Notes on contraposing conditionals

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Abstract

The contraposing conditional 'If A then C' is defined by the conjunction of $A > C$ and $\neg C > \neg A$, where $>$ is a conditional of the kind studied by Stalnaker, Lewis and others. This idea has recently been explored, under the name 'evidential conditional', in a sequence of papers by Crupi and Iacona and Raidl, and it has been found of independent interest by Booth and Chandler. I discuss various properties of these conditionals and compare them to the 'difference-making conditionals' studied by Rott, which are defined by the conjunction of $A > C$ and not $\neg A > C$. I raise some doubts about Crupi and Iacona's claim that contraposition captures the idea of evidence or support.

"... es ist manchmal, als wollte Längstverblühtes
und Längstentblättertes aus der Jugendzeit
noch einmal zum Leben erwachen."[1]

Alexius Meinong, letter to Hans Vaihinger, 21 May 1911

1. Contraposing conditionals. The following idea has been studied, apparently independently and in somewhat different ways of presentation, by Vincenzo Crupi and Andrea Iacona in "The Evidential Conditional" (2019b) and Richard Booth and Jake Chandler (in work elaborating on Booth and Chandler 2020). Crupi and Iacona use a system-of-spheres possible worlds semantics à la Lewis (1973), Booth and Chandler the belief-state revision framework of Darwiche and Pearl (1997). Eric Raidl (2019) provided the first completeness proof for the 'evidential conditional' using selection functions in the style of Chellas (1975). Raidl, Crupi and Iacona (2020) provide a nicer axiomatic system and spell out all the details of the completeness proof.

Let us speak about *contraposing conditionals* (a term used by Jake Chandler) and base our discussion on this definition:

[1] "... it is sometimes as if something long-withered and long-defoliated from our early days wanted to return to life again."

(CPC) $A \triangleright C$ iff $A > C$ and $\neg C > \neg A$.

Here $A > C$ should be read as 'If A, then C'. It is an ordinary ('suppositional') conditional of the kind that has been studied in conditional logic since the pioneering works of Stalnaker and Lewis. Crupi and Iacona call a rule very much like (CPC) the 'Chrysippus Test'[2] and claim that it "seems required in order to preserve the intuition that A must support C" and that it "characterizes the evidential interpretation" according to which "a conditional is true just in case its antecedent provides evidence for its consequent" (2019b, pp. 2, 5). Booth and Chandler (2020, Proposition 12), who read '$A \triangleright C$' as 'A is taken to support C', derive CPC from another, more fundamental definition of support involving iterated belief changes,[3] coupled with the idea that these belief changes apply the method of restrained revision (Booth and Meyer 2006).

In most of the following I suppose (similarly to Crupi and Iacona as well as Booth and Chandler) that the base conditional $>$ satisfies something very much like Lewis' conditional logic **VW** (i.e., **VC** without Centering). The distinctive property of $\triangleright$ is that it satisfies Contraposition, essentially by definition. Two other very important properties are that it satisfies *neither* Strengthening of the Antecedent (aka Monotonicity) *nor* Weakening the Consequent (aka Right Weakening, RW):

(Mon) If $A \triangleright C$ and $B \vdash A$, then $B \triangleright C$. Strengthening the Antecedent

(RW) If $A \triangleright C$ and $C \vdash B$, then $A \triangleright B$. Weakening the Consequent

2. Valid and invalid inference schemes for contraposing conditionals. The following is all very simple. I just check, with a **VW**-like background logic for $>$, the validity of a few principles, and give counterexamples, using linear orders of possible worlds, to a few other principles. Possible worlds here are really partial possible worlds (only very few facts that matter are represented), and they will be identified with valuations (thus the models are 'injective'). The notation '$pqr \prec pq\bar{r}$', for example, means that pqr (the world at which p, q and r are all true) is closer to the evaluation world than/more plausible than/more normal than, $pq\bar{r}$ (the world at which p and q are true and r is false). $A > C$ is true at a world/accepted iff the closest/most plausible A-worlds are all C-worlds.

Crupi and Iacona have addressed the following principles (with the exception of Disjunctive Rationality) in their earlier paper "Three Ways of Being Non-Material" (2019a). Chandler mentions some of these principles in his emails. Perhaps the main point worth highlighting in advance is that contraposing conditionals fail to satisfy Rational

[2] Personally, I think that Sanford (1989, pp. 68–69) is right when he states: "[C.I.] Lewis's account of strict implication is ... as close as it can be to the account of the conditional we attribute to Chrysippus."

[3] Here it is: A is taken to *support* B in belief state Ψ, in symbols $A \triangleright B$, if and only if $B \in (\Psi * \neg B) * A$. This reading was given to me by Jake Chandler in personal communication. In the paper quoted Booth and Chandler actually use a somewhat different terminology and say that A *strictly overrules* $\neg B$ (in Ψ) iff $B \in (\Psi * \neg B) * A$. Condition (30) in Rott (1986, p. 353), EV3 in Chandler (2013, p. 390) and RT_G^- in Andreas and Günther (2019, p. 1241) are similar definitions.

Monotony, but satisfy the weaker condition of Disjunctive Rationality. For $>$, Disjunctive Rationality is known to be strictly stronger than Negation Rationality if non-injective models are allowed. The models of Crupi and Iacona are non-injective: there may be different worlds that satisfy exactly the same sentences (cf. Lehmann and Magidor 1992, Freund 1993, Freund and Lehmann 1996).

Possible principles for $\rhd$:

(And)	If $A \rhd B$ and $A \rhd C$, then $A \rhd B \wedge C$.	Conjunction in the Consequent
(Or)	If $A \rhd C$ and $B \rhd C$, then $A \vee B \rhd C$.	Disjunction in the Antecedent
(CMon)	If $A \rhd C$ and $A \rhd B$, then $A \wedge B \rhd C$.	Cautious Monotonicity
(NRat)	If $A \rhd C$ and not $A \wedge \neg B \rhd C$, then $A \wedge B \rhd C$.	Negation Rationality
(DRat)	If $A \vee B \rhd C$ and not $A \rhd C$, then $B \rhd C$.	Disjunctive Rationality
(RMon)	If $A \rhd C$ and not $A \rhd \neg B$, then $A \wedge B \rhd C$.	Rational Monotonicity
(Cut)	If $A \wedge B \rhd C$ and $A \rhd B$, then $A \rhd C$.	Cautious Cut
(CEq)	If $A \rhd B$, $B \rhd A$ and $A \rhd C$, then $B \rhd C$.	Reciprocity

It is readily checked that

- And, Or, CMon, NRat and DRat are valid, while
- RMon, Cut and CEq are invalid.

(And) Suppose that (a) $A > B$, (b) $\neg B > \neg A$, (c) $A > C$ and (d) $\neg C > \neg A$. We want to show that (e) $A > B \wedge C$ and (f) $\neg(B \wedge C) > \neg A$. But (e) follows from (a) and (c) and And for $>$, and (f) follows from (b) and (d), by Or and LLE for $>$.

(Or) Suppose that (a) $A > C$, (b) $\neg C > \neg A$, (c) $B > C$ and (d) $\neg C > \neg B$. We want to show that (e) $A \vee B > C$ and (f) $\neg C > \neg(A \vee B)$. But (e) follows from (a) and (c) and Or for $>$, and (f) follows from (b) and (d), by And and RW for $>$.

(CMon) Suppose that (a) $A > C$, (b) $\neg C > \neg A$, (c) $A > B$ and (d) $\neg B > \neg A$. We want to show that (e) $A \wedge B > C$ and (f) $\neg C > \neg(A \wedge B)$. But (e) follows from (a) and (c) and CMon for $>$, and (f) follows from (b) and RW for $>$.

(NRat) Suppose that (a) $A > C$, (b) $\neg C > \neg A$, and *either* (c) not $A \wedge \neg B > C$ *or* (d) not $\neg C > \neg(A \wedge \neg B)$. By RW for $>$, (b) excludes (d), so (c) is the case. We want to show that (e) $A \wedge B > C$ and (f) $\neg C > \neg(A \wedge B)$. But (e) follows from (a) and (c) and NRat for $>$, and (f) follows from (b) and RW for $>$.

(DRat) Suppose that (a) $A \vee B > C$, (b) $\neg C > \neg(A \vee B)$, and *either* (c) not $A > C$ *or* (d) not $\neg C > \neg A$. By RW for $>$, (b) excludes (d), so (c) is the case. We want to show that (e) $B > C$ and (f) $\neg C > \neg B$. But (e) follows from (a) and (c) and DRat for $>$, and (f) follows from (b) and RW for $>$.

(RMon) Consider the linear order $pq\bar{r} \prec \bar{p}\bar{q}\bar{r} \prec p\bar{q}r$. Here we have $p \rhd q$, not $p \rhd \neg r$ (because not $r > \neg p$) and yet not $p \wedge r \rhd q$ (because not $p \wedge r > q$).

(Cut) Consider the linear order $pqr \prec \bar{p}\bar{q}r \prec p\bar{q}\bar{r}$. Here we have $p \wedge q \rhd r$, $p \rhd q$ and yet not $p \rhd r$ (because not $\neg r > \neg p$).

(CEq) Consider the linear order $pqr \prec \bar{p}\bar{q}r \prec \bar{p}q\bar{r}$. Here we have $p \rhd q$, $q \rhd p$, $p \rhd r$ and yet not $q \rhd r$ (because not $\neg r > \neg q$).

3. Disjunctive Rationality is stronger than Negation Rationality. DRat implies NRat also in the context of contraposing conditionals: substitute $A \wedge \neg B$ and $A \wedge B$ for A and B, respectively, in DRat and use LLE and RW for $>$ in order to derive NRat.

Moreover, it can also be shown that DRat for $\rhd$ is strictly stronger than NRat for $\rhd$ provided that we allow non-injective models with only a partial ordering of the worlds (Crupi and Iacona work with non-injective models with a weak ordering of the worlds). Consider the language based on three propositional variables p, q and r and a model with five worlds $W = \{pq\bar{r}^{(1)}, pq\bar{r}^{(2)}, p\bar{q}r, \bar{p}qr, \bar{p}\bar{q}\bar{r}\}$. Here $pq\bar{r}^{(1)}$ and $pq\bar{r}^{(2)}$ are two distinct worlds that satisfy p and q but not r. Assume that W is only partially (and not weakly) ordered by $\prec$ and that we just have the transitive closure of $\bar{p}\bar{q}\bar{r} \prec p\bar{q}r \prec pq\bar{r}^{(1)}$ and $\bar{p}\bar{q}\bar{r} \prec \bar{p}qr \prec pq\bar{r}^{(2)}$. This model violates Disjunctive Rationality, since we have $p \vee q \rhd r$, but neither $p \rhd r$ nor $q \rhd r$ (since neither $p > r$ nor $q > r$; notice that $pq\bar{r}^{(2)}$ is a minimal p-world and $pq\bar{r}^{(1)}$ is a minimal q-world). However, the model satisfies Negation Rationality. Suppose that $A \rhd C$ but not $A \wedge \neg B \rhd C$. From the former assumption, we have $A > C$ and $\neg C > \neg A$. By RW for $>$, we get $\neg C > \neg(A \wedge \neg B)$ and $\neg C > \neg(A \wedge B)$. From the latter assumption, we have either not $A \wedge \neg B > C$ or not $\neg C > \neg(A \wedge \neg B)$. We conclude that not $A \wedge \neg B > C$. We need to show that $A \wedge B > C$. If A is such that the set $[\![A]\!]$ of worlds satisfying it has a unique minimal element, this follows directly from $A > C$ and not $A \wedge \neg B > C$. So suppose that $[\![A]\!]$ has two minimal elements (there are no sets with more minimal elements in this model). Then $[\![A]\!]$ must be a subset of $\{pq\bar{r}^{(1)}, pq\bar{r}^{(2)}, p\bar{q}r, \bar{p}qr\}$. Since $A > C$ but not $A \wedge \neg B > C$, the set $[\![A \wedge \neg B]\!]$ must include one of $pq\bar{r}^{(1)}$ and $pq\bar{r}^{(2)}$ and in fact, since these two worlds are linguistically indistinguishable, both of them. But then $[\![A \wedge B]\!]$ is a non-empty subset of the minimal elements of $[\![A]\!]$, and thus $A \wedge B > C$, as desired.[4]

4. The impact of modularity. Crupi and Iacona as well as Booth and Chandler work with models that feature weak orders (aka total preorders) on possible worlds, that is, asymmetric orders $\prec$ for which $u \prec v$ implies that either $u \prec w$ or $w \prec v$. This condition is called modularity.[5] It implies that $>$ satisfies Rational Monotonicity. However, we have seen that while Negation Rationality and Disjunctive Rationality transfer from $>$ to $\rhd$, this transfer fails for RMon. On the other hand, RMon for $>$ has not been used for the derivation of any of the above valid principles for $\rhd$. So what is the impact of modularity or RMon?

[4]This proof is an adaptation of the proof in Lehmann and Magidor (1992, p. 18) to the case of contraposing conditionals.

[5]Other names for the same property are ‘negative transitivity’, ‘virtual connectedness’ or ‘almost-connectedness’.

Eric Raidl (2019) showed that it is the following condition:

(CV*) If $A \rhd A \wedge C$ and not $A \rhd A \wedge \neg B$ and $\neg A$ is a truth/belief, then $A \wedge B \rhd A \wedge B \wedge C$.

Principle CV* is a restricted version of the condition called ($\gg$8) in Rott (2019), the restriction being that $\neg A$ is a truth/belief. The unrestricted condition ($\gg$8) is valid for difference-making conditionals, but invalid for contraposing conditionals.

The possible-worlds interpretation of CV* is presented by Raidl; it presumes Lewis's requirement of Centering (there is only one world closest to w, namely w itself).

Let us now use the belief-change semantics using the Ramsey Test for $>$ and verify that CV* is valid, assuming that the belief set *Bel* is consistent. Suppose that (a) $A > A \wedge C$, (b) $\neg(A \wedge C) > \neg A$, that *either* (c) not $A > A \wedge \neg B$ *or* (d) not $\neg(A \wedge \neg B) > \neg A$ and that (e) $\neg A$ is a belief. We want to show that (f) $A \wedge B > A \wedge B \wedge C$ and (g) $\neg(A \wedge B \wedge C) > \neg(A \wedge B)$. Using (e), the assumption that *Bel* is consistent and closed and AGM's Preservation postulate, we find that $\neg A \in Bel \subseteq Bel * \neg(A \wedge B)$, so case (d) is impossible. Thus we may use (a) and (c) to infer (f), by Ref,[6] And and Rational Monotonicity for $>$. On the other hand, we have $\neg A \vee \neg C > \neg A \vee \neg B$, by (b) and RW for $>$, and $\neg B > \neg A \vee \neg B$, by Ref and RW for $>$, so we get (g), by DRat for $>$.

Two more remarks here. First, the Centering condition for systems of spheres, which is common in truth-conditional possible worlds semantics, is not acceptable from a belief revision point of view. Agents don't have beliefs about everything, so there are several possible worlds that are doxastic candidates for being the actual world (Grove 1988 vs. Lewis 1973). However, whenever A and C are beliefs in a consistent belief set *Bel*, then $A > C$ is accepted, due to the Ramsey Test and AGM's Preservation postulate.[7] Here we presuppose that *Bel* is consistent. A weakened version of Preservation will actually do: if $A \in Bel$, then $Bel \subseteq Bel * A$.

Second, can we express that A is a belief in the language using just conditionals? It can be expressed with the help of the difference-making conditional 'not $\bot \gg A$', but it is not expressible, as far as I can see, with (the belief-revision interpretation of) the contraposing conditional $\rhd$, according to which $\bot \rhd A$ is trivially true, for instance.[8] One way of making it expressible would be to introduce a designated (non-conditional) sentence Bel expressing the strongest of the agent's beliefs (Bel is true at all minimal worlds, throughout the innermost sphere). Then we would have: A is believed if and only if $\mathsf{Bel} \rhd A$ is accepted in the belief state.

[6] Ref (for 'Reflexivity') is the axiom scheme $A > A$.

[7] Incidentally, one of my main reasons for introducing the Relevant Ramsey Test in Rott (1986)—then called the 'Strong Ramsey Test'—was that I wanted to block this and-to-if inference.

[8] Jake Chandler has informed me that he and Richard Booth have proved that the fact that A is a belief is not expressible in terms of contraposing conditionals. In a brand-new draft paper, Eric Raidl (2020) has shown how to remedy the situation by introducing an extra belief modality into the object language.

5. The same logic? Vincenzo Crupi and Andrea Iacona prove that their logic of 'the evidential conditional' in "Three Ways of Being Non-material" (2019a), which is based on a probabilistic degree-of-assertability semantics, satisfies And, Or, CMon and NRat, but it does not satisfy RMon, Cut and CEq (they don't mention DRat or CV*). In "The Evidential Conditional" (2019b), they have a truth-conditional system-of-spheres semantics for 'the evidential conditional', which—they claim—"implies exactly the same pattern of validities and invalidities" (p. 19) as the former semantics. This claim is stated without proof. Crupi and Iacona don't mention the principles just listed in their later paper. However, the above checks have indeed lent inductive support to their claim that the two semantics generate the same logic.

Much more general results on the non-probabilistic evidential conditional with completeness proofs are reported by Raidl (2019; 2020) and Raidl, Crupi and Iacona Raidl et al. (2020).

6. Dual principles. It can be seen from the verifications above that And and Or are essentially *dual* to each other in the context of contraposing conditionals, so Or might even be labelled And^d.[9] 'Dual' here means: take the contraposition of each sentence, use logical equivalents where appropriate and relabel metavariables in order to avoid unnecessary negations. Given the definition of $\rhd$ (and the substitutability of logical equivalents), duals to each other have the same validity status. In exactly the same way, Monotonicity and Right Weakening are duals for contraposing conditionals. There are similar duals to the other validities and invalidities mentioned above:

(CMon^d) If $A \rhd C$ and $B \rhd C$, then $A \rhd B \vee C$.

(NRat^d) If $A \rhd C$ and not $A \rhd \neg B \vee C$, then $A \rhd B \vee C$.

(DRat^d) If $A \rhd B \wedge C$ and not $A \rhd B$, then $A \rhd C$.

(CV*^{d}) If $A \vee C \rhd C$ and not $\neg B \vee C \rhd C$ and C is a belief/truth, then $A \vee B \vee C \rhd B \vee C$.

(RMon^d) If $A \rhd C$ and not $\neg B \rhd C$, then $A \rhd B \vee C$.

(Cut^d) If $A \rhd B \vee C$ and $B \rhd C$, then $A \rhd C$.

(CEq^d) If $A \rhd B$, $B \rhd A$ and $C \rhd A$, then $C \rhd B$.

Principle CMon^d is listed and shown to be valid for contraposing conditionals by Raidl (2019, his name for it is 'RCMon'). One of the simplest and most interesting of the dual principles is DRat^d, which may deserve a name in its own right. Let us call it *Conjunctive Rationality*, CRat. It plays an important role as condition ($\gg$1) for difference-making conditionals in Rott (2019).

It is clear that the dual variants are just as valid or invalid as their respective originals.

- CMon^d, NRat^d, DRat^d and CV*^{d} are valid, while
- RMon^d, Cut^d and CEq^d are invalid.

[9] This was already noted by Raidl (2019).

If one wants direct verifications, they are again easy:

(CMon^d) Suppose that (a) $A > C$, (b) $\neg C > \neg A$, (c) $B > C$ and (d) $\neg C > \neg B$. We want to show that (e) $A > B \vee C$ and (f) $\neg(B \vee C) > \neg A$. But (e) follows from (a) by RW and CMon for $>$, and (f) follows from (b) and (d) by CMon and LLE for $>$.

(NRat^d) Suppose that (a) $A > C$, (b) $\neg C > \neg A$, and *either* (c) not $A > \neg B \vee C$ *or* (d) not $\neg(\neg B \vee C) > \neg A$. By RW for $>$, (a) excludes (c), so (d) is the case. We want to show that (e) $A > B \vee C$ and (f) $\neg(B \vee C) > \neg A$. But (e) follows from (a) and RW for $>$, and (f) follows from (b) and (d), by LLE, NRat and RW for $>$.

(DRat^d) Suppose that (a) $A > B \wedge C$, (b) $\neg(B \wedge C) > \neg A$, and *either* (c) not $A > B$ *or* (d) not $\neg B > \neg A$. By RW for $>$, (a) excludes (c), so (d) is the case. We want to show that (e) $A > C$ and (f) $\neg C > \neg A$. But (e) follows from (a) and RW for $>$, and (f) follows from (b) and (d), by LLE, DRat and RW for $>$.

(CV$*^d$) We again use the belief-change semantics using the Ramsey Test for $>$, assuming that the belief set *Bel* is consistent. Suppose that (a) $A \vee C > C$, (b) $\neg C > \neg(A \vee C)$, that *either* (c) not $\neg B \vee C > C$ *or* (d) not $\neg C > \neg(\neg B \vee C)$ and that (e) C is a belief. We want to show that (f) $A \vee B \vee C > B \vee C$ and (g) $\neg(B \vee C) > \neg(A \vee B \vee C)$. We have $A \vee C > B \vee C$, by (a) and RW for $>$, and $B > B \vee C$, by Ref and RW for $>$, so we get (f), by DRat for $>$. On the other hand, we can use (e), the assumption that *Bel* is consistent and closed and AGM's Preservation postulate, and we find that $C \in Bel \subseteq Bel * (\neg B \vee C)$, so case (c) is impossible. Thus we may use (b) and (d) to infer $\neg C \wedge (\neg B \vee C) > \neg(A \vee C)$, by Rational Monotonicity for $>$. But the latter implies (g), by Ref, LLE, And and RW for $>$.

(RMon^d) Consider the linear order $pqr \prec \bar{p}\bar{q}r \prec p\bar{q}\bar{r}$. Here we have $p \rhd q$, not $\neg r \rhd q$ (because not $\neg r > q$) and yet not $p \rhd q \vee r$ (because not $\neg(q \vee r) > \neg p$).

(Cut^d) Consider the linear order $\bar{p}\bar{q}\bar{r} \prec \bar{p}qr \prec pq\bar{r}$. Here we have $p \rhd q \vee r$, $q \rhd r$ and yet not $p \rhd r$ (because not $p > r$).

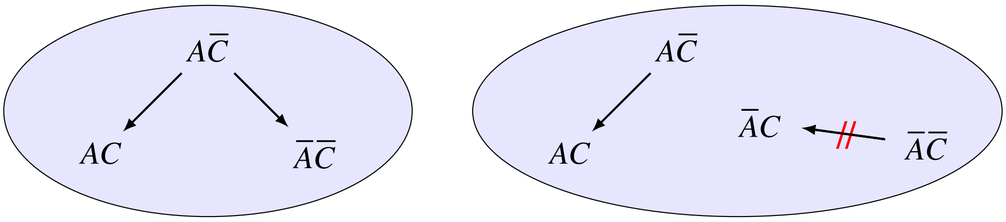
(CEq^d) Consider the linear order $pq\bar{r} \prec \bar{p}\bar{q}\bar{r} \prec p\bar{q}r$. Here we have $p \rhd q$, $q \rhd p$, $r \rhd p$ and yet not $r \rhd q$ (because not $r > q$).

7. Comparison with difference-making conditionals. 'Difference-making conditionals' as introduced in Rott (2019) were not meant to be compounds of other conditionals, but intrinsically contrastive connectives (like the explanatory connectives 'because' or 'since'). However, if one wants to go for a definition of difference-making conditionals in terms of ordinary ('suppositional') conditionals, it looks like this

(DMC) $A \gg C$ iff $A > C$ and not $\neg A > C$.

First of all, we can convince ourselves of the fact that $A \gg C$ and $A \rhd C$ are logically independent.[10] The arrows and crossed-out arrows in Fig. 1 indicate the kind of semantic information that contraposing conditionals and difference-making conditionals

[10]There is yet another kind of conditional in Rott (2019) that is called *dependence conditional*. It is



$A \rhd C$ $\qquad\qquad$ $A \gg C$

Figure 1: Constraints imposed on relations between possible worlds by evidential conditionals and difference making conditionals. Read the edge $A\overline{C} \longrightarrow AC$, for instance, roughly as 'for every world satisfying $A\overline{C}$ there is a more plausible world satisfying AC.' Arrows are pointing downwards to more plausible worlds. The negated edge $\overline{A}\overline{C} \not\longrightarrow \overline{A}C$ should be understood as saying 'there is a world satisfying $\overline{A}\overline{C}$ such that there is no more plausible world satisfying $\overline{A}C$.' It may be thought of as pointing either upwards or sideways.

encode. Utilising this information, we can show that even within the rather restricted class of *linear orderings* of possible worlds there are situations in which the acceptability conditions of contraposing and difference-making conditionals differ: neither is stronger than the other (cf. Fig. 2):

Case 1: Consider the linear order $pq \prec p\overline{q} \prec \overline{p}\overline{q} \prec \overline{p}q$. Here we have $p \gg q$ (because $p > q$ and not $\neg p > q$) but not $p \rhd q$ (because not $\neg q > \neg p$).

Case 2: Consider the linear order $\overline{p}q \prec \overline{p}\overline{q} \prec pq \prec p\overline{q}$. Here we have $p \rhd q$ (because $p > q$ and $\neg q > \neg p$) but not $p \gg q$ (because $\neg p > q$).

The intuitions behind the two kinds of conditionals are clearly distinct. Suppose that A implies C. This is sufficient for making the contraposing conditional $A \rhd C$ true. In one sense, A certainly "supports" C. For the difference-making conditional, however, A needs to be a reason that makes a difference for C. If an agent believes C anyway, even on the assumption of $\neg A$, she would not assert (nor accept?) $A \gg C$. More specifically, contraposing conditionals of the form $A \rhd \top$ (with $\top$ denoting a tautology) are universally true, while the corresponding difference-making conditionals $A \gg \top$ are universally rejected.

Though clearly different, there are a few features contraposing and difference-making conditionals have in common.

7.1. Similarities. Both kinds of conditionals happily violate the And-to-If Inference (also known as 'Conjunctive Sufficiency' or 'Conjunction Conditionalisation') and

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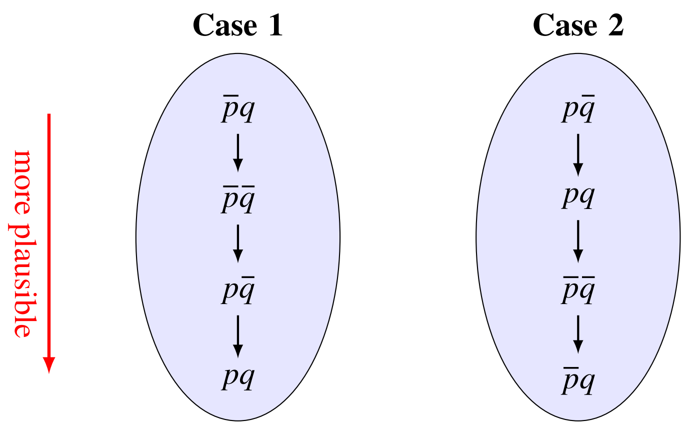
defined by

(DPC) $A \gg C$ iff $A > C$ and $\neg A > \neg C$

and must of course not be confused with the conditional defined by the similarly-looking condition CPC.



$p \gg q$
but not $p \rhd q$

$p \rhd q$
but not $p \gg q$

Figure 2: Two cases showing that evidential conditionals and difference making conditionals are logically independent, even within the restricted class of linear orders of possible worlds. Here pq, $p\bar{q}$ etc. stand for single (partial) worlds.

Right Weakening. This is not to be regarded as bad behaviour. Rott (2019) even called this violation 'the hallmark of relevance'. The reasons for the violation of RW are different. For contraposing conditionals it is just a corollary to the non-monotonicity of ordinary ('suppositional') conditionals. For difference-making conditionals, it is the (metalinguistic) negation in the second part of the defining clause.

Both contraposing and difference-making conditionals violate RMon, Cut and CEq, but this does not seem to reveal a common ideology behind them.

Contraposing and difference-making conditionals agree in that they validate And and Conjunctive Rationality, which are labelled (d26) and ($\gg$1), respectively, in Rott (2019). (But note that their 'duals', Or and Disjunctive Rationality, are not valid for difference-making conditionals; for $\gg$ there is no such duality.) They also agree about the validity of CV*, which follows from ($\gg$8), as mentioned above.

Another property that contraposing conditionals and difference-making conditionals have in common is that, surprisingly, 'If A, then A and C' is *strictly weaker* than 'If A, then C' (and thus they violate what logico-linguists working on generalized quantifiers have called 'Conservativity', cf. Keenan and Stavi 1986, p. 275, and van Benthem 1986, pp. 8, 77). For both contraposing and difference-making conditionals, however, 'If A, then A and C' can be complemented by a conditional involving a disjunction to get the full strength of 'If A, then C'. For contraposing conditionals, we have:[11]

(E$^+$) $A \rhd C$ iff ($A \rhd A \wedge C$ and $A \vee C \rhd C$).

[11] The right-to-left direction of condition E$^+$ appears as axiom E in Raidl (2019).

This is in a peculiar way similar to difference-making conditionals which satisfy the following condition (see Rott 2019):

($\gg$2a) $A \gg C$ iff ($A \gg A \wedge C$ and $A \gg A \vee C$).

Besides the violation of And-to-if and Right Weakening, perhaps most important among the similarities is the fact that the content of ordinary, non-contraposing, de-relevantised, suppositional conditionals can be expressed by the new conditionals in exactly the same way. Let us devote an extra subsection for this point.

7.2. Recovering $>$ from $\rhd$ and $\gg$. This is the common way of recovering the ordinary conditional $>$ from both the relevantised conditional $\gg$ *and* the evidential conditional $\rhd$:

(SC) $A > C$ iff $A \ggcurly A \wedge C$ or $A \wedge C$ is a truth/belief.

Here '$\ggcurly$' is a placeholder for '$\rhd$' or '$\gg$'. Compare Rott (2019, p. 18), Raidl (2019, p. 6) and Crupi and Iacona (2019b, p. 16, with credits to Raidl), with 'belief' applying to the first and 'truth' applying the the other papers. The idea of SC can be used in completeness proofs. Raidl calls it a 'backtranslation' from $\ggcurly$ to $>$ and in fact uses the object-language backtranslation formula $(A \rhd A \wedge C) \vee (A \wedge C)$. The latter is not expressible in the language of Rott (2019) that does not allow compounds of conditionals. In the latter's belief-change framework, SC is to be read as quantifying over acceptance/belief in all belief states and the 'or' in it is a metalinguistic connective.

In order to get a good feel for the similarities and differences, I now sketch the proof for SC, given either CPC or DMC, in terms of a Ramsey Test semantics based on AGM-style belief revision.

The direction from the right-hand side to the left-hand side is easy. On the one hand, $A \ggcurly A \wedge C$ is stronger than $A > A \wedge C$ by definitions CPC and DMC, and $A > A \wedge C$ implies $A > C$, by RW for $>$. On the other hand, the fact that $A \wedge C$ is in the belief set Bel implies that the revision by A will not lose any belief, by AGM's Preservation postulate (presupposing that Bel is consistent). So C will remain in $Bel * A$ which gives us $A > C$, by the Ramsey Test. (Notice that for the last argument it would be sufficient to replace the assumption $A \wedge C \in Bel$ by the weaker assumption that $A \notin Bel$ and $A \supset C \in Bel$.)

For the direction from the left-hand side to the right-hand side, we have to distinguish between CPC and DMC:

- For CPC: Suppose that $A > C$ and not $A \rhd A \wedge C$. That is, (a) $C \in Bel * A$ and (b) either $A \wedge C \notin Bel * A$ or $\neg A \notin Bel * \neg(A \wedge C)$. From (a), we get by Inclusion (AGM*3) $C \in Cn(Bel \cup \{A\})$, and thus $A \supset C \in Bel$. Therefore $\neg A \in Cn(Bel \cup \{\neg(A \wedge C)\})$. The first disjunct of (b) is excluded, by (a) and Success and Closure. So $\neg A \notin Bel * \neg(A \wedge C)$. We conclude that not $Cn(Bel \cup \{\neg(A \wedge C)\}) \subseteq Bel * \neg(A \wedge C)$, and, by Success and Closure again, that not $Bel \subseteq Bel * \neg(A \wedge C)$.

Thus, by Preservation (AGM*4), $A \wedge C \in Bel$.

- For DMC: Suppose that $A > C$ and not $A \gg A \wedge C$. That is, (a) $C \in Bel * A$ and (b) either $A \wedge C \notin Bel * A$ or $A \wedge C \in Bel * \neg A$. From (a), we get by Inclusion (AGM*3) $C \in Cn(Bel \cup \{A\})$, and thus $A \supset C \in Bel$. The first disjunct of (b) is excluded, by (a) and Success and Closure. So $A \wedge C \in Bel * \neg A$. By Success and Closure, we conclude that $\bot \in Bel * \neg A$, which means that A is a doxastic necessity. By Inclusion (AGM*3), it follows that $\bot \in Cn(Bel \cup \{\neg A\})$, i.e., by Closure, $A \in Bel$. Since we also have $A \supset C \in Bel$, we conclude by Closure that $A \wedge C \in Bel$.

So we have verified that SC is in fact suitable for both contraposing and difference-making conditionals.

Actually, a slightly different version of SC was used in Rott (2019, p. 17), namely

$$A > C \quad \text{iff} \quad A \gg A \wedge C \text{ or } (A \text{ is a doxastic necessity and } C \text{ is a belief}).$$

The reason for opting for this definition was that I wanted to maximise the range of application of the main clause ($A \gg A \wedge C$) and minimise that of the side clause. This condition stresses that the acceptability of $A > C$ and that of $A \gg A \wedge C$ differs only in the limiting case when A is doxastically necessary. I also mentioned (in a footnote) an equivalent alternative definition with the side clause '$\neg A$ is not a belief, but $A \supset C$ is a belief'. Notice that the clause 'A is a doxastic necessity and C is a belief' is much stronger, and that the clause '$\neg A$ is not a belief, but $A \supset C$ is a belief' is much weaker, than the side clause in SC. An inspection of the proof above reveals that the weaker, but not the stronger variant of the side clause is suitable for contraposing conditionals as well.[12]

Incidentally, in 'Definable conditionals', Raidl (in press) does not use SC for $\gg$, but the alternative definition of $A > C$ in terms of 'not $\neg A \gg \neg A \vee C$' that is given in Rott (2019, p. 18).

7.3. Dissimilarities. Contraposing conditionals also satisfy Or, CMon, NRat and DRat. Difference-making conditionals do not satisfy these principles, but only counterparts using de-relevantised conditionals:

(Or^c) If $A \gg A \wedge C$ and $B \gg B \wedge C$ and $A \vee B \gg A \vee B$, then $A \vee B \gg (A \vee B) \wedge C$.

(CMon^c) If $A \gg A \wedge C$ and $A \gg A \wedge B$, then $A \wedge B \gg A \wedge B \wedge C$.

(DRat^c) If $A \vee B \gg (A \vee B) \wedge C$ and not $A \gg A \wedge C$, then $B \gg B \wedge C$.

(RMon^c) If $A \gg A \wedge C$ and not $A \gg A \wedge \neg B$, then $A \wedge B \gg A \wedge B \wedge C$.

Just a quick verification of DRat^c which is not mentioned in the paper. Suppose that $A \vee B \gg (A \vee B) \wedge C$, i.e., (a) $A \vee B > (A \vee B) \wedge C$ and (b) not $\neg(A \vee B) > (A \vee B) \wedge C$. Using RW, we get from (a) that $A \vee B > C$, and thus that either $A > C$ or $B > C$, by

[12] However, the non-belief status of a sentence is not expressible in truth-conditional semantics.

DRat for $>$. Thus either $A > A \wedge C$ or $B > B \wedge C$. On the other hand, (b) implies that not $\neg A \wedge \neg B > \bot$, by LLE and RW for $>$. By RW and CMon for $>$, this implies that neither $\neg A > \bot$ nor $\neg B > \bot$. This in turn implies that neither $\neg A > A \wedge C$ nor $\neg B > B \wedge C$, by Ref, And and RW. In sum, either $A \gg A \wedge C$ or $B \gg B \wedge C$.

8. Conclusion: formal properties vs. intuitive support. Given that contraposition has long been supposed to be a paradigmatically *in*valid inference scheme for conditionals, the logic of contraposing conditionals is surprisingly well-behaved. It is certainly much more well-behaved than the logic of difference-making conditionals defined by the Relevant Ramsey Test (that I had called the 'Strong Ramsey Test' in Rott 1986). It validates not only And, CRat and CV*, but also Or, CMon, NRat and DRat. The latter four principles are not valid for $\gg$. So *formally* contraposing conditionals are nicer indeed than difference-making conditionals.

However, I think that the *intuitive* motivation for the relevance/dependence idea encoded in difference-making conditionals is stronger than the intuitive motivation for contraposition. I also think the relevance/dependence idea is a better fit for the notion of support than contraposition. In Rott (1986), I took inspiration from Gärdenfors's (1980) notion of explanation and Spohn's (1983) notion of reason for my analysis of conditionals in terms of the Relevant Ramsey Test. An important part of the intellectual background to Gärdenfors's and Spohn's papers were debates in the philosophy of science as to whether a good explanation should require that the explanans makes the explanandum highly probable (Hempel) or whether it should require that the explanans raises the probability of the explanandum (Salmon, Jeffrey). Both Gärdenfors and Spohn endorsed, in different ways, the raising-the-doxastic-status approach, but neither of them had conditionals in mind. To the best of my knowledge, the analysis given in Rott (1986) was the first contrastive, positive-relevance interpretation *of conditionals*, and it still strikes me as very natural—at least for one kind of conditionals that has wide currency in natural language.[13] On the other hand, once we have given up the traditional conception that conditionals are monotonic (i.e., that they satisfy Strengthening the Antecedent), I find it very hard to come up with a good intuitive motivation for contraposition.[14] And even if one is ready to accept contraposition, one needs to have a clear argument showing that contraposition in fact encodes the ideas of evidence and support.

Which conditionals capture the notion of evidence or support better? Let us try to clarify the situation by an example. Suppose an infectious disease breaks out with millions of cases, and consider the following two scenarios concerning a certain treatment

[13] A very interesting use of probabilistic positive relevance for conditionals is made in Douven (2016, chap. 5). Douven conjoins the positive relevance criterion with a high probability criterion (in reference to a certain threshold value) and studies the logic of such conditionals.

[14] On the close relation between Monotonicity and Contraposition in a context in which Right Weakening is presupposed, see Freund, Lehmann and Morris (1991) and Lehmann and Magidor (1992, pp. 180–181, 200–202).

that has been developed to combat the disease.

Scenario 1: Almost all—more precisely, 99 percent—of the people infected were administered the medicine, and 92 percent of those have recovered. However, only few 10 percent of the persons who did not receive the medicine have recovered.

Scenario 2: Only very few—more precisely, 0.9 percent—of the people infected were administered the medicine. But fortunately, most people end up recovering anyway. It turns out that within the group of people who got the medicine, 89 percent have recovered, and within the group who did not get it, even 93.5 percent have recovered.

Now compare the two scenarios. Suppose we know that Ann has contracted the disease, but we don't know whether she received the medicine. In Scenario 1, we have excellent reasons to say: 'If Ann received the medicine, she has recovered.' The fact that Ann received the medicine would clearly support, or be evidence for, the fact that she recovers. It would also make recovery very likely. In scenario 2, it is doubtful, to say the least, whether we are ready to assert the same conditional. True, it is very likely that Ann recovered, but having received the medicine would not support recovery, and would not be evidence for it either. In fact, having got the medicine would even be somewhat unfavourable to recovery. To the extent that we feel justified in saying 'If Ann received the medicine, she has recovered', the assertion is more like an *even-if* conditional ('concessive conditional') than an evidential conditional.

The two stories could have been told in qualitative, non-probabilistic terms right from the start. But for convenience, let us adopt the simple rule that a possible world v is more plausible than another possible world w if and only if the probability of v is much higher than the probability of w—for concreteness say, more than eight times as high. Let p stand for 'Ann received the medicine' and q for 'Ann recovered'. Then a little calculation shows us that the scenarios are captured by the following linear orders on possible worlds:[15]

Scenario 1: $pq \prec p\bar{q} \prec \bar{p}\bar{q} \prec \bar{p}q$.

Scenario 2: $\bar{p}q \prec \bar{p}\bar{q} \prec pq \prec p\bar{q}$.

These are exactly the two cases illustrated in Fig. 2. In the first scenario, we have $p \gg q$, but not $p \rhd q$. In the second, we have $p \rhd q$ but not $p \gg q$. Comparing this with our intuitive judgments, we find that $\gg$ gets the examples right while $\rhd$ gets them wrong. Of course I cannot exclude that there are examples with the reverse result.[16]

[15] In scenario 1, the probabilities are approximately $Pr(pq)$ = 91.1 percent, $Pr(p\bar{q})$ = 7.9 percent, $Pr(\bar{p}q)$ = 0.1 percent and $Pr(\bar{p}\bar{q})$ = 0.9 percent. In scenario 2, the probabilities are approximately $Pr(pq)$ = 0.8 percent, $Pr(p\bar{q})$ = 0.1 percent, $Pr(\bar{p}q)$ = 92.7 percent and $Pr(\bar{p}\bar{q})$ = 6.4 percent.—Nothing depends on the specific numbers of this example.

[16] When I was about to finish these notes, Vincenzo Crupi offered a potential example in private communication.

But at the time of writing, it seems to me that difference-making conditionals capture the idea of support or evidence (or positive relevance or connexion) just better than contraposing conditionals. The question to be answered by the advocates of contraposing conditionals, I think, is this: Why exactly should we use the 'Chrysippus Test' in the first place?[17]

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