Halvorson, The Logic in Philosophy of Science

Neil Dewar, MCMP, LMU Munich

Central Division APA, 29 February 2020

1. Ramsey sentences

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$$T: \quad \exists x(x = m)$$

$$T^R: \quad \exists y \exists x(x = y)$$

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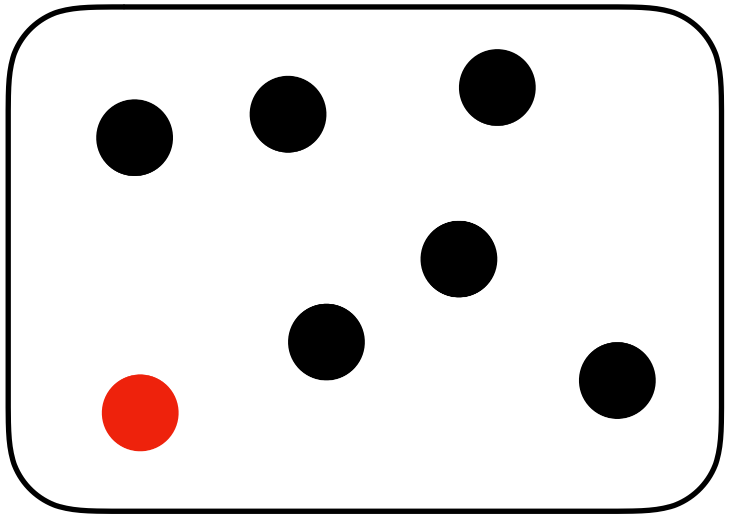

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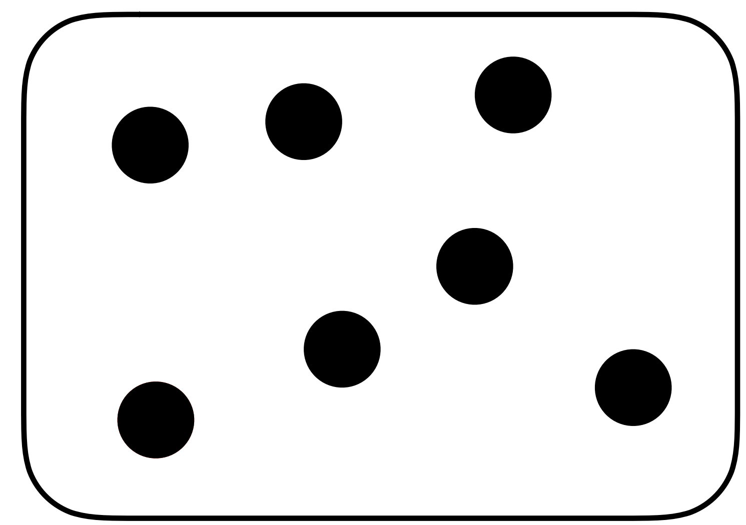

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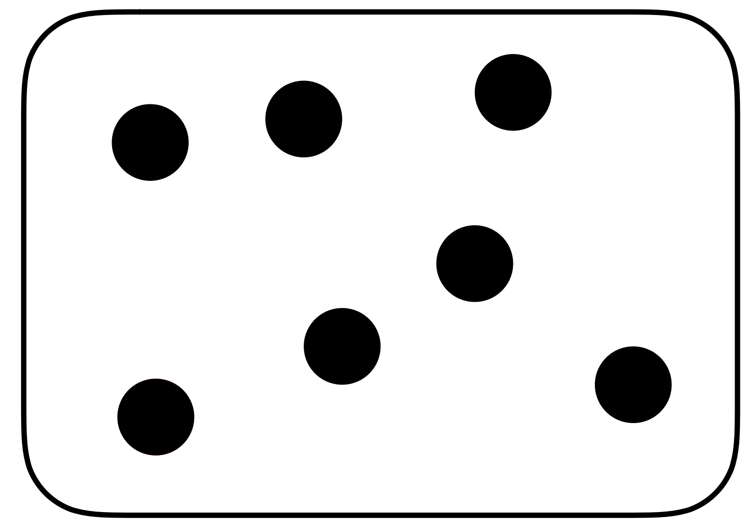

“We can see that T and T^R are not intertranslatable (or definitionally equivalent). Nonetheless, there is a sense in which mathematicians would have no qualms about passing from T^R to the more structured theory T. Indeed, once we’ve established that the domain X is nonempty . . . we could say “let m be one of the elements of X.” This latter statement does not involve any further theoretical commitment over what T^R asserts.” (p. 248)

1. Ramsey sentences

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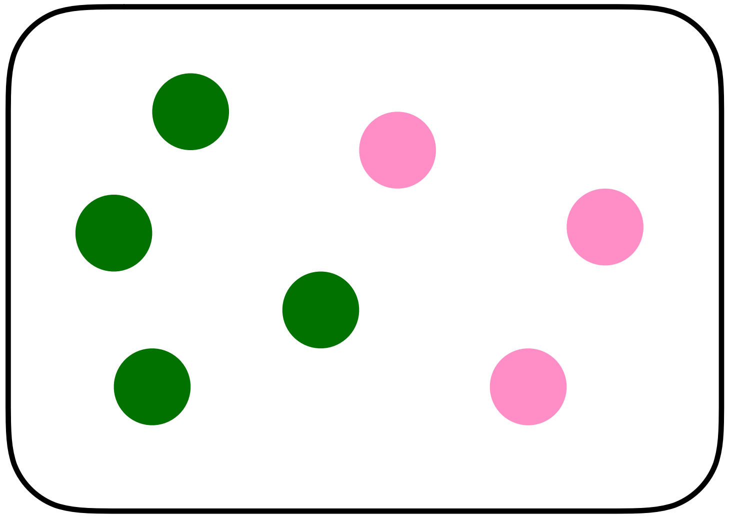

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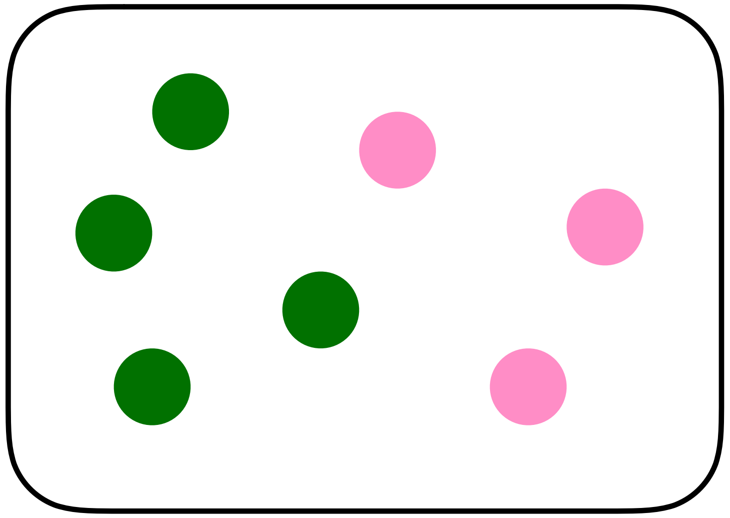

$$F : T_H \to T_H$$
$$F(L) = R$$
$$F(R) = L$$

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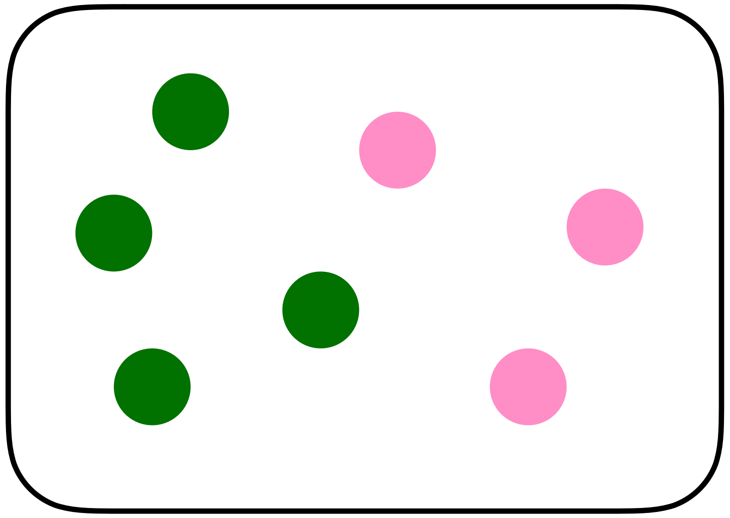

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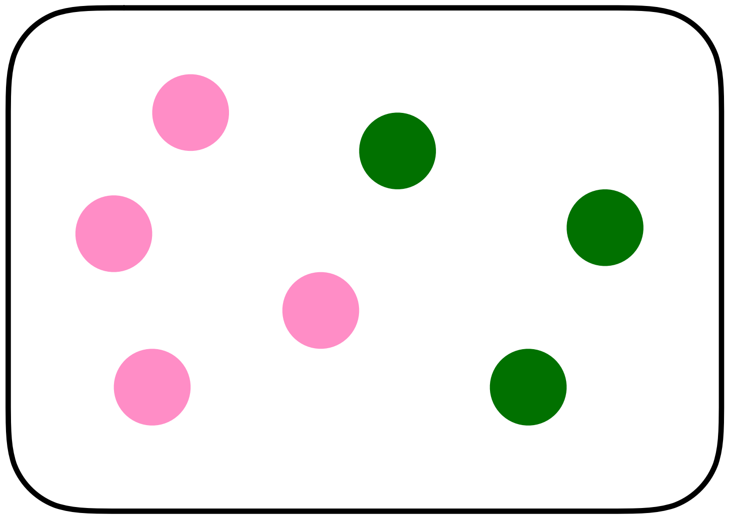
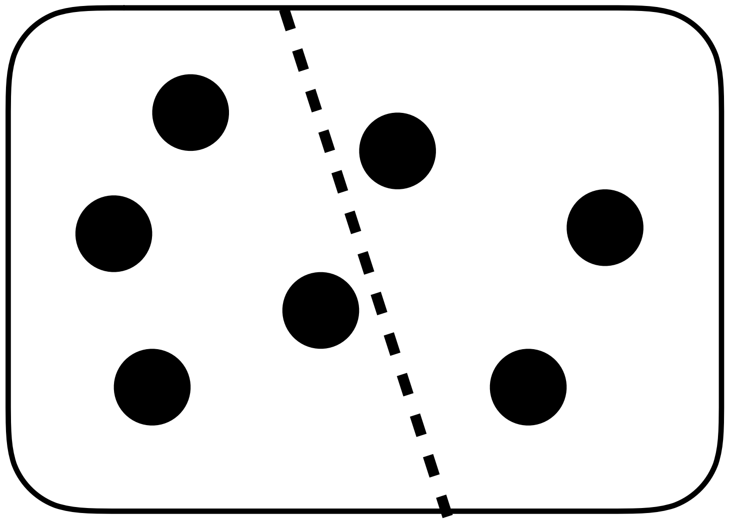


1. Ramsey sentences

$T_C : \forall x Cxx$

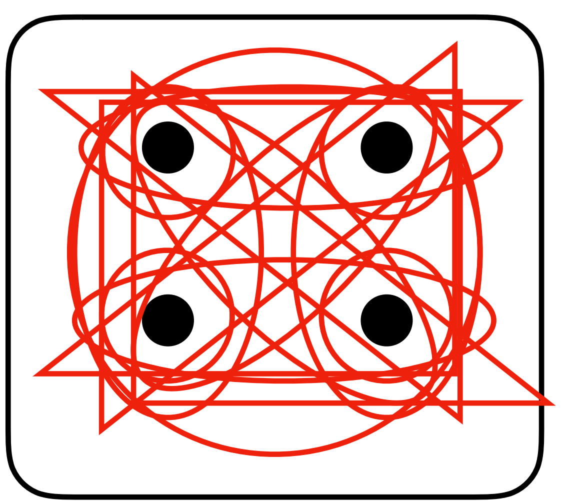
$\forall x \forall y (Cxy \to Cyx)$

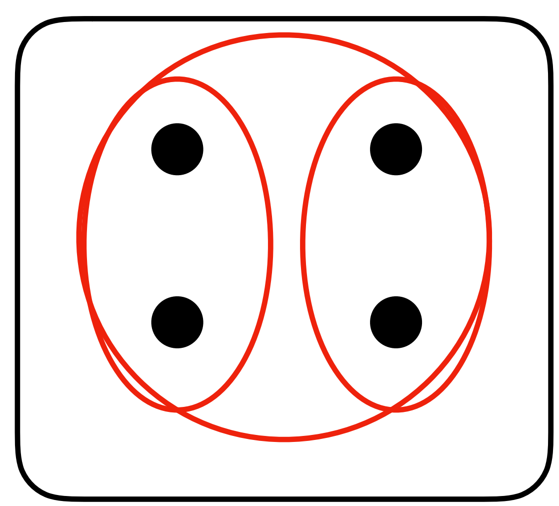
$\forall x \forall y \forall z (Cxy \wedge Cyz \to Cxz)$



1. Ramsey sentences

- A brief reminder:
  - In the *full* semantics for second-order logic, the second-order quantifiers range over *all* subsets of the domain (or Cartesian powers of the domain)
  - In *Henkin* semantics for second-order logic, the second-order quantifiers only range over *definable* subsets of the domain (or Cartesian powers thereof)





1. Ramsey sentences

$$T_H^R = \exists X \exists Y(\forall x(Xx \vee Yx) \wedge \forall x \neg(Xx \wedge Yx))$$

$$T_C^R = \exists Z(\forall x Zxx \wedge \forall x \forall y(Zxy \rightarrow Zyx) \wedge \forall x \forall y \forall z(Zxy \wedge Zyz \rightarrow Zxz))$$

1. Ramsey sentences

$$T_H^R = \exists X \exists Y(\forall x(Xx \vee Yx) \wedge \forall x \neg(Xx \wedge Yx))$$

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- On *full* semantics, $T_C^R \equiv T_H^R$

1. Ramsey sentences

$$T^R_H = \exists X \exists Y(\forall x(Xx \vee Yx) \wedge \forall x \neg(Xx \wedge Yx))$$

$$T^R_C = \exists Z(\forall x Zxx \wedge \forall x \forall y(Zxy \rightarrow Zyx) \wedge \forall x \forall y \forall z(Zxy \wedge Zyz \rightarrow Zxz))$$

- On *full* semantics, $T^R_C \equiv T^R_H$
  - …but $\vDash T^R_H$ and $\vDash T^R_C$!

1. Ramsey sentences

$$T_H^R = \exists X \exists Y(\forall x(Xx \vee Yx) \wedge \forall x \neg(Xx \wedge Yx))$$

$$T_C^R = \exists Z(\forall x Zxx \wedge \forall x \forall y(Zxy \rightarrow Zyx) \wedge \forall x \forall y \forall z(Zxy \wedge Zyz \rightarrow Zxz))$$

- On *full* semantics, $T_C^R \equiv T_H^R$
  - …but $\vDash T_H^R$ and $\vDash T_C^R$!
- On Henkin semantics, $T_H^R \vDash T_C^R$

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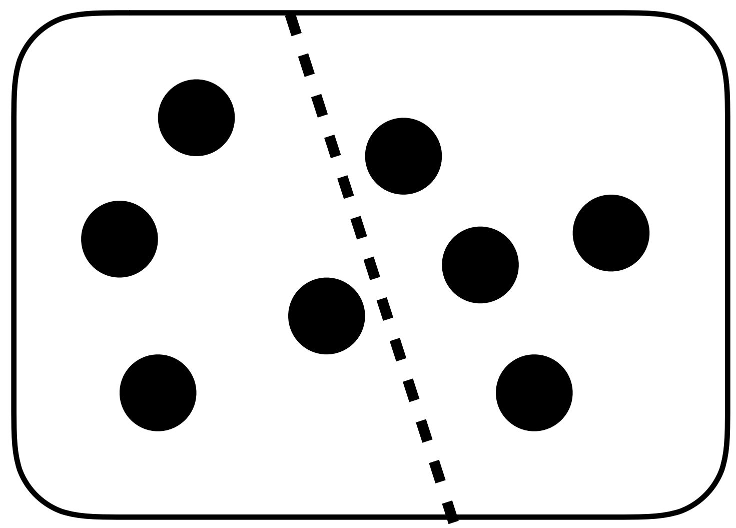
- On *full* semantics, $T_C^R \equiv T_H^R$
  - …but $\vDash T_H^R$ and $\vDash T_C^R$!
- On Henkin semantics, $T_H^R \vDash T_C^R$
  - …but $T_C \nvDash T_H^R$!

1. Ramsey sentences



2. Counting possibilities

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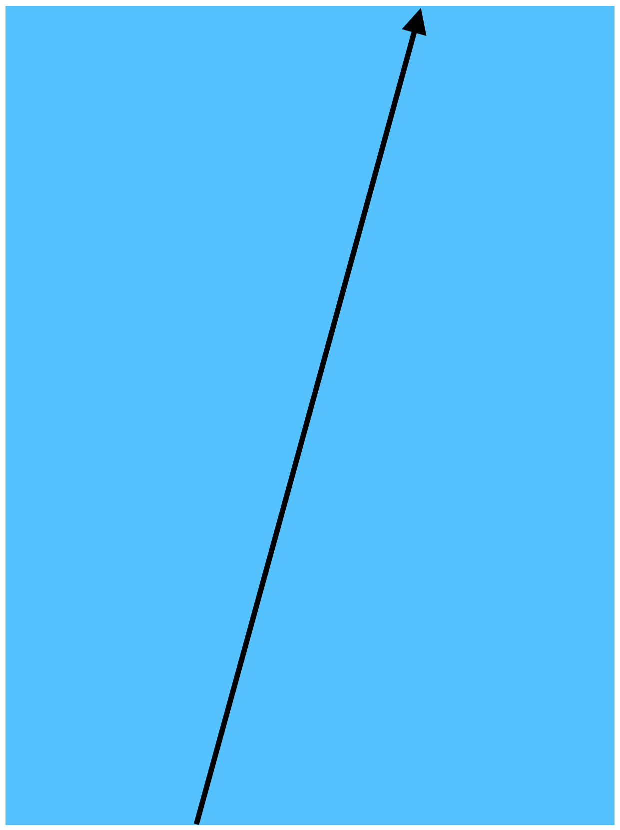

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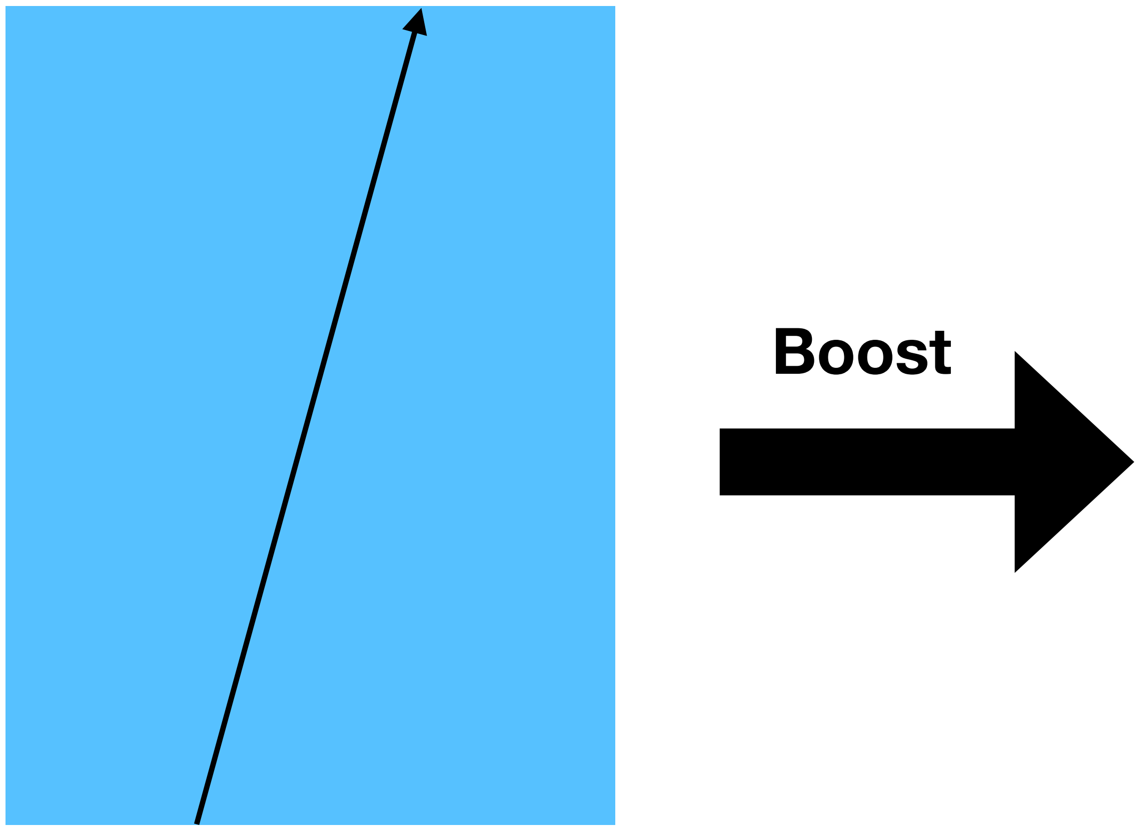

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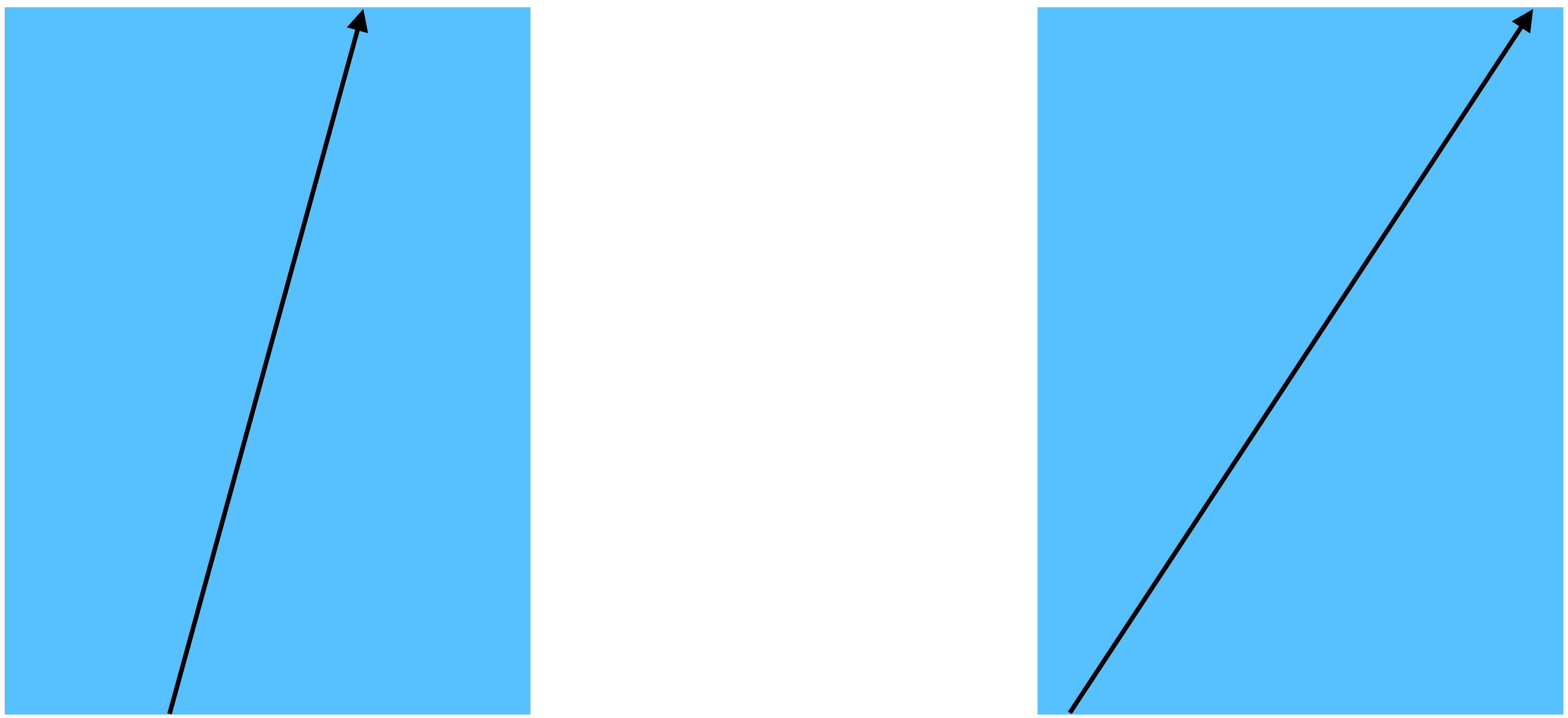

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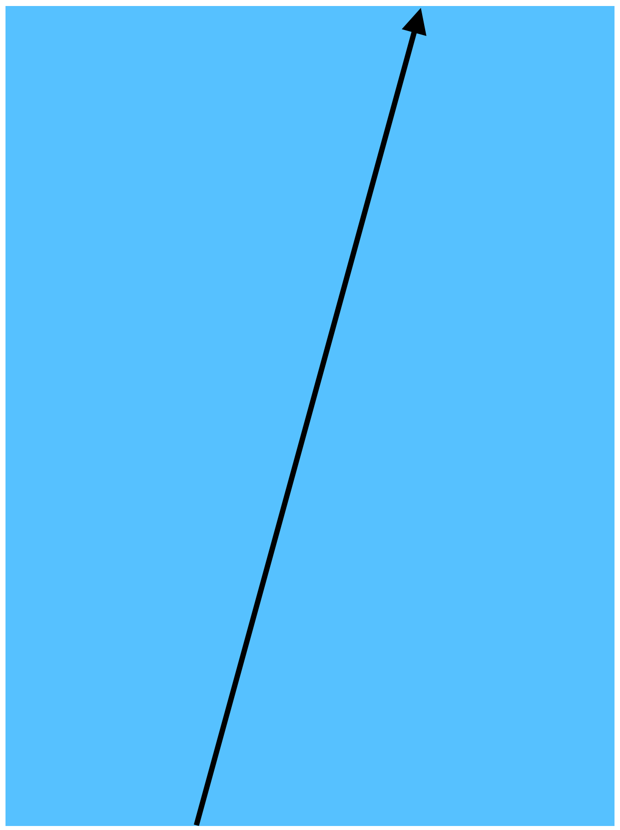

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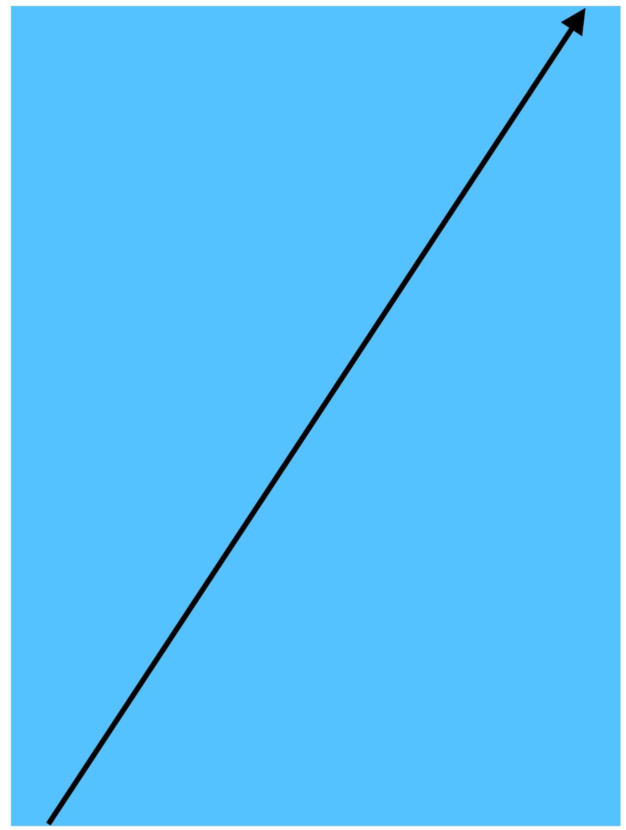
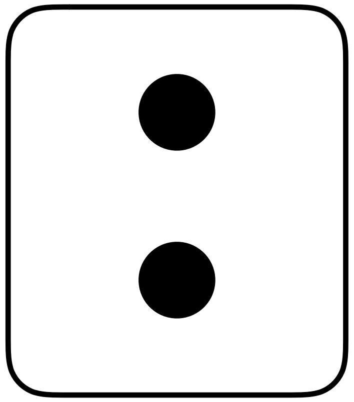
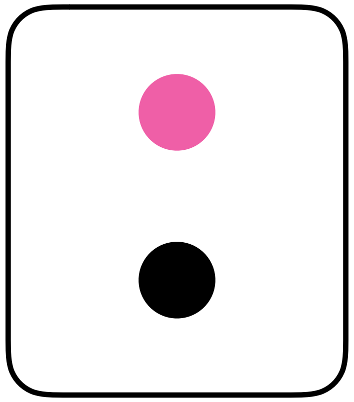


2. Counting possibilities

$$T_0 : \exists x \exists y (x \neq y \wedge \forall z (z = x \vee z = y))$$

$$T : \exists x \exists y (x \neq y \wedge \forall z (z = x \vee z = y)) \wedge \exists x Px \wedge \exists x \neg Px$$





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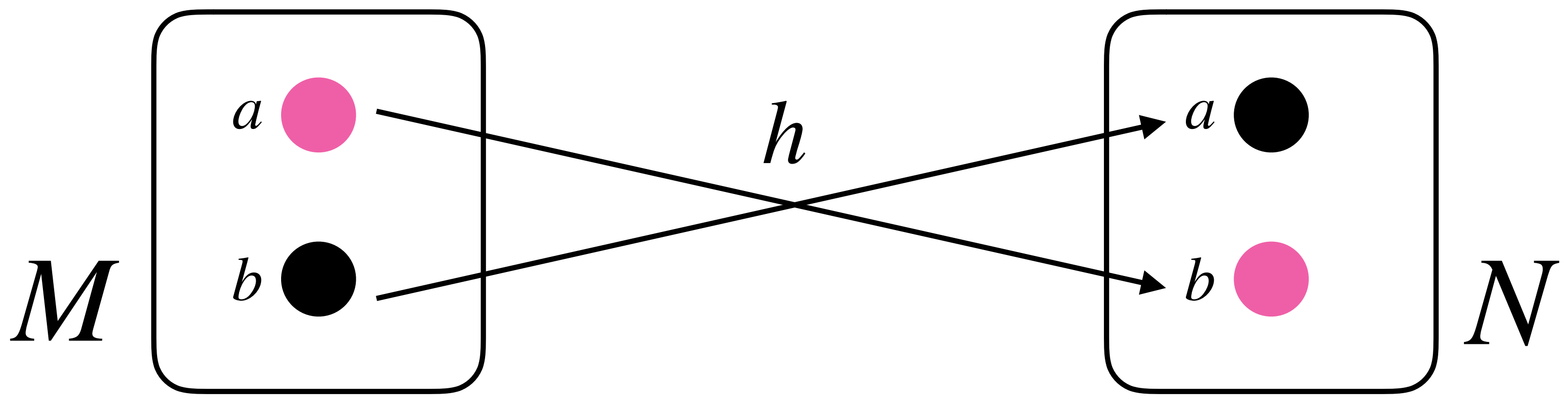

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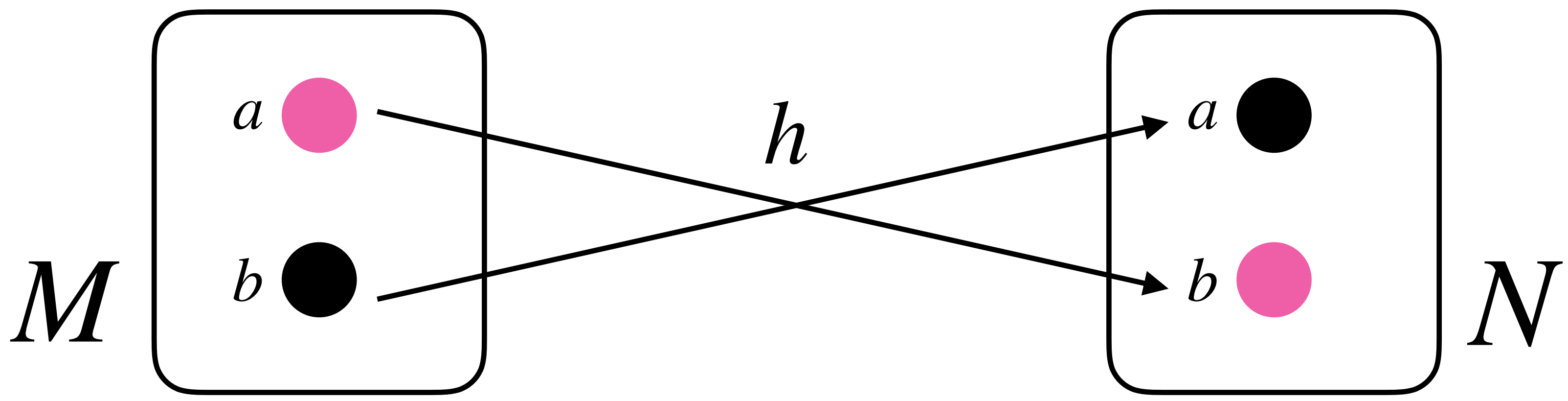

$$I : T_0 \to T$$

$$I^* : \mathrm{Mod}(T) \to \mathrm{Mod}(T_0)$$

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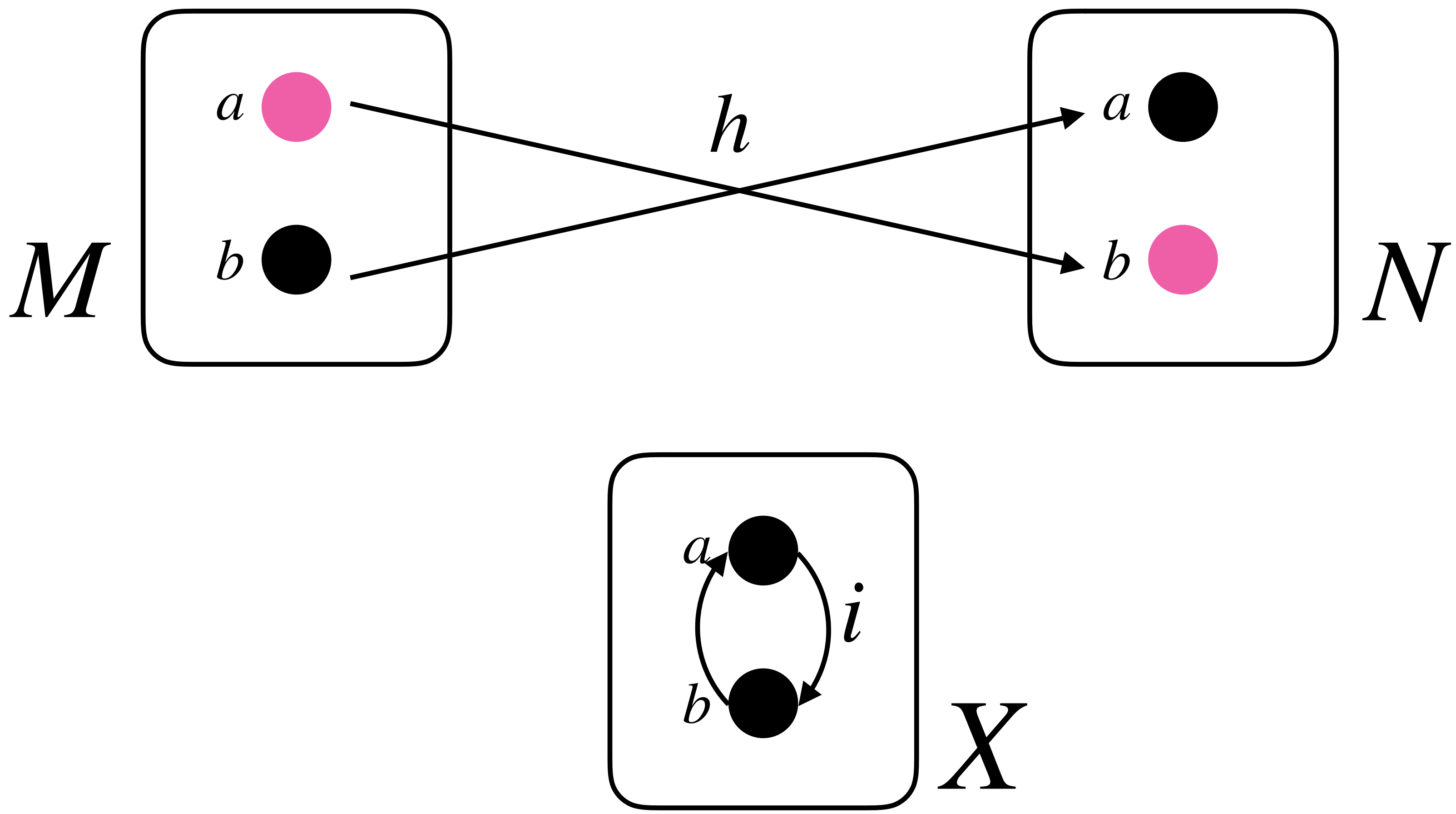

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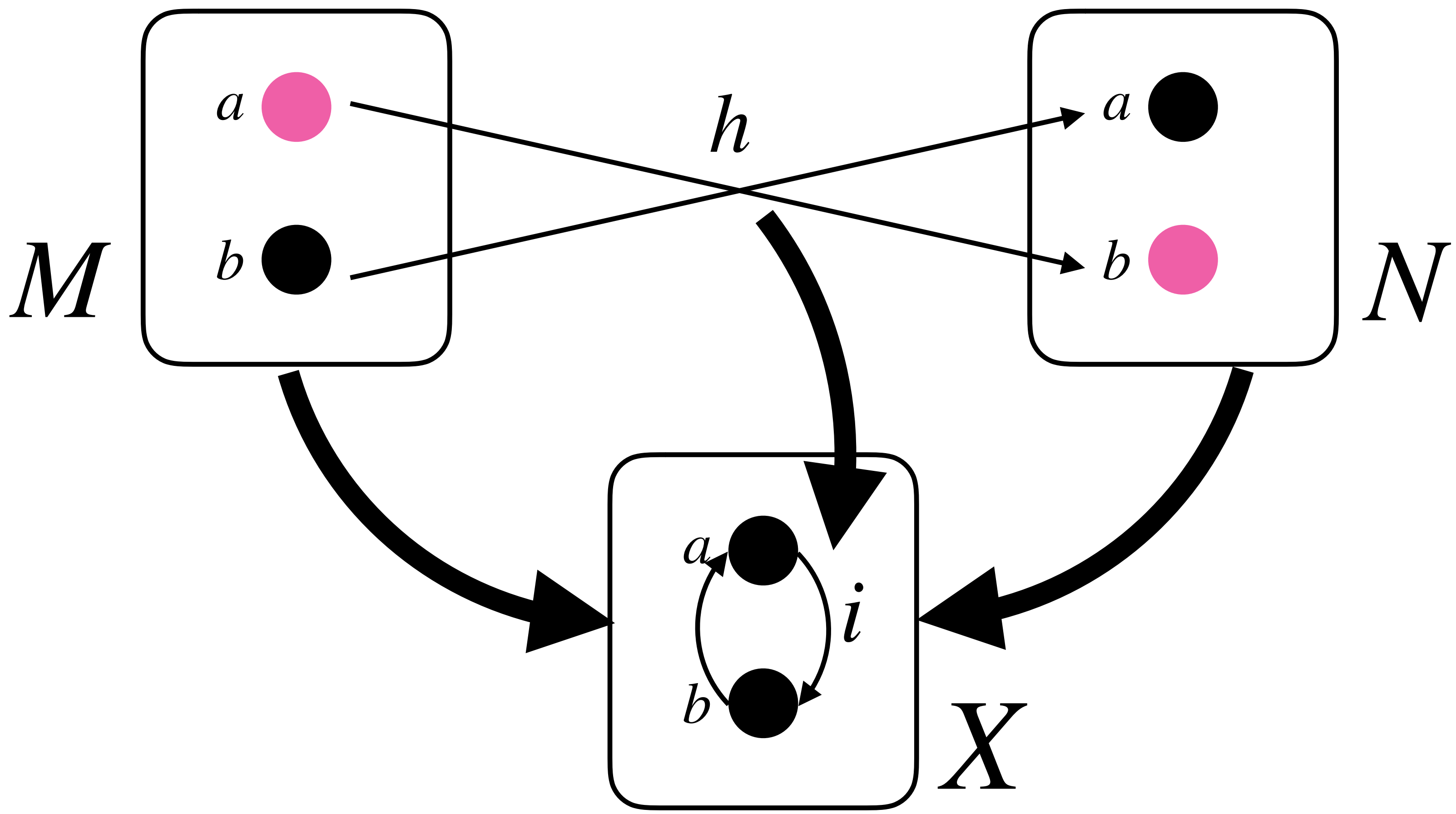

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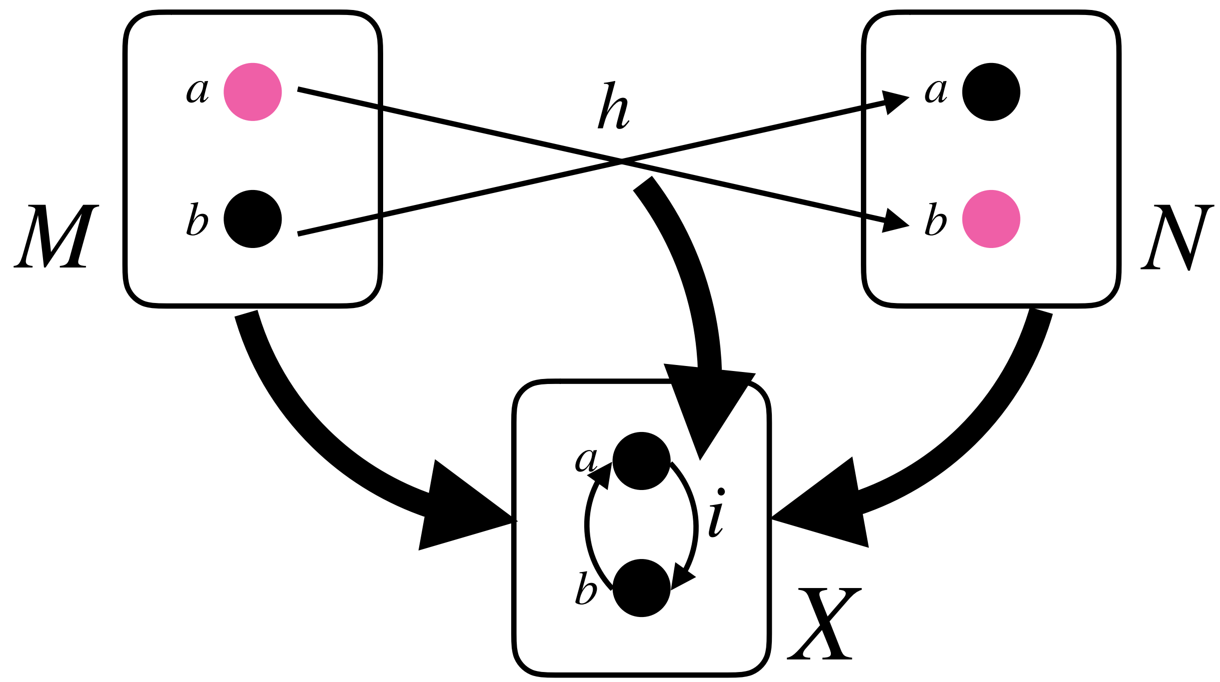
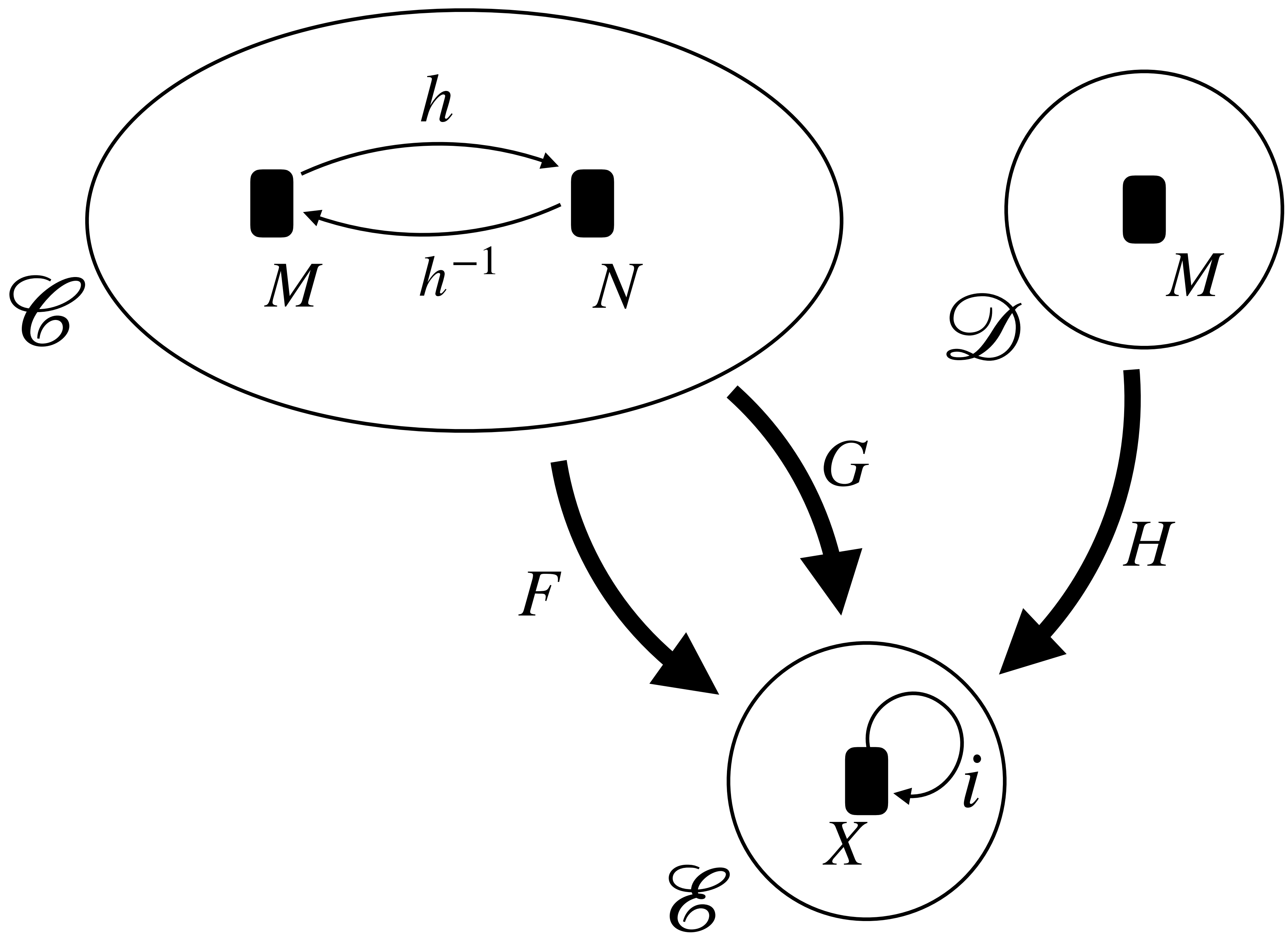


“Recall, though, that functors map identity morphisms to identity morphisms. Hence, if the isomorphism $h : M \to N$ is considered to be an identity (as the shiftless seem to do), then it would follow that I^*h is the identity morphism. Thus, contra the shiftless, we cannot identity M and N and forget that there was a nonidentity isomorphism $h : M \to N$. If we do that, then we won’t be able to see how the theory T is related to the theory T_0.”

2. Counting possibilities



2. Counting possibilities

“A theory T is indifferent to the question of the identity of its models. In other words, if M and N are models of T, then T neither says that M = N nor that M/ = N. The only question T understands is: are these models isomorphic or not?” (p. 259)

“…the philosopher of science shouldn’t say things about Mod(T) that are not invariant under categorical equivalence, nor should they argue over questions—such as “how many models does T have?”—whose answer is not invariant under categorical equivalence.” (p. 260)

2. Counting possibilities

“Now, I don’t disagree with this claim [that synonymous sentences express the same proposition]; I only doubt its utility. If you give me two languages I don’t understand, and theories in the respective languages, then I have no way of knowing whether those theories pick out the same propositions.” (p. 271)

“It is not even clear, granted meanings, when we have two and when we have one; it is not clear when linguistic forms should be regarded as synonymous, or alike in meaning, and when they should not. If a standard of synonymy should be arrived at, we may reasonably expect that the appeal to meanings as entities will not have played a very useful part in the enterprise.” (Quine, 1948)

2. Counting possibilities

“It is only if we resist the attempt to press together the totality of forms (which here offer themselves to us) into a final metaphysical unity, into the unity and simplicity of an absolute “world-basis”, that their authentic concrete content and their concrete abundance reveal themselves to us. ... The naive realism of the ordinary worldview ... always, of course, falls into this mistake. From the totality of possible reality-concepts, it separates some individual one out and erects it as norm and exemplar for all the rest.” (Cassirer, 1921)

3. Putnam’s paradox

“…let me be completely clear about my view of the argument: it is absurd. This version of Putnam’s argument is not merely an argument for antirealism, or internal realism, or something like that. This version of the argument would prove that all consistent theories should be treated as equivalent: there is no reason to choose one over the other.” (p. 263)

3. Putnam's paradox

"Metaphysical realism…is, or purports to be, a model of the relation of any correct theory to all or part of THE WORLD. …Let us set out the model in its basic form. In its primitive form, there is a relation between each term in the language and a piece of THE WORLD (or a kind of piece, if the term is a general term). … there has to be a determinate relation of reference between terms in L and pieces (or sets of pieces) of THE WORLD, on the metaphysical realist model … What makes this picture different from internal realism (which employs a similar picture within a theory) is that (1) the picture is supposed to apply to all correct theories at once…; and (2) THE WORLD is supposed to be independent of any particular representation we have of it—indeed, it is held that we might be unable to represent THE WORLD correctly at all…" (Putnam, 1977)

3. Putnam’s paradox

“But how does the metatheorist’s language get a grip on the world? … Now, Putnam might claim that it is not he, but the realist, who thinks that the world is made of things, and that when our language use is successful, our names denote those things. So far I agree. The realist does think that. But the realist can freely admit that even he has just another theory, and that his theory cannot be used to detect differences in how other people’s theories connect up with the world.” (pp. 264–265)

3. Putnam's paradox

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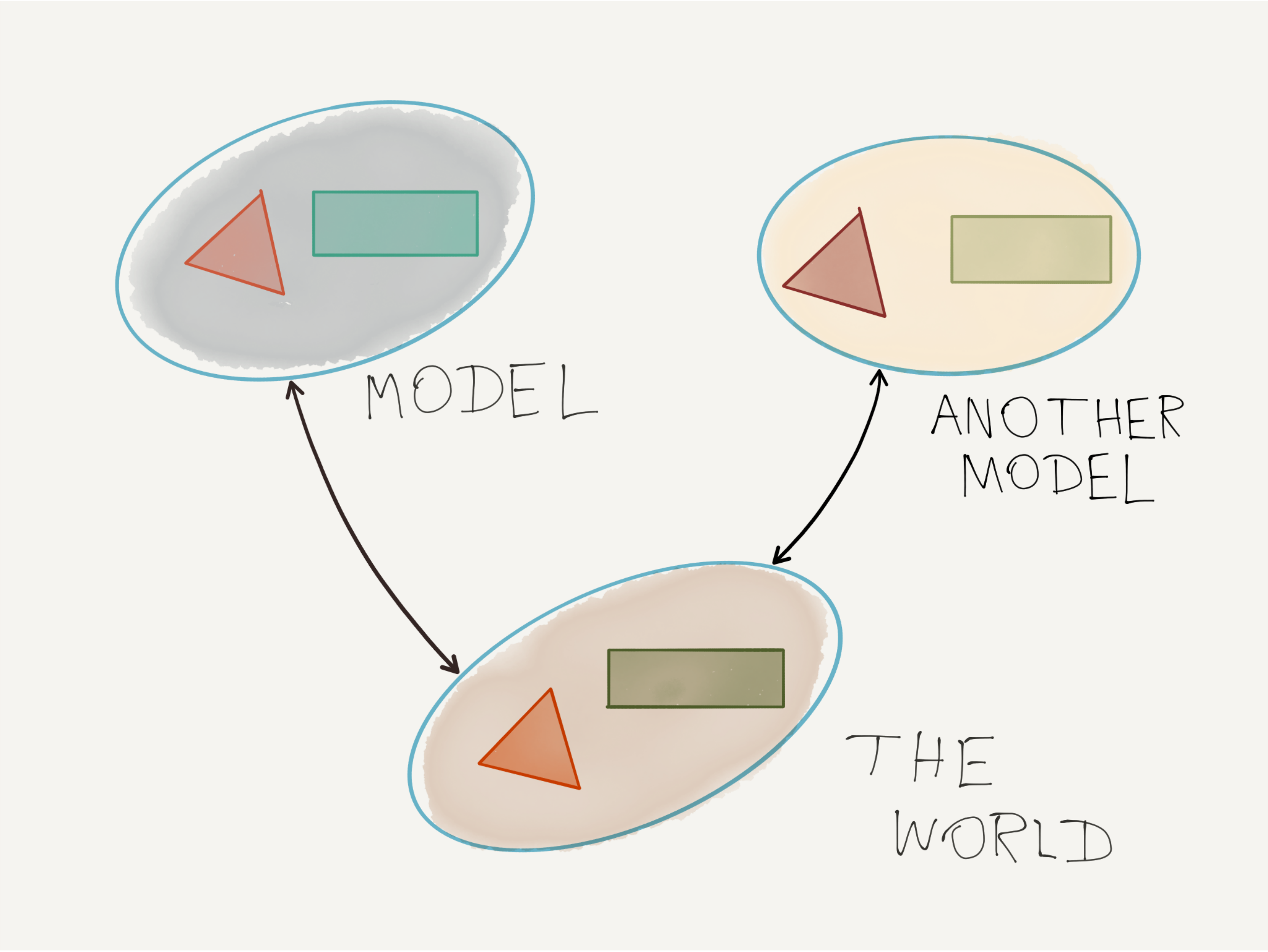
“…the picture typically presented to us is that logical semantics deals with mind-independent things (viz. set-theoretic structures), which can stand in mind-independent relations to concrete reality, and to which we have unmediated epistemic access. Such a picture suggests that logical semantics provides a bridge over which we can safely cross the notorious mind-world gap. But something is fishy with this picture. How could logical semantics get any closer to “the world” than any other bits of mathematics?” (pp. 164–165)

“logical semantics is ... wait for it ... just more mathematics. As such, while semantics can be used to represent things in the world, including people and their practice of making claims about the world, its means of representation are no different than those of any other part of mathematics.” (p. 15)

3. Putnam's paradox



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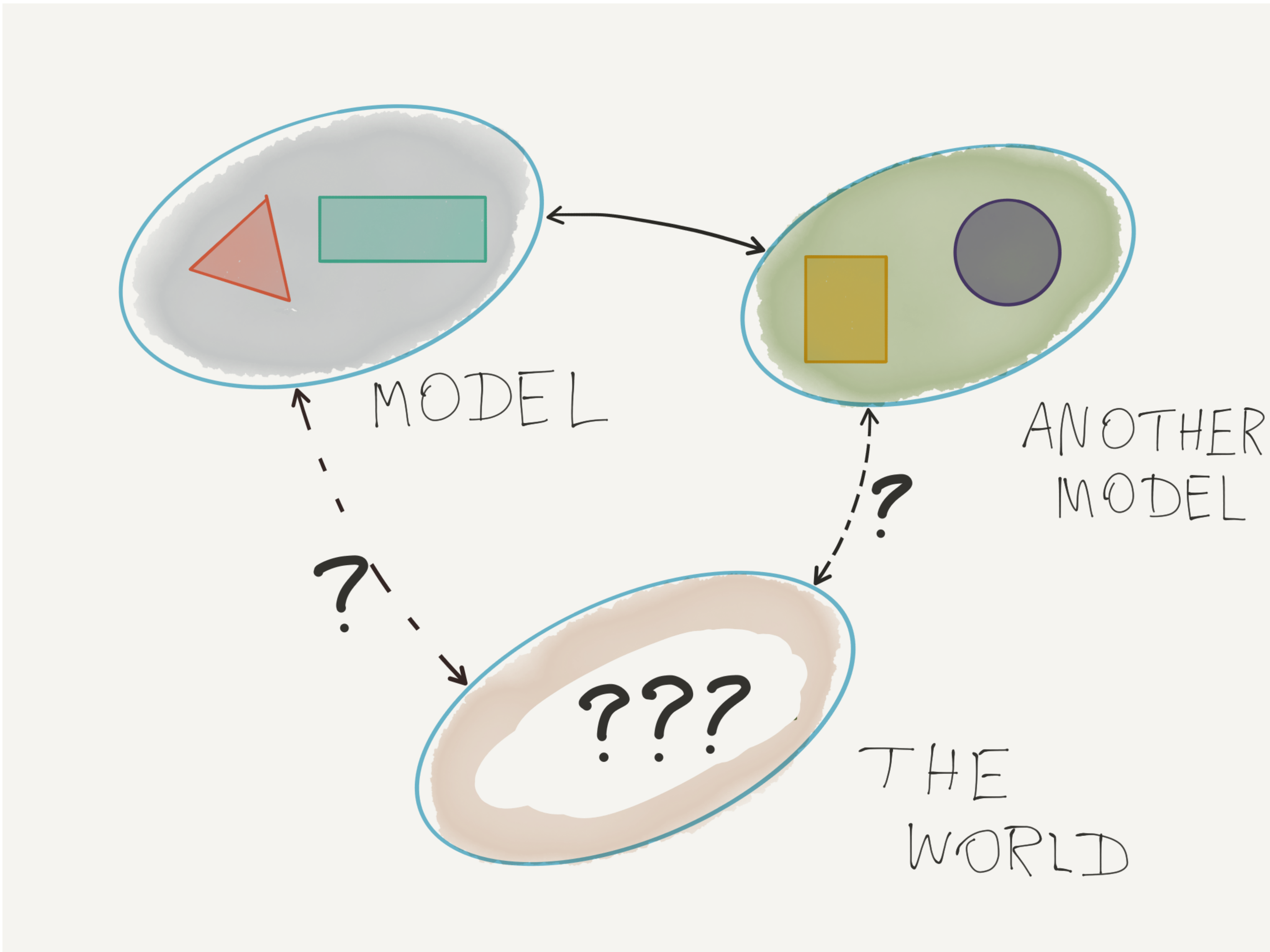

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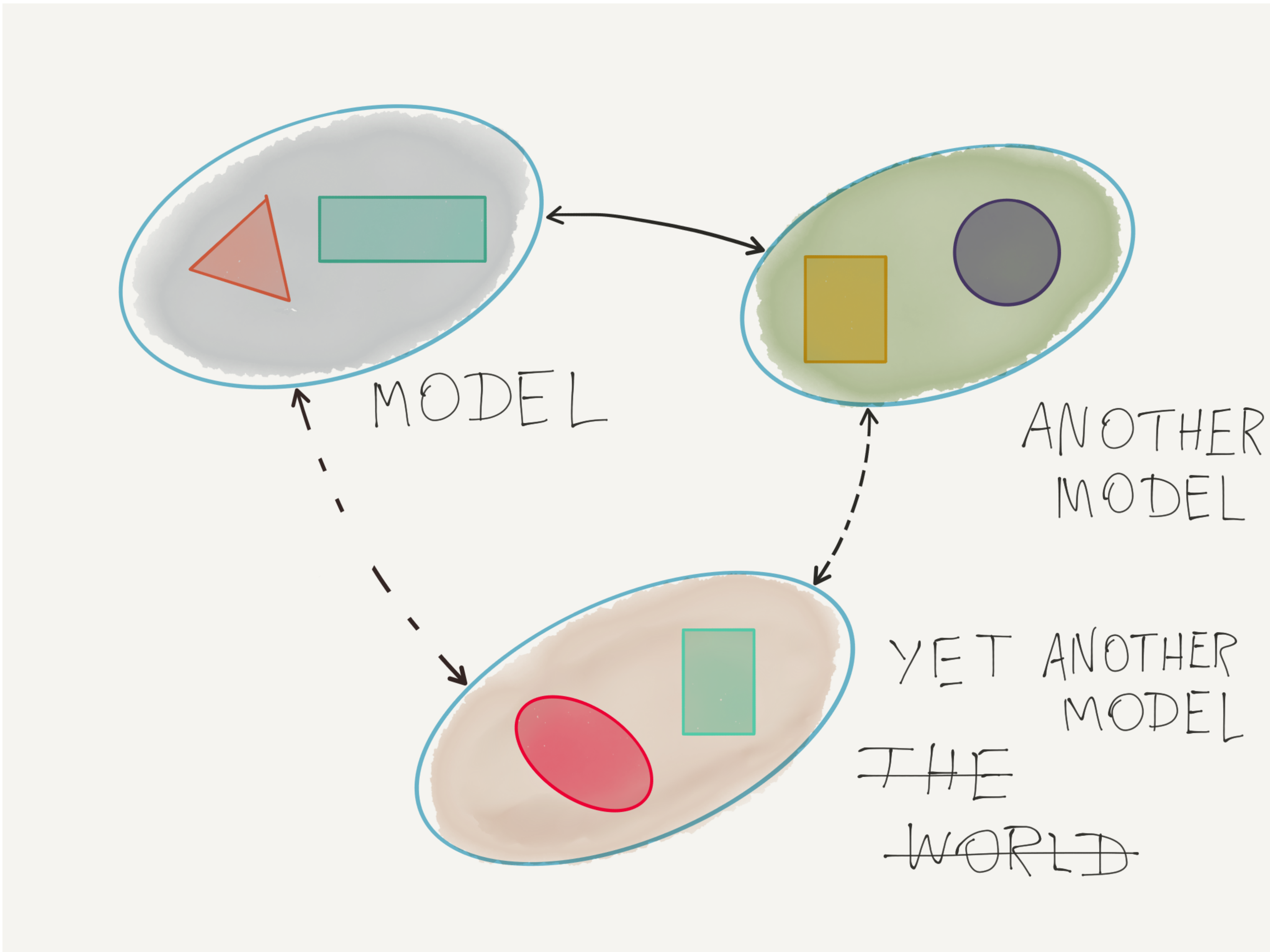

3. Putnam's paradox

"…one can "Tarski-ize" any theory that is constructed in the standard way—that is, with the help of a metalanguage of suitable strength, and also containing the extra-logical resources of the language of the theory T under discussion, one can supplement T with a theory of reference for it, T′; and can do so in such a way that whenever θ is a theorem of T, the statement that θ is true is a theorem of T′. Thus, … the Tarskian theory of reference and truth in a rather serious sense trivializes the desideratum put forward by the realists. … The point can be restated this way: The semantics of reference and truth (for a given theory) is itself a theory." (Stein, 1989)

4. Realism and equivalence

Niels 😐

Mette 😀

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Niels

😐

$$c \neq d$$

$$\forall x(x = c \vee x = d)$$

Mette

😀

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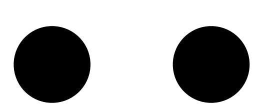
😐

$$c \neq d$$
$$\forall x(x = c \lor x = d)$$



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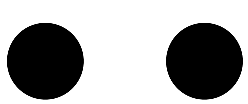

Mette

😀

$$\forall x \neg(xPx)$$
$$\forall x \forall y \forall z(xPy \land yPz \to xPz)$$
$$Ac \land Ad$$
$$c \neq d$$
$$\forall x(Ax \to (x = c \lor x = d))$$
$$\exists z(cPz \land dPz \land \forall y(\neg Ay \to y = z))$$

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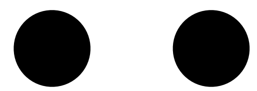

Mette

😀

$$\forall x \neg(xPx)$$
$$\forall x \forall y \forall z(xPy \wedge yPz \rightarrow xPz)$$
$$Ac \wedge Ad$$
$$c \neq d$$
$$\forall x(Ax \rightarrow (x = c \vee x = d))$$
$$\exists z(cPz \wedge dPz \wedge \forall y(\neg Ay \rightarrow y = z))$$

where $\forall x(Ax \leftrightarrow \neg \exists y(yPx))$

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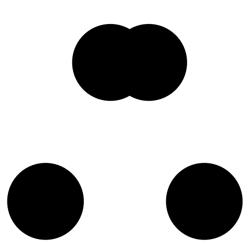

Mette

😀

$$\forall x \neg (xPx)$$
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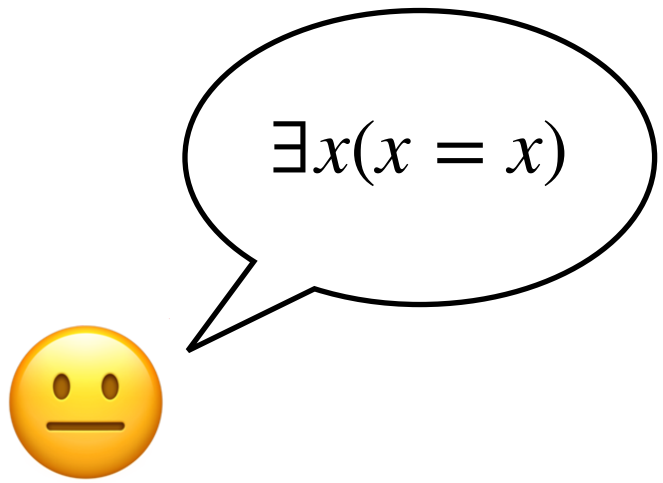

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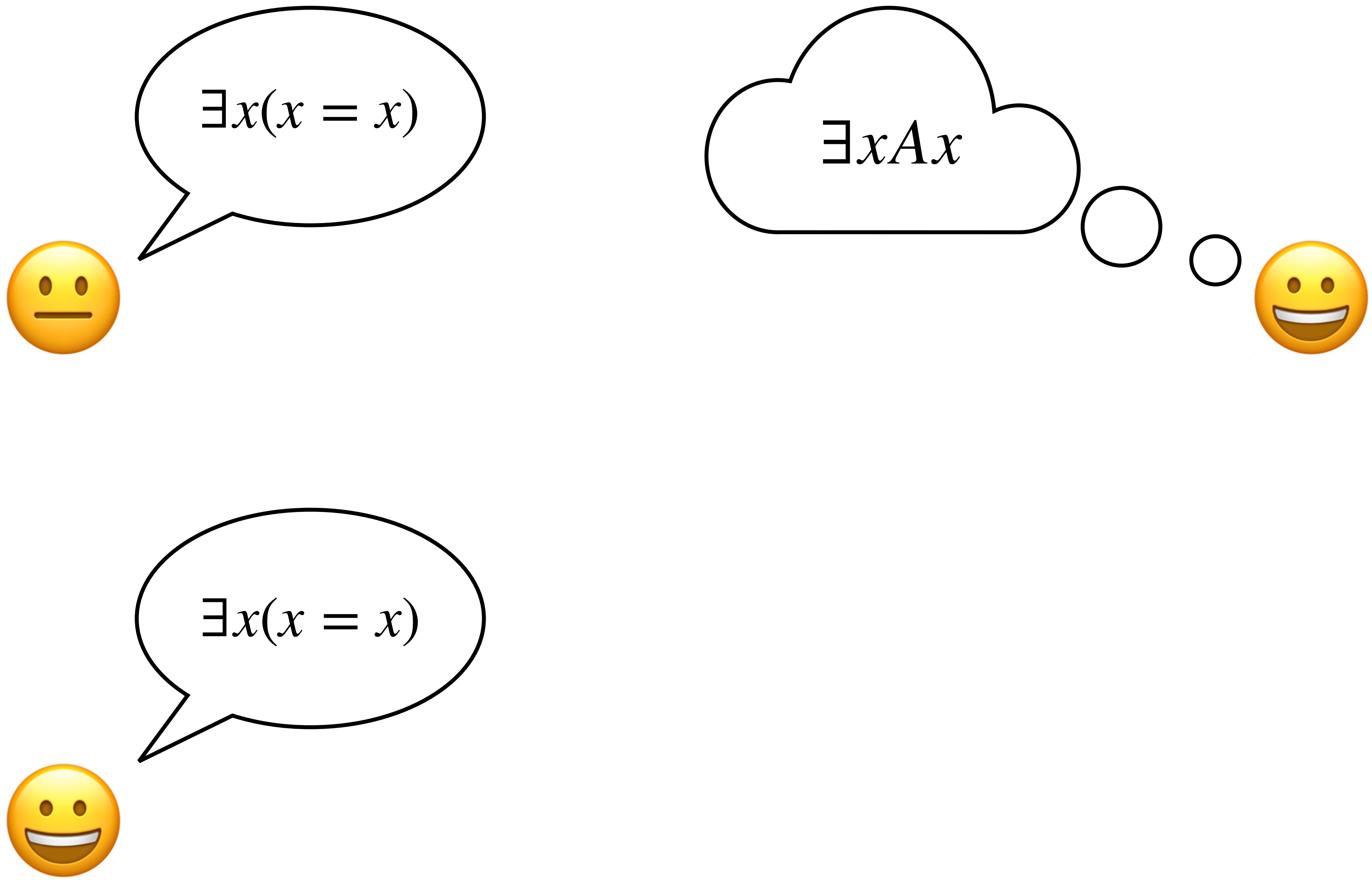

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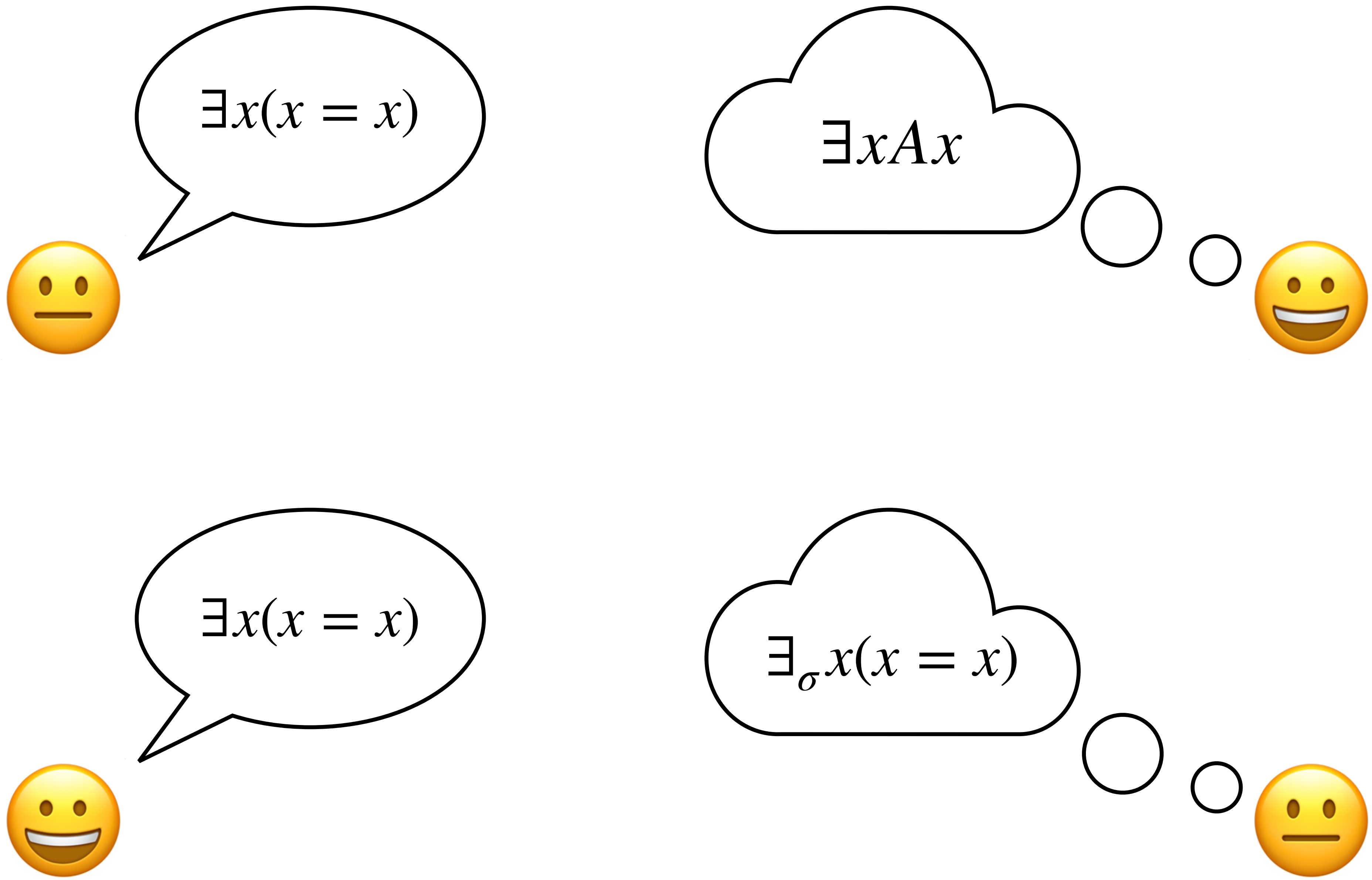

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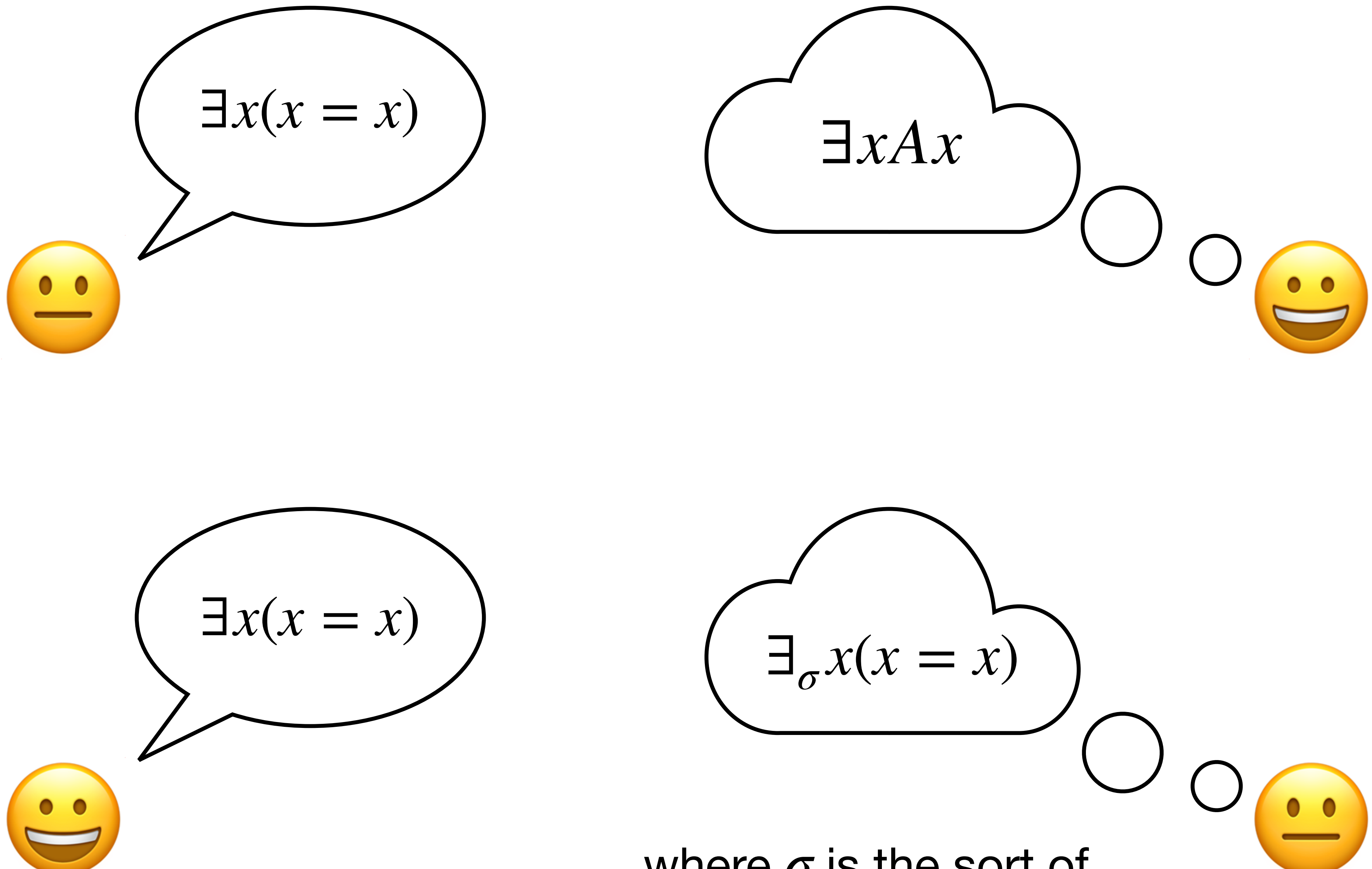

where σ is the sort of two-element multisets (quotients of pairs)

4. Realism and equivalence

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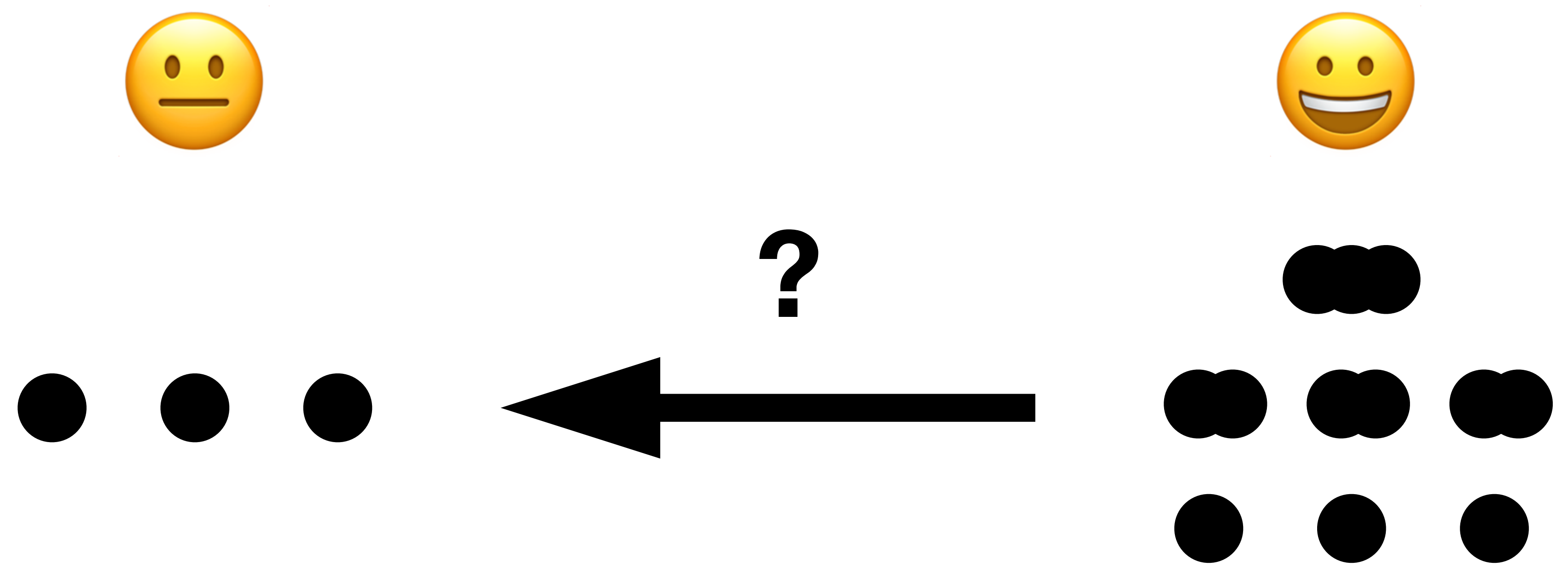

4. Realism and equivalence

$$\begin{tikzcd} \forall x \arrow[r, bend left] \arrow[r, phantom, "\textbf{?}"] & \forall X \arrow[l, bend left] \end{tikzcd}$$

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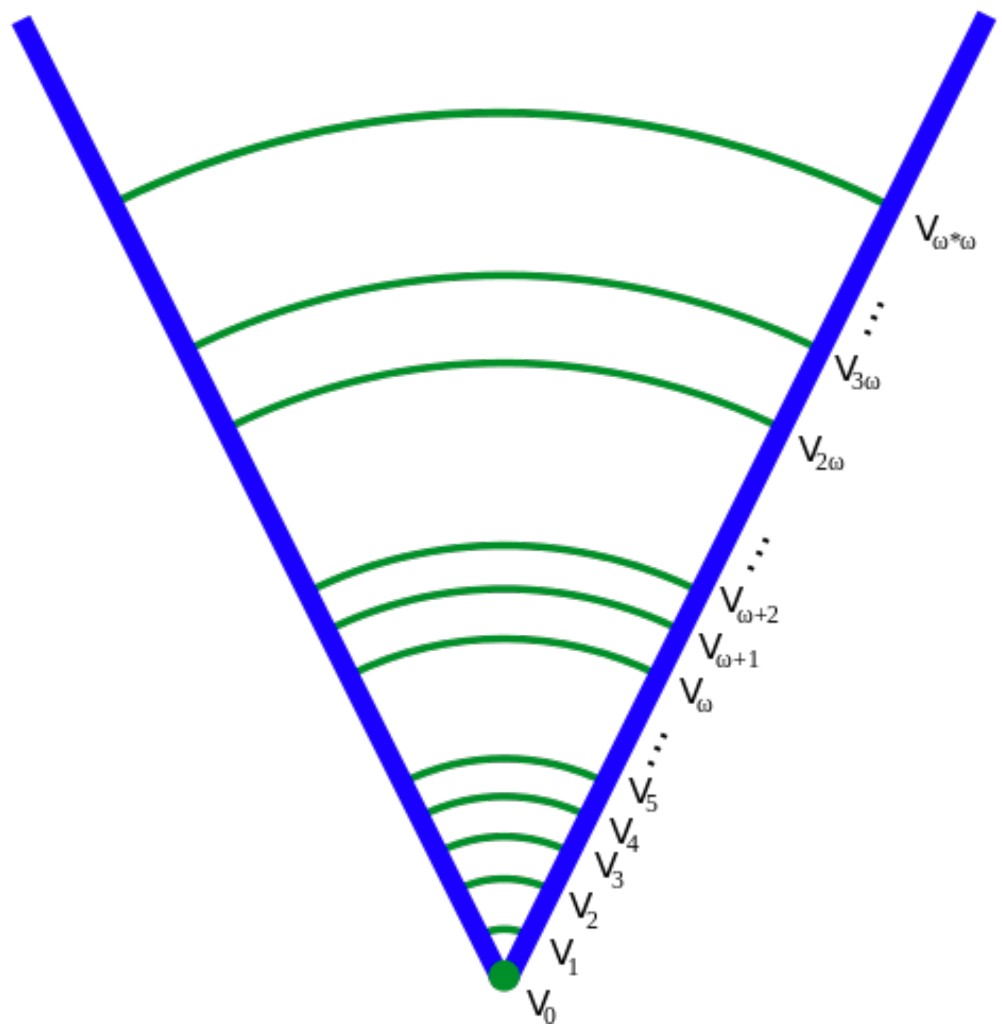

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