A conceptual frame for giving a physical content to the uncertainty principle and the quantum state

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Abstract

We have defined a physical event as an invariant replaced structural form, possessing a finite space-time region which emerges as a unification of the uncertainty principle and the quantum state. To achieve this concept, we have first unearthed and then criticized two related conjectures relying on the foundation of the current physics which are that: (1) there is an operational-based prior space-time frame in which physical events evolve continuously; and (2) the events are understood based on a pure two-valued logic of rational-reason. We have concluded that the presented conceptual frame supports (i) a time operator having a domain $(0, 2\pi)$. In addition, it implies that: (ii) the dynamical variables of a physical event are the functional faculties as the content of the structural form of the event; and (iii) every physical event needs a finite time for realization.

Keywords: Continuous kinematical and dynamical descriptions, Quantum state, Uncertainty principle, Physical event, Structural form, Rational reason

1. Introduction

Monumental improvements in physics have usually resulted from a philosophical perspective on its foundation. Interpreting the motion of physical systems in phase-space and the idea of (the principle of) action has given rise to analytical mechanics. A strict and robust examination of both measurement and observation led A. Einstein to a new comprehension of space-time, and thus, to the theory of special relativity; and in contrast to the prevalent view at that time, assuming the disintegration of energy discreetly led M. Planck to reach his formula of black body radiation, which is also the origin of quantum theory.

. In a similar manner, after W. Heisenberg and E. Schrödinger published their works [1, 2, 3] on the dynamics of quantum systems, a debate about the physical content of the Schrödinger wave function arose among physicists, which again had a philosophical character and still maintains its vitality. Also related to this debate, many studies have been dedicated to giving a reasonable and satisfactory foundation to quantum mechanics, especially to its fundamental features, such as the quantum state of a system, the measurement process and the uncertainty principle [4]. When the quantum information field emerged, these issues gained a vital importance; in particular, the dynamic of the measurement process and the entanglement of (the quantum state of) physical systems became cornerstones for the computations in quantum circuits and quantum information respectively. Being directly related with the measurement process of obtaining information through a quantum circuit, the uncertainty principle has again attracted attention to itself in the context of error-disturbance [5, 6] and entropic measurement [7, 8, 9]. In addition, the compatibility of the quantum-state assignments of parties and the entanglement problem have highlighted the physical content of the quantum state as a problem about the foundation of quantum information [10, 11, 12].

. In the literature, since the works on the uncertainty principle have been positioned from an operational (definition) perspective [13], they have unavoidably given rise to either a measurement based [1, 5, 6] or a statistical interpretation of it [14, 15, 16]. *Over time, as a result of these attempts, the dynamical relation of the uncertainty principle with the quantum state has been neglected; thus,leaving and understanding of the physical content to the quantum state as an unresolved issue.* Even if the quantum state has taken a wider content in the scope of quantum information and computation theory, it is still devoid of a consistent physical content such that it is considered either as having a physical reality [17, 18] or as merely a tool showing our lack of information about the system [19]; other works have not taken the dynamics of the uncertainty principle into account [20, 21, 22]. Consequently, a consensus has not been reached on the physical content of either the quantum state or the uncertainty principle [13]. More importantly, what is missing in these studies is that the quantum state and the uncertainty principle have not been considered together. In other words, the fact that both quantum state and uncertainty principle exist within a physical event has been overlooked. Indeed, a consideration of these two phenomena together is possible only if we base them on the concept of a physical event. Therefore, as the problem of this paper, we will interpret them together based on the concept of a physical event to obtain a physical content of them.

. In the present paper, as our main motivation, a conceptual frame of a physical event is constructed which connects the quantum state and the uncertainty principle physically and unifies them in a consistent interpretation. To achieve this aim, Sec.2 first unearths and then criticizes two inveterate conjectures underlying the controversial arguments about both quantum state and uncertainty principle. One of the conjectures is that "*there is an operational-based prior space-time frame in which events evolve continuously*", and the other, related with the former, is that "*events are understood based on a pure two-valued logic of rational-reason*". After constructing this conceptual frame, Sec.3 presents the physical content of the quantum state and the uncertainty principle as connected. Sec.4 summarizes the results and discusses related subjects in the scope of the new conceptual frame, such as the time operator and the realization of a physical event.

2. Two conjectures relying on the foundation of physics

A complete and satisfactory description of the state of a physical event, in the classical vein, should fulfill the demanded information of what exactly happens at any point in space at any instant of time [23]. This very exhaustive statement implies a continuous description of the physical event into the space-time domain (or frame) covered by its evolution. At the same time, such continuity reveals a continuous causal description of that event in terms of the related differential equation in such a way that the future-state of the event in space-time is determined by a past-domain of space-time. Through this causal description, whenever one asks where and when the event positions, which is the demand of its realization, it is said that it is exactly there-place in space and there-instant in time which can be called a kinematical realization. In addition, this demand requires a continuous space-time realization; that is, the realization of the event must be well-defined over a continuous pattern. Hence, the realization is continuous and kinematical. Whenever one asks what that event is, from its continuous kinematical realization, the given answer is that it is, as an aggregation of matter, right there in space-time without giving any reference to its other space-time region. Furthermore, in the first place, if this aggregation of matter has existed in an interval of time, it is also existent in an arbitrary portion of this interval. Thus, we can say that the wholeness of a speck of matter is immune to the division of time. In the second place, an arbitrary portion of that aggregation of matter shows exactly the same dynamical and existential property as its whole, which means an arbitrary division of the volume, so space does not affect the characteristic property of matter [24]. Hence, the lapse of time flows per se and space is a kind of locus for matter. This affair of matter against space-time is indeed an implication of its continuous and kinematical realization. Summarizing this description, we want to rephrase it in two suppositions for the sake of clarity: a continuous kinematical description of an event in nature presupposes (i) *a prior existent of space-time*; and (ii) *a physical event is an amount of matter which takes place exactly right there in space-time.* One can straightforwardly infer from this perspective that an aggregation of matter, or a physical event settles in principle into the space-time frame independently from the dynamics of that frame. Therefore, one always explains the dynamics of a physical event necessarily by the changes in its position in the flow of time; thus the continuous kinematical description becomes the foundation of the dynamics of the event. Another, but the most important, result is that one cannot construct from this description the inductive or causal character of the dynamics of physical events due to both not referring to another region of space-time and the immunity of the events to the division of space-time. Thus, we say that the dynamics of a physical event is conceptually unconnected to its continuous kinematical description. Indeed, if one contemplates in light of this type of realization and describes a physical event at every point of space-time without referring to both its past and its future, the event stays alone on its own right into that space-time point, and thus its dynamics is fully uncertain. On the other hand, there is a rigid causal character of the dynamics in the language of the continuous path equation of the event. We indeed ascribe a mere continuous dynamics to the event by its path function such that, at every point of space-time, it has a well-defined momentum which is continuous over the path. In addition, the explanation of a physical event in the context of its dynamics is given in terms of its space-time coordinates and interaction with the environment, which is again given in term of space-time coordinates. Therefore, the description of the event from the perspective of its dynamics is constructed on the basis of continuous space-time coordinates. As a result, regarding the observer's measurement of space-time coordinates, the kinematics of the event is conceptually uncertain in respect to continuous dynamics of the event, because it is impossible for the observer to say that the event is right there in space-time so long as it is evolving continuously. It is surprisingly seen that there is a mutually exclusive relation between the continuous kinematical description and the continuous dynamical description. Throughout this classical description, on one side, there is motion or the field of the interaction as an event, and on the other side, there is space-time as a stage. However, when relativity theory is taken into account, the description must be changed in some manner. According to (special or general) relativity theory, space-time has a relational character and takes its shape according to the distribution of matter through it. It is essential to emphasize that the revolution from the absolute to the relational conception of space-time is a consequence of *the operational definition* [25]. Another remarkable implication of the theory is that the matter is not an additional something, say a substance, existing in space-time. Indeed, as can be seen from the definition of the differential interval of two space-time events, $ds^2 = g_{uv}dx^u dx^v$, the metric tensor of the gravitational field underwrites the conception of both length and time-interval of a physical event. Therefore, by taking into account the operational definition of space-time, one can conclude that matter does not have a meaning in its own right without the geometrical structure of space-time. It can be thus inferred that neither matter nor space-time has a privileged and per se meaning . From this fact one can also confidently jump to the characteristic relation between matter and space-time, which is that extension is the essence of both matter and space-time. We now ask whether or not relativity theory changes the continuous kinematical (and dynamical) description. From the perspective of relativity theory, we leave the notion of absolute space-time and any kind of privileged relationship between matter and space-time. However, it would be deception if one thought that *the notion of "a prior existence of space-time" is accrued from the notion of "absolute space-time"* because the issue is not if there is a prior existence of space-time on its own right, but how we take it into account of the description of a physical event. Therefore, whenever one determines the event in light of the operational definition, one always observes it right there in space-time from one's reference frame. This precisely explains why we describe the dynamics of the event in terms of space-time coordinates. In addition to this, since we hold the notion of matter to be immune to an arbitrary division of space-time in the context of relativity theory, it is evident to say that the notion of an operational-based prior existence of space-time remains valid. In conclusion, we want to point out that relativity theory has not changed the conceptual description given above except for the fact that the geometrical character of space-time is connected with the state of matter. It is thus important to grasp how a physical event seems through the windows of these two classical theories. In this case, we define a physical event in a way that:

(i) it is of an aggregation of matter which is meaningful in term of space-time coordinates, giving rise to a continuous dynamics of these coordinates; and

(ii) it is immune to an arbitrary division of space-time coordinates, giving rise to a continuous kinematical description.

In this picture, a physical event is a passive geometrical structure which reacts to its environment through space-time and curiously enough, this point of view has been retained in the foundations of quantum theory. Nevertheless, the mutually exclusive character of the kinematics and dynamics have been revealed in quantum theory and both Bohr's complementary argument and the uncertainty principle have been considered as the declaration of that mutual exclusivity (see, e.g., [26, 27]). However, the truth is that both Heisenberg and Bohr had made their comments on the foundation of quantum theory relying upon the classical definition of an event just given above. While Bohr had reached his complementary principle, the operational definition had led Heisenberg to remove the notion of orbital from atom theory and to reach a statistical interpretation of the uncertainty principle [1].
Another conjecture underlying the foundation of physics is that events in nature can be interpreted, and thus understood, based on a mere two-valued logic of rational reason, which will be abbreviated hereafter as *the fallacy of rational reason.* What we mean here deliberately by term "*rational reason*" is that:

(i) it is based on the three fundamental laws of logic which are the principle of identity, $((\forall x)(x = x))$, the law of contradiction, $(\neg(p \wedge \neg p))$ and the law of excluded middle, $(p \vee \neg p)$; and

(ii) the transitive property of an equivalence relation $\mathbf{R}$ is always valid, i.e., if $p\mathbf{R}q$ and $q\mathbf{R}r$ then $p\mathbf{R}r$ [28].

We want to rephrase the principle of identity as *the reflexive property of the events*, law of contradiction as *the impermeable notion of the events* and the transitive property as *the ratio-nal relation of the events* which is the cause of why we use rational reason. The lack of time in the laws and transitive property serves as the main property of rational reason, which is that the elements of rational reason have solid distinctive notions, i.e., they conserve their contents during any logical process. According to the continuous kinematical description of physical events, an aggregation of matter is right there and only right there with immunity to the division of space-time and without referring to any other region of space-time. These properties of events provide the rational reason for taking them as solid geometrical structures existing per se at every point of space-time. For the sake of concreteness, we can see, for example, how such a perception manifests itself on the interpretations of the well-known double-slit experiment. Firstly, one slit is open and a pattern of electrons is detected on a screen in terms of intensity, say I_1. Next, the other, but not the first, slit is open and the same detection for electrons is recorded as I_2. In the third case, we want to predict the pattern of the electrons on the screen again in terms of intensity if both slits are open. According to the conceptualization of rational reason, electrons are an aggregation of matter residing in space-time, or better said, are passive geometrical structures enclosed by a finite volume of space flowing through time, and they conserve their (external passive geometrical) character during their evolution through space-time. Therefore, in the third scenario, the intensity of the resultant pattern should be a ratio-nal accumulation of those of the previous scenarios: $I_3 = I_1 + I_2$. In other words, the constituents (I_1 and I_2) already existed in accordance with the continuous kinematical description, before the existence of the whole (I_3), and they constitute the whole as a synthesis, keeping their physical character. It is crucial to understand how rational reason achieves this result: (1) according to the reflexive property, electrons are the same electrons during their evolution; and (2) taking the continuous kinematical description into account, they satisfy the distinctive notions of events; and thereby, consequently, (3) they obey the law of the excluded middle. Under these conditions, it is straightforward to suppose that:

proposition-1. if the electrons-α pass through the first slit and hit the P-region of the screen, and

proposition-2. if the electrons-β pass through the second slit and hit the P-region of the screen then

result: the intensity $I_3(P)$ at the P-region is electrons-α + electrons-β,

which is a direct implication of *the rational relation of events*. However, the double-slit experiment tells us that this implication is not true. The revision we make in favor of the experiment is to abandon the notion of (point) particle of electrons and assign them wave-property to explain the result in case of opening both slits. But when we improve the experiment such that the electrons are released from the source one by one, we detect each time the electron as a speck on the screen, from which we surely understand that an electron is not a kind of ubiquitous wave just like a wave propagating through the water. As is seen from this example, the concepts of both the continuous kinematical description and the fallacy of rational reason usually intertwine in the explanation of any physical event, and thus result in a duality about the nature of physical events like the wave-particle duality in the double-slit experiment.

Curiously enough, the authors of the EPR argument [29] also grounded their physical reality criterion upon continuous kinematical description when they criticized the completeness of quantum theory. In their paper, the authors stated the physical reality criterion as such [29]: "*if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to one) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity* (pp. 777)". After the authors had stated their first inference, they claimed to *show that the assumption of the completeness of* Ψ*-function, together with the criterion, leads to contradiction.* For this purpose, they have first assumed two systems, I and II, which interact with each other for an interval of time and whose states have been known before interaction. With the aid of the Schrödinger equation, they have presented two wave functions of the composite system according to two observables, A and B, of the first subsystem respectively as the following:

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2)\mu_n(x_1)$$
$$\Psi(x_1, x_2) = \sum_{s=1}^{\infty} \varphi_s(x_2)\nu_s(x_1),$$

where x_1 and x_2 stand for the variables describing the subsystems, and $\mu_n(x_1)$ is the eigenfunction of A with eigenvalue a_n and $\nu_s(x_1)$ of B with eigenvalue b_s. One can take the observable A as the momentum and B as the position of the first subsystem. Now, according to the wave packet reduction, if the observable A is measured and is detected with the value a_m then, after measurement, the first system is left in the state $\mu_m(x_1)$ and the second system in the sate $\psi_m(x_2)$. On the other hand, if the observable B is measured resulting with the value b_r, then this time the first system is realized in the state $\nu_r(x_1)$ and the second one in the sate $\varphi_r(x_2)$. Bearing in mind that $\psi_m(x_2)$ is the eigenfunction of the momentum operator and $\varphi_r(x_2)$ of the position operator of the second system, the authors have made the crucial comment [29]:

> "*As a consequence of two measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system. This is, of course, merely a statement of what is meant by the absence of an interaction between the two systems. Thus,* ***it is possible to assign two different wave functions*** *(in our example ψ_m and φ_r)* ***to the same reality*** *(the second system after the interaction with the first)*" (pp. 778) [boldfaced belonging to us].

We can now give the EPR argument, which is that: (1) having started with the completeness of Ψ-function, and (2) according to the physical reality criterion, and since (3) (the authors have inferred that) ψ_m and φ_r represent the same reality, then one can predict both the momentum and position of the second system with certainty, and thus, they are real simultaneously. This would seem to be an appropriate place to shed light on how continuous kinematical description has been used in this conclusion. First of all, the physical reality criterion states that a physical quantity is real and always real if it is predicted with certainty. This is indeed nothing but only the expression of the continuous kinematical (or dynamical) description of the physical quantities. What is more, the two different wave functions, that is ψ_m and φ_r, have been assigned to the same reality with the aid of the operational definition, which again relies on the continuous kinematical description as stated earlier. In the first place, when the observable A is measured and consequently the second system reduces to the state ψ_m, it must be assumed that the momentum of the second system is real and always real. In the second case, when the observable B is measured and thus the second system falls into the state φ_r, similarly the position of the second system must be regarded as real and always real. This is the background of how the two different eigenfunctions become representations of the same reality. It is also the ground of locality, which entails that every interaction must be local due to the assumption that every physical entity is some aggregation of matter having a geometrical structure. We want to emphasize the fact that the two different eigenfunctions belong to the same reality is not only a direct consequence of the non-disturbing measurement but also the continuous kinematical description, because there is no way to conclude that, after measurements, both the eigenfunctions represent the same reality without accepting the continuous kinematical (and dynamical) description. However, there are, for example, entangled systems disproving the EPR argument [30]. This is also evidence against the ground of the physical reality criterion, i.e., the concept of the continuous kinematical description, and thereby the generality of locality.

2.1. The criticisms of the conjectures and restatement of the physical event

The two conjectures examined above come unavoidable into any description of a physical event because our minds acquire them naturally throughout our experiences over time. However, as it has been shown above, any description of a physical event relying on these conjectures brings us to a duality within the nature of the event even though there is no such duality in the event itself. If indeed there had been, then there would not have been, on one hand, motion in the view of the kinematical description and realization of events in the view of the dynamical description on the other hand. After all, *we observe, for example, the emission spectrum of atoms many times and each time as the same; and we detect the behavior of an electron, for example, in an electromagnetic field many times and each time, again, as the same. In addition to these facts, there are also some (side of) events that did not existed before, but exist now for a time, and will disappear a while later(e.g. the position of events, the colour of events, etc.). This progress is actually the necessarily condition for talking about the existent of a physical event. Therefore, there are factual (and some permanent) events in nature surviving throughout the physical processes, i.e., with respect to their dynamics. In this case, we have to remove the continuous kinematical description from the ground of the expression of a physical event in favor of these factual and permanent events.* The main characteristic property of this description is the immunity of a physical event to the division of space-time. We should abandon this supposition and state that: *it is impossible to preserve the notion of* *the factual and permanent events against the arbitrary division of space-time.* The other characteristic property is that a physical event is an aggregation of matter having meaning in terms of space-time. This supposition forces us to conclude that a physical event is nothing but an amount of matter characterized by space-time coordinates. *If one goes a step further, the event seems to be a local entity having a nonfunctional (or passive) geometry. It was this supposition which allowed us to reach the rational accumulation of the intensities in the double-slit experiment scenario.* Giving wave character to the electrons is nothing but disproving this supposition because whenever we append the wave character to the electrons, we destroy at the same time their nonfunctional geometrical structure, and thereby they gain a functional geometrical structure, ensuring destructive and constructive inference when they come into interaction. Therefore, we must also leave this supposition and should state that: **a physical event is a structural form**. The term "structural form" used here signifies a shape whose geometry is both determined and organized by the structural (or dynamical and physical) properties of the event; thus, it is the source of the external geometrical shape. Every physical event has this structural form, which is needed for the objective reality of the events. Otherwise, one will have to ascribe physical properties of a physical event to its interaction with its surroundings, which means the rejection of the objective reality. For example, according to orthodox interpretation of quantum theory, experiments demonstrate whether electrons behave like particles or waves. This idea implies that the physical properties of electrons depend on their interaction with their surroundings.

We want to emphasize the fact that a physical event is neither a pure geometrical shape nor a mere accumulation of matter and, by keeping its structural form, its external geometry is identified by its structural properties which are functional. Due to their functionality, we call the structural properties *the functional faculties.* Now, if one unifies the two statements above, a physical event obtains a definition with respect to its kinematics, which is that a physical event is a structural form possessing a finite region of the space-time. However, this definition is still deficient because it is a kind of frozen snapshot which does not include the dynamics of the event. If there is no dynamics, there is no event, and hence no the perception of the event because the dynamics is the source of the functional faculties of the event. For instance, it is the dynamics giving existence, and thus meaning, to the wave property of electrons. To make the dynamics inherent in the nature of the event, we must give reference to the past and the future of the event.

This revision then implies that *a physical event is not an amount of matter taking place exactly right there in space-time.* Once the event includes the references to its past and its future, we confront something replaced, that is, there is something repeating somehow. We do not here synthesize the kinematical and the dynamical properties of the event; they are already inherent in the event. Furthermore, they are not separated in the nature of the event itself such that one has no reality without the reality of the other. Therefore, replacement is the existence condition of a physical event. In other words, replacement is analytic to the nature of a physical event. We can now ask whether space-time is a physical event or not. *It is clear that space-time is not a physical event because neither space nor time has functional faculties per se, and they are not observable as bare fact. They are always ascribed to physical events for explanation. If this is so, do we destroy the unprivileged relationship between space-time and matter that was stated before? The absence of a priority between space-time and matter is stated with respect to the observer, and therefore, it has meaning from the perspective of the operational definition.* However, here we are not observing a physical event but trying to give it the conception best suited to its concreteness, kinematics and dynamics. From the perspective of the observer, there is a realized event and this event appears to the observer as having a finite region of space-time. Therefore, from the view of the operational definition, there is still no privileged relationship between the (realized) event and space-time. Thereby, we conclude that space-time is the mode (or condition) of our observation, not the condition in which the physical event survives.

According to this revised description of the physical event, can we still preserve the character of the rational reason? Since we put away the view of a physical event being a passive geometrical structure, we cannot preserve the impermeable notion of the event, that is, we cannot apply the law of contradiction any more in general. Whenever, for example, an electron interacts with another one, the electron does not preserve its geometrical structure during the interaction due to interference. This is why the linear summation of intensities is not valid but that of the probability amplitudes is valid. Therefore, the law of contradiction should be restricted, at least in the scale of elementary particles. In addition, as pointed out earlier, the lack of time in the propositions of logic is crucial in the logical process of the rational reason. It is the lack of time, for example, which allows us to think that, if the momentum of a physical event is real by predicting with certainty then it is always real (in the lapse of time). Thus, in order to stay consistent with the nature of physical events, we should state the propositions about the events as the function of time, if, for example, they are related to the dynamics of the events. Finally, the current rational reason is based on the continuous kinematical description and does not include the dynamical character of the events. However, the kinematics of physical events has no meaning without their dynamics, as stated above. Therefore, a revision of the current basis of rational reason is naturally needed. For such a purpose, new logical systems have been offered by taking the dynamical properties of the events into account [31, 32].

3. The conceptual frame for the foundation of the quantum theory

Our main concept is the physical event, which has been derived in the criticisms of the conjectures such that: *it is a replaced structural form possessing a finite region of space-time*. Without losing this definition, one can now stand as an observer inside the event and seek how a physical event appears. First of all, from the reference frame of the observer outside the event, observing a physical event means that the structural form is a realization having a finite space-time. Therefore, we, as an observer outside the event, always observe the structural form which has become realized. Secondly, taking into account the replacement character of the structural form, and if we stay with the realized structural form as an observer inside the event, we recognize that:

(i) A physical event is the result of the realization of a structural form having dynamical properties which are about realizing itself. We call this aspect not-yet realized (in short, NY-Realized), referring to the past of the event.

(ii) At the same time, a physical event is the source of the subsequent realization of the structural form having now kinematical properties. We call this aspect not-anymore realized (in short, NA-Realized), referring to the future of the event.

This is the general conceptual frame of the physical event, and we have depicted it in Fig.1. However, this concept still does not fulfill the requirement of those events being subject to physical science, because the investigation of a physical event in the context of physics is possible only if some invariant(s) is (are) inherent in its dynamical nature such as the conservation of energy

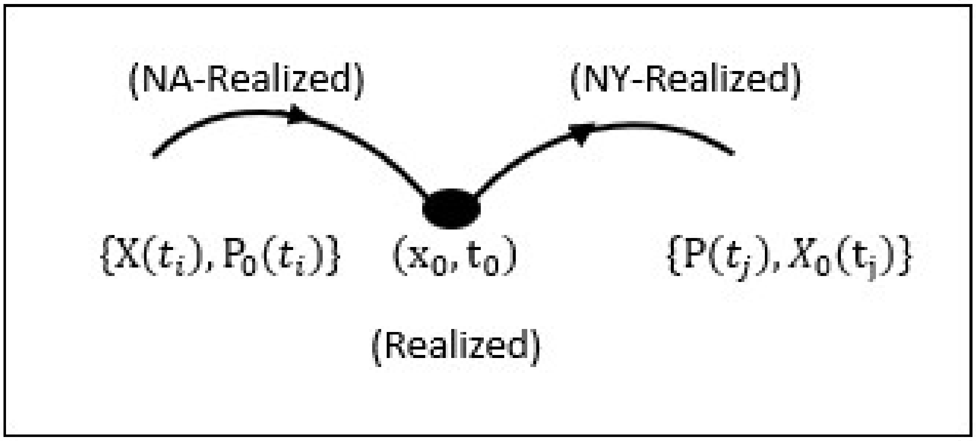

Figure 1: In this figure, we take place on the realized structural form of the physical event which has the region of space-time (x_0, t_0), and we try to perceive the event without losing its given definition. In this case, we recognize, for example, a moving physical object such that it is the realized structural form with both definite momentum variable and unspecified position variable (*NY-Realized aspect*), and at the same time it is the source of subsequent realization of the structural form with both definite position variable and unspecified momentum variable (*NA-Realized aspect*). We attribute to NY-Realized aspect the pair of the dynamical variables $\{X(t_i), P_0(t_i)\}$ in order, and to NA-Realized aspect $\{P(t_j), X_0(t_j)\}$ in order. The dynamical variables X and P correspond to the position variable and the momentum variable, respectively.

and momentum. Hence, if we restrict the physical events to those of physics, an exhaustive and comprehensive definition of a physical event can be given as follows:

Definition 1. *A physical event is an invariant replaced structural form, possessing a finite region (or extension) of space-time.*

The picture given in Fig.1 can be considered also for this definition. Ignoring the time dynamical variable, we describe, as an example for the sake of concreteness, the motion of a physical event (object) as follows. It seems from the perspective of the NA-Realized that the dynamical variable momentum has to be determined so that the evolution of the structural form should result in the present (realized) state of the physical event, that is, the space-time localization, (x_0, t_0) which is the possessed region of the space-time of the (realized) event. Therefore, we attribute to the NY-Realized aspect the pair of dynamical variable $\{X, P_0\}$ in order, where subscript "0" shows the determined character of P. Similarly, from the perspective of NA-Realized, this present (realized) structural form should be the source of the subsequent realization of the structural form such that it has a specified position variable with an unspecified momentum variable. Thus, we reasonably attribute to the NY-Realized aspect the pair of the dynamical variable $\{P, X_0\}$ in order, where the subscript "0" shows the determined character of X. We will return to this point when arguing the uncertainty principle.

Without losing the general scope of quantum theory, energy conservation can be taken as the general invariant inherent in the nature of physical events. Hence, the two aspects of the physical event are interconnected with energy conservation. One can characterize this invariant by a variational principle. The NA-Realized and NY-Realized aspects of a physical event reveal naturally the non-commutative character of the evolution of a physical event. The justification of this interpretation can be seen in terms of Liouville's equation:

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{1}{i\hbar}[A, H],$$

where A is a physical variable. Indeed, if the physical variable does not explicitly depend on time, then its evolution manifests itself according to the invariant of the event, that is, Hamiltonian of the event, which consists of the non-commutativity character.

In the scope of quantum theory, what concept can be attributed to this notion of the structural form? Our first requirement is that it should unify the dynamical and kinematical perspective of the physical event in itself. In other words, both the NY-Realized and NA-Realized aspects of the structural form should merge into the concept in such a way that the structural form preserves its unity. Our second requirement is that the concept not force us to the continuous kinematical description. Because, according to this description as stated in Sec.2, a physical event is meaningful only in terms of space-time coordinates (x_0, t_0), and it is immune to the indefinite division of space-time. This approach, in addition to giving an operational definition to the event, forces us to consider that the event is a continuous kinematical and dynamical realization of an aggregation of matter, which results in a duality in the nature of the event. Furthermore, from the window of this approach, the dynamical and kinematical properties of the physical event have to be real in the course of a continuous evolution. However, this induction contradicts with the fundamental principle of quantum theory, that is, Planck's hypothesis that the absorption and emission of energy are discontinuous. Therefore, we have to refrain from such a continuous realization of the physi-

cal properties of the event. It then seems that one of the possible candidates satisfying these two requirements is the density matrix of a quantum system. Indeed, one can represent the density matrix in the different representations (or descriptions) which have the same eigenvalues, and these representations are connected to each other with a unitary transformation[33]. As is well known, the space of the position variable is connected to the space of the momentum variable by Fourier transformation, that is, a unitary transformation. Hence, *the invariant replaced structural form can be interpreted as the density matrix of the event, which is also the quantum state.*
We now proceed to embed the eigenstates and eigenvalues of physical observables into this picture from a ontological perspective. It has been customary to interpret the eigenvalues as the only realized values of the corresponding observables [4, 34]. In other words, a physical operator realizes itself only in one of these eigenvalues. It is obvious that this is an imposition of the measurement perspective. However, we have stated before that the measurement perspective relies on the operational definition, which is fraught with the conjectures given in Sec.2. For this reason, in addition to other evidence such as the polarization of light, we reject this interpretation. In the case of the polarization of light, we assume a monochromatic plane wave moving to the positive z-direction and being polarized in the xy-plane by making an angle θ in the x-direction. We measure the intensity of the wave at various values by means of an analyzer according to how its optical axis is positioned. If its optical axis makes the same angle as that of the wave, we then measure the intensity in full. This means that the components of the polarization of light along the x-direction and y-direction manifested themselves simultaneously as a whole in the measurement. Therefore, to comprehend such situations in the picture, the eigenvalues of an observable should not be interpreted according to the measurement perspective. Instead, they are the contents of the possible eigenstates which can be entitled as modes. Hence, we can make the following definition for the eigenvalues:

Definition 2. *The eigenvalues of a physical observable are the contents of the modes of the observable contained in the structural form .*

According to this definition, an invariant replaced structural form can contain a superposition of the eigenstates, and thus, the eigenvalues of an observable. This definition is compatible with the reality of a physical event[21]. Similar to the polarization of light, for example, the superposition of the modes of the spin operator S_z with equal weight gives new modes either along the x-direction or y-direction.

From this perspective, the functionality of dynamical variables imposes itself as a fact inherent in the structural form. This is the specific behavior of dynamical variables, for instance, the vectorial behavior. As another example, the functionality of energy enables the destructive and instructive inferences of electrons. Thus, we cannot assume physical objects, just like an aggregation of matter having a passive geometrical shape. The modes contained in the invariant replaced structural form determine, for example, both the energy extension and its functionality when the structural form becomes realized. This conclusion is consistent with the interpretation of the energy-time uncertainty presented in Refs. [35, 36]. As a special area of physical events, we can ask what the physical objects are, in particular, the elementary particles like protons and electrons. The permanent and stable existence of the elementary particles is their main character. Definition 1 of a physical event tells us that the elementary particles are an invariant replaced structural form having a finite region (or extension) of space-time. In addition, if we look at Planck hypothesis, $E = hv \Rightarrow E = h/T \Rightarrow ET = h$, it seems that the realization of the energy, or equivalently, a complete replacement of the structural form needs a finite time interval which has already been stated in definition 1. Indeed, the hypothesis is a counter-factual declaration of physical events against the continuous kinematical description. According to the hypothesis, we can interpret the frequency as the quantitative determination of energy. In addition, since the extended energy of the structural form cannot be "*nothing*", the space occupied by the (realized) structural form cannot also be arbitrarily small without losing the (realized) structural form. it is now clear that an elementary particle can be defined as follows:

Definition 3. *An elementary particle is a permanent wave packet, or equivalently, a permanent frequency packet such that its energy extension is defined by its frequency which is a superposition of the modes of the energy contained in the invariant replaced structural form. In short, if* v *is the frequency corresponding to the finite time that is needed for the realization of the energy, then an elementary particle is a uniform constantly replacement of the structural form with the frequency* v.

This definition is consistent with the general form of the wave packet in the literature, which is $\psi(q,t) = A(x,t)exp(iS(q,t)/\hbar)$. The extended energy as the realization of structural form is not a mode of the energy observable, but is the superposition of the modes contained in the structural form. Furthermore, since the replacement of the structural form emerges according to the invariant, that is, the conservation of energy, then the realization of the structural form should be characterized by the trace function such that:

Definition 4. *Realization of a structural form is trace of the invariant, $Tr(H\rho) = \langle E \rangle$, where density matrix ρ plays the role of the structural form.*

According to this definition, the extended energy as the realization of the structural form is the expectation value of the energy variable; thus, we can reinterpret Planck's hypothesis in context of the realization of the invariant replaced structural form again, like $\langle E \rangle T = h$, for an elementary particle. The time appearing in this reinterpretation is the finite time needed for a complete realization of the structural form. In other words, it is the finite time possessed by a physical event, which was stated in definition 1 of the event.

This conceptual frame is sufficient to merge the main questions, presented in the introduction, into a consistent interpretation. They were these: i) what is the physical status of the uncertainty principle?, and ii) what is the physical status of the quantum state? We have already determined the physical status of the quantum state (i.e., density matrix) as the invariant replaced structural form expressed in definition 1. Now, we wish to express how the uncertainty principle plays a role in the structural form. To this aim, we focus on the picture given in Fig.1. We have described that: (i) from the perspective of the NY-Realized, a physical event is the result of the realization of an invariant replaced structural form having a determined momentum on the one hand; and at the same time, (ii) from the perspective of the NA-Realized, it is the source of the subsequent realization of the invariant replaced structural form having a determined position, on the other hand. A determined variable can be represented, in general, by a positive operator-valued measure (POVM). We have attributed to the former perspective the pair of the dynamical variables $\{X, P_0\}$ in order, and to the latter the pair of the dynamical variables $\{P, X_0\}$ in order. If these two variables were compatible with each other, then their orders would seem exactly the same from both perspectives, that is, they would be $\{X_0, P_0\}$ and $\{P_0, X_0\}$, respectively, from the description perspective. In other words, they would become realized compatibly (or simultaneously). However, according to the description of the physical event, this is impossible. This impossibility should be read as a counter-factual declaration of physical events against the continuous dynamical description.

During a complete replacement, the structural form unifies in itself both perspectives according to its invariant. This unification happens by means of a unitary transformation. Therefore, we conclude that the uncertainty between the position and momentum variables arises as a physical character of the invariant replacement of the structural form. In short, contrary to common sense[13], the uncertainty is not due to the measurement but is ontological. This idea does not exclude measurement error, but rather, it implies a universal measurement error within incompatible measurements. In other words, it implies that, independent of measurement, there is no simultaneous continuous kinematical and dynamical description of a physical event if we do not want to lose the notion of factual and permanent events. One can read the second term on the right side of the Robertson-Schrödinger inequality [37, 38],

$$\Delta X^2 \Delta P^2 \geq \frac{Cov^2(X,P)}{4} + \frac{|\langle [X,P] \rangle|^2}{4}$$

as the ontological uncertainty entering into the measurement error.

How does the measurement seem from this conceptual frame? In the first place, we underline the fact that an observation is an action of the observer from a reference frame outside a physical event. This means that the observer tries to observe the realization of the structural form, i.e., $Tr(H\rho) = \langle E \rangle$. In the second place, if the observer attempts to learn the content of the structural form, the observer first interacts with the event by means of an experiment and then observes the realization of the event. Therefore, taking $\{\Pi_k, k \in I\}$ as a set of POVMs, one measurement is the realization $Tr(H\Pi_i\rho)$; thus, the measurement process is inherently statistical. In addition, if we would present the density matrix of a physical event as $\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$, where the pure states $\{|\psi_i\rangle\}$ are not necessarily orthogonal, then we would describe the state of the event statistically because this density matrix is a mixture of reality (the sates $\{|\psi_i\rangle\}$) and the observer's lack of information (the probabilities p_i) about the structural form of the event. However, the structural form of a physical event in itself is a pure state because if it is not a pure state then it does not have an invariant which unifies the kinematical and dynamical perspective of the event. In short, a physical event does not have two different structural forms simultaneously. Furthermore, if the states $\{|\psi_i\rangle\}$ are not orthogonal then we cannot apply the law of excluded middle categorically to the interaction of the event with its environment because the notions (the states) are not distinct (orthogonal).

4. Conclusion

We have presented the uncertainty principle and the quantum state as two essential problems of quantum theory. We have resolved the problems on the ground of a unification of the kinematical and dynamical characters of physical events and then unified them as an invariant replaced structural form. We wish to underline the result that the uncertainty between the position and momentum variables is inherent in the nature of the structural form. An understanding of the nature of this structural form requires the revision of the ratio-nal reason. Because the current ratio-nal reason is based on the operational definition, which relies on the continuous kinematical description, which always forces us to determine the content of a physical event from the perspective of the operational definition by means of abstract concepts. We need a new logic to unearth the content of the structural form (see, for example [31, 32]).

Another important result of our conceptual frame is that space-time is created as a result of the realization of the invariant replaced the structural form. This does not contradict with the results of relativity theory about space-time because relativity theory determines a physical event from an operational perspective; in other words, its object is not the structural form ρ but the realization of the structural form, $Tr(H\rho)$, which has an inborn finite space-time region. One can design an experiment to measure this finite time of realization [39]. Therefore, the absence of privilege between the matter and space-time is preserved from the operational perspective.

The notion of a (one) complete replacement of structural form in our conceptual frame supports the possibility of a time operator[40] having a domain $[0, 2\pi]$, which is another result of our conceptual frame. The time parameter-t used in classical mechanics is based on the perspective of the operational definition which has a domain $(-\infty, \infty)$. However, treating this time parameter-t as a canonical variable is fraught with dynamical problems such as the impossibility of finding a Hamiltonian for the enlarged system consisting of the time as the (n+1)th canonical coordinate [41].

The most remarkable result of our conceptual frame is that a physical event, as an invariant replaced structural form, has a functional nature; in other words, it is not an accumulation of matter with a passive geometrical shape, but it is an active shape such that its dynamical properties organize in unity, the so called structural form. These organized dynamical properties manifest themselves by means of the invariant. Two aspects, NY-Realized and

NA-Realized, of the structural form suggest a non-commutative geometry as long as one considers microscopical scale of physical events. The reason behind the need for non-commutative geometry is related to the following question: why do the things around us seem as if they are continuous and stable objects? If one considers a motionless object having $1gr$ mass and attains, for the sake of simplicity, the Hamiltonian $mc^2 I$, then its realization is $Tr(mc^2 I\rho) = mc^2 = \langle E \rangle$. The time needed for a complete realization of the object is $T = h/\langle E \rangle \approx 10^{-39} sec$. This explains why the things around us seem continuous, and as passive and stable geometrical objects. In this example, we assumed the motionless object as an aggregation of matter having a passive geometrical structure, which means that the structure of the object is non-functional. Therefore, non-commutativity is about whether or not the functional faculties of a physical event manifest their functional character. If the realization period of a physical event and the interaction time of the event with its environment are at the same scale, we then observe the effect of the functional character of the variables.

[1] W. Heisenberg, "Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik," *Zeitschrift für Physik*, vol. 43, p. 172–198, 1927.

[2] W. Heisenberg, "Über quantentheoretische umdeutung kinematischer und mechanischer beziehungen," *Zeitschrift für Physik*, vol. 33, p. 879–893, 1925.

[3] E. Schrödinger, *Collected Papers on Wave Mechanics.* Blackie and Son, 1928.

[4] B. d'Espagnat, *Conceptual Foundations of Quantum Mechanics.* Perseus Books, 1999.

[5] M. Ozawa, "Uncertainty relations for noise and disturbance in generalized quantum measurements," *Annals of Physics*, vol. 311, pp. 350–416, 2004.

[6] P. Bush, P. Lahti, and R. F. Werner, "Proof of heisneberg's error-disturbance relation," *Physical Review Letters*, vol. 111, 2013.

[7] I. Białynicki-Birula and J. Mycielski, "Uncertainty relations for information entropy in wave mechanics," *Journal of Communications in Mathematical Physics*, vol. 44, 1975.

[8] D. Deutsch, "Uncertainty in quantum measurements," *Physical Review Letters*, vol. 50, 1983.

[9] H. Maassen and J. B. M. Uffink, "Generalized entropic uncertainty relations," *Physical Review Letters*, vol. 60, 1988.

[10] C. M. Caves, C. A. Fuchs, and R. Schack, "Conditions for compatibility of quantum-state assignments," *Physical Review A*, vol. 66, 2002.

[11] J. S. Bell, "On the einstein podolsky rosen paradox," *Physics Physique Fizika*, vol. 1, 1964.

[12] N. D. Mermin, "Extreme quantum entanglement in a superposition of macroscopically distinct states," *Physical Review Letters*, vol. 65, 1990.

[13] J. Hilgevoord and J. Uffink, "The uncertainty principle," in *The Stanford Encyclopedia of Philosophy* (E. N. Zalta, ed.), Metaphysics Research Lab, Stanford University, winter 2016 ed., 2016.

[14] E. Wigner, "The problem of measurement," *American Journal of Physics*, vol. 31, 1963.

[15] H. Margenau, "Measurements in quantum mechanics," *Annals of Physics*, vol. 23, 1963.

[16] M. Paty, "Are quantum systems physical objects with physical properties?," *European Journal of Physics*, vol. 20, 1999.

[17] M. Pusey, J. Barrett, and T. Rudolph, "On the reality of the quantum state," *Nature Physics*, vol. 8, 2012.

[18] D. Frauchiger and R. Renner, "Quantum theory cannot consistently describe the use of itself," *Nature Communications*, vol. 9, 2018.

[19] N. D. M. C.Fuchs and R. Schack, "An introduction to qbism with an application to the locality of quantum mechanics," *American Journal of Physics*, vol. 82, 2014.

[20] A. Auffèves and P. Grangier, "Contexts, systems and modalities: A new ontology for quantum mechanics," *Foundation of Physics*, vol. 46, 2016.

[21] P. G. Lewis, D. Jennings, J. Barrett, and T. Rudolph, “Distinct quantum states can be compatible with a single state of reality,” *Physical Review Letters*, vol. 109, 2012.

[22] J. Combes, C. Ferrie, M. S. Leifer, and M. F. Pusey, “Why protective measurement does not establish the reality of the quantum state,” *Quantum Studies: Mathematics and Foundations*, vol. 5, 2018.

[23] E. Schrödinger, *Nature and The Greeks and Science and Humanism.* Cambridge University Press (Canto Edition), 1996.

[24] A. N. Whitehead, *Science and The Modern World.* The New American Library, 1949. An elaborate examination of the continuity can be found in ref.[23, p. 130-143].

[25] P. W. Bridgman, *The Logic of Modern Physics.* The Macmillan Company, 1958.

[26] N. Bohr, “The quantum postulate and the recent development of atomic theory,” *Nature*, vol. 121, p. 580–590, 1928.

[27] N. Bohr, “Can quantum-mechanical description of physical reality be considered complete?,” *Physical Review*, vol. 48, pp. 696–702, 1935.

[28] I. Stewart and D. Tall, *The Foundations of Mathematics.* Oxford University Press, 2 ed., 2015.

[29] A. Einstein, B. Podolsky, and N. Rosen, “Can quantum-mechanical description of physical reality be considered complete?,” *Physical Review*, vol. 47, no. 10, 1935.

[30] J. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, “Experimental test of quantum nonlocality in three-photon greenberger–horne–zeilinger entanglement,” *Nature*, vol. 403, 2000.

[31] G. Birkhoff and J. V. Neumann, “The logic of quantum mechanics,” *Annals of Mathematics*, vol. 37, 1936.

[32] Y. Koç, *Nazari Mantık’ın Esasları (Foundations of the Ir-rational Logic of Theoria).* Cedit Neşriyat, 2013. The author does not use the term "Ir-rational" opposed to the term "rational", but he use it for those concepts

(or physical events) which do not obey the symmetric and transitive properties of the equivalence relation.

[33] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information.* Cambridge University Press, 2002.

[34] A. Peres, *Quantum Theory: Concepts and Methods.* Kluwer Academic, 2002.

[35] J. Hilgevoord, "The uncertainty principle for energy and time," *American Journal of Physics*, vol. 64, 1996.

[36] J. Hilgevoord, "The uncertainty principle for energy and time. ii," *American Journal of Physics*, vol. 66, 1998.

[37] H. P. Robertson, "The uncertainty principle," *Physcal Review*, vol. 34, pp. 163–164, 1929.

[38] E. Schrödinger, "Zum heisenbergschen unschärfeprinzip," *Proc. Prussian Acad. Sci.*, vol. 19, 1930.

[39] R. Moreira, A. Carvalho, M. Mendes, M. Moreno, J. Ferraz, F. Parisio, L. Acioli, and D. Felinto, "Toward an experimental test for the finite-time wave function collapse," *Optics Communications*, vol. 426, 2018.

[40] J. Hilgevoord, "Time in quantum mechanics," *American Journal of Physics*, vol. 70, no. 3, 2001.

[41] H. Goldstein, C. Poole, and J. Safko, *Classical Mechanics.* Addison-Wesley, 3 ed., 2002.