# Worldly imprecision

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Physical theories often characterize their observables with real number precision. Many non-fundamental theories do so needlessly: they are more precise than they need to be to capture the physical matters of fact about their observables. A natural expectation is that a truly fundamental theory will require its full precision in order to exhaustively capture all of the fundamental physical matters of fact. I argue against this expectation and I show that we do not have good reason to expect that the standard of precision set by successful theories, or even by a truly fundamental theory, will match the granularity of the physical facts.

**1. Introduction.** Suppose you have been tasked with measuring the value of my height at some particular instant in time. You might proceed by asking me to stand up straight against a wall at that instant, making a mark just above the top of my head with a very fine tipped pen, and measuring the distance between the mark and the floor with a meter stick. Were you to do this you would likely find that the mark on the wall falls between two millimeter markings on your meter stick, say the seventh and eighth millimeter markings between the 95th and 96th centimeter markings. Having already measured one full length of the meter stick, you would come to the conclusion that I am $1.957 \pm 0.001$ m tall.

The precision of this measurement can obviously be improved. If only one additional decimal place of precision is required you could simply obtain a rule with finer markings. With the aid of an electron microscope you could determine the value with nine or ten decimal places of precision. In fact, on first inspection it seems that the only limit to the precision with which my height can be accurately determined is the resolution provided by currently available technology. In order for the *only* limit on the precision to come from such pragmatic factors, there must be a physical fact of the matter not just about the tenth decimal place of my height but also about the $n$th decimal place for any arbitrary $n$. If there is some level of precision beyond which there is no longer such a physical matter of fact, that marks a principled, not merely pragmatic, limit on the precision with which my height can be measured.

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It turns out that there is such a principled limit, at least if we impose plausible conditions on the semantics for the ordinary language term "height". By my height we plausibly mean the distance between the floor and the highest point on my head, and so a measurement of my height is a measurement of that distance. Consider the determination of the position of the highest point on my head.[1] Suppose we can all agree which cells are part of me and which electrons are parts of those cells and which are not. Determining which of those electrons happens to be the furthest from the floor at a given instant requires exactly determining each of their positions at that instant. But this is precisely the sort of thing that quantum mechanics indicates that there will not be a physical matter of fact about because the top of my head is a complicated superposition of many quantum mechanical particles.[2] A real number provides more precision than is required to exhaust the physical facts about my height.

This conclusion tells against the expectation that there are only pragmatic limitations to the precision with which quantities such as height can be measured. One might think that this is a peculiarity arising from scientific investigation of a term whose meaning is restricted to the realm of ordinary language. This turns out not to be the case. The same phenomenon, a mismatch between the standard of precision in our theories and the granularity of the physical facts, is a commonplace feature of our physical theorizing. Empirically successful theories can display such a mismatch, and I will argue that fundamental theories can exhibit such a mismatch as well.

The argument proceeds as follows. In Section Two I argue that the mismatch between the precision of our theories and the facts about the world they are designed to capture can be found in many aspects of our physical theorizing. In the third section I consider how to identify the standard of precision that is natural for a given theory. This discussion leads to the articulation of a collection of distinct standards of precision, each of which might be natural for some class of theories. In Section Four I consider how to identify the granularity of a collection of physical facts that a theory is designed to capture. This process relies on experimental observation, and I identify general features of experimental practice which demonstrate that the success of a theory at matching the observational evidence does not require a match between the standard of precision in the theory and the granularity of the physical facts. In the fifth section I consider the view that in order for a theory to be fundamental, its standard of precision must match the granularity of the fundamental physical facts. As in the case of empirical success,

[1] Issues similar to those that follow obviously also affect the determination of the position of the floor, but this won't be of consequence for my argument.

[2] I develop the argument for this claim in more detail in Section Two.

I argue that fundamentality does not require such a match. The final section contains concluding remarks.

**2. Worldly Imprecision.** I have argued that the physical facts concerning height have a granularity less sharp than the standard of precision provided by real numbers. Many find this observation surprising, which motivates the following question: where does the expectation that one should be able to make meaningful statements about my height with real number precision come from? In this section I will argue that it is the practice of physics that has led us to this expectation.

Consider, for example, the elementary physics problem of determining the vertical displacement from equilibrium as a function of time $y(t)$, of a block of mass, $m$, suspended from a spring with spring constant, $k$. The dynamics of classical mechanics holds that this displacement is determined by the equation,

$$m\frac{d^2y(t)}{dt^2} = -ky(t), \tag{1}$$

which is solved by,

$$y(t) = A\cos(\omega t + \phi) \;\text{ for }\; \omega = \sqrt{\frac{k}{m}}. \tag{2}$$

Note that once $m$ and $k$ are fixed, and $A$ and $\phi$ are determined by the initial conditions of the system at $t_0$, this solution assigns a real number to the value of the displacement around the equilibrium position for all times $t$. As our theoretical representation of the displacement takes the form of a real number, we are led naturally to expect that the physical matters of fact about the actual displacement will come with a granularity that matches the real numbers. However, as in the case of my height, consideration of the quantum mechanical nature of the particles constituting the block show that there is no physical matter of fact about the position of the edge of the block with real number precision.

Suppose we define the position of the edge of the block to be the position of the lowest electron constituting the block. Then on most interpretations of quantum mechanics, that there is no real number fact about the position is obvious from any one of a number of different observations, such as that position measurements do not yield exact position eigenstates. Bohmian mechanics presents an apparent counterexample as it assigns definite positions to each of the electrons in the complicated superposition that constitutes the edge of the block. However, the measurement of any one of the electrons instantaneously influences the positions of all of the others. As a result, any attempt to determine whether or not a given electron is the lowest will affect the positions of all of the others, and thus there is not a unique, identifiable lowest point, independent of how one goes about the measurement process. At a certain level of resolution, there is simply no longer a physical matter of fact for the measurement to track. Just as we found in the height case, the theory is more precise than it needs to be to exhaust the physical matters of fact about the displacement of the block. The facts about the world are imprecise with respect to the natural standard of precision in the theory. Let's call this phenomenon *worldly imprecision*.

Instances of worldly imprecision are related to, but distinct from, failures of a view that Teller has recently called measurement accuracy realism (Teller 2018). According to measurement accuracy realism, there is a fact of the matter about the exact physical value of measured quantities, and thus there is an objective fact about the accuracy of any given measurement. Teller argues that in many cases measurement accuracy realism is false, and there is no objective fact concerning the accuracy of our measurements. He defends this view by arguing that there is not a fact of the matter concerning the physical value of the measured quantity. The failure of there to be such a fact results from reference failure of the statements about the physical quantity, which in turn result from the idealizations that go into our theoretical articulation of the quantity. As Teller explains, "Accuracy realism fails because of reference failure, and reference fails because of a fact that we too easily let drop out of view: the ubiquitous idealizations of our theoretical accounts of the world" (Teller 2018, p. 288).

The reference failure that Teller argues for is on full display in the case of our spring. By modelling the displacement of the spring with Eq. (1), we have adopted many idealizations. We have assumed, for example, that the response of the spring is perfectly linear, even though any real spring will have non-linearities in its response, however slight. We have also assumed that the gravitational field in which the spring is oscillating is perfectly uniform, even though the actual field is slightly stronger when the mass is closer to the earth because of the $1/r^2$ form of the gravitational force law. And we could, of course, go on.[3] Reference failure ensues, according to Teller, because deidealizing requires that we specify a run-away list of additional conditions. The temperature of the room in which the spring is located will affect the form that the non-linearities in its response will take. The exact form of the non-uniformity in the gravitational field depends on the specific location on the surface of the earth where the experiment takes place. The term "the displacement of the block at time $t$" fails to refer because we have not

[3]Many of the idealizations tacitly adopted by modelling the spring with Eq. (1) give rise to larger contributions to the displacement than those mentioned here. For reasons that will become clear below, I have chosen to focus on small effects.

sufficiently specified the conditions involved in the deidealizations to pick out any one particular quantity with a determinate value. Teller is pessimistic about the prospects for us ever being able to fully specify a sufficient set of conditions to completely deidealize, and thus he thinks that reference failure for quantities runs rampant and measurement accuracy realism generically fails.

Cue the inevitable rejoinder. Perhaps we aren't the kinds of agents that can actually do it, but surely in principle it is possible to completely deidealize. After all, for a particular mass on a particular spring at a particular time, there are facts of the matter about where they are located, the prevailing conditions at that location, the material constitution of the mass and the spring, and every other causally relevant factor for the determination of the displacement of the block. The reference of "the displacement of the block at time $t$" fails not because the displacement isn't the kind of thing that can have a determinate value, but rather because it is ambiguous between the different exact values it takes on for different specifications of the causally relevant conditions. What we mean by "the displacement of the block at time $t$" is just the value that one would arrive at if they were actually able to execute this process for the conditions that obtain when and where the experiment is actually conducted.

I think that Teller is right to point out the importance of deidealization in fixing the reference of statements picking out quantities and I share his pessimism about our capability to completely deidealize. What I want to emphasize is that not all idealizations behave the same with respect to the determinacy of the reference of terms specifying quantities like "height" and "the displacement of the block at time $t$". There are idealizations that result from neglecting small effects. These are the kinds of effects that the imagined interlocutor of Teller is sure we can fill in, at least in principle. But there are other idealizations whose role is to frame the problem. The perfectly localized point masses and perfectly sharp edges of blocks of classical mechanics are examples of idealizations of this second kind. These idealizations do not make small, neglected contributions to the value of the quantity. Rather, they make terms like "the displacement of the block at time $t$" the kind of quantity that takes a real number value for the other deidealizations to correct the value towards. Worldly imprecision occurs when reference fails due to idealizations of this second kind. Even if we were the kinds of agents that could fix all of the causally relevant conditions for fixing all of the classical mechanical details in the first group, the reference of "the displacement of the block at time $t$" would still fail, because of the framing idealizations in the second group.

Suppose we begin to make more and more precise measurements of the displacement of the block as we did in the case of my height. Executing the deidealizations of classical mechanical effects with large contributions becomes important first and then we can proceed to deidealizations that contribute at the next level of precision. As we proceed, eventually we arrive at a level of precision where the principled limits from instances of worldly imprecision become relevant. It is crucial to note that these limits can arise *before* we have completed the process of deidealizing all of the relevant classical mechanical effects. The semantic difficulties with "top of my head" and "edge of the block" arising because of their constitution from quantum mechanical particles become relevant at a precision of approximately $10^{-11}$ m, the scale of the Bohr radius.[4] Many idealizations arising from classical mechanical effects will be relevant before we reach this level of precision and as such are significant for the determinacy of the reference of "height" and "the displacement of the block at time $t$". Other classical mechanical effects give rise to contributions right around the $10^{-11}$ m threshold. The non-uniformity of the gravitational field close to the earth's surface due to the $1/r^2$ form of the force law provides an example.[5] But some classical mechanical idealizations give rise to smaller effects than the $10^{-11}$ m threshold at which the semantic difficulties with "edge of the block" arise. The gravitational influence of Pluto on the displacement of the block is likely one. The gravitational influence of a speck of dust in the 10,087th most distant galaxy from ours certainly is. Some classical mechanical effects are so small that the framing idealizations that we rely on to pose the problem give out before they make a difference.

The phenomenon we have identified here is not a peculiarity of classical mechanics and its treatment of position observables. Examples where other classical mechanical observables exhibit worldly imprecision can be readily constructed. The phenomenon we have identified here is also not a peculiarity of the relationship between classical and quantum mechanics. Instances of worldly imprecision also arise for observables in thermodynamics when we consider limits arising from statistical mechanics, for classical electrodynamics when we consider limits from quantum field theory, and as we will see in detail in Section Four, they arise in quantum electrodynamics from limitations coming from the Standard Model of particle physics. The basic ingredients required for worldly imprecision to obtain occur throughout our

[4] The Bohr radius is the most likely distance between the electron and nucleus of a hydrogen atom in its ground state. The current best measured value is $r_B = 5.29177210903(80) \times 10^{-11}$ m. The exact standard of precision will depend on the particular material constitution of the block as different materials can have significantly different surfaces.

[5] For reasonable values of the parameters in the problem, numerical evaluation of the effect shows that it influences the displacement on the order of $10^{-9}$ m or $10^{-10}$ m over the course of one period of oscillation.

physical theorizing. The framing idealizations in higher level theories give out before the full precision of the characterization of the observables they give matters. In many cases, there is more precision in the higher level theories than is required to say everything that there is to say about their observables.

**3. Standards of Precision.** When worldly imprecision occurs, the following condition fails to obtain:

> **Match**: the standard of maximal precision in the theory matches the granularity of the physical facts concerning the theory's observables.

Of course, a given theory might employ one standard of maximal precision for some of its observables, and a different standard for others. Match should be read as requiring that the relevant standard matches the relevant facts in each case. There are two ingredients that go into making precise what ought to count as failures of Match; the standard of precision in the theory, and the granularity of the physical facts. Before proceeding, it is worth thinking in more detail about both of these ingredients.

The standard of precision of a given theory is determined by the mathematical structure of the theory and the relations that the theory posits between its observables. Our classical mechanical treatment of the mass on a spring, captured in Eq. (1) and Eq. (2), stipulates a collection of relations obtaining between the observables $m$, $y$, $k$, $A$, and $\phi$. This collection of relations establishes a standard of precision that is maximal for the theory. In particular, by modelling the problem with Eq. (1) and Eq. (2), we tacitly adopt the following standard of maximal precision:

> $\mathbf{MP}_{\mathbb{R}}$: A statement about a quantity $Q$ is maximally precise if and only if it ascribes a real number $d \in \mathbb{R}$ to that quantity.

Some ascriptions of values to quantities are less than maximally precise with respect to this standard. For example, one can take the value of $m$ to be 1/2 kg. This is an ascription of a rational number to $m$. We frequently pass between the ascription $m = 1/2$ kg and the ascription $m = 0.5\bar{0}$ kg, as they can be used interchangeably for the purposes of some mathematical manipulations. But by taking $m = 1/2$ kg we typically mean something different than $m = 0.5\bar{0}$ kg. In particular, by taking $m = 1/2$ kg, we typically mean that $m = 0.5d_1d_2\ldots$ kg and that the decimal places after the 5, $d_1$, $d_2$, and so on, are uncertain. Understood in this way, the ascription $m = 1/2$ kg is less precise than the $\mathrm{MP}_{\mathbb{R}}$ standard. Ascriptions of natural numbers, such as $m = 1$ kg, are even less precise than ascriptions of rational numbers.

One can also make statements that are more precise than $\mathrm{MP}_{\mathbb{R}}$. If a theory ascribes hyperreal numbers to its observables, for example, then its statements are more precise than $MP_{\mathbb{R}}$. The hyperreal numbers, $^*\mathbb{R}$, contain the real numbers as a subset and the order relation on the real numbers is a subset of the order relation on the hyperreal numbers, establishing a clear sense in which ascriptions of hyperreals to observables are more precise than the standard of maximal precision set in $MP_{\mathbb{R}}$.[6] The real numbers are used throughout our physical theorizing and for this reason, $MP_{\mathbb{R}}$ might seem to be the natural or perhaps even inevitable standard. But we can make statements more precise than ascriptions of real numbers to quantities, and we can also make statements that are less precise. This shows that the standard of precision in a theory is a modelling choice that we make when we develop the theory. We can cast theories with standards of precision more sharp than $MP_{\mathbb{R}}$ by allowing for ascriptions of hyperreals to quantities, and we can cast theories with standards of precision less sharp than $MP_{\mathbb{R}}$ by restricting ascriptions of values to natural or rational numbers.[7]

The other ingredient in Match is the granularity of the facts about the physical observables that the theory aims to capture. I will presuppose, as I think is common in recent discussions of the metaphysics of quantity, that there are physical facts about the values that quantities possess. This presupposition commits us to some form of realism about quantities. In particular, it seems to involve commitment to the view that quantitative properties exist.[8] On this view, electrons really have properties like mass and charge, and there are physical facts about the values of these properties. This in turn involves commitment to the existence of an objective fact about how fine-grained the properties are as well. If there is a fact about the value a physical quantity takes, there are is an additional fact about the granularity of that value.

For Match to be satisfied, the standard of precision that we input into a theory's characterization of its observables must exactly agree with the granularity of the physical facts concerning those observables. Cases of worldly imprecision amount to failures of Match because they involve situations where the standard of precision that we input into the theory is more fine-grained

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[6] A clear introduction to the hyperreal numbers can be found in (Keisler 2012).

[7] Natural numbers and rational numbers are also real numbers, so one might be tempted to argue that the ascription of a natural or a rational number makes the standard of maximal precision that of the real numbers. That would be a mistake. When we restrict ascriptions of values to natural or rational numbers, there are less ascriptions possible than there would be if we allowed for ascriptions of any arbitrary real number, not just the real numbers that also happen to be naturals and rationals. This is the sense in which the standards obtained by restricting to the naturals or the rationals are more coarse than $MP_{\mathbb{R}}$.

[8] Realist views of quantities that exhibit this commitment can be found in (Byerly and Lazara 1973; Swoyer 1987; Mundy 1987). This stands in tension with at least some forms of operationalism and conventionalism about quantities. For helpful discussion, see (Tal 2017).

than the physical facts about the observables that the theory provides an account of. Consider a toy theory whose only statement is the following one: "at time $t$, there are 12 books on my desk". This is an ascription of natural number to a quantity, and so the standard of maximal precision in the theory is that of the natural numbers. If we suppose that there are only whole books on my desk at any given time, then the granularity of the physical facts about my books is correctly matched by the standard of precision in our toy theory, and hence Match is satisfied. If we used a distinct toy theory consisting of the statement "at time $t$, there are $12.\bar{0}$ books on my desk" to capture the same collection of physical facts, then we have a failure of Match due to an instance of worldly imprecision. By adding the additional decimal places we have changed the standard of precision to $\mathrm{MP}_{\mathbb{R}}$. But the additional decimal places of precision in the new theory do not track anything present in the physical situation. This is also what happens in the case of my height and the spring.

Failures of Match need not result from instances of worldly imprecision, though. To see this, suppose that the physical domain we are interested in representing is one continuous spatial dimension. If the observables we are interested in representing with our theory are the distances between the points along this dimension, our theory will need to use the precision of the real numbers in order to satisfy Match, no matter what system of units we work in. But now suppose that the spatial dimension has a minimum length. If we choose a system of units in which that minimum length takes a real number as its value, then our theory will not satisfy Match. All of the physical distances between the points are multiples of the minimum length, and hence the full granularity of the physical facts can be matched with theoretical statements that ascribe natural numbers to the distances. Similarly, we typically work in a system of units where the value of the spin of an electron along a particular direction is $\pm\hbar/2$, a real number. Here again, Match fails because of the system of units we have chosen to work in, not because of worldly imprecision.

In several of these examples, we have stipulated what the facts about the granularity of the physical observables are for the purposes of illustration. But in the context of scientific investigation, this typically is not given to us for free. Rather, the granularity of the physical facts is something that we set out to discover about the world, as we did in the case of spin. The best tool we have to discover such facts is to represent the domain in question theoretically and determine whether or not our theory successfully represents the domain in question by making observations of the domain. In the next section we will consider how the success of a theory bears on the issues discussed above.

**4. Success and Precision.** Scientific realists often reason from the success of a theory to the reference of its theoretical terms, its approximate truth, and to agreement between aspects of its structure and the structure of the world. I have argued that the standard of precision that we input into a theory is one element of its structure. So one might argue that because a theory is successful, we are warranted in inferring that it satisfies Match. That is, one might argue that the following principle is true:

> **Success:** For a theory to be successful, the standard of maximal precision in the theory must match the granularity of the physical facts concerning the theory's observables.

In this section I will argue that Success is false.

One reasonable metric for the success of a theory is how well it matches the available empirical data. Our classical mechanical treatment of the mass on the spring is successful with respect to this metric. Measurements of the displacement agree with the solution provided in Eq. (2) within the associated uncertainties. If Success were true, this would lead us to believe that Match was satisfied in this case. Since the standard of precision in the theory in this case is $\mathrm{MP}_{\mathbb{R}}$, this would mean that the granularity of the facts about the displacement matches the precision of the real numbers. But the argument given in Section 2 shows that this is not the case. Something goes wrong in the inference from the success of a theory to the satisfaction of Match.

We can readily identify what has gone wrong: when we evaluate the match between a theory and some empirical data, we often pass to a more coarse standard of precision than the one involved in the statement of Match. There are two basic features of the comparison between theory and experiment that force us to adopt this more coarse standard. First, measurements have associated uncertainties and so the measurement of an observable is typically more coarse than the granularity of the facts about the observable. And second, in order for Eq. (2) to tell us anything at all about $y(t)$, we first need to fix $m$, $k$, $A$, and $\phi$. In order to do so we need to measure their values, and these measurements will also have associated uncertainties. Because of these uncertainties, the precision of the theoretically determined value for $y(t)$ will not be maximal with respect to $\mathrm{MP}_{\mathbb{R}}$. The comparison between the measured and theoretical values thus involves a standard of precision less sharp than the one that is maximal for the theory. This is what goes wrong in the inference from success to the satisfaction of Match. The agreement between theory and observation that counts as the success of the theory occurs at a level of precision more coarse than the one that is maximal for the theory, and hence the one relevant to Match.

The two basic features of comparison between theory and experiment leading to this conclusion are not particular to the case of the spring. They are generic features of scientific practice. Consider the comparison between theory and experiment in the case of the anomalous electron magnetic moment, an observable that functions as a high precision test of quantum electrodynamics. The electron's magnetic moment is a property of electrons when they are exposed to an external magnetic field. In this case, the current best theoretical and measured values are as follows:[9]

$$\mathrm{a_e}\ \text{(theory)} = 0.00115965218178(77) \tag{3}$$

$$\mathrm{a_e}\ \text{(experiment)} = 0.00115965218073(28) \tag{4}$$

Agreement between theory and experiment to twelve decimal places is one of the highest precision successes that has been achieved in the history of our physical theorizing. But as in the case of the spring, we have passed to a standard of precision much more coarse than the one that comes along naturally with the theory in this case. Again we have a case where the testing and confirmation of theories that ensures us they are successful is more coarse than the one involved in Match, and so once again we have a failure of Success.

The uncertainty associated with the experimental value of $a_e$ depends on the details of the technique used to conduct the measurement. In the first measurement revealing a non-zero value of $a_e$,[10] the uncertainty was in the fifth decimal place, and in the current best value it is uncertain in the 12th decimal place, a rate of improvement of approximately one order of magnitude per decade. The precision of the theoretical value has also improved over time, roughly keeping pace with the measured value. It is determined by perturbative calculations in quantum electrodynamics and as such individual orders of perturbation theory must be calculated and then summed to determine the theoretical value. This evaluation gives terms that decrease in magnitude as one proceeds to higher orders of perturbation theory. The complexity of the calculation also increases with increasing order, and the current state of the art allows for the calculation of five orders of perturbation theory which are summed to yield the theoretical value.

This process of perturbative evaluation leads to a number of sources of uncertainty in the theoretical value. While the integrals contributing to the first three orders of perturbation theory can be treated analytically, at fourth

[9] The theoretical value reported here is the one reported in (Aoyama, Hayakawa, Kinoshita, and Nio 2012), and the experimental value is from (Hanneke, Fogwell, and Gabrielse 2008). Helpful discussion of the details of the theoretical calculations and the measurements of these values can be found in (Koberinski and Smeenk forthcoming).

[10] (Kusch and Foley 1948)

and fifth order the integrals become more complicated and some require numerical evaluation, which introduces uncertainty in the resulting value. The perturbation series is an expansion in powers of the fine-structure constant and so in order to determine the theoretical value of $a_e$ we need to input the measured value into the calculation, just as we needed to input measured values of quantities like the mass and the spring constant in the case of the spring. The uncertainty associated with the measured value of the fine structure constant is in fact the dominant uncertainty in the current best theoretically determined value of $a_e$. These limitations on the precision of the theoretical value are pragmatic. With more computing power, the numerical error from fourth and fifth order could be reduced, and perhaps eventually the contributions from sixth order will be determined. And of course, the fine-structure constant will eventually be measured with even more precision.

But there are also principled limitations to the precision of the theoretical determination of $a_e$. Quantum electrodynamics is expected to contain a Landau pole – a finite energy scale at which the renormalized coupling becomes infinite – and for this reason it is best understood as an effective field theory. Treating a quantum field theory as an effective field theory results in limits on the precision with which it characterizes its observables. In this case, the limitation is an exceedingly small one, far beyond the level of precision with which the success of the theory is demonstrated. A second principled limit to the theoretical value comes from the perturbative evaluation used to determine the value of $a_e$. Terms early in the expansion get smaller in magnitude with increasing order. However, this pattern is eventually expected to stop, with the terms eventually growing in magnitude with the result that the sum of the infinite collection of terms diverges. To obtain a finite result, the series must be truncated at some finite order of perturbation theory.[11] These effects yield principled limits to the value of $a_e$ in quantum electrodynamics, though they are very small compared to the current precision frontier.

There are other contributions to the anomalous electron magnetic moment that are external to quantum electrodynamics. This theory was originally developed to describe the coupling between the photon field and the electron field, and can be generalized to include the coupling of the photon field to the muon and tauon fields as well. Eventually it was realized that some of these fields also couple to other fields in the Standard Model and experience the strong and weak nuclear interactions as well as electromagnetism. These couplings give rise to weak and hadronic processes that give small, but non-

[11] See (Fraser 2018) and (Blinded for Review) for further discussion.

zero contributions to $a_e$:[12]

$$\mathrm{a_e}\ \text{(weak)} = 0.00000000000002973(52) \tag{5}$$

$$\mathrm{a_e}\ \text{(hadronic)} = 0.000000000001685(22) \tag{6}$$

The hadronic contribution, which is slightly larger than the weak contribution, is reflected in the theoretical uncertainty of the value reported in Eq. (3).[13]

The observation that there are hadronic and weak contributions to $a_e$ is sufficient to show that quantum electrodynamics exhibits worldly imprecision. When we treat the problem of determining $a_e$ theoretically in quantum electrodynamics, we make both of the kinds idealizations that we found operative in the spring case. First, there are the higher order terms of perturbation theory that are neglected when the perturbation series is truncated at a given order. By neglecting these terms, we do not include transitions in the fields that are part of quantum electrodynamics and which contribute non-trivially to $a_e$. To deidealize, additional orders of perturbation theory must be calculated and added to the value.

But there are also the idealizations of the second kind that are involved in posing the problem. In this case, the structural idealization is that the electron magnetic moment comes from the coupling of the electron field to the photon field alone. This idealization is the analog of the assumption that the block has a boundary that can be viewed as arbitrarily sharp. That is to say, the anomalous electron magnetic moment of an electron field that only couples to the photon field is like a block with an edge that can be made arbitrarily sharp. There is no such block, or set of coupled fields in the world. In the world the anomalous electron magnetic moment comes from an electron field that couples to a $\mathrm{W}^{\pm}$ and $\mathrm{Z}^0$ fields as well as the Higgs field. In this case the idealization makes no difference for the first 10 decimal places of $a_e$, but then the weak and hadronic couplings become relevant. Beyond the level of precision where these couplings become relevant, there is simply no longer a physical fact about the value of the anomalous electron magnetic moment from the coupling of the electron field to the photon field alone. Quantum electrodynamics is more precise than it needs to be to say everything that there is to say about its observables. One can continue calculating orders of

[12] These values are reported in (Mohr, Newell, and Taylor 2016).

[13] There is also a contribution from the coupling of the electron field to the Higgs field, though this is smaller than the leading order weak and hadronic contributions. Many scenarios for physics beyond the Standard Model would make small additional contributions to $a_e$, making discrepancies between the best theoretical and measured values a potential signal of new physics.

QED perturbation theory, but eventually this becomes like calculating the gravitational influence of the speck of dust in the 10,087th galaxy over on our spring. The framing of the problem has already given out by the time these effects become relevant. The boundary giving rise to worldly imprecision makes it the case that corrections from the effective nature of QED, and the divergent nature of its perturbative expansion, fall beyond the limit of worldly imprecision.

For a theory to be successful, it need not satisfy Match. We have seen that in the demonstration that a theory is successful, we pass to a different standard of precision than the one involved in Match. Our measurements are less sharp than the granularity of the facts about the world they reveal. And the processes used to get theoretical values out of our theories necessitate a retreat to a more coarse standard of precision than what is maximal in the theory. If one wants to argue in favor of Match, they need a different strategy. In the next section, I will consider what I think is the last resort for defenders of Match.

**5. Fundamentality and Precision.** I have argued that the facts about the world are imprecise with respect to the standards of precision set by many of our physical theories. In each case where I have argued that worldly imprecision obtains, I did so by appealing to limitations arising from a more fundamental theory. In the case of classical mechanics, I appealed to quantum mechanics. In the case of thermodynamics, I appealed to statistical mechanics. And in the case of quantum electrodynamics I appealed to the Standard Model. A natural thought when presented with this collection of observations is that if we had a truly fundamental theory, a theory that exhaustively characterized a complete minimal basis for everything that there is,[14] then it would not exhibit the phenomenon of worldly imprecision. In other words, one might expect that the following principle is true:

> **Fundamental:** For a theory to be fundamental, the standard of maximal precision in the theory must match the granularity of the physical facts concerning the theory's observables.

Like Success, I think that Fundamental is false. In order to show that Fundamental is false, it will be helpful to restate it in the following equivalent form:

> **Fundamental:** For a theory to be fundamental, (i) the standard of maximal precision in the theory must be at least as sharp as

[14] I will proceed with this intuitive, but admittedly contentious, understanding of fundamentality. Whether or not the arguments below depend on one's particular account of fundamentality is not a question I will pursue in this paper, though it merits further investigation.

> the granularity of the physical facts concerning the theory's observables, and (ii) the granularity of the physical facts concerning the theory's observables must be at least as sharp as the standard of maximal precision in the theory.

This is equivalent to the first statement of Fundamental because (i) and (ii) are both satisfied if and only if Match is satisfied.

First consider clause (i). It holds that in order for a theory to be fundamental, its statements about its observables must be at least as sharp as the granularity of the facts about those observables. This, I claim, is true. The domain of a fundamental theory is the collection of fundamental physical facts. If a theory's statements about the fundamental physical facts are less precise than the granularity of those facts, then there are more precise truthful statements to make about the fundamental physical quantities. When this is the case, the theory fails to be complete, and hence fails to be fundamental. So clause (i) is a reasonable condition on fundamentality.

Now consider clause (ii). It holds that in order for a theory to be fundamental, the granularity of the facts about its observables must be at least as sharp as its statements about its observables. Unlike clause (i), this is not a reasonable condition on fundamentality. To see this, suppose that it doesn't obtain. Then the maximally precise statements of the theory are more precise than the fundamental physical facts. But this doesn't tell against the completeness of the theory. Rather, it means that the theory is more precise than it needs to be to say everything that there is to say about the fundamental physical facts: a theory about the fundamental level satisfying (i) but not (ii) exhibits worldly imprecision. But in this case, the worldly imprecision does not come from limitations from some more fundamental theory. The fundamental physical facts simply are less sharp than the maximally precise statements of the theory.

To put the point a different way, clause (ii) is not about the completeness of the theory, but rather how concise the theory is in its characterization of its observables. When clause (ii) fails to obtain, some of the structure in the theory, some of its precision, is surplus to representational requirements. To insist that clause (ii) is necessary for a theory to be fundamental is to insist that a fundamental theory contains no surplus structure of a particular kind. But we use theories with different kinds of surplus structure all of the time, and no one ever complains that makes such theories non-fundamental. The surplus structure at issue in this case, surplus precision, is admittedly somewhat different than the standard cases, so it is perhaps worth illustrating with a concrete example.

Consider a candidate fundamental theory which includes as one of its observables the mass of the electron, $m_e$. Suppose that the electron mass is in fact one of world's fundamental physical properties, and that the natural standard of maximal precision for $m_e$ in our candidate fundamental theory is $MP_{\mathbb{R}}$. The principle Fundamental says that for this theory to be genuinely fundamental, two things must obtain. First, it must be the case that the granularity of the physical facts about $m_e$ are not more sharp than $MP_{\mathbb{R}}$. So long as the physical matters of fact do not have the granularity of the hyperreals, or some other granularity sharper than $MP_{\mathbb{R}}$, this first condition will be satisfied. Second, Fundamental says that in order for our theory to be genuinely fundamental it must be the case that the standard of maximal precision in the theory is not more sharp than the granularity of the physical facts about $m_e$. Since we are supposing that the standard of maximal precision in the theory is $MP_{\mathbb{R}}$, in order for this second condition to obtain, there must be physical matters of fact about the 50th decimal place of $m_e$ and the $10^{500}$th decimal place, and more generally, $d_N$ for arbitrarily large values of $N$:

$$m_e = d_0.d_1 d_2 d_3 \ldots d_{10^{50}} \ldots d_{10^{500}} \ldots d_N \ldots.$$

If we deny that clause (ii) of Fundamental is necessary for a theory to be fundamental, things come out differently. Of course, since we are still committed to clause (i), it will still need to be the case that the granularity of the physical facts about $m_e$ are not more sharp than $MP_{\mathbb{R}}$. If they were, the theory would fail to be complete, and as a result, it would fail to be fundamental. But when we deny that (ii) is necessary for a theory to be fundamental, we open the possibility that even though the standard of precision in the theory is $MP_{\mathbb{R}}$, the granularity of the facts about $m_e$ are less sharp than this standard. That is, our theory might be fundamental even though there are physical facts about $m_e$ up to some decimal $d_W$ and no fact about the subsequent decimal places:[15]

$$m_e = d_0.d_1 d_2 d_3 \ldots d_{10^{50}} \ldots d_{10^{500}} \ldots d_W \ldots.$$

This is what happens in the other cases of worldly imprecision introduced above. Our theories assign real numbers to their observables, and at some point in the decimal expansion, the subsequent decimal places cease to give us any additional information about the physical matters of fact concerning the observable in question. The argument of this section has shown that the same thing is possible even in a truly fundamental theory. Why should the failure of there to be a physical fact about the $10^{500}$th decimal place of $m_e$

[15]Where this $d_W$ occurs in the expansion will depend on the system of units in which one chooses to work. But if there is such a $d_W$ in some system of units, then there will be one in every system of units which is a rescaling of the first.

be taken to tell against the fundamentality of a theory that assigns a real number to $m_e$? I just can't see any good reason. There is nothing about the notion of fundamentality that requires it.

**6. Conclusion.** We are accustomed to thinking that some aspects of a theory structurally correspond to aspects of the world itself. We are similarly accustomed to thinking that some aspects of a theory are surplus and do not correspond to structural aspects of reality. Though not frequently thought of in this way, the standard of maximal precision that we adopt in a theory is one aspect of a theory's structure. Once this is realized, we can consider whether or not we have good reason to expect that Match is satisfied. I have argued that the demonstration that a theory is successful does not typically bear on whether Match is satisfied or not. I have given positive reason to doubt that it is satisfied in some of our non-fundamental theories, and that a move to a fundamental theory does not provide grounds to think the situation will be any different in that context.

Edward Purcell once quipped that "There's not enough carbon in the universe to print out the value of one classical variable" where by a classical variable he meant a variable with maximal precision in the sense of $\mathrm{MP}_{\mathbb{R}}$ (Rabi et al. 1985, p. 48). Considerations of the sort Purcell suggests serve to illustrate just how far removed real number precision is from our epistemic practices. When we ascribe real number precision to physical quantities we are using an exceptionally rich structure. Fundamental physical facts might come structured so richly, but for all we know, they do not. For this reason, it strikes me as well worth considering the possibility that much of the structure that we employ when we ascribe real numbers to quantities is in fact surplus structure.

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