has been applied. What if something similar is going on in the EPR case? The EPR-type statistical scenario, just like the coin case, is describable in terms of the conjunctive product of two frequency spaces, each of them characterizing a well-defined statistical phenomenon: the measurement choices and outcome events in the left and right wing of the experiment, respectively. Statistics (9)–(18), and correlations (19)–(22) in particular, are relative to the conjunctive relation of *simultaneous* measurements. What if in the EPR case this counts as an illegitimate conjunctive relation, not being an element of the admissible set $\mathscr{C}$? What if an admissible conjunctive relation corresponds to a different kind of pairing of the measurements in the two wings (Fig. 6)? First we will demonstrate that such an explanation is logically possible.

Let $\mathscr{F}_a$ and $\mathscr{F}_b$ denote the frequency spaces describing the statistics of the left and right wings of the EPRB experiment, respectively. $\mathscr{F}_a$ and $\mathscr{F}_b$, characterized by relative frequencies (9)–(13), are visualized in Fig. 7 (a)–(b):

- the members of the ensembles $\mathscr{E}_a$ and $\mathscr{E}_b$—the particular events in the left and right wings respectively—are represented as points of two copies of the unit square[4]
- outcome functions o_a and o_b assign to the points of the corresponding squares the event types instantiated; thereby, a_1, a_2, A_1, A_2 and b_1, b_2, B_1, B_2, together with their Boolean combinations, are seen as subsets of the respective unit squares
- relative frequencies (9)–(13) are represented by the areas of these subsets.

The conjunctive relation of events in the two wings, $c : \mathscr{E}_a \to \mathscr{E}_b$, can now be thought of as an area preserving bijection between the two unit squares. As a result of this bijection, sets a_1, a_2, A_1, A_2 and b_1, b_2, B_1, B_2 will be arranged as subsets of one single unit square, such that the areas and subset relations in the two original squares are all preserved. The resulting single square structure is what represents the conjunctive product of $\mathscr{F}_a$ and $\mathscr{F}_b$. Fig. 7 (i)–(iii) depict three different possible conjunctive products:

(i) portrays the outcomes in the two wings as statistically independent,

$$p(A_i \wedge B_j) = p(A_i)\, p(B_j) \tag{27}$$

(ii) portrays the outcomes as perfectly (anti)correlated,

$$p(A_i \wedge B_j) = 0 \tag{28}$$

(iii) corresponds to the simultaneous correlations (17)–(18).

As it can be read off from Fig. 7 (i)–(iii), other than having different values for correlations of outcomes all three conjunctive products agree on (9)–(16), the remaining part of the simultaneous statistics. It is easy to verify that, as opposed to (17)–(18), both (27) and (28), supplemented with (9)–(16), satisfy the Clauser–Horne inequalities (23)–(26). As is known from Fine's (1982a) version of Bell's theorem, the satisfaction of the Clauser–Horne inequalities is not only necessary but also sufficient for the existence of a local hidden variable model for 2×2-type correlations.[5] Therefore, the statistics described by conjunctive products (i) and (ii), by contrast with the simultaneous correlations described by (iii), can be given a local hidden variable model. This proves that in the particular EPR scenario we consider the conjunctive relation can be modified in a way that the statistics relative to the modified ensemble admit a local hidden variable explanation.[6]

[4] Alternatively, one can take a finite partition of the unit square into small cells of equal size, and identify the ensembles with the finite collections of these cells.

[5] It must be noted that the Clauser–Horne inequalities are only sufficient in conjunction with the following supplementary inequalities (as also noted by Fine 1982a, 293):

$$p\left(A_i \wedge B_j | a_i \wedge b_j\right) \leq p(A_i|a_i)$$
$$p\left(A_i \wedge B_j | a_i \wedge b_j\right) \leq p\left(B_j|b_j\right)$$
$$p(A_i|a_i) + p\left(B_j|b_j\right) - p\left(A_i \wedge B_j | a_i \wedge b_j\right) \leq 1$$

These however follow from (12)–(16) and thus are respected by all three conjunctive products.

[6] A local hidden variable model provides a common causal explanation. However, after modifying the conjunctive relation the pairs of events in the two wings matched together may no longer be spatially separated and so explaining the statistics relative to the modified ensemble may also invoke direct causal connection between the two wings. Nevertheless, what we have demonstrated here is that an explanation purely in terms of common causes is also available.

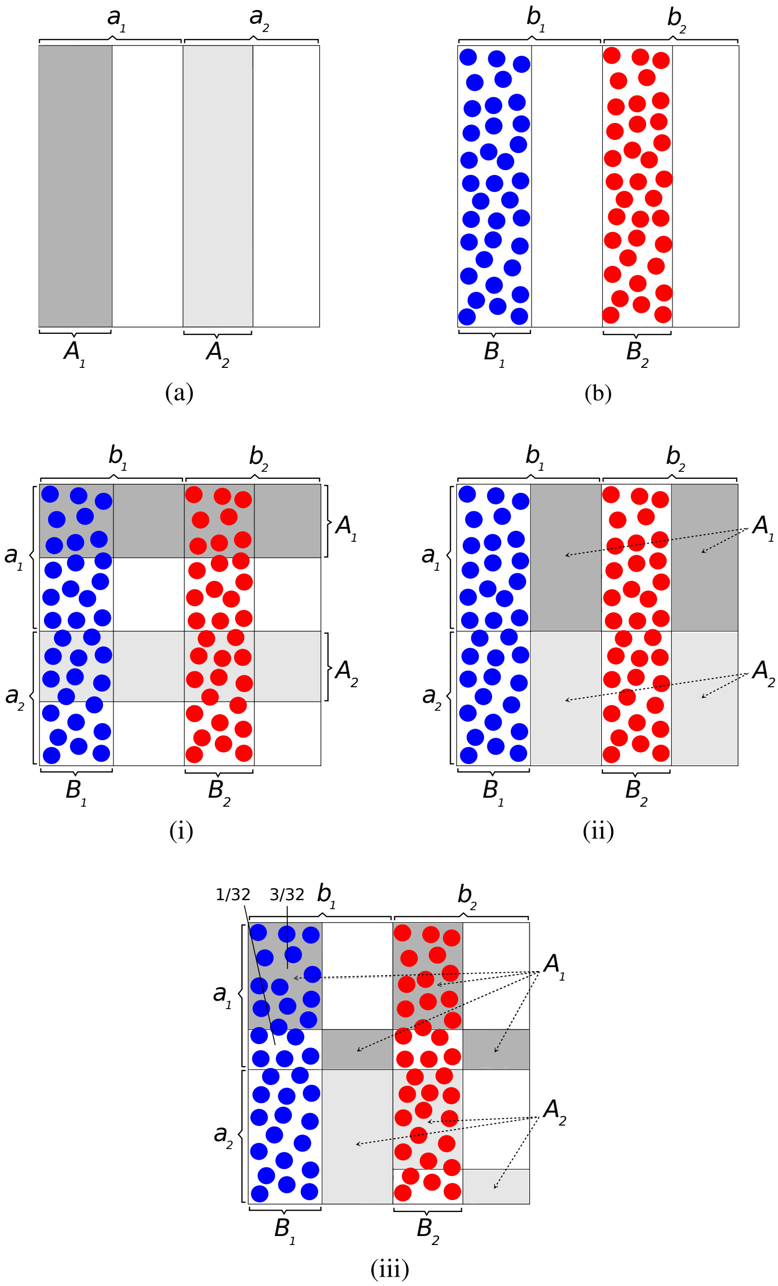

Figure 7: *Description of the EPR statistics in terms of frequency spaces. (a) and (b) correspond to the statistics of the right and left wings of the experiment, respectively. (i)–(iii) correspond to different joint descriptions of the statistics of the two wings taken together—different conjunctive products of frequency spaces in (a) and (b). (i) portrays the outcomes in the two wings as statistically independent, (ii) portrays the outcomes as perfectly (anti)correlated, (iii) corresponds to the simultaneous correlations violating the Clauser–Horne inequalities*

No matter what conditions an admissible conjunctive relation should satisfy in general, one might argue however that in the EPR case we have good physical ground to believe that the simultaneous measurements stand in an admissible conjunctive relation, that is, statistics (19)–(22) express real correlations. For, the macroscopic events of measurement outcomes are produced by *particles* hitting the detectors; a single run of the EPR experiment consists of an emission of a pair of particles and the detection of the *same* pair. Therefore, the conjunctive relation of simultaneous measurements in the two wings is physically grounded in the fact that the simultaneous outcomes A_i, B_j of the chosen measurements a_i, b_j are produced by particles *coming from the same emission*.

Notice however that this only justifies considering the statistics of *simultaneous* measurements if the velocities of particles from the same emission are equal. It must be emphasized that we have no direct access to particle velocities in a quantum experiment—neither emission events, nor particle trajectories are tracked. In fact, in real EPR-type experiments the pairs of particles are matched together on the basis of their time of arrival at the detectors, their detections being coincident (Clauser and Shimony 1978; Aspect et al. 1981; Larsson 2014).[7] That is, the pairing of measurement events in two wings happens on the basis of the *assumption* that particles from the same emission have identical velocities. However, within the context of a hidden variable theory such an assumption can be questioned. For if there is a hidden variable—for instance, characterizing hidden properties of the particles—whose value predetermines the outcome of the measurements, then it is quite plausible to assume that this hidden variable also predetermines the behavior of the particles in general, including their velocities. One can easily imagine this dependency to be such that particles coming from the same emission will have different velocities and hence arrive at the detectors in different "runs" of the experiment at different times (Fig. 8).

Let me call such a local hidden variable model—in which, again, particles from the same emission have different velocities depending on the value of the hidden variable—a *time-delayed model*.[8] The properties of a time-delayed model not only depend

[7] There do exist experiments in which particle pairs can be identified independently of their detections. However, in none of these experiments they succeeded to ensure spatial separation of the two wings (Larsson 2014, section 3).

[8] The thought that the particles' time of arrival at the detectors may depend on the hidden variable is not new. This is the main idea of hidden variable models based on the so-called coincidence loophole (Fine 1981, 1982b; Pascazio 1986; Larsson and Gill 2004). In these models one builds on the fact that in real EPR-type experiments pairs of detection events in the two wings are counted as "coincident" if they happen within a given time window; if the retardation of one detection with respect to the other is larger then the prescribed amount the data is simply ignored and will not contribute to the statistics. If, as it is assumed in the models in question, there is a systematic connection between the value of the hidden variable and the detections that are counted as coincident, then the statistics over the subensemble of coincident detections may exhibit violations of Bell's inequalities even though the statistics over the original ensemble, being produced by a fully local mechanism, adhere to them.

There is another type of "loophole" in Bell's theorem related to the fact that the statistical data is given as a temporal series of outcomes. In the original Bell analysis events in different runs of the experiment are treated as independent. However, since different runs of an EPR-type experiment are timelike separated, earlier values of outcomes, settings and the hidden variable can in principle influence the later ones. For example, if the consecutive measurement choices in one wing are correlated, one can imagine a mechanism whereby the local measurement apparatus predicts the current remote measurement choice by registering the earlier ones. This way the remote measurement setting could influence the local outcome, resulting in a violation of Bell's inequalities, by means of a fully local mechanism. This problem is sometimes referred to as the memory loophole (Hess and Philipp 2001; Barrett et al. 2002; Gill et al. 2002; Gill 2003a, 2003b).

While both the coincidence and memory loopholes are related with the role of time in the EPR–Bell problem, and thereby they have some similarities with the idea of the time-delayed models discussed here, it must be emphasized that the main thought behind the time-delayed models is different from the problems

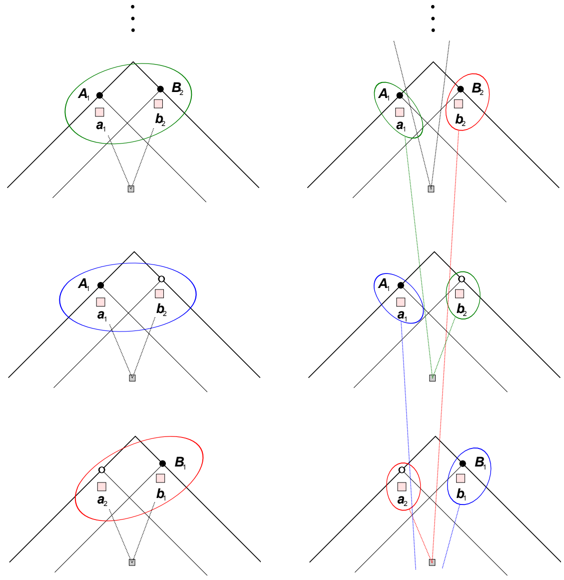

Figure 8: *Giving different velocities to particles coming from the same emission changes the conjunctive relation and thereby the statistical ensemble*

on the statistical features of the given scenario, like (9)–(18), but also depend on the detailed time series of data that gives rise to this statistics. We will now sketch how to construct such a model.

As the foregoing analysis shows, one can modify the conjunctive relation of the simultaneous EPR statistics (9)–(18) such that the statistics relative to the modified conjunctive relation admit a local hidden variable model. Consider this modified conjunctive relation $\tilde{c} : \mathscr{E}_a \to \mathscr{E}_b$ and the corresponding hidden variable model with hidden variable λ. All a time-delayed model has to do is supplementing variable λ, whose value determines the outcomes of measurements performed on a pair of particles from the same emission, with information about when, in which "runs" of the experiment, the two particles are measured. This is done by specifying for each emitted particle pair, characterized by a certain value of λ, the time t of its emission and the velocities $v_{a,}, v_b$ of the two particles. Accordingly, introduce the extended hidden variable $\tilde{\lambda} = (\lambda, t, v_{a,}, v_b)$. A time-delayed model will consist of a series of $\tilde{\lambda}$-values, parametrized by particle emissions.

The only constraint on a time-delayed model is that the values of additional variables $t, v_{a,}, v_b$ be set in a way that the pairs of measurements on particles from the same emission correspond with the pairs of measurements established by conjunction relation $\tilde{c}$—relative to which the original hidden variable λ has been introduced. This is easily fulfilled. Suppose that the detectors' distance from the source is l, and the experiment lasts from time t_0 to t_1. The time series of observed data is given by functions $\tau_a : \mathscr{E}_a \to [t_{0,}, t_1]$ and $\tau_b : \mathscr{E}_b \to [t_{0,}, t_1]$, specifying the time coordinates of detection events (outcomes) in the left and right wings, respectively. Take a pair of

discussed in the literature. The reason is that all existing hidden variable models assume that it is the statistics of the *simultaneous* measurements that is to be explained by a local mechanism, while the essence of the time-delayed model idea is to *question* that the simultaneous statistics exhibit real correlations in need of explanation.

events $e \in \mathscr{E}_a, \tilde{c}(e) \in \mathscr{E}_b$ matched together by conjunctive relation $\tilde{c}$. The emission time t of particles producing the corresponding outcome events must be somewhere between t_0 and $min\{\tau_a(e) - l/c, \tau_b(\tilde{c}(e)) - l/c\}$, ensuring that the particles' velocity is not greater than c, the speed of light. Under this constraint, set t arbitrarily. Then the velocity variables are assigned the following values:

$$v_a = \frac{l}{\tau_a(e) - t} \tag{29}$$

$$v_b = \frac{l}{\tau_b(\tilde{c}(e)) - t} \tag{30}$$

To obtain the corresponding value of $\tilde{\lambda}$, supplement $t, v_{a,}, v_b$ with a λ-value that pertains to outcome combination $o_a(e), o_b(\tilde{c}(e))$ in the original hidden variable model. Specifying $t, v_{a,}, v_b$ and λ for all event pairs $(e, \tilde{c}(e)), e \in \mathscr{E}_a$ in this manner, we will have a series of $\tilde{\lambda}$-values which reproduces the EPR statistics, relative to conjunction relation $\tilde{c}$.

A time-delayed model thus provides a local hidden variable explanation for the EPRB scenario. The model is not intended to be physically realistic; in fact, it has certain features that will evidently make it physically uninteresting. One such feature is the explicit dependence of the hidden variable $\tilde{\lambda}$ on l, the detectors' distance from the source, determined by the experimenter's free choice. This is a version of a conspiratorial dependence. Another concern may be that the values of $t, v_{a,}, v_b$ are in fact constrained by robust physical principles—related with the type of particles involved, the mechanism of particle emission, the conservation laws thought to apply, etc.—that are not incorporated in the model.

The point of the time-delayed model idea, however, is not its physical relevance. The point is that according to the time-delayed model EPR correlations (19)–(22) are *not real* correlations. According to the time-delayed model the unexplainable EPR statistics is obtained by applying an *inadmissible* conjunctive relation. What makes a conjunctive relation admissible? Before answering this question the very concept of correlation is simply meaningless, and the EPR–Bell problem in quantum mechanics cannot even be formulated. The next section attempts to provide possible answers to this crucial question.

4 Possible Answers

Answer 1

Consider the jumpy pairing in Fig. 2. The first reaction one might have is this. It is of course no big deal to set the pairing in the appropriate way, achieving maximal correlation, once you already know what the outcomes are. But if someone, before the experiment is performed, could predict that relative to the jumpy pairing in Fig. 2 the outcomes would be maximally correlated, we would be very much surprised and would cry out for an explanation of this *correlation*. One might thus suggests that what is wrong about the pairing in Fig. 2 is that its specification was dependent on the outcomes themselves. The admissibility of a conjunctive relation requires, the suggestion might go, that the corresponding ensemble $\mathscr{E}'' \subseteq \mathscr{E} \times \mathscr{E}'$ should be able to be determined *independently* of what the "content" of the particular events in $\mathscr{E}$ and $\mathscr{E}'$ is; that is, without referring to the outcome function o.

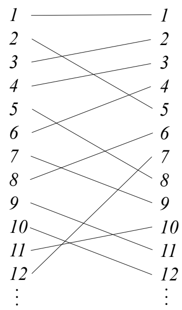

Figure 9: *The pairing in Fig. 2 can be given as a function between numbers, independently of what the outcomes are*

Notice, however, that it is no problem to specify the pairing in Fig. 2 independently of the outcomes. The pairing itself can be given, for example, as a function between numbers (or any sort of things by which we can label the elements of $\mathscr{E}$ and $\mathscr{E}'$), without ever referring to Heads and Tails,[9] as depicted in Fig. 9. The question is: does this pairing, specified independently of the outcomes, correspond to an *admissible* conjunctive relation?

We have strong intuition that if someone can predict a maximal correlation, without previously knowing the outcomes, then the corresponding pairing must be an admissible conjunctive relation. Thus, the possibility of learning about a correlation, without observing the outcomes, must be a *sign* of the correlation being real. (We shall return to this intuition below.) But this by no means can serve as a definition. A correlation and the corresponding conjunctive relation may very well be real and meaningful even if *we learn* the fact of correlation, and the corresponding conjunctive relation, by observing the pattern of outcomes itself. Indeed, imagine you receive a laboratory record with the data depicted in Fig. 1 (a), from an experiment of coin tosses *already performed*. Your task is to determine whether there is any regularity, in need of explanation, in the pattern. You are going to look for correlations under various different pairings. Now, it would be absurd to require that the *way you learn* whether there is anything to explain cannot depend on the outcomes, that is on the data itself you are given.

Answer 2

One might propose that the difference between pairings in Fig. 1 (b)–(d) versus Fig. 2 is related with the problem of induction. The first three patterns of pairing can be meaningfully regarded as being subject to inductive generalization, while the jumpy pairing cannot. So if the next outcome on the left is Heads, then in case of patterns (b)–(d) I am able to predict, on the basis of the pairing, what a future outcome on the right is going to be; while no such a prediction is available in case of the jumpy pairing. I certainly cannot be sure whether my prediction will turn out to be correct or not—due to the problem of induction. Nevertheless, the conjunctive relation in the first three cases at least *generates* such a prediction.

I believe, however, that this idea reflects a misunderstanding with regard to the problem of induction. For not only the prediction of outcomes but also *extending the*

[9]Here we assume that a token event can be individuated without referring to the event types it falls under. Such an identification seems possible, for example, by referring to the spatiotemporal location of the token events.

pairing itself is an act of generalization laden by the problem of induction—the pairing being a rule read off from a finite pattern. The available finite parts of the parings in patterns (b)–(d) tell us nothing about how to extend these assignments for yet unknown, unpaired outcomes, exactly like in the case of the jumpy pairing—due to the problem of induction. Therefore, we have no more ground to make predictions about future outcomes in patterns (b)–(d) then in case of the jumpy pairing.

The problem of induction concerns the task of generalizing a finite rule, be that is a list of Heads and Tails or a pattern of assignments, to yet unknown cases. The problem of the admissibility of the conjunctive relation is not related with this task. Our problem is to determine what counts as an admissible assignment *in the given, finite pattern* of outcomes. That is, our problem is to determine what a possible finite pattern of pairing is that we are to extend in the act of inductive generalization.

Answer 3

Perhaps the reason why we are inclined to regard one pattern of pairing as being subject to inductive generalization, while not the other one, is simply because the assignments in question are of different complexity. Presumably, the jumpy pairing of Fig. 2 will be a rather complex assignment as it is generated by random outcomes. Perhaps one could try to characterize the admissibility of the conjunctive relation in terms of the complexity of the corresponding mappings of numbers (Fig. 9).

The problem with this suggestion is twofold. First, the mathematical theory of computability/recursive functions used for such a characterization only applies to functions defined on an infinite domain; that is, to functions of type $\mathbb{N} \to \mathbb{N}$ (Immerman 2016). While we always have a finite amount of data and therefore the corresponding mappings are of $\{1,2,...,n\} \to \{1,2,...,n\}$. Nevertheless, even if the abstraction of an infinite experiment is accepted, there is a more fundamental difficulty. The problem is that complexity can only be characterized in terms of assignments defined between *numbers*. Characterizing the conjunctive relation in terms of complexity is thus only possible if we have already assigned numbers to the particular events in question (cf. Fig. 2 versus Fig. 9). Notice, however, that such an assignment raises exactly the same kind of problem as the problem of the admissibility of pairings itself. For on what grounds do we assign this number as opposed to that one to a particular event? There is a canonical order of the natural numbers, but is there a canonical order of particular events in an $\mathscr{E}$ and $\mathscr{E}'$?

One might think that the temporal order of events (in a given frame of reference) can serve as a basis for such a natural order. The particular example of the coin tosses indeed seems to suggest such an idea: in Fig. 1–2 the results of tosses are depicted in their temporal order, and it is this order relative to which the assignments in Fig. 1 (b)–(d) count as "simple," whereas the one in Fig. 2 counts as "jumpy" and "complex." But now imagine a different ordering procedure. Suppose that we place two identical counters at each end of the corridor. The readings of the counters are controlled by the neighboring tosses. The counter on the left when the neighboring tosses come up Heads steps forward over the even numbers; and when the neighboring tosses come up Tails it steps forward over the odd numbers. The counter on the right does the same but with interchanged roles of Heads and Tails. The readings of the two counters provide the events on the two sides with orders that in general differ from their temporal order. Notice that relative to this new numbering the "jumpy" pairing in Fig. 2, conceived as an assignment between numbers, becomes the identity function $\{1,2,...,n\} \to \{1,2,...,n\}, i \mapsto i$, the simplest assignment possible (Fig. 10). Now,

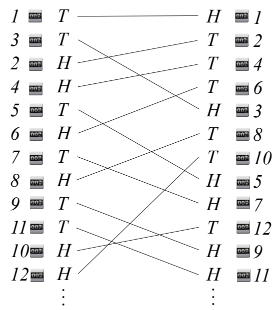

Figure 10: *The counters placed on the two sides are governed by the outcomes of the neighboring tosses. The counter on the left steps forward over the even numbers when the neighboring tosses come up Heads; and it steps forward over the odd numbers when the neighboring tosses come up Tails. The counter on the right does the same but with interchanged roles of Heads and Tails. Relative to the numbering provided by the counters the "jumpy" pairing in Fig. 2, as an assignment between numbers, becomes the identity function*

why would be the counter that we call clock in any way privileged among the possible counters, with regard to what counts as a "simple" or " complex" assignment?

Answer 4

Consider the statement of the Criterion of Reality appearing in the famous paper of Einstein, Podolsky and Rosen on the incompleteness of quantum mechanics:

> If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. (EPR 1935, 777)

Apply this criterion to the prediction of the value of a certain correlation relative to a given conjunctive relation. Suppose we can be sure, before the experiment in question is performed, that certain events will be maximally correlated with respect to a certain pairing. (An example of this is that the perfect (anti)correlation of the simultaneous measurement outcomes in the EPRB experiment, with parallel measurement directions in the two wings, can be predicted by means of quantum mechanics.) Then, according to the criterion, this correlation must be physically real. If so, then there must exist some elements of physical reality corresponding to the conjunctive relation relative to which the correlation is understood. The conjunctive relation then must be physically realized, it must be singled out by some physical matters of fact. This is I think what lies behind the intuition, formulated in Answer 1, that the admissibility of the conjunctive relation is ensured once we are able to specify the conjunctive relation, and the value of correlation relative to it, without observing the outcomes.

This consideration suggests a general condition under which a conjunctive relation could be seen as admissible. So far we conceived the pairings as abstract assignments $c : \mathscr{E} \to \mathscr{E}'$ (cf. Proposition 2). The intuition about predictability suggests that such an abstract assignment can be seen as an admissible conjunctive relation just in case it is physically realized, if there exist some physical matters of fact, some elements of reality that single out this assignment.

Indeed, as long as the jumpy pairing in Fig. 2 is just an *abstract* assignment, a *conceived* pairing, the correlation corresponding to it doesn't seem to be a real correlation, and there is nothing in need of explanation about that pattern. However, once we physically attach the counters described in Answer 3 to the outcomes of tosses, the readings of the counters will physically realize the jumpy pairing in the sense that the same counter readings on the two sides establish an objective similarity relation between the events related by the jumpy assignment. If we now ask what the causal explanation for this correlation is, we have a clear answer: it is the behavior of the counters, governed by the neighboring tosses, that explains this correlation.

Similarly, the suggestion goes, the conjunctive relation corresponding to the simultaneous pairs of tosses is admissible because the identical readings of two clocks placed at the two ends of the corridor establish an objective, physical similarity relation between the simultaneous pairs of tosses.[10] In this case, the readings of the clocks constituting the conjunctive relation will not play a role in the causal explanation of the correlation relative to the simultaneous pairs (if there is correlation), since the behavior of the clocks is causally independent from the tosses. Nevertheless, in case we observe simultaneous correlation, that correlation, being physically real, would cry out for explanation.

To give a third type of example for a physically realized conjunctive relation, imagine that the outcomes of tosses obtain in the following way. A bag contains two biased coins, one of them is two-headed, the other one two-tailed. We draw a coin, take it to the left, we take the other coin to the right, and we toss the two coins at the same time. We then put the coins back in the bag, or take another, similar bag, and repeat the procedure. This experiment will lead to maximal correlation relative to the conjunctive relation of the simultaneous pairs of tosses as depicted in Fig. 1 (b). Now repeat this procedure in a way that the coins drawn from the same bag are tossed at different times on the two sides. No matter what the exact pattern of the delays of tosses is, there will be maximal correlation between the outcomes of tosses performed with those pairs of coins that come from the same bag. Here, the conjunctive relation relative to which the maximal correlation is understood is physically realized by the facts as to which coins are drawn from the same bag. Notice that this was the basic idea behind the time-delayed models of the EPR scenario where the conjunctive relation of measurements events in the two wings was physically realized by the pairs of particles that come from the same emission.

The main problem with the "physical realization" idea, however, is that it is way too general and vague so as to serve as a demarcating condition. What, exactly, counts as a "physically realized" relation, as opposed to an "abstract," "conceived" one? Suppose that two identical counters, whether ones operating as described in Answer 3, two clocks or any devices outputting numbers, are placed at the two ends of the corridor. The counter readings provide the events on each side with an order relation, and they also establish a similarity relation between events on the opposite sides, defined by the identical counter readings. Both these relations are physically real insofar as they are physically implemented by the readings of the counters. Now, on the basis of these re-

[10] It must be emphasized that the physical realizability of the simultaneous pairing has nothing to do with the issue of whether simultaneity is factual or conventional, in the sense as it arises with regard to the problem of distant clock synchronization. What gives physical objectivity to the simultaneous pairing is not that the local clocks in question are synchronized or that they measure the "same" (temporal) order—in fact no such an assumption has to be made. What is important here is that the counters that we call clocks—synchronized or not—are identically constructed and therefore there is a physical correspondence between their readings, based on their similarity. This physical correspondence is what grounds the reality of the simultaneous pairing.

lations one can *define* various further notions, such as "the 22nd toss on the right side," "the toss on the left *succeeding* the toss on the right," etc. Are these notions physically realized, or merely "abstract," "conceived" ones? I don't see any reason why to regard the relation "the toss on the left succeeding the toss on the right, with respect to the counter readings" in any way less physically real than the relation "tosses with the same counter readings." But if so, one can then define all the pairings in Fig. 1–2 in terms of real physical relations, even the unwanted jumpy one: just number the elements of $\mathscr{E}$ and $\mathscr{E}'$ according to the order provided by the local counter readings, and then define the pairing in question by employing the corresponding $\{1,2,...,n\} \to \{1,2,...,n\}$ function. In this way all pairings in Fig. 1–2 become physically realized, unless there are constraints about which $\{1,2,...,n\} \to \{1,2,...,n\}$ functions count as *admissible*. This leads us back to our initial question posed in Answer 1: what condition should an "abstract" pairing satisfy so as to count as admissible?

5 Philosophical Upshot

Discussion of the problem of distant simultaneity shows that one can find no objective criterion under which two distant events could be regarded as simultaneous (in a given reference frame) (Salmon 1977). The simultaneity relation is essentially conventional. As Winnie (1970) demonstrated, relativity theory can be reformulated in a way that leaves "simultaneity" as an unset, free parameter, while all the empirical predictions of the theory still follows. In this sense simultaneity can be seen as a kind of "gauge parameter," much on a par with the potentials in electrodynamics (Rynasiewicz 2012); it does not correspond to anything factual.

The situation seems much similar in case of the problem of distant correlation. The notion of statistical correlation refers to, as its essential conceptual component, the relation "in conjunction" between (distant) events. What is the criterion under which two events, perhaps as members of two larger collections of events, can be regarded as occurring "in conjunction"? As we have seen, there is no clear answer to this question. Is then the specification of the conjunctive relation *merely* a matter of *convention*? If the relation "in conjunction" is purely conventional then there exists no fact of the matter as to the value of statistical correlation, as to which events are correlated and which ones are independent.

This would not be without philosophical consequences. Just like simultaneity, the notion of correlation/regularity has deeper metaphysical connotations. In our thinking, temporal order and the past-present-future-type classification of events are intertwined with existence, with the metaphysical notions of becoming, determinateness, etc. Similarly, correlation is not simply a matter of assigning a statistical parameter to a collection of events, but it is supposed to have a deeper metaphysical significance. For, correlation is a sign of causal connection. Throughout my foregoing analysis implicit was the idea that a real correlation is something in need of causal explanation—an intuition coming from the philosophical principle we call the Principle of the Common Cause (Hofer-Szabó et al. 2013). But what remains of this idea if it turns out that correlation is essentially conventional in character? Does the underlying causal structure of the world, that we suppose to explain correlations, is also a matter of convention? For something that is determined by of our choice cannot be a sign of, cannot be explained by, something that is independent from our choice.

To put this problem into a different perspective, consider contemporary debates over laws of nature. Much of the discussion is centered around Humean supervenience, the thesis that everything in the world supervenes on the spatiotemporal distribution of fundamental physical facts, on the Humean mosaic. According to this thesis, every meaningful concept that features in our description of the world must be expressible in terms of the Humean mosaic. Nomic concepts famously fail to pass this censorship and hence are eliminated from the Humean picture of the world—modalities, dispositions, necessary connections, causal powers, governing laws, etc. all must go. It has been held since Hume's observations that what remains after the cleansing of the nomic are the regularities, "constant conjunctions" of non-nomic facts. In Hume's words:

> But philosophers, who carry their scrutiny a little farther, immediately perceive, that, even in the most familiar events, the energy of the cause is as unintelligible as in the most unusual, and that we only learn by experience the frequent *Conjunction* of objects, without being ever able to comprehend anything like *Connexion* between them. (Hume 2007, p. 51)

Inherent to this idea is the conviction that at least the "frequent conjunction of objects" is something that we can learn by experience; in other words, the regularity of non-nomic facts is something that does supervene on the Humean mosaic. But is this really so? How can "frequent conjunction" be a matter of empirical/physical fact if "conjunction" has no unambiguous empirical/physical content, but can only be set by convention? What remains of the Humean picture of the world if it turns out that just like modalities, dispositions, necessary connections, causal powers, governing laws, etc., regular connection is also not a real thing?

Acknowledgment

This work has grown out from discussions with László E. Szabó on his physicalist account of scientific theories (Szabó 2017), in which the semantics of a theory is analyzed in terms of the "distant correlation" of facts of the world and "facts" of the theory. I wish to thank László for these discussions. I am also grateful to Gábor Hofer-Szabó and Balázs Gyenis for valuable comments on earlier versions of the paper.

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