Modality in Physics

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# Introduction

This review concerns the notions of physical possibility and necessity as they are informed by contemporary physical theories and the reconstructive explications of past physical theories according to present standards. Its primary goal is twofold: first, to motivate and introduce a range of accessible issues of philosophical relevance around these notions; and second, to provide extensive references to the research literature on them. Although I will have occasion to comment on the direction and shape of this literature, pointing out certain lacunae in argument or scholarly attention, I intend to advance no overriding thesis or point of view, aside from the selection of issues I deem most interesting.

I have grouped these issues into four categories by the domain of physical phenomena to which they pertain, with two or three issues in each, as follows.

1. Those that arise across physical theories of various sorts (sections [1–4):](#_bookmark0)
   1. the modal commitments of accepting a physical theory, especially through the lens of structural realism and anti-realism (section [2);](#_bookmark1)
   2. the notion of physical possibility seemingly invoked in variational principles throughout physical theory (section [3);](#_bookmark2) and
   3. the role of physical possibility and necessity in accounts of symmetry in physics (section [4).](#_bookmark3)
2. Those in thermodynamics and its relation to statistical physics (sections [5–7):](#_bookmark4)
   1. the possibilities allowed or forbidden by attributions of (ir)reversibility to thermodynamic processes, and how these possibilities interface with those allowed or forbidden by statistical mechanics (section [6);](#_bookmark5) and
   2. the sense in which the adiabatic accessibility relation in the Lieb-Yngvasson axiomatic formulation of thermodynamics is a modal accessibility relation (section [7).](#_bookmark6)
3. Those in spacetime theory, especially in general relativity (sections [8–11):](#_bookmark7)
   1. the interpretation and ontology of spacetime events (section [9);](#_bookmark8)
   2. the delineation of the physically possible worlds according to general relativity, viz., the *physically reasonable* spacetimes (section 10); and
   3. definitions and representations of (in)determinism (section [11).](#_bookmark10)
4. Notions of possibility and actuality in the interpretation of quantum theory (sections [12–14),](#_bookmark11) in particular:
   1. in the many-worlds interpretation (section [13)](#_bookmark12) and
   2. in the modal interpretation (section [14).](#_bookmark13)

I have not included any focused discussion of the role of modality in physical law,1 or of modal reasoning with physical theories, for these are covered elsewhere in this handbook. (But, see the discussion in section [2](#_bookmark1) for some allusions.)

I have also not included much discussion of modality in quantum gravity [(Muntean 2015)](#_bookmark94) or in experimental physics. That’s not to say that there are no interesting issues there. For instance, assumptions about what kinds of events are possible in a particle collider such as the Large Hadron Collider guides which data is discarded through hardware filtering before it even reaches analysis—a necessity when too much data is produced to even record (P[erovic](#_bookmark98) [2011).](#_bookmark98) Thus such experiments are laden by the modal assumptions of theories physicists decide to probe [(Karaca 2017).](#_bookmark68) But these modal commitments could be understood through those of theories and models of experiment, data, and data acquisition [(Suppes 1966;](#_bookmark110) [Karaca](#_bookmark69) [2018),](#_bookmark69) and so don’t obviously consistent a separate category. Indeed, the next section discusses issues in the modal commitment of physical theory.

# General Issues: Introduction

Each modern physical theory posits a collection of mathematics models that represent a range of physical phenomena or states of affairs within that theory’s scope. Usually, each model of a theory represents some way or ways for that phenomena or state of affairs to be—its representational capacities [(Fletcher 2018).](#_bookmark56) This observation alone raises several general issues for modality in physical theory.

# General Issues: Physical Theory and Modal Commitment

One quite general question that arises in the interpretation of physical theory is to what modal facts one commits when one commits to such a theory [(Smeenk and Hoefer 2015:](#_bookmark109) §4). For example, what does it mean to be a realist about physical possibilia that are not actual? Ought the realist thus commit themselves? Ought one commit thus to realism?

Constructive empiricists, while recognizing the need for modal reasoning in science (and physics in particular), abjure any realist commitment (v[an Fraassen 1980:](#_bookmark114) Ch. 5.4). Instead, they seek only to use modal talk as a means for commitment about observable phenomena. Some progress on an account of modal reasoning in science (and, again, physics in particular) without presupposing realist commitment has been made [(Muller 2005;](#_bookmark91) [Fletcher 2019),](#_bookmark57) but most prefer to accept some measure of realism.

Whether this belief beyond observables forms a mere modicum or a mound depends on who you ask. Some accept realist commitments to particular models, but not to the relations between models that represent modal structure [(Dieks 2010;](#_bookmark41) [Brading 2011).](#_bookmark21) Others are realists about physical laws, letting their preferred account of how laws ground physical necessity—of which there are a great many—discharge their modal commitments [(Carroll](#_bookmark33) [2016).](#_bookmark33) Still others take modal structure to be real and irreducible, understood as arising from causal powers [(Esfeld 2009;](#_bookmark50) [Esfeld and Lam 2011)](#_bookmark52) or on their own [(Ladyman 1998;](#_bookmark71) [French](#_bookmark59) [and Ladyman](#_bookmark59) [2003;](#_bookmark59) [Berenstain and Ladyman](#_bookmark20) [2012;](#_bookmark20) [French and McKenzie](#_bookmark60) [2012;](#_bookmark60) [Ladyman](#_bookmark74) [2018).](#_bookmark74) Intellectual skirmishes between these different camps have focused on whether careful attention to the features of physical theories themselves and how they are successfully applied rules in favor or against particular metaphysical theses—see, for example, [Ladyman (2000),](#_bookmark72) [Monton and van Fraassen](#_bookmark90) [(2003),](#_bookmark90) [Ladyman](#_bookmark73) [(2004),](#_bookmark73) [French and McKenzie](#_bookmark60) ([2012),](#_bookmark60) and [Esfeld](#_bookmark51) [(2018).](#_bookmark51)

# General Issues: Variational Principles

Most discussions of the modal commitments of a physical theory assume a binary classification of putative models—possible, or not—and that explanations involving that theory use only a single model of the theory representing the phenomena explained. The widespread use of *variational principles* across physics seems to challenge both of these convictions. Roughly speaking, a variational principle is a rule for finding solutions to an equation—that is, models of a theory—that requires some property of the solution to be locally extremized—minimized, or maximized, compared with similar putative solutions. To do so, one considers putative solutions some of whose properties are variations—that is, perturbations—of the target solution and demonstrates that this target is in fact a true solution in virtue of it minimizing or maximizing a certain quantitative property. For example, Fermat’s principle, one of the earliest used in physics, states that a light ray traveling from one point in space to another travels the path that minimized its duration of travel. Modern Hamiltonian and Lagrangian mechanics, and Hamilton-Jacobi theory, also use such principles, focusing instead on extremizing a quantity call the *action* of a solution (Y[ourgrau and Mandelstam](#_bookmark119) [1960;](#_bookmark119) [Rojo and Bloch 2018).](#_bookmark106) They are used throughout classical and quantum physics, including their statistical versions, spacetime theory, and thermodynamics.

The first challenge these principles present to the straightforward view of modal commitment is that when a particular model obtained using them represents some actual physical phenomena, it does so in virtue of the relations that model has to *other* models. In other words, it violates what [Butterfield (2004*b*](#_bookmark32)) calls the (weak) truthmaker principle, that what is actually true should supervene on the actual facts. Although there is extensive debate within metaphysics about the nature of such a principle, the challenge presented here is different in kind from those usually considered, which focus on the logical and metaphysical dimensions of propositions and facts.

The second challenge is that some of these principles consider variations which yield mathematical models that are not *themselves* models of the theory considered. This is in fact a double challenge, and corresponds to what [Butterfield (2003;](#_bookmark30) [2004*b*](#_bookmark32);[*a*](#_bookmark31)) calls the third grade of modal involvement. Its first aspect is a magnification of the first challenge: not only does a model representing phenomena seem to do so in virtue of other models, but ones not even possible according to the theory. How can what’s physically true supervene on what’s physically impossible? The second aspect of this challenge is that because the variational principles quantify over models representing physically impossible phenomena, the principles seem both to commit to their possibility in some sub-physical or metaphysical sense, but not in the physical sense that the theory’s models delineate.

[Butterfield](#_bookmark31) [(2004*a*](#_bookmark31)) argues that one can meet the first challenge first by showing that variational principles of the sort described can be understood as an infinite conjunction of counterfactual conditionals, so that the usual analyses of such conditionals compatible with the truthmaker principle can be applied. Alternatively, one can argue that the principles, though practically useful, are also equivalent to standard differential equations of motion, which make no appeal to counterfactual or counterlegal possibilities. (Cf. section [10’s](#_bookmark9) discussion of modal properties of spacetimes used to define which spacetimes are physically possible.) They are thus dispensable in principle, so that one may take an instrumentalist attitude towards them. This insight, which also applies to the second challenge [(Butterfield](#_bookmark32) [2004*b*](#_bookmark32)), seems to be a more satisfactory resolution of the challenge because it does not hinge on perhaps controversial assumptions about the semantics of counterfactual and counterlegal conditionals and better reflects the practice of mathematical physics, in which it is quite common to solve a problem by the output of an evinced algorithm that takes as input the solution to an entirely *different* problem. But, one might instead *embrace* the different levels of modal commitment in these principles as providing an impetus for new metaphysics (T[erekhovich 2018).](#_bookmark111)

# General Issues: Symmetries

A bijective transformation on a space may preserve certain properties of its elements, and when it does, it is said to be a symmetry on that space of those properties. For example, consider the property of being a model of a particular theory. A bijective transformation that preserves this property is said to be a symmetry of the theory. Similarly, such a transformation that acts as an isomorphism on a model is said to be a symmetry of that model. Symmetries can be discrete, such as time-reversal, or continuous, such as spatial and temporal translation.

There is a voluminous literature on the role of symmetry in physics [(Brading et al.](#_bookmark22) [2017);](#_bookmark22) here I confine the discussion to two related remarks. The first is that having a symmetry is a modal property of a model of a theory: it is defined essentially in terms of its relations with other models. For instance, whether a particular mode of a thermodynamic process is time-reversal invariant depends not just on the process itself, but what other processes are considered possible [(Uffink](#_bookmark112) [2001:](#_bookmark112) §3)—cf. section [6.](#_bookmark5) So, debates about the symmetries of models of physical theories, such as that over the time-reversal, are debates about ineliminably modal content of the theories.

The second remark is that the modal content of a physical theory can play a role in its interpretation, including its ontology, by way of its symmetries. In a word, what is the metaphysical nature of the modal features of symmetries? Should they be understood dispositionally, or in other terms (F[rench 2017;](#_bookmark58) [Livanios 2018)?](#_bookmark78) Some have argued as well that the modal structure of symmetries makes them more fundamental than, hence explanatorily prior to, any conservation laws that hold just in case such a symmetry does [(Lange 2007).](#_bookmark75)

# Thermodynamics: Introduction

Thermodynamics is the branch of physics concerned with heat, the relation of heat’s interchange with mechanical work and other forms of energy, and auxiliary concepts pertaining to these relations, such as temperature and entropy. From its inception, its concerns were as much with practical engineering problems about the efficiency of engines, the generation of electricity, and the construction, design, and maintenance of machines as with scientific laws governing the phenomena in its domain of application [(Uffink 2007).](#_bookmark113) Far from an historical curiosity, this mixed genealogy is both apparent and important in the interpretation of the theory today, including certain of its modal features.

# Thermodynamics: (Ir)reversibility

Thermodynamics put a set of constraints on the possible processes, or sequences of state change, of a thermodynamic system. Perhaps most famous of these is its Second Law. Although, as [Uffink (2001)](#_bookmark112) remarks, there is surprising difficulty stating exactly what the law demands and prohibits, what is relevant here is that most accounts of the law understand part of its content as a statement of, or at least logically entailing, the impossibility of certain processes. For instance, one tentative statement of the Second Law is that the entropy of any isolated thermodynamic system cannot decrease. Thus processes for which the entropy remains the same are also possible in reverse, while processes for which the entropy increases are not also possible in reverse. The former are accordingly called *reversible*, and the latter *irreversible*.

Because thermodynamics is a physical theory, one might just construe this notion of possibility as physical possibility *simpliciter*. The situation is not so straightforward, however, because of the theory’s mixed genealogy adumbrated above. If thermodynamics is a theory born from engineering concerns, to what extent are its claims about what is (im)possible conditioned on seemingly contingent facts about human agents and their abilities to manipulate thermodynamic systems? Answering this question depends on a careful analysis of the scope of classical thermodynamics and its relation to other physical theories with overlapping scope, in particular statistical mechanics.

Classic thought experiments, such as Maxwell’s demon [(Earman and Norton 1998;](#_bookmark48) [1999),](#_bookmark49) may seem to suggest a positive answer. Maxwell envisaged a box of gas partitioned into two, with a small frictionless trap door in the partition operated by an intelligent demon. The demon only lets fast gas molecules through the door from one side, and only slow molecules from the other. Operating in this way, the demon increases the pressure of the gas on the first side, and decreases the pressure of the gas on the second. It is easy to show that, although the system involved is seemingly isolated, this process decreases its total entropy, in violation of the Second Law. Moreover, the classical mechanics on which kinetic theory is based is time-reversal invariant (cf. section [4).](#_bookmark3)

If this analysis is right, then the Second Law marks as impossible processes which are possible in statistical mechanics. Now, from a certain perspective this may not be so surprising: one expects that newer, more fundamental theories supplanting older, less fundamental theories will contradict them on what’s possible. As discussed in section [10,](#_bookmark9) for example, general relativity seemingly allows for spacetime to have “holes” and causal irregularities of various sorts that are impossible according to pre-relativistic spacetime theory. But it does raise the question of how exactly one should understand the type of modal claim made by the Second Law. One answer is thermodynamics does not concern physical possibility per se, but some more limited notion, such as physical possibility conditioned on the physical circumstances within the scope of thermodynamics, or perhaps some notion of *practical* possibility more closely aligned with human (engineering) capabilities.

But if the analysis is wrong, then the physical possibilities that statistical mechanics seemingly entails must be curtailed instead. There has been a huge range of responses along these lines—see [Earman and Norton (1998;](#_bookmark48) [1999)—but](#_bookmark49) some of the most remarkable demand an accounting of the workings of the demon in information-theoretic terms [(Norton](#_bookmark96) [2005;](#_bookmark96) [Maroney 2](#_bookmark87)009). Such accounting would connect physical possibility more tightly with what might have been understood as engineering concerns, namely, constraints on the processing of information. If this is right, it would provide a surprising connection way in which some aspects of physical possibility have precise expression in terms of practical possibility.2

# Thermodynamics: Adiabatic Accessibility

One way to gain traction on the content and scope of thermodynamics is axiomatize the theory. This is not just a pipe dream of philosophers quaintly enamored with logical empiricism, but a project that mathematical physicists take to be worthwhile. To focus on one example most relevant to the topic of modality in physics, [Lieb and Yngvason](#_bookmark77) [(1999)](#_bookmark77) have provided such an axiomatization, for which they received the Levi Conant Prize of the American Mathematical Society in 2002, based on the binary relation of *adiabatic accessibility* between states. The interpretation of “adiabatic” is somewhat non-standard—see [Uffink](#_bookmark112) [(2001:](#_bookmark112) §11) for a discussion—but one can more or less understand it as a relation that holds between two states when the second can be reached from the first without adding any energy (except perhaps that which can be obtained from a weight falling in a uniform gravitational field). Their work is too rich to discuss in any detail here, but one feature worth emphasizing is that, among other properties, the relation is reflexive and transitive, but in general not symmetric. Importantly, the non-symmetry of this relation entails the existence of irreversible processes.3

Beyond questions of the nature of the modality thus invoked (and described in section [6),](#_bookmark5) this theory also suggests an unexplored connection with modal semantics. Recall that a “possible world” model of a modal language consists of a set *W* (of “worlds”), a binary accessibility relation *R* on that set, and a Boolean valuation *v* on worlds and sentences *φ* of the language such that *v* assigns “true” to ◊*φ* at *w* ∈ *W* if and only if for some *wj* ∈ *W* accessible from *w* (i.e., *wRwj*), *v* assigns “true” to *φ* at *wj*.

The states of a thermodynamic system in the theory of [Lieb and Yngvason](#_bookmark77) [(1999)](#_bookmark77) are like *W* and their adiabatic accessibility like *R*, so the corresponding modal operator ◊ could be interpreted as “it is possible to adiabatically transform the system so that …”.4 In particular, adiabatic accessibility satisfies reflexivity and transitivity, yielding the modal logic **S4** [(Burgess](#_bookmark27) [2009).](#_bookmark27)5 It would be interesting to investigate whether this analogy is strong enough to support a semantics for modal reasoning within the theory along the lines suggested by [Muller (2005).](#_bookmark91)

# Spacetime Theory: Introduction

Modern approaches to spacetime theory (e.g., by [Friedman](#_bookmark61) [(1983))](#_bookmark61) represent spacetime and matter using the mathematics of differential geometry, in particular a four-dimensional smooth manifold *M* to which one assigns fields Φ1*, …,* Φ*m*. A four-dimensional smooth manifold is a structured space of points such that the space around each point is a chunk of R4, where R is the real line. Each point represents an event, an occurrence or configuration of matter that has no temporal or spatial extension. Different spacetime theories—e.g., Newtonian, special relativistic, and general relativistic—employ somewhat different types of fields, but they all use fields to give extended collections of points temporal duration and spatial volume, and to represent the presence of matter at events.

# Spacetime Theory: Ontology

What is the ontological status of events, and their relation to matter? Is one somehow prior to, or more fundamental than, the other? The opposed positions of substantivalism and relationalism take, respectively, spacetime and matter to be the one prior and more fundamental. (The names arise from an historical tradition of understanding this question in terms of the appropriate category of existence for space: substance or relation?) Whichever position one takes, one must account for the modal properties of spacetime, and events in particular. Traditional arguments about this topic focusing on whether temporally, spatially, or velocity shifted universes—cf. section [4—constitute](#_bookmark3) distinct possibilities can be couched in modal terms [(Dasgupta 2015](#_bookmark38)). (Discussion of the other modern classic argument bearing on this question, the hole argument, I will postpone for section [11.)](#_bookmark10) But this is so even confining attention just to how a spacetime model represents a physically possible spacetime. For instance, in the models of general relativity, certain smooth curves (the timelike ones) are picked out as the possible histories, or worldlines, of massive particles. (These are the ones that every observer would agree are traveling slower than the speed of light.) Note that

the modal character of the assertions (i.e., the reference to possibility) is essential. It is simply not true … that all images of smooth, timelike curves are, in fact, the worldlines of massive particles. The claim is that, as least so far as the laws of relativity theory are concerned, they could be. [(Malament 2012:](#_bookmark80) 121)

Generally, each model can equally well represent many different states of affairs, where particles are present or not; but one can add distinguished curves to the models to make them less idealized [(Fletcher 2018).](#_bookmark56)

Both substantivalists and relationalists must account for these representational properties, or else modify the theory. Subantivalists generally have an easier time meeting this demand, for they hang the modal properties on the events themselves. Relationalists must decide how they these modal properties accrue to matter. [Sklar (1974)](#_bookmark107) raises the possibility, in the context of articulating a defense against Kant’s incongruent counterparts argument against relationalism, of what has come to be known as modal relation(al)ism, the view that the relationalist should understand the events represented by the spacetime manifold as encoding the possible relations among the possible material bodies. [Brighouse (1999)](#_bookmark25) and [Belot](#_bookmark19) [(2011)](#_bookmark19) explicate and defend this position, especially in the light of technical results by [Manders (1982).](#_bookmark86) Yet objections come from substantivalists and relationalists alike [(Huggett 2006;](#_bookmark66) [Field 1984;](#_bookmark53) [Earman 1989:](#_bookmark43) Ch. 6.12).

# Spacetime Theory: Physically Reasonable Spacetimes

One of the ways that general relativity differs from its predecessor theories of space, time, and gravitation is that the properties of spacetime can differ much more significantly from model to model—that is, from one physically possible universe, or portion of a universe, to another. Some of these differences are extreme enough that physicists declare spacetimes with them “physically unreasonable”—they deny that they are genuinely physically possible after all. If one takes the models of a physical theory to represent the physical possibilities it allows, then this debate concerns the very *content* of the theory. That is, the question of which spacetime properties are physically (un)reasonable *is* the question of what’s physically possible in general relativity.6

Most attention has focused on two broad classes of suspect properties: spacetime holes, and causal irregularities [(Manchak 2011;](#_bookmark81) [2013](#_bookmark82): §4). One type of such hole involves seemingly “missing” events from spacetime. For a simple example, take a spacetime manifold *M* and remove a point from it. All the possible objects and matter that could have coincided at that event no longer can: it represents an “end” or “edge” to spacetime itself. But in this case it seems as though the spacetime “ends” prematurely. Such spacetimes are called *extendable*, and one proposal is that extendability is not a physically reasonable property for a spacetime to have [(Earman](#_bookmark44) [1995:](#_bookmark44) 32). However, general relativity is not just a cosmological theory, but also a theory of more isolated spatiotemporal phenomena or states of affairs that have no pretension to representing an entire world. It’s unclear why extendable models, when considered as more limited states of affairs, are physically unreasonable, although one must be careful about the inferences one draws from such models. Moreover, [Manchak](#_bookmark84) [(2016*b*](#_bookmark84); [2017)](#_bookmark85) points out that some extendable spacetimes only have extensions may have other physically unreasonable properties. This puts different proposals for physically unreasonable properties in tension with one another.

Other types of holes cannot simply be filled in, because there is no consistent way to add events while extending the spacetime model’s fields Φ*i* to those added events. Because those fields involve the very structures that make events spatiotemporal, there is no sense in which any added points could represent the same sort of events as the others. Such spacetimes are *singular*, meaning (roughly) that there are curves representing the worldines of particles that end abruptly, without any means of continuing [(Earman 1995:](#_bookmark44) Ch. 2). Yet some singularities, such as the past singularities of the Big Bang, are deemed by many not to be physically unreasonable. Although debate, proposals, and counterexamples abound, there seems to be no consensus on which sorts of singularities are physically reasonable, and which not [(Manchak 2011;](#_bookmark81) [2013:](#_bookmark82) §4.1).

The other class of suspect properties centers on the existence of *closed causal curves*, which are widely accepted as representing a sort of time travel in general relativity [(Earman](#_bookmark44) [1995:](#_bookmark44) Ch. 6). The worldine of events representing a particle’s history for such a curve is such that the time *line* for the particle is in fact a time *loop*. The claim that all physically reasonable spacetimes contain no such time travel is sometimes known as the chronology protection conjecture [(Hawking](#_bookmark63) [1992).](#_bookmark63) Although physicists have given some evidence and arguments for why time travel is not physically reasonable, like with holes there are also counterarguments and troves of purported counterexamples ([Manchak 2013:](#_bookmark82) §4.2).

These complications have led to some to advocate for a simpler but much farther-reaching condition, such as a version of the so-called Cosmic Censorship Conjecture: “All physically reasonable spacetimes are globally hyperbolic” [(Wald 1984:](#_bookmark115) 304). Globally hyperbolic spacetimes are, in a sense, those spacetimes of general relativity whose properties are most like those of special relativity. (See [Wald](#_bookmark115) [(1984:](#_bookmark115) 201) for a precise definition.) In particular, while they rule out many seemingly physically unreasonable spacetimes, they seem to rule out much more besides [(Manchak 2013).](#_bookmark82)

Others have recently observed that some of the candidate physically (un)reasonable properties are themselves modal, since whether they obtain for a spacetime depends on what other spacetimes are considered physically (un)reasonable [(Manchak](#_bookmark83) [2016*a*](#_bookmark83)). Allowing for such candidate properties seems to imply that one may not be able to give an explicit definition of the physically reasonable spacetimes, as that term appears in the definition of these properties. But some modal spacetime properties are extensionally equivalent to non-modal properties, so an explicit definition could be recovered if one replaced any of the former by the latter. For example, [Hawking (1969)](#_bookmark62) proved that the existence of a global time function in a spacetime is equivalent to stable causality. The former property is the existence of a consistent assignment of times that respects the causal order of events. Stable causality is a modal property of spacetime that holds when the spacetime does not allow for time travel, and the same for small perturbations thereof. But what count as legitimate perturbations themselves depend on what spacetimes are physically reasonable. However, it is unclear why an explicit definition is really necessary. Using modal conditions to delineate the physically (un)reasonable spacetimes is still compatible with them playing a role in an implicit definition thereof.

# Spacetime Theory: (In)determinism

Among the important modal properties that a spacetime model might have is *determinism* [(Earman](#_bookmark45) [2007;](#_bookmark45) [Hoefer](#_bookmark65) [2016:](#_bookmark65) §4). Roughly, a spacetime model is said to satisfy *Laplacian determinism* in a spacetime theory if and only if whenever that model and another agree about the facts represented at a particular time-slice, they agree on all other facts represented. In a word, the state of the world at a time determines, in the theory, the state of the world for all other times. It is a modal property because whether it obtains at a model depends not just on the model itself, but what other models are in the theory considered.

There are many variants on the definition of determinism, which modify what it means for models to agree on the facts and which facts are involved [(Earman](#_bookmark42) [1986:](#_bookmark42) Ch. II). In particular, these variants and the debate surrounding them has been escalated through determinism’s invocation in the influential hole argument [(Belot 1995*a*](#_bookmark17); [Leeds 1995](#_bookmark76)). According to that argument [(Earman and Norton 1987;](#_bookmark47) [Norton 2018),](#_bookmark97) one considers two isometric relativistic spacetimes with the same manifold *M* but different metric fields *g* and *g*˜, each of which assigns temporal duration and spatial volume to collections of events. However, those considered are not entirely different: they in fact assign the same temporal durations and spatial volumes outside of some extended collection (the “hole”) of events *O* ⊂ *M* —in fact, they are identical outside of that collection. Moreover, there is a smooth bijective map *ψ* : *M* → *M* that acts as the identity outside of *O*, and within *O* maps regions of events with certain durations and volumes according to *g* to ones with identical durations and volumes according to *g*˜. Indeed, *ψ* shows that the only differences between *g* and *g*˜ are the identities of the points to which they assign magnitudes, not the magnitudes themselves. A substantivalisist—cf. section [9—seemingly](#_bookmark8) must maintain that spacetime model with *g* is distinct from the spacetime model with *g*˜ because they assign different such magnitudes to collections of points in *O*. Yet the facts of both models are compatible with the facts on any time-slice outside of *O*. Thus, the argument concludes, the manifold substantivalist is committed to indeterminism for metaphysical, rather than physical, reasons.

One influential line of response to this argument has focused on the definition of determinism, insisting that, for spacetime models to “agree” on the facts, it is not necessary for them to be identical, but only necessary that they be isomorphic, i.e., that there exists a smooth bijection *ψ* : *M* → *M* of the sort described above [(Butterfield 1988;](#_bookmark28) [1989;](#_bookmark29) [Brighouse](#_bookmark23) [1994).](#_bookmark23) This led to an edifying exchange of ideas in the 1990s about the nature of determinism and whether this response was sufficient to defuse the tensions of the hole argument [(Belot 1995*b*](#_bookmark18); [Brighouse 1997;](#_bookmark24) [Melia 1999),](#_bookmark88) with some later skeptical re-evaluations [(Brighouse](#_bookmark26) [2008).](#_bookmark26) Thus the modal properties of spacetime are more relevant for spacetime ontology than they might at first seem.

For some, this suggests that a different structure would be appropriate to capture indeterminism in a single model, such as branching space-times [(Placek et al.](#_bookmark105) [2014;](#_bookmark105) [Mu¨ller and](#_bookmark89) [Placek 2018;](#_bookmark89) [Placek 2019).](#_bookmark103) In such models, one captures the idea of an “open” future—i.e., future-indeterminism with past-determinism—by having the manifold *M* literally branch into alternatives. [Belnap (1992;](#_bookmark14) [2012)](#_bookmark16) gives a version of this for Newtonian spacetimes, motivated in part also from the seemingly indeterministic and non-local outcomes of quantum experiments—see also [Kowalski and Placek](#_bookmark70) [(1999),](#_bookmark70) [Placek](#_bookmark99) [(2000),](#_bookmark99) [Belnap](#_bookmark15) [(2002),](#_bookmark15) [Mu¨ller](#_bookmark92) [(2002),](#_bookmark92) and [Placek](#_bookmark100) [(2002).](#_bookmark100) Since then, versions have been constructed for special relativity [(Wron´ski and Placek 2009;](#_bookmark118) [Placek 2010;](#_bookmark101) [Mu¨ller 2013)](#_bookmark93) and general relativity [(Placek 2014).](#_bookmark102) Part of the significance of this program, revealed in the face of criticism (e.g., by [Earman](#_bookmark46) [(2008)),](#_bookmark46) is formalization of the alternative “Aristotelian” concept of indeterminism involving an open future [(Placek and Belnap 2012).](#_bookmark104)

# Interpretations of Quantum Theory: Introduction

Quantum theory deserves a more extended development of its basic concepts than I can provide here, where I only recount the basic ideas relevant for the issues in modality discussed in sections [13](#_bookmark12) and [14.](#_bookmark13) (For a brief introduction to the formalism of quantum mechanics, with many suggestions for further study, see [Ismael (2015).)](#_bookmark67) Recall, then, that in elementary quantum mechanics, the state space of a physical system is represented by a vector in a Hilbert space, and an observable for that system is represented by a Hermitian operator. If the system is not being measured, its time evolution is described by a smooth, one-parameter family of unitary operators—rotations in the Hilbert space, essentially—acting on its initial state. But if an observable is measured, the state projects, generally discontinuously and stochastically, onto one of the eigenstates of the observable whose eigenvalue represents the outcome of the measurement, with probability given by the Born rule. The inclusion of this “projection postulate” provides both an explicit mechanism for how experiments with quantum systems seem to yield definite outcomes, but also a conceptual puzzle: how are measurements supposed to be different from any other physical process? An interpretation of quantum theory, besides giving a description of what the world would be like if the theory were true, also must solve this so-called “measurement problem” [(Myrvold 2018:](#_bookmark95) §4). Some of these interpretations in particular involve explicit invocations of modality.

Interpretations of quantum mechanics in which modality plays an important role include some versions of the many-worlds interpretation and, unsurprisingly, the modal interpretation. Both of these interpretations propose to *eliminate* the projection postulate, so that measurement can be understood not as discontinuous changes of the state of the measured system, but as ordinary dynamical evolution of that system interacting with the measurement device. However, the way they do and their consequent connections with modality are different.

# Many-Worlds Interpretations of Quantum Theory

The many-worlds interpretation attempts to explain how, without adding any extra formal ingredients, one can recover the use of probability, the semblance of definiteness to the outcomes of experiments and, indeed, human experience itself. This is needed because if a system were to interact with a measurement device according to the standard (non-measurement) dynamics, their states would become entangled, in general consisting of a *sum* of terms in each of which the system is in some eigenstate of the measurement operator and the measurement device is in a state indicating as such. Apparently, the state has “split” into many worlds, each in which there a measurement device with a definite outcome for the experiment.

What then is the status of these splitting worlds? [Wilson (2013;](#_bookmark116) [2015)](#_bookmark117) has recently suggested that *each* of them is in fact a metaphysically possible world, not just some “part” of one metaphysically possible world in which the system and measurement device are entangled. This in turn has implications for the understanding of objective probability in the theory. See also [Conroy (2018)](#_bookmark34) for an actualist alternative and [Skyrms (1976)](#_bookmark108) for skepticism about relating the many-worlds interpretation to modal metaphysics.

# Modal Interpretations of Quantum Theory

In contrast with many-worlds interpretations, modal interpretations *add* structure to quantum theory after removing the projection postulate. In particular, they interpret the usual quantum state as purely modal—it describes merely what properties are *possible* for the system—and add a second state that represent what properties are *actual* for the system. This is a list of the actual properties of the system at a time and so is in general distinct from the dynamical quantum state. The structure of quantum theory prevents this list from being as comprehensive as it would be for systems described classically [(de Ronde et al.](#_bookmark40) [2014).](#_bookmark40) There are many different proposal for what these actual properties should be, how they should be determined, and whether one’s attitude should be empiricist [(Dieks](#_bookmark41) [2010)](#_bookmark41) or realist ([de Ronde 2010)](#_bookmark39) towards the purely modal quantum state, but they all share the aforementioned basic structure [(Lombardi and Dieks 2017).](#_bookmark79)

Like with the adiabatic accessibility relation in axiomatic thermodynamics discussed in section [7,](#_bookmark6) one can interpret the standard quantum dynamics as providing a kind of accessibility relation between possible states, albeit one of a much more complicated structure, since the accessibility relations (i.e., the quantum states) are not determined by the actual state. It would be interesting to develop a sort of dynamic modal logic to capture this.7

# Notes

1 This includes the somewhat subtle question of to what extent the laws themselves are necessary [(Darrigol](#_bookmark37) [2014).](#_bookmark37)

2 [Uffink (2001:](#_bookmark112) §3) considers a third notion of possibility according to which “the claim that such a process [proscribed by the Second Law] is impossible, becomes a statement that transcends theoretical boundaries.” But it isn’t clear to me why this isn’t just the notion of physical possibility haughtily immunized against disconfirmation. It is hard to see any basis for a claim that the Second Law is an analytic or a priori truth.

3 [Uffink](#_bookmark112) [(2001)](#_bookmark112) also claims that, despite this, their formulation of the Second Law is compatible with time-reversal invariance, although [Henderson (2014)](#_bookmark64) has more recently disputed this.

4 As states are not worlds in general but small fragments or parts of worlds, this perhaps brings the analogy closer to truthmaker semantics [(Fine 2017).](#_bookmark54)

5 For “simple” systems, [Lieb and Yngvason](#_bookmark77) [(1999)](#_bookmark77) also demand that they satisfy the “Comparability Hypothesis,” namely that the adiabatic accessibility relation is total. For such systems, this would yield the modal logic **S4.3**. Cf. the comments of [Earman](#_bookmark42) [(1986:](#_bookmark42) 99–100) regarding the modal strength of physical necessity.

6 This analysis distinguishes being “physically reasonable,” as a physical-modal attribute, from being “physically significant,” as an epistemic-modal attribute [(Fletcher 2016).](#_bookmark55) Some of the exegetical puzzles for the physics literature on these issues may arise from conflating the two—cf. [Earman (1995:](#_bookmark44) 80).

7 There is already a modal interpretation of quantum logic [(Dalla Chiara 1977;](#_bookmark35) [Dalla Chiara and Giuntini](#_bookmark36) [2002),](#_bookmark36) but the proposal here is for a *dynamic* logic that fits with the modified structure of modal interpretations of quantum theory.

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