Nomic Vagueness

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Abstract

If there are fundamental laws of nature, can they fail to be exact? In this paper, I consider the possibility that some fundamental laws are vague. I call this phenomenon *nomic vagueness*. I propose to characterize nomic vagueness as the existence of borderline lawful *worlds*. The existence of nomic vagueness raises interesting questions about the mathematical expressibility and metaphysical status of fundamental laws.

For a case study, we turn to the Past Hypothesis, a postulate that (partially) explains the direction of time in our world. We have reasons to take it seriously as a candidate fundamental law of nature. Yet it is vague: it admits borderline (nomologically) possible worlds. An exact version would lead to an *untraceable arbitrariness* absent in any other fundamental laws. However, the dilemma between nomic vagueness and untraceable arbitrariness is dissolved in a new quantum theory of time's arrow.

Keywords: vagueness, exactness, higher-order vagueness, semanticism, epistemicism, imprecise probabilities, laws of nature, objective probabilities, time's arrow, Past Hypothesis, entropy, fundamentality, Humeanism, anti-Humeanism, density matrix

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1 Introduction

Vagueness is often regarded as a linguistic phenomenon. Many predicates we use in everyday contexts do not have determinate boundaries of application. Is John bald when he has exactly 5250 hairs on his head? There are determinate cases of baldness, but there are also many borderline cases of being bald. In other words, predicates such as baldness are indeterminate: there are borderline individuals (such as John) such that it is indeterminate whether they are bald.[1] Moreover, the boundaries between baldness and borderline baldness are also indeterminate. Hence, there do not seem to be sharp boundaries *anywhere*. The phenomenon of vagueness gives rise to many paradoxes (such as the sorites) and serious challenges to classical logic.

It is natural to expect that, at the level of fundamental physics, vagueness should disappear. That is, fundamental predicates and laws should be exact and not vague.[2] The expectation is supported by the history of physics and the ideal that physics should deliver an objective and precise description of nature. All the paradigm cases of candidate fundamental laws of nature are not only simple and universal, but also *exact*, in the sense that, for every class of worlds (or class of solutions), fundamental laws either determinately apply or determinately fail. Suppose the fundamental laws are Newton's equation of motion $F = ma$ and law of universal gravitation $F = Gm_1m_2/r^2$: there is no ambiguity or vagueness about whether a certain class of worlds (described in terms of trajectories of point particles with Newtonian masses) satisfies the conjunction.[3]

[1]There are subtleties here about how best to characterize vagueness. For reviews on vagueness and the sorites, see Sorensen (2018) and Hyde and Raffman (2018).

[2]Here, we assume that there are fundamental laws of nature and it is the aspiration of physics to discover them. At the level of non-fundamental physics, and in the special sciences, the ideal of exactness may still be important but not absolute.

[3]It is plausible that the laws of effective field theories (EFT) are not determinate, since they do not specify a fundamental scale. However, it is controversial whether EFT should be regarded as a fundamental theory. See Miller (2019) for a discussion about imprecision in EFT. Miller's proposal is best construed as a theory of ontic imprecision, not ontic or nomic vagueness, as his examples of physical quantities can be represented by set-valued functions. As such they are unlike the cases with

Nomic exactness—the ideal (roughly) that fundamental laws are exact, determinate, and non-vague—supports an important principle about the mathematical expressibility of fundamental laws. If some fundamental laws were vague, it would be difficult to describe them mathematically in a perfectly faithful way that genuinely respects their vagueness and does not impose sharp boundaries anywhere. The kind of mathematics we are used to, built from a set-theoretic foundation, does not lend itself naturally to model the genuine fuzziness and indeterminacy of vagueness. One could go further: the language of mathematics and the language of fundamental physics are supposed to be exemplars for the "ideal language," a language that is exact, precise, and free from vagueness, suggested in Frege's *Begriffsschrift*, Russell's logical atomism, and perhaps Leibniz's *characteristica universalis*. The successful application of mathematical equations in formulating physical laws seems to leave no room for vagueness to enter into a fundamental physical theory.[4]

Little has been written about the connection between vagueness and laws of nature. The topic is philosophically and scientifically important, with ramifications for philosophy of language, metaphysics, philosophy of science, and foundations of physics. What does it mean for a fundamental law to be vague? Is nomic vagueness intelligible? If so, are there examples of vague fundamental laws that may obtain in a world like ours? What would nomic vagueness mean for the metaphysical status and mathematical expressibility of fundamental laws? How does it relate to ontic vagueness or semantic vagueness? This paper is an attempt to address some of those questions.

First (§2), I propose an account of nomic vagueness, distinguish it from approximations and subsystem fuzziness, and discuss its implications for nomic possibility and necessity, its connections to semanticism and epistemicism about vagueness, and to Humeanism and anti-Humeanism about laws of nature.

Second (§3), I focus on the case of the Past Hypothesis, the postulate that (roughly) the universe started in a special macrostate of low entropy (a specification of macroscopic properties). Given its role in explaining various arrows of time in our world, we have reasons to take it seriously as a fundamental law or at least an *axiomatic postulate* in physics that is on a par with fundamental laws. Yet, macrostates are vague. Even when we specify an exact level of entropy, the Past Hypothesis remains vague: there will be borderline lawful worlds. An exact version of the Past Hypothesis (which I call the Strong Past Hypothesis) contains an objectionable kind of arbitrariness not found in any other fundamental laws or dynamical constants—its exact boundary leaves no trace in the material ontology, resulting in a gap between the nomic and the ontic. It violates a theoretical virtue that I call *traceability*.

The case study highlights a dilemma between nomic vagueness and untraceability. In §4, I suggest that, under some conditions, the dilemma is dissolved in a natural way. The conditions are realized in a new quantum theory of time's arrow where there is

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fuzzy boundaries we have in mind here. Another alleged case concerns the measurement axioms of orthodox quantum theory, which I discuss in §2.5.

[4] See Keefe and Smith (1996)§1 for discussions about vagueness and fundamental properties in physics.

an appropriate link between the macrostate and the micro-dynamics. Surprisingly, far from making the world fuzzy or indeterminate, quantum theory can restore exactness in the nomological structure of the world.

2 Nomic Exactness and Nomic Vagueness

In this section, I propose an account of exactness and vagueness of the fundamental laws.

2.1 What They Are

First, we review some features of vagueness in ordinary language predicates. As we discuss later, they have analogues in nomic vagueness. The paradigmatically vague predicates include ones such as *being bald*, *being tall*, *being red*, *being a child*, and *being a heap*. Following Keefe and Smith (1996)§1, we summarize their common features:

(Borderline) Vague predicates (apparently) have borderline cases.

To be a borderline case is to be some object or state of affairs for which the predicates do not determinately apply. John with exactly 5250 hairs on his head is a borderline case of *being bald*. The parenthetical qualifier 'apparently' allows for views such as epistemicism, according to which borderline cases do not really exist. On epistemicism, there *seems* to be borderline cases of baldness because we lack precise knowledge about where the determinate boundaries are.

(No Sharp Boundary) Vague predicates (apparently) do not have well-defined extensions.

A precise extension of *being bald* and a precise extension of *being non-bald* would pick out a precise boundary between the two. Suppose anyone with 6000 or more hairs is non-bald and anyone with fewer than 6000 hairs is bald. Then John would fall under the extension of *being bald*. Alex with exactly 6000 hairs would fall under the extension of *being non-bald*, but if Alex loses just one more hair she would fall out of *being non-bald* and fall into *being bald*.

(Sorites) Vague predicates are susceptible to sorites paradoxes.

It is easy to generate sorites paradoxes on vague predicates. For example, we can start from a case that is determinately bald (having no hair) and proceed to add one hair at a time, argue that at no point can adding one hair make the difference between being bald and being non-bald, and come to the absurd conclusion that no number of hairs will make one non-bald.

(Higher-order Vagueness) Vague predicates (apparently) come with higher-order vagueness.

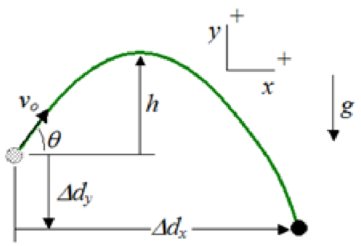

Figure 1: The motion of a projectile under Newtonian mechanics. Picture source: Wikipedia.

Whenever there are (apparent) borderline cases, there are (apparent) borderline borderline cases, and (apparent) borderline borderline borderline cases. This is known as the phenomenon of higher-order vagueness. If it is indeterminate where to draw the line between *being bald* and *being non-bald*, it is natural to think that it is indeterminate where to draw the line between *being bald* and *being borderline bald*, and between *being non-bald* and *being borderline non-bald*, and so on. In other words, it seems inappropriate to draw a sharp line *at any level*. This is part of the genuine fuzziness we are interested in below.

Higher-order vagueness is a challenge to any formal and precise models of vagueness. Even on some degreed theory of vagueness, there will be an exact boundary between maximal determinateness and less-than-maximal determinateness and exact boundaries around any determinate degree of vagueness, which not only seems unfaithful to the phenomena of higher-order vagueness but also amplifies it further. The same point applies to fuzzy, imprecise, "vague" probabilities that are treated in terms of set-valued probabilities. After all, a set of probability measures is still too precise to faithfully represent the phenomenon of higher-order vagueness.[5]

It is natural to expect that, when we reach the fundamental level of reality, everything is perfectly exact and non-vague. In particular, we expect that there is no vagueness in the fundamental physical ontology of the world (the fundamental physical objects and their properties) or in the fundamental nomological structure of the world (the fundamental laws of physics).

How should we understand the exactness of paradigm fundamental laws of nature? Let us start with the familiar case of Newtonian mechanics with Newtonian gravitation. The theory can be formulated as a set of differential equations that admit a determinate set of solutions. Those solutions will specify all and only the possible histories compatible with Newtonian equations ($F = ma$ and $F = Gm_1m_2/r^2$); each solution represents a nomologically possible world of the theory.

For example, let us consider the projectile motion illustrated in Figure 1. Suppose

[5]See Rinard (2017) for insightful arguments against using set-valued probabilities to model imprecise probabilities (IP). See Williamson (1994) for related arguments against degreed theories of truth and indeterminacy. Rinard's argument is relevant to our discussion of the Past Hypothesis. Even if we use a probability distribution or a set of probability distributions concentrated on some macrostate, it is still too precise. To genuinely respect higher-order vagueness, we can replace a set of probability distribution with a vague "collection" of probability distributions, where some distributions will be borderline members of the "collection." Membership turns out to be vague.

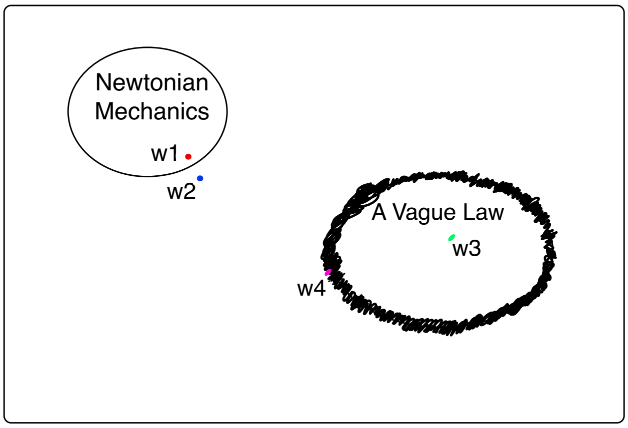

Figure 2: An exact law and a vague law represented in modal space.

that the projectile has unit mass m and the gravitational acceleration is g (we simplify the example by ignoring the rest of the world). We can specify the history of the projectile with the initial height, velocity, maximum height, and distance traveled. There will be a determinate set of histories compatible with Newtonian equations. For any history of the projectile, it is either determinately compatible with the equations or determinately incompatible with the equations. And the same is true when we fully describe the example by accounting for all the massive bodies in the world.

In terms of possible worlds (or models, if one dislikes possible worlds): if W represents the space of all possible worlds, then Newtonian mechanics corresponds to a proper subset in W that has a determinate boundary. Let us call that subset the *domain* of Newtonian mechanics. For any possible world $w \in W$, either w is contained in the domain of Newtonian mechanics or it is not. For example, in Figure 2, $w1$ is inside but $w2$ is outside the set of worlds delineated by Newtonian mechanics. In other words, $w1$ is nomologically possible while $w2$ is nomologically impossible if Newtonian laws are true and fundamental. This suggests that we can capture an aspect of nomic exactness in terms of domain exactness:

Domain Exactness A law L is domain-exact if, for any world $w \in W$, there is a determinate fact about whether w is contained inside L's domain of worlds.

I propose that we understand vagueness as the failure of exactness. Hence, we can use the above analysis of domain exactness to understand domain vagueness. A law L is domain-vague if it fails to be domain-exact.

Domain Vagueness A law L is domain-vague if, for some world $w \in W$, there fails to be a determinate fact about whether w is contained inside L's domain of worlds.

Domain vagueness can occur if the domain of L has a vague boundary. In Figure 2, a domain-vague law is characterized by a "collection" of worlds with a fuzzy boundary. Just as a cloud does not have a clear starting point or a clear end point, the fuzzy "collection" of worlds does not delineate the worlds into those that are clearly compatible and those that are clearly incompatible with the law. For example, $w3$ is clearly contained inside the domain of the vague law, since it is so far away from the fuzzy boundary; but $w4$ is not clearly contained inside the domain of the vague law, and neither is it clearly outside.

Domain vagueness has features similar to those of ordinary-language vagueness:

- A domain-vague law (apparently) has borderline worlds that are not determinately compatible with it.
- A domain-vague law (apparently) does not have well-defined extensions in terms of a set of models or a set of nomological possibilities.
- A domain-vague law is susceptible to sorites paradoxes. We can start from a world that is determinately lawful, proceed to gradually makes small changes to the world along some relevant dimension, and eventually arrive at a world that is determinately unlawful. But no particular small change makes the difference between lawful and unlawful.
- A domain-vague law (apparently) comes with higher-order domain-vagueness. Whenever there are borderline worlds, there are borderline borderline worlds, and so on. It seems inappropriate to draw a sharp line anywhere. This reflects the genuine fuzziness of domain vagueness.[6]

Domain exactness and domain vagueness capture the kind of nomic exactness and nomic vagueness we care about in this paper. We will use them to understand some case studies in the following sections.

However, there is another kind of nomic vagueness that results from the failure of a different kind of nomic exactness. They are much more controversial. Even when L is domain-exact, it may be vague for other reasons. It may assign *vague quantities* (or vague objective probabilities) to some world, some parts of the world, or some set of worlds. First, L may assign vague properties. For example, a (kinematic) law can assign a vague quantity of mass to each massive particle. The vague properties would exemplify a kind of ontic vagueness at the fundamental level.[7] Many people find ontic vagueness unintelligible, let alone ontic vagueness at the fundamental level of physical objects and properties. Notwithstanding some impressive progress on this topic (for example, see Barnes (2010)), it is controversial whether ontic vagueness is possible in the fundamental ontology.

[6]For similar reasons suggested before, a set-valued approach would still be too sharp to model the vagueness here. For example, for a domain-vague law L, one could group the possible worlds into three sets: possible, borderline possible, and impossible. Alternatively, one could assign exact numerical degree of possibility from the continuous range between 1 (maximally possible) and 0 (maximally impossible). But neither would respect the phenomenon of higher-order vagueness or the genuine fuzziness we are interested in here.

[7]See Miller (2019)'s proposal for indeterminate fundamental quantities in the context of EFT. The quantities he consider can be perfectly exact by using set-valued quantities. So it is different from the vagueness we have in mind here.

Second, L may assign vague objective probabilities to a determinate set of worlds. This may be possible if we understand objective probabilities (e.g. in Boltzmannian statistical mechanics and Bohmian mechanics) in terms of *typicality*.[8] Typicality can be a vague notion: what is typical and what is atypical, even in a specific context, do not have exact threshold values. If typicality is vague, it comes with higher-order vagueness and cannot be completely captured by set-valued measures. A set of measures still has a sharp boundary—too precise to capture the fuzziness of higher-order vagueness.

The existence of vague fundamental properties and vague objective probabilities (or typicalities) are controversial. Fortunately, they are not central to our arguments below. Hence, in what follows, I will focus primarily on domain exactness and domain vagueness. In §3, we show that the Past Hypothesis, if it is true and if it can be regarded as a fundamental law, is an instance of a vague fundamental law of nature. It exemplifies domain vagueness: the "collection" of worlds compatible with the Past Hypothesis does not have a sharp boundary.

2.2 What They Are Not

To better understand nomic exactness and nomic vagueness, it would be helpful to say what they are not. Nomic exactness and vagueness concern the application of fundamental laws to complete world histories (histories of the universe) stated in terms of the physically fundamental quantities. For example, when we say that the conjunction of $F = ma$ and $F = Gm_1m_2/r^2$ is (domain-)exact, we mean that any complete microscopic history of the world, described in terms of the positions and masses of all particles, is either determinately lawful or determinately unlawful. Implicitly here, we assume that the fundamental properties only consist in the positions and masses of particles. If there were any other fundamental quantities, such as charges, then the conjunction may not be the complete theory of the world. A world with masses, charges, and positions may still obey the conjunction of $F = ma$ and $F = Gm_1m_2/r^2$ and thus lawful with respect to them. But if the total force is not exhausted by gravitational force, then in general the world will not strictly obey the conjunction of $F = ma$ and $F = Gm_1m_2/r^2$ but some other laws, such as the conjunction of $F = ma$ and $F = Gm_1m_2/r^2 + k_eq_1q_2/r^2$.

Nomic exactness and nomic vagueness do not directly concern the application of fundamental laws to *partial* world histories, such as histories of some subsystems. In many subsystems, they are governed by effective laws—laws that are only approximately true about certain kind of subsystems. For example, when a subsystem is not completely isolated from its environment (the rest of the universe), there may be forces between objects in the subsystem and objects in the environment that are negligibly small but nonzero. In that case, we can, for all practical purposes, treat the subsystem as if it were a closed system and apply the fundamental laws (such

[8]See Goldstein (2012). Fenton-Glynn (2019) offers an account of imprecise chances in the best-system theory. However, he acknowledges that his theory falls into the category of exact laws and chances (with exact set values) and those imprecise values are not vague.

as the gravitational law and Newton's equation of motion) to the subsystem, but with the understanding that the laws are only approximately true. In such cases, the fundamental laws are determinately false about the subsystem. That is not the kind of nomic vagueness we are interested here. Similarly, when a world is quantum mechanical, it may still be approximated to some degree by classical mechanics. In that case, it is not vague whether the actual laws are classical. In a quantum world, the laws are determinately not classical and determinately quantum mechanical.

Moreover, some subsystems may have fuzzy or vague boundaries. Suppose we want to consider a table as a subsystem. Where does it begin and where does it end? Not only are its physical boundaries hard to pin down because of sorites-type reasoning, it is also constantly bombarded by air molecules and radiation so that the table is constantly gaining and losing particles. It brings out another kind of vagueness in the laws: when applying fundamental laws to subsystems, it may be vague what counts as the subsystem. For all practical purposes of calculating the table's influence on my computer, it does not matter as long as the boundaries are within some reasonable margin, because the difference would be very small. (What counts as a reasonable margin is also a vague matter.) But there is a kind of vagueness nonetheless, just not the kind of vagueness we are interested in here.

The above kinds of vagueness, which arise from the applications of fundamental laws to subsystem, disappear when we apply the fundamental laws to the universe (or its history) as a whole. For the universe, we no longer need to worry about influences from the environment or the fuzzy boundaries, as there is nothing outside the universe. It is also reasonable to think that the universe has a determinate (albeit possibly dynamically changing) boundary.

2.3 Semanticism and Epistemicism

Let us turn to some connections between nomic vagueness and two main views about vagueness—semanticism and epistemicism.

According to semanticism, vagueness exists because of certain indeterminacy in our language. Consider John who has exactly 5250 hairs on his head. It is indeterminate whether John is bald. A possible interpretation is that it is neither true nor false that John is bald. This conflicts with the intuitive principle of bivalence that every declarative sentence has only one of the two truth-values: {True, False}. The sentence John is bald has neither truth-value, and is an instance of a truth-value gap.[9] A much discussed version of semanticism (due to Fine (1975)) is called supervaluationism, according to which a sentence is determinately true (or super-true) if it is true under all admissible precisifications. It arguably preserves higher-order vagueness as what counts as admissible is vague.

Semanticism can accommodate nomic vagueness if it arises from the semantic indeterminacy of scientific terms. Let us consider a hypothetical vague law:

L1 The universe is small.

[9]Some versions of semanticism add intermediate truth-values between True and False.

Suppose (though it is obviously unrealistic) L1 is a fundamental law that prescribes nomic necessity: the nomic possible worlds are all and only those worlds that are small. However, it is a domain-vague law because the predicate *small* is vague. What is the extension of smallness? It has no sharp boundary. We can give it a semantic interpretation, which will have interesting consequences for nomic modality. If it is indeterminate whether some worlds are small, then it seems that it would be indeterminate whether they are nomologically possible. On a supervaluationist analysis: a world is determinately lawful (hence determinately nomologically possible) according to L1, if it is compatible with all admissible precisifications of L1. However, if we want to respect higher-order vagueness, admissibility should be understood as vague. Hence, it is unclear how to express L1 in a mathematical language that is perfectly precise and perfectly faithful to higher-order vagueness. A set of "smallness measures" would not do, for it still has sharp boundaries. Neither would a degreed version of smallness. L1 may be mathematically inexpressible. (Even though L1 is obviously unrealistic, two versions of the Past Hypothesis (§3.2) are similar to L1 in their domain-vagueness.)

According to epistemicism, vagueness exists because of our ignorance of the sharp boundaries in the extensions of vague predicates. On this view, there really is a number of hairs that is the minimum for being non-bald. One might appeal to facts about the world, our language, and social conventions as the supervenient base for where the sharp boundary lies (see Williamson (1994)). A consequence of epistemicism is that even though the exact boundaries exist, we can never come to know where exactly they are (Keefe and Smith (1996)).

Applying epistemicism to nomic vagueness, one can maintain that a law is vague due to our (ineliminable) ignorance of what the law really is. There is an exact law—perhaps the true fundamental law—that is hidden from us. There is no need to give up bivalence or any other intuitive principles.[10] In fact, there are no borderline worlds. Vagueness is only in the appearances. Epistemicism may be less attractive in philosophy of language, but one can argue that it is more attractive in the case of nomic vagueness. While it may be implausible to think that something about the world, language, and convention determine the sharp boundary between bald and non-bald, it may be plausible to think that the world has an objective nomological structure that is perfectly sharp and precise, such that every fundamental law is exact, leaving no ambiguities about what is nomologically possible. Epistemicism would also ensure the mathematical expressibility of fundamental laws.

One's intuition in favor of an epistemicist understanding of nomic vagueness may be stronger if one has anti-Humean sympathies (which we discuss below). Nevertheless, it is important to analyze the nature of nomic vagueness on a case-by-case basis. As we see in §3, for the vague Past Hypothesis, the epistemicist understanding conflicts with a reasonable demand for traceability and non-arbitrariness of nature.

[10] It is worth noting that, for ordinary-language vagueness, Raffman (1994) developed a non-epistemic approach that also preserves bivalence and classical logic.

2.4 Humeanism and Anti-Humeanism

If nature suggests there are vague fundamental laws, we should make room for them in our metaphysical accounts, and not the other way around. This way, the possibility of nomic vagueness and any actual cases can serve as important data points in the ongoing debate about which metaphysical theory of lawhood is correct. Nevertheless, one's view about the metaphysics of laws—Humeanism or anti-Humeanism—can influence one's attitude towards nomic vagueness.

As a first approximation, Humeanism and anti-Humeanism disagree about the relative fundamentality of laws and material ontology. On Humeanism, laws are nothing but certain summaries of local facts in spacetime (the Humean mosaic). The summaries have to meet certain standards such as simplicity and informativeness, and only the system that best balances the various standards provides us with the true laws of nature. Being summaries of the mosaic, the laws are *metaphysically explained* by the mosaic. So, the mosaic is metaphysically prior to the laws. In contrast, on anti-Humeanism, laws are part of the fundamental facts of the world. They explain and constrain the patterns on the mosaic. They are as fundamental as the mosaic itself, and both can be fundamental.

While nomic vagueness initially seems to favor Humeanism over anti-Humeanism, the situation is not clear-cut. Consider the following plausible principle:

Fundamental Exactness All the fundamental facts of the world are exact.

There is no problem to maintain fundamental exactness on Humeanism even if some fundamental laws are vague. After all, any laws (and hence any vague laws) are supervenient on the mosaic. Even if the best system is "contaminated" by vagueness, the Humean mosaic does not have to be. The fundamental facts can be exact while the best summaries are vague. For anti-Humeans, vagueness in the fundamental laws may violate the principle of fundamental exactness. If certain vague laws are among the fundamental facts, then not all fundamental facts are exact. Hence, if that principle is a defeasible desideratum, then all else being equal, any actual or possible instance of nomic vagueness would favor Humeanism over anti-Humeanism.

More specifically, on a particular version of anti-Humeanism—the governing view developed by (among others) Armstrong (1983), Tooley (1977), and Maudlin (2007)—laws govern how things are. If governing is an exact notion: a state of affairs is either permitted or banned, then laws cannot be vague. (Perhaps the anti-Humeans can invoke a notion of vague governance for which nomic vagueness is allowed.) If Fundamental Exactness is desirable, then anti-Humeans have motivations to adopt an epistemic approach to nomic vagueness, according to which there is an exact law that is epistemically hidden from us.

However, nomic vagueness may conflict with some versions of Humeanism, such as the one proposed by Lewis (1986), according to which the predicates in the best system refer to perfectly natural properties. The paradigm cases of such properties are fundamental microscopic properties such as mass and charge. The requirement for perfect naturalness avoids the trivialization problem ($\forall xFx$) that worried Lewis. Unfortunately, the kind of properties that occur in vague fundamental laws, such as *having a particular level of entropy* (see §3), are not always perfectly natural—they supervene on certain microscopic properties. They fall under what Cohen and Callender (2009) call "supervenient kinds." Hence, Lewis's Humeanism needs to be revised if such laws are fundamental. Recently, several descendants of Lewis's version of Humeanism have been developed: Loewer's (2007b) package-deal account, Cohen and Callender's (2009) better-best-system account, and Eddon and Meacham's (2015) account. They relax the requirement that terms in the best system have to be perfectly natural. (One might reasonably worry about the theoretical costs that arise from the revisions.) On the revised versions of Humeanism, it is natural to adopt a semanticist approach to nomic vagueness, according to which vagueness comes from the semantic indeterminacy of scientific terms that occur in the relevant best system.

Lewis's original proposal may already contain some resources to accommodate nomic vagueness. The standards for simplicity and informativeness are themselves vague, and the trade-off metric is not determinate. So it could be vague which system is "the best." Lewis *hopes* Nature is kind to us so that there is a clearly best choice such that the vagueness is irrelevant. However, it may not accommodate all cases of nomic vagueness. First, Nature may not be kind to us in that way (such as in the case of the Past Hypothesis). Second, the best system itself may contain vague terms such as entropy.

The issues about semanticism vs. epistemicism, Humeanism vs. anti-Humeanism, and how they interact with nomic vagueness have important consequences. In order to get a concrete understanding of nomic vagueness, we now turn to some cases.

2.5 Vagueness in the Quantum Measurement Axioms?

No physical theory has inspired more discussions about indeterminacy than quantum theory. It has been argued that ontic vagueness is a feature implied by quantum theory (see, for example, Lowe (1994), French and Krause (2003)). However, we now have realist theories of quantum mechanics such as Bohm's theory, GRW collapse theory, and Everett's theory that make quantum mechanics precise (see Myrvold (2017) for a review). In those precise theories, ontic vagueness disappears: there is no indeterminacy in the fundamental ontology or fundamental dynamics. The world can be described as a universal quantum state evolving deterministically (or stochastically) and in some cases guiding and determining precise material objects moving along (deterministically or stochastically), all of which are exact.

However, textbook versions of quantum mechanics seems to suggest a genuine case of nomic vagueness. We focus on the dynamical laws here.[11] Textbook versions suggest that quantum theory contains two kinds of laws: one linear, smooth, and deterministic evolution of the wave function (the Schrödinger equation), and the other stochastic jump of the wave function triggered by measurements of some

[11] For the related issue of "quantum metaphysical indeterminacy" that arises from considerations of realism about quantum observables, see Calosi and Wilson (2019) and the references therein.

system (collapse postulates). However, it is unclear what counts as a measurement, and hence it is unclear when the two dynamical laws apply. For any precise definition we can give, say in terms of the scale or the size of the system, we can imagine a slightly smaller scale or a slightly larger scale that may also work. For any precise boundary between the measured system and the measuring apparatus, we can imagine a somewhat different line that includes the measuring apparatus as part of the measured system, and some outside apparatus as the measurer. So there seems to be no principled way in drawing the boundary between the system and the apparatus and hence no principled definition of the measurement process.

Bell, in his paper "Against Measurement," speaks out against such vagueness in the fundamental axioms of quantum mechanics:

> What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system...with a Ph.D.?....The first charge against 'measurement', in the fundamental axioms of quantum mechanics, is that it anchors there the shifty split of the world into 'system' and 'apparatus'.

Bell's objection is that textbook quantum mechanics is too vague. Even suppose terms such as 'measurement' have determinate cases, it is hard to imagine there to be a sharp split between systems that are measurers and systems that are measured. Hence, there will be histories of the wave function that count as borderline possible or borderline borderline possible, and so on.

The vagueness problem has been resolved in the precise formulations of quantum mechanics of Bohm, GRW, and Everett, making quantum mechanics less worrisome as a case for ontic vagueness or nomic vagueness. (These theories are also predictive and explanatory in ways that the vague axioms are not. In particular, the theories can explain why the vague measurement axioms work for all practical purposes.) In order to find a more realistic case of nomic vagueness, we must look elsewhere.

3 A Case Study of Nomic Vagueness: The Past Hypothesis

In this section, we provide a more realistic case of nomic vagueness that arises from considerations of the arrow of time.

3.1 Temporal Asymmetry and the Past Hypothesis

In a world governed by (essentially) time-symmetric dynamical laws such as classical mechanical equations, quantum mechanical equations, or relativistic equations, it is plausible to think that the time-asymmetric regularities (such as the tendency for entropy to increase and not decrease) cannot be derived from dynamical laws alone.

What else should be added? An influential proposal suggests we postulate a special initial condition: the universe was initially in a low-entropy macrostate, one with a high degree of order. This is now called the Past Hypothesis (PH).[12] Assuming PH (and an accompanying Statistical Postulate (SP) of a uniform probability distribution over possible microstates compatible with the low-entropy macrostate), most likely the universe's entropy increases towards the future and decreases towards the past. The presence of PH, and the absence of a corresponding Future Hypothesis at the other end of time, explains the wide-spread temporal asymmetry. There are several versions of the Past Hypothesis, of varying strengths, which we discuss in §3.2 and §3.3. What is important for our purpose in this section is that PH narrows down the choices of the initial microstate of the universe (a maximally fine-grained description of its physical state): they have to be compatible with some special macrostates (coarse-grained descriptions of the physical state) with low entropy.

To qualify as a case study for nomic vagueness, PH needs to be considered as a possible candidate for a fundamental law of nature. Its nomic status has been taken seriously in the literature. For example, as Feynman (2017)[1965] wrote:

> Therefore I think it is necessary to add to the physical laws the hypothesis that in the past the universe was more ordered, in the technical sense, than it is today—I think this is the additional statement that is needed to make sense, and to make an understanding of the irreversibility.

Making a similar point, Goldstein et al. (2019) also emphasized that PH is an interesting kind of law:

> The past hypothesis is the one crucial assumption we make in addition to the dynamical laws of classical mechanics. The past hypothesis may well have the status of a law of physics—not a dynamical law but a law selecting a set of admissible histories among the solutions of the dynamical laws.

This goes against the idea that all laws are dynamical (laws about temporal evolution). See also Albert (2000), Callender (2004), and Loewer (2007a). What are the reasons for interpreting PH as a law? Moreover, why take it as a *fundamental* law? There are some highly suggestive considerations:

1. The Second Law of Thermodynamics is a non-fundamental law that is scientifically explained (in part) by PH (Feynman (2017)§5 and Albert (2000)).

2. The counterfactual arrow of time depends on nomic facts. Treating PH as a law of nature provides a good explanation for the counterfactual arrow: PH severely constrains the possibility space such that typical (in the sense made

[12]The Past Hypothesis was originally proposed in Boltzmann (2012)[1896] and discussed in Feynman (2017)[1965]. A geometric version was proposed by Penrose (1979). For recent discussions, see Albert (2000), Loewer (2016), Callender (2011), North (2011), Lebowitz (2008), Goldstein (2001), and Goldstein et al. (2019). For critical discussions, see Earman (2006).

precise by SP) histories exemplify counterfactual dependence in one temporal direction only.[13]

3. There is an abundance of physical records about the past but no such records about the future. Having PH as a law of nature explains that. It also explains why such an asymmetry is not accidental or extremely unlikely (Albert (2000)§5).

4. If one is a Humean about laws of nature, i.e. one thinks that laws of nature are theorems of the best system that balances simplicity and informativeness, then one has another argument for the fundamental nomological status of PH. The system containing PH and SP as theorems is way more informative (explaining the wide-spread time asymmetries) than the system without them and is only slightly more complex (Callender (2004) and Loewer (2012)).

Taken together, they seem to outweigh the idea that PH cannot be a fundamental law because it is not dynamical (it dictates not the temporal evolution but only a boundary condition of the universe). To show that it is a fundamental law, we make explicit the hidden premises in 1-3: the explanans of those phenomena should be nomologically necessary, there are no better explanations of those phenomena, and PH is not explained further by other laws. For a non-mathematical and non-logical proposition, its nomic necessity and non-explainability by other laws are together good indicators for its *fundamental* nomological status. So we have good reasons to infer that PH is a fundamental law. They are by no means knock-down arguments, and a rational person can reasonably disagree. But that would come with certain costs. If one does not accept that PH is a fundamental law, then one faces a dilemma. Either PH is nomologically contingent (perhaps as an accidental initial condition of the actual world) or it is explained by other fundamental laws.

For the first horn: if PH is contingent, what explains the *nomological necessity* of the Second Law of Thermodynamics, the counterfactual arrow of time, and the asymmetry and reliability of records? Take the Second Law for example. We know by experiences that it is impossible to build a perpetual motion machine of the second kind, no matter where, when, and how hard you try. This strongly indicates that the Second Law (which forbids it) is nomologically necessary.

One response is that the apparent temporal asymmetries are the results of our world starting in an exact initial microstate that *happens to be* in a low-entropy macrostate. The initial microstate (plus the dynamical laws) explain all the time-asymmetric regularities. This proposal faces difficulties. First, the exact initial microstate is nomologically contingent. The Second Law, then, would become nomologically contingent, since on this view it obtains because of an accidental fact. In contrast, a nomologically necessary PH would make the Second Law nomologically necessary. Second, specifying the exact microstate requires an extraordinary amount of detailed information (such as the positions and momenta of all particles), making the explanans extremely complicated. Other things being equal, we should prefer a

[13] Lewis (1979)'s time-symmetric proposal for counterfactual dependence is not sufficient due to reasons discussed in Elga (2001) and Loewer (2007a).

scientific explanation with simpler explanans. In contrast, PH is much simpler, since it contains only coarse-grained information about the initial condition.

For the second horn, there is some progress in developing further explanations of PH, but success is not guaranteed. For example, Carroll and Chen (2004) developed a multiverse model with unbounded entropy in which "baby universes" are spontaneously created in low-entropy states. In such a model, PH is not a fundamental postulate but a local initial condition induced by time-symmetric dynamical laws that are more fundamental. However, although theoretically possible, it is far from clear whether the actual dynamical laws produce such "baby universes" and produce them in sufficiently low-entropy states.

These considerations provide defeasible reasons to think that, given our current knowledge, it is acceptable to postulate that PH is a fundamental law in our world. It is also acceptable to some "minimal" anti-Humeans. If the central anti-Humean intuition is just that laws don't supervene on the mosaic, why think only dynamical laws can be laws? A "minimal" anti-Humean would be perfectly happy to accept, on scientific grounds, a fundamental boundary-condition law such as the PH (or any boundary condition that earns its status as a law). Moreover, even if one is not willing to accept or take seriously the possibility that PH is a fundamental *law*, one may still accept that, given its nomic necessity and underivability from other laws, PH enjoys an axiomatic status in the fundamental theory of the world. As such, its vagueness has the same ramifications for the nomic modalities and the mathematical expressibility of the fundamental theory in which PH is an axiom.

3.2 Vagueness of the Weak Past Hypothesis

Given the considerations discussed in §3.1, if PH is vague, then we have defeasible reasons to think that nomic vagueness could exist in our world. Is PH vague? To begin, let us consider the following version of PH that is sometimes proposed:

Super Weak Past Hypothesis (SWPH) At one temporal boundary of space-time, the universe has very low entropy.

SWPH is obviously vague. How low is low? The collection of worlds with "low-entropy" initial conditions has fuzzy boundaries in the space of possible worlds. Hence, if SWPH were a fundamental law, then we would have nomic (domain) vagueness.

However, SWPH may not be detailed enough to explain all the temporal asymmetries. For example, in order to explain the temporal asymmetries of records, intervention, and knowledge, Albert (2000) and Loewer (2016) suggest that we need a more specific condition that narrows down the initial microstates to a particular macrostate. One way to specify the macrostate invokes exact numeral values for the macroscopic variables of the early universe. Let S_0, T_0, V_0, D_0 represent the exact values (or exact distributions) of (low) entropy, (high) temperature, (small) volume, and (roughly uniform) density of the initial state. Consider the following version of PH:

Weak Past Hypothesis (WPH) At one temporal boundary of space-time, the universe is in a particular macrostate M_0, specified by the macroscopic variables S_0, T_0, V_0, and D_0.

WPH is a stronger version of PH than SWPH. By picking out a particular (low-entropy) macrostate M_0 from many macrostates, WPH more severely constrains the initial state of the universe. WPH is also more precise than SWPH. (Some may complain that the WPH is too strong and too precise.) Unfortunately, WPH is still vague. The collection of worlds compatible with WPH has fuzzy boundaries. If WPH were a fundamental law, then we would still have nomic (domain) vagueness: there are some worlds whose initial conditions are borderline cases of being in the macrostate M_0, specified by the macroscopic variables S_0, T_0, V_0, and D_0.

The vagueness of WPH is revealed when we connect the macroscopic variables to the microscopic ones. Which set of microstates realizes the macrostate M_0? There is hardly any sharp boundary between those that do and those that do not realize the macrostate. A macrostate, after all, is a coarse-grained description of the physical state. As with many cases of coarse-graining, there can be borderline cases. (To connect to our discussion in §2.1, the vagueness of macrostates is similar to the vagueness of *being bald* and *being a table*.) In fact, a macrostate can be vague even when it is specified with precise values of the macro-variables. This point should be familiar to those working in the foundations of statistical mechanics.[14] However, it is worth spelling out the reasons to understand where and why such vagueness exists.

Let us begin by considering the case of temperature, a macroscopic variable in thermodynamics. Take, for example, the macrostate of having temperature $T = 273.15\text{K}$ (i.e. 0°C or 32F). It is sometimes suggested *without qualifications* that temperature just is average kinetic energy, giving the impression that temperature is exact (because average kinetic energy is exact). The oversimplification is harmless for all practical purposes. However, in our case the qualifications matter. In fact, temperature is vague, even when we use a precise number such as $T = 273.15\text{K}$. Moreover, it is overdetermined that it is vague.

According to kinetic theory of gas, temperature has a microscopic meaning. For example, the temperature of an ideal gas in equilibrium is proportional to its average (translational) kinetic energy. In symbols:

$$\bar{E} = \frac{3}{2} k_B T_k \tag{1}$$

where $\bar{E}$ represents the average kinetic energy of the gas molecules, k_B is the Boltzmann constant, and T_k represents the thermodynamic temperature of the gas. Assuming that the collection of gas molecules is an exact notion, and that each molecule has an exact value of (translational) kinetic energy, then the average kinetic energy of the gas is an exact quantity, which equals the sum of kinetic energies divided by the

[14]Commenting on the vagueness of the macrostate boundaries, Loewer (2007a) writes, "Obviously, the notion of *macro state* is vague and there are many precisifications that would serve the purposes of statistical mechanics." Goldstein et al. (2019) write, "there is some arbitrariness in where exactly to 'draw the boundaries."'

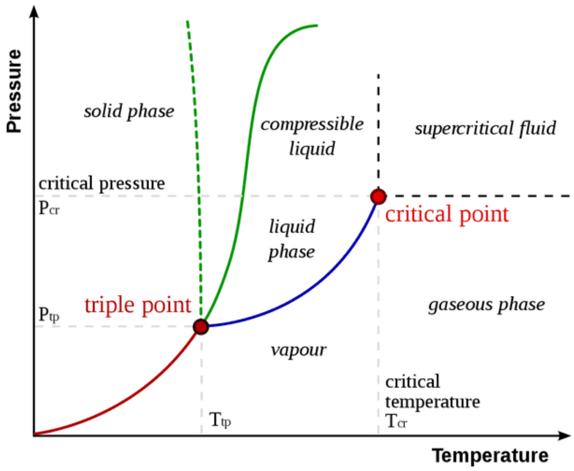

Figure 3: A phase diagram of the triple point of water. Picture source: Wikipedia.

number of molecules. The constant $\frac{3}{2}$ is obviously exact. If k_B has an exact value, then T_k also has an exact value, for the ideal gas at equilibrium. In this case, for certain ideal gasses in equilibrium, they will have the exact temperature $T = 273.15$K.

However, vagueness enters from at least two sources: (1) the Boltzmann constant and (2) the notion of *being in equilibrium*. The upshot is that having temperature $T = 273.15$K is vague and admits borderline cases: for some gasses in the world, it is not determinate whether they are in the macroscopic state of having temperature $T = 273.15$K.

First, the Boltzmann constant, k_B, does not seem to have an exact value known to nature (unless we commit to untraceable arbitrariness to be explained in §3.3). k_B is a physical constant different from those that occur in the dynamical laws, such as the gravitational constant G in $F = Gm_1m_2/r^2$. Unlike G in the law of gravitation, k_B is a *scaling constant*, playing the role of bridging the microscopic scales of molecules and the macroscopic scales of gas in a box. In this sense, k_B is like the Avogadro number N_A. Just as there is no sharp boundary between the microscopic and the macroscopic, there is unlikely a sharp boundary between different values of any macroscopic variable. Historically, k_B is a measured quantity with respect to the triple point of water, a particular state of water where the solid, liquid, and vapor phases of water can coexist in a stable equilibrium (see Figure 3). The triple point also serves as a reference point for $T = 273.15$K. But the picture is highly idealized and assumes a sharp transition that would naturally stand for *the triple point*. In fact, for any body of water in the real world, there is no single *point* that is aptly named the "triple point." At best, there is a quick but smooth transition that only becomes a sudden jump in the infinite limit (e.g. as the number of particles goes to infinity), which does not obtain in the real world.Given the smoothness of the transition, there is no sharp boundary to draw between which states count as $T = 273.15$K and which states do not. And the same issue likely carries over to any phase transition or critical point we encounter. That is the first source of vagueness.[15]

Second, the notion of being in equilibrium is not exact. To apply Equation (1) and calculate the temperature of some real gas in front of us, we need to adjust the equation and account for any differences between the ideal gas law and the real gas. Suppose that can be done without introducing any additional vagueness. To apply the corrected equation, it still needs to be the case that the gas is in thermal equilibrium.

However, some gasses are borderline cases of being in thermal equilibrium. For any gas in a box, thermal equilibrium is the "most likely" state. It is a state that (roughly) requires that the positions of the gas molecules to be evenly distributed in the box and their velocities conform to a particular Gaussian distribution (the Maxwell-Boltzmann distribution). However, the uniform distribution in positions and the Gaussian distribution in velocities obtain in the infinite limit (as the number of gas molecules goes to infinity) and almost never in the real world (when the gas only has a finite number of molecules). For example, a gas of 100 billion molecules almost never has exactly 50 billion on the left half of the box and 50 billion on the right half, just as an unbiased coin flipped 100 billion times almost never produces exactly 50 billion heads and exactly 50 billion tails (it only approaches 50/50 as the number of flips tends to infinity). So, to avoid making equilibrium an "unlikely state," we accept the modification that a gas is in equilibrium if its gas molecules are *more or less* uniformly distributed in positions and *more or less* of the Gaussian distribution in velocities. Hence, a gas of 100 billion molecules can be in equilibrium when exactly 50 billion is in the left half of the box and exactly 50 billion is in the right half of the box; but it can stay in equilibrium if there is one more molecule on the left and one fewer on the right; and it can still stay in equilibrium if there are two more on the left and two fewer on the right; and so on.[16] But when does the gas stop being in equilibrium and start being in non-equilibrium? What is the exact meaning of "more or less" in the modified definition? We can use the strategy in §2.1 and run a *sorites argument* here similar to the one about baldness. Hence, there are real gasses in the world that have the required average kinetic energy $\bar{E}_T$ but nonetheless are borderline cases of being in equilibrium. Thus, such gasses are borderline cases of having temperature $T = 273.15\text{K}$.[17]

Since neither the Boltzmann constant or the notion of being in equilibrium is exact, it is overdetermined that the macrostate of $T = 273.15\text{K}$ is vague. The same goes for any other particular temperature, and any particular level of entropy, pressure, and so on. We have similar reasons to think that almost every other macroscopic variable,

[15] One could of course stipulate an exact value for k_B. This is actually done recently, at the 26th meeting of the General Conference on Weights and Measures, to define k_B with an exact value instead of referring to it as a measured quantity. See www.bipm.org/en/CGPM/db/26/1. We should understand the redefinition as a practical instruction for how to calculate things. But as for the constant k_B known to nature, if it has an exact value, it would contain untraceable arbitrariness. See §3.3.

[16] To fully describe thermal equilibrium, we need to coarse-grain *more finely* into smaller cells than just two halves, and we need to consider momentum degrees of freedom. But the point made above easily generalizes.

[17] A moment's reflection suggests that invoking "local equilibrium" does not help; it faces the same problem of vagueness.

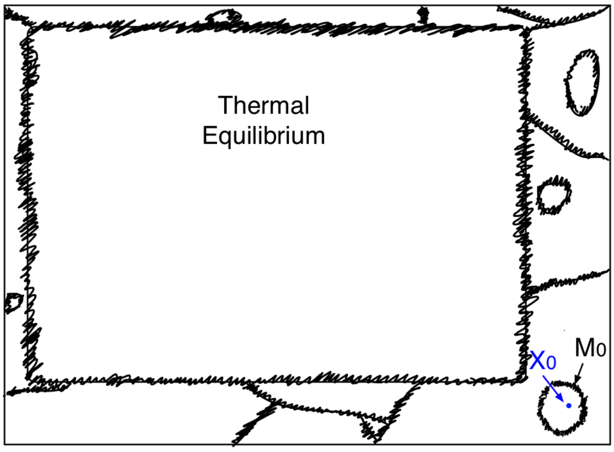

Figure 4: A diagram of phase space where macrostates have fuzzy boundaries. The macrostate M_0 represents the initial low-entropy condition described by WPH. X_0 is the actual initial microstate. The picture is not drawn to scale.

as used in thermodynamics and scientific practice, is vague. Hence, we have good reasons to think that WPH is vague: there are some worlds whose initial conditions are borderline cases of being in the macrostate M_0.

There is a more systematic way to think about the vagueness of the thermodynamic macrostates in general and the vagueness of M_0 in the WPH. In the Boltzmannian account of classical statistical mechanics, macrostates and microstates can be understood as certain structures on phase space (Figure 4).

- Phase space: in classical mechanics, phase space is a 6N-dimensional space that encodes all the microscopic possibilities of the system.
- Microstate: a point in phase space, which is a maximally specific description of a system. In classical mechanics, the microstate specifies the positions and the momenta of all particles.
- Macrostate: a region in phase space in which the points inside are macroscopically similar, which is a less detailed and more coarse-grained description of a system. The largest macrostate is thermal equilibrium.
- Fuzziness: the partition of phase space into macrostates is not determinate; the macrostates have fuzzy boundaries. Their boundaries become exact only given some choices of the "C-parameters", including the size of cells for coarse-graining and the correspondence between distribution functions and macroscopic variables.
- Entropy: $S(x) = k_B \log|M(x)|$, where $|\cdot|$ denotes the standard volume measure in phase space. Because of Fuzziness, in general, the (Boltzmann) entropy of a system is not exact.

We can translate WPH into the language of phase space: at one temporal boundary of space-time, the microstate of the universe X_0 lies inside a particular macrostate M_0 that has low volume in phase space.

Fuzziness is crucial for understanding the vagueness of the macrostates. Without specifying the exact values (or exact ranges of values) of the C-parameters, the macrostates have fuzzy boundaries: some microstates are borderline worlds for certain macrostates. The fuzzy boundaries of M_0 illustrate the existence of borderline microstates. There will be a precise identification of macrostates with sets of microstates only when we exactly specify the C-parameters (or their ranges). In other words, there is a precise partition of microstates on phase space into regions that are macroscopically similar (macrostates) only when we make some arbitrary choices about what the C-parameters are. In such situations, the WPH macrostate M_0 would correspond to an exact set Γ_0 on phase space, and the initial microstate has to be contained in Γ_0.

However, proponents of the WPH do not specify a precise set. A precise set Γ_0 would require more precision than is given in statistical mechanics—it requires the specific values of the coarse-grained cells and the specific correspondence with distribution functions. (In the standard quantum case to be discussed in §4.2, it also requires the precise cut-off threshold for when a superposition belongs to a macrostate.) The precise values of the C-parameters could be added to the theory to make WPH into a precise statement (which we call the Strong Past Hypothesis in the next section). But they are nowhere to be found in the proposal (and rightly so).[18]

Some choices of the C-parameters are clearly unacceptable. If the coarse-graining cells are too large, they cannot reflect the variations in the values of macroscopic variables; if the coarse-graining cells are too small, they may not contain enough gas molecules to be statistically significant. Hence, they have to be macroscopically small but microscopically large (Albert (2000) p.44(fn.5) and Goldstein et al. (2019)). However, if we were to make the parameters (or the ranges of parameters) more and more precise, beyond a certain point, any extra precision in the choice would seem completely arbitrary. They correspond to how large the cells are and which function is the correct one when defining the relation between temperature and sets of microstates. That does not seem to correspond to any objective facts in the world. (How large is large enough and how small is small enough?) In this respect, the arbitrariness in precise C-parameters is quite unlike that in the fundamental dynamical constants. (In §3.3, we discuss their differences in terms of a theoretical virtue called 'traceability.') Moreover, not only do we lack precise parameters, we also lack a precise set of permissible parameters (hence no exact ranges of values for the C-parameters). There do not appear to be sharp boundaries for *being determinately large enough*, *being determinately determinately large enough*, and so on. This is the phenomenon of higher-order vagueness that often comes with first-order vagueness.

The vagueness here is appropriate since macroscopic variables only make sense when there are enough degrees of freedom (such as a large number of particles). In

[18] For example, see descriptions of SWPH and WPH in Goldstein (2001), Albert (2000), and Carroll (2010).

practice, however, such vagueness rarely matters: there will be enough margins for error such that to explain the thermodynamic phenomena, which are themselves vague, we do not need the extra exactness. The vagueness disappears *for all practical purposes*. Nevertheless, WPH is a genuine case of nomic vagueness and it is a possibility to take seriously.

3.3 Untraceable Arbitrariness of the Strong Past Hypothesis

Given the expectation (which may be more strongly held if one is an anti-Humean) that fundamental laws should be exact, it would be natural to consider the possibility of an exact version of the Past Hypothesis. That is, there is an exact law known to nature, and the vagueness of WPH is only epistemic: there is, in fact, a precise set Γ_0 with exact boundaries on phase space that stands in for the initial macrostate.

Let us now consider the exact version of PH:

Strong Past Hypothesis (SPH) At one temporal boundary of space-time, the microstate of the universe is in Γ_0, where Γ_0 corresponds to a particular admissible precisification of M_0.

Unlike WPH, SPH is exact. As such, it is mathematically expressible. However, as we explain below, SPH violates a plausible feature that every other fundamental law and dynamical constant satisfies: SPH is "untraceable." The exact boundary of Γ_0 does not "leave a trace" in typical worlds compatible with it. Hence, SPH is arbitrary in a way that other exact fundamental postulates in physics are not. Moreover, it widens the gap between the ontic and the nomic. Other things being equal, that seems to make it less appealing among proposals for understanding the Past Hypothesis as a fundamental postulate.

On the epistemic interpretation of vagueness, there is in fact an exact number of hairs, n, that turns someone from being bald to being non-bald. But the number n is not known to us. In fact, it cannot be known to us in any way. Similarly, there are in fact exact boundaries of the macrostate M_0, represented by the set of microstates Γ_0 on phase space. The exact set can be picked out only by the unhelpful description "the set that is invoked by the SPH." Which set it is is unknown and likely unknowable by empirical investigations (as we explain below). However, many things that are true of nature may be unknown or unknowable to us, as a consequence of certain physical laws. There are examples of in-principle limitations of knowledge in well-defined physical theories such as Bohmian mechanics and GRW collapse theories (Cowan and Tumulka (2016)). Moreover, we may not know the exact values of the fundamental constants and the fundamental dynamical laws (if not forever then at least for a long time). Hence, knowledge and knowability about the precise boundaries of Γ_0 cannot be the issue, for that may also arise for other fundamental laws and dynamical constants that we think are fine. Neither is the problem that the postulate of a precise set Γ_0 would be a brute fact that is not explained further. Every other fundamental law or dynamical constant is supposed to be also brute and not explained further (in the scientific sense and not in the metaphysical sense of explanation).

What sets the arbitrariness of SPH apart from that of the dynamical constants and other fundamental laws is its *untraceability*. The exact boundaries of SPH are typically untraceable. There are infinitely many ways to change the boundaries of Γ_0 that do not lead to any differences for most worlds SPH deems possible. Hence, Γ_0 does not leave a trace in most worlds compatible with it.

Let us make this notion of traceability more precise. It is plausible that the objective features of the world are reflected in the changes in the properties of particles, the field configurations, the mass densities, the space-time geometry, and so on. Such changes do not have to be measured or measurable by human beings. But for the familiar fundamental laws and their dynamical constants, typical changes in their exact values will be "felt" by the matter distributions (or some other part of the fundamental ontology excluding the fundamental laws) in the nomologically possible worlds. That is, there are some worldly features in the fundamental ontology that are sensitive to typical changes in the "nomology." For example, any changes in the gravitational constant G will be felt by the massive objects and will change (however slightly or significantly) the motion of planets around stars, the formation of galaxies, and the distribution of fundamental matter. On a closer scale, it affects how exactly my vases shatter when they hit the ground. In other words, there should be *some traces in the material ontology* of the world. If the value of G had been different, the material ontology would have been different. We can capture this idea modally as changes in the nomological status (from possible to impossible) or the objective (conditional) probability (e.g. from 0.8 to 0.3 given prior histories) of the world. We formulate the following condition on traceability:

Traceability-at-a-World A certain adjustable parameter O in the physical law L is *traceable at world w* if any change in O (while holding other parameters fixed) will result in some change in the nomological status of w with respect to L, i.e. from possible to impossible or from likely to unlikely (or some other change in the probabilistic measures).

We are treating "adjustable parameter" in a loose sense. For example, in the case of Newtonian theory of gravitation with $F_G = Gm_1m_2/r^2$, we can adjust it in the following (independent) ways:

- Change the constant G= 6.67430 to G′=6.68 (in the appropriate unit);
- Change division by r^2 to division by $r^{2.001}$;
- Change the multiplication by m_1 to multiplication by $m_1^{1.00001}$.

All such changes are *traceable at typical worlds* that satisfy Newton's law of motion and law of universal gravitation. For a typical Newtonian world whose microscopic history h is a solution to the Newtonian laws, h will not be possible given any of those changes. In other words, it will change a typical history h from nomologically possible to nomologically impossible with respect to Newtonian theory of gravitation. Here, we are interested in traceability at most worlds that are allowed by L. This is because there may be cases in which for "accidental" reasons two different values of O may produce the same world w in exact microscopic details. So, a change of the value of O will not change w from possible to impossible or change its probability. Such cases would be atypical. The relevant property is this:

Traceability A certain adjustable parameter O in the physical law L is *traceable* if O is traceable at most worlds allowed by L.[19]

If some degree of freedom O is traceable at most worlds, then at most worlds (typically) the value of O can be determined to arbitrary microscopic precision. Then, normally, the more information we know about the actual world the more precise we can determine the value of O. However, what matters is not our epistemic access. For typical worlds compatible with L, if it is deterministic, most worlds will only admit one value of O. That is the case for the gravitational constant G.

Similarly, the laws and dynamical constants of Maxwellian electrodynamics are traceable; those of Bohmian mechanics are traceable; those of Everettian quantum theory are traceable; those of special and general relativity are traceable. In those theories, there is a tight connection between the nomic and the ontic. Typically the precision of the laws leaves traces in the material ontology.

A stochastic theory such as GRW presents an interesting wrinkle. The GRW theory postulates two fundamental constants: the collapse rate λ and the collapse width σ. Consider just the collapse rate λ that describes the probability of collapse (per-particle-per-unit-time). Since it is a probabilistic theory, the same history of quantum states can be compatible with distinct values of λ. What λ does is to provide a probability measure (together with a slightly-modified Born-rule probability measure) that tells us which collapse histories are typical (or very probable) and which are not. However, each micro-history receives zero measure. It is the macro-history (considered as a set of micro-histories that are macroscopically similar) that can receive positive probabilities. Hence, in a stochastic theory, we should understand the appropriate change of nomological status as changes in the probabilistic measure of the macro-history that the micro-history realizes.

Although familiar laws in physics are traceable, SPH is not. To see this, consider Γ_0 and another set Γ_0' that has slightly different boundaries (see Figure 5). Suppose both are admissible precisifications of M_0 and both include the actual initial microstate X_0 as a member. Then the world starting in microstate X_0 is compatible with SPH and another law SPH' that slightly alters the boundaries of Γ_0. Moreover, this is the case for typical worlds compatible with Γ_0: at most worlds compatible with Γ_0, slightly altering the boundaries of Γ_0 will not make a difference to the nomological status of

[19]Here "most" is with respect to some natural measure in the state space such as the Lebesgue measure in phase space or normalized surface area measure in Hilbert space. The threshold size for a set to qualify as most or typical is vague. As such, there may be borderline cases of traceable parameters. This is to be expected, as traceability is supposed to be a theoretical virtue; like other theoretical virtues, it can be vague. But the examples we encounter here are clear-cut.

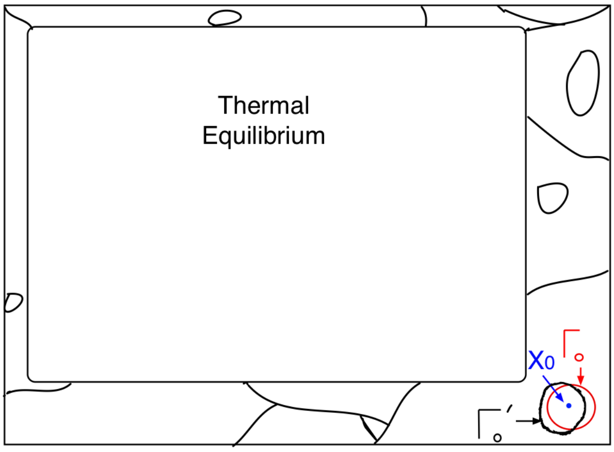

Figure 5: A diagram of phase space where macrostates have exact boundaries. Γ_0 and Γ_0' are two admissible precisifications of M_0. The actual initial microstate X_0 lies inside both.

the worlds. (At some atypical worlds very close to the boundaries of Γ_0, altering the boundaries will take them from being possible to being impossible or vice versa.) For most worlds inside Γ_0, there will be infinitely many changes to the boundaries of Γ_0 that do not affect the probabilities of those worlds. We discuss this more in §3.4.

Hence, SPH is not traceable.[20] And there lies the key difference between SPH and other fundamental laws and constants. The former is arbitrary in a way the latter are not: SPH has untraceable arbitrariness. For traceable laws and constants such as G, their values may be arbitrarily precise; their values are not explained further. However, they still respect a close connection between the nomic and the ontic: their exact values are reflected in the material ontology. That is not the case of SPH; the exact boundaries of Γ_0 outrun the ontic; the exact choice of Γ_0 is not reflected in the material ontology. *Other things being equal, we should minimize the gap between the ontic and the nomic.* (To emphasize: this is different from the gap between the nomic and what's epistemically accessible, for plenty of facts about the material ontology may forever lie beyond our epistemic ken.)

It is implausible that we can appeal to super-empirical virtues to pin down Γ_0. Take for example the theoretical virtue of simplicity. It is unlikely that there will be a simplest precisification of M_0, just as it is unlikely there is a simplest choice of the coarse-graining size (or other C-parameters). Furthermore, those theoretical virtues are themselves vague. In cases where Nature is kind to us, there may be a choice that is by far the simplest (or best balances various virtues) that their vagueness makes no difference. However, although we may have *faith* in Nature's kindness, we have no *reason* to think that SPH is such a case.

[20]Because of higher-order vagueness, the same will be true for a disjunctive version of SPH that says that the initial microstate belongs to a determinate set of precisifications, such as: X_0 is in Γ_0 or Γ_0' or Γ_0'' or Γ_0'''.

Endorsing SPH leads to endorsing some untraceable arbitrariness in Nature. Although it is not impossible Nature acts in this strange way, if every other fundamental postulate and dynamical constant in physics seems to respect traceability, conservativeness suggests that we try to keep it if we can. We should respect the tight connection between the nomic and the ontic by not letting in untreaceable arbitrariness.

Interestingly, although traceability may seem like a novel theoretical virtue, it explains our different attitude towards the quantum measurement axioms (§2.5). Many philosophers of physics are unfriendly towards a fundamental yet vague quantum measurement axiom, but some of them will be much happier with a fundamental WPH. Both are vague. What can be a principled reason that distinguishes the two cases? In the case of WPH, its exact alternative (SPH) with precise boundaries is untraceable. In the case of the quantum measurement axiom, its exact alternative is in fact *traceable*: different cut-offs in the law will typically lead to differences in the fundamental material ontology. All else being equal, if we can avoid nomic vagueness without committing untraceable arbitrariness, we should prefer an exact alternative. But if we can do it only if we commit untraceable arbitrariness, then a fundamental yet vague law is perfectly acceptable.

Nevertheless, for people who want to avoid nomic vagueness at all costs, they can still choose SPH over WPH. Therefore, a more neutral way to summarize our findings so far is that we face a dilemma: either accept vague fundamental laws such as SWPH and WPH, or violate traceability by adopting SPH.[21] There is no free lunch in Nature; either way we have to pay.

3.4 Contrast with One-Parameter Chance Hypotheses

One might worry that we have proven too much. The Past Hypothesis and the Statistical Postulate are surely similar to the other chance hypotheses in the following sense. PH+SP assign a probability to the macroscopic histories, and in general different choices of the presicification of M_0 will make a difference to the probability of the macro-histories. But in the familiar example of the coin case, with different hypotheses given by different weights to the bias towards heads, such as 50-50 and 51-49, any small difference in the biases (chancy hypotheses) will assign a different likelihood to the actual frequencies of the outcomes: HTHTTTHTHH... Hence, if we use Bayesian updating, assuming that we start with uniform priors of the competing chance hypotheses, exactly one hypothesis will receive the highest

[21] Traceability is different from Lewis's thesis of Humean supervenience (HS): no differences in the nomology without there being some differences in the mosaic. For example, they do not imply each other. For one direction, we note that it is logically possible that SPH is the best summary of the mosaic in virtue of being the simplest system (and tied with the rest on other best-system criteria). Yet SPH can still fail traceability, for the reasons discussed earlier. For the other direction, we note that it is logically possible that there are non-Humean GRW-type laws—GRW1 and GRW2—that are stochastic and have different collapse rates λ_1 and λ_2. Since they are non-Humean, they can obtain with the same mosaic. Yet the collapse rate can still be traceable: different choices of the collapse rate can assign different probability measures over typical worlds (their macro-histories).

posterior probability after the full run. Surely the same can be said about PH+SP and the precise version SPH?

However, they are different in a crucial way. Two stochastic theories with difference chances for the coin landing heads will be confirmed to different degrees after a finite sequence of coin flips. Here is an example:

- Theory 1: the chance of coin landing heads is 0.5.
- Theory 2: the chance of coin landing heads is 0.5005

Suppose the actual outcomes are: HTHHHTHTHTHH (more heads than tails). The two theories are not empirical equivalent with respect to the actual outcomes. Assuming Theory 2 assigns a (slightly) higher probability to the actual sequence than Theory 1 does, the actual sequence of coin flips confirms Theory 2 (slightly) more than Theory 1. Assuming equal priors, then Theory 2 deserves higher posterior probability than Theory 1.

However, that's not always the case for different hypotheses about the precise boundaries of M_0, the macrostate of PH. Consider again Γ_0, the precisification of the initial macrostate used in SPH. Let S_1 and S_2 be two regions next to the boundary of Γ_0, such that the following are also admissible ways of drawing the boundaries of the initial macrostate.

- Theory 3: the initial phase point lies in $\Gamma_0 \bigcup S_1$, with uniform probability over the combined region.
- Theory 4: the initial phase point lies in the union of $\Gamma_0 \bigcup S_2$, with uniform probability over the combined region.

Suppose the actual outcomes can be stated as a sequence of pointer readings; call that E. Now, Theory 3 and Theory 4 may agree on the probability of E, the actual sequence of pointer readings, as long as the following two ratios are equal:

- i) the measure of microstates realizing E divided by the measure of $\Gamma_0 \bigcup S_1$
- ii) the measure of microstates realizing E divided by the measure of $\Gamma_0 \bigcup S_2$

In this case, given all the evidence in the actual universe, the two theories are confirmed to the same degree, i.e. there are no confirmatory differences between Theory 3 and Theory 4 with respect to the actual evidence E. If we gather all the evidence in the world, there will still be many versions of SPH that are confirmed to exactly the same degree. Hence, the exact boundary of M_0 cannot be discovered empirically even if we have access to every relevant fact in the world. We may be able to narrow down to some Γ's that are most probable, but unlike in the coin case, no amount of data can pin down the precise boundary of the PH macrostate. Moreover, the set of Γ's that are most probable will include candidate precisifications of M_0 that are inadmissible. But admissibility is itself vague. So, no amount of data can pin down even a precise set of boundaries for M_0.

The key difference with the coin case is that we consider different chance hypotheses only along one dimension of variation (e.g. the chance is 0.4, or 0.5, or 0.6). Different chance hypotheses generate different likelihoods for the actual outcomes. The same is true for different hypotheses about the GRW collapse rate. But in the SPH case there are many dimensions of variation such that different precisifications of M_0 may generate the same likelihood probabilities for the actual outcomes. Hence, the arbitrariness in SPH is untreaceable even probabilistically, while the usual stochastic theories with precise chances are traceable.

4 Nomic Exactness without Untraceable Arbitrariness

4.1 Motivations

As we discuss in the previous sections, we have reasons to take seriously the possibility that PH is a fundamental law of nature, and we also have reasons to think that an exact version of of (SPH) leads to untraceable arbitrariness. So we seem to face a dilemma: either accept nomic vagueness, or accept a theory that disrespects traceability. Let us go further and get a better understanding of the trade-off and under what conditions the dilemma may be dissolved.

The origin of the untraceable arbitrariness can be seen in the *macroscopic* character of PH. It is a constraint on the initial macrostate of the universe, and indrectly on the microstate. Given a particular microstate, it screens off any dynamical influence of the macrostate, although the macrostate will still be highly informative. It is therefore a possibility that the untraceable arbitrariness (and consequently the vagueness) may be eliminated if certain conditions are met, such as when the macrostate is given a microscopic role. In this section, we present a concrete realization of such conditions. We show that nomic vagueness can be eliminated without the objectionable kind of arbitrariness in theories where we appropriately connect PH to the fundamental micro-dynamics. We appeal to a new class of quantum theories for a time-asymmetric universe in which the fundamental quantum state is mixed rather than pure and in which the PH is replaced with another postulate called the Initial Projection Hypothesis. The result is an initial low-entropy condition that is as traceable as other fundamental laws or dynamical constants. This provides a surprising twist to our story: far from making the world fuzzy or indeterminate, quantum theory holds the key to restore nomic exactness without untraceable arbitrariness.

4.2 The Standard Quantum Case: Still Vague

In what follows, we will briefly explain the standard framework of the Boltzmannian account of quantum statistical mechanics (Goldstein et al. (2019)). It has the following ingredients:

- Hilbert space: Hilbert space is a vector space equipped with inner product structure that encodes all the microscopic possibilities (possible worlds) of the system (or the universe as a whole).

- Microstate: a vector in Hilbert space, a maximally specific description of a system.[22]
- Macrostate: a subspace in Hilbert space in which the quantum states contained within are macroscopically similar, which is a less detailed and more coarse-grained description of a system. The Hilbert space is orthogonally decomposed into subspaces.
- Fuzziness: the decomposition of Hilbert space into macrostates is not determinate; the macrostates have fuzzy boundaries. Their boundaries become exact only given some choices of the C-parameters, including the size of cells for coarse-graining, the correspondence between distribution functions and macroscopic variables, and the cut-off threshold for macrostate inclusion.
- Entropy: $S(\psi) = k_B \text{dim} \mathscr{H}$, where dim denotes the dimension counting in Hilbert space and $\mathscr{H}$ is the subspace that contains most of ψ. Higher-dimensional subspaces tend to have higher entropies. Because of Fuzziness, in general, the (Boltzmann) entropy of a system is not exact.

We can translate the Weak Past Hypothesis in the language of Hilbert space:

Quantum Weak Past Hypothesis (QWPH) At one temporal boundary of space-time, the wave function of the universe is in a particular macrostate M_0, where M_0 is the low-entropy macrostate characterized by the Big Bang cosmology.

The Statistical Postulate would take the following form:

Quantum Statistical Postulate (QSP) At one temporal boundary of space-time, the probability distribution is the uniform one (with respect to the normalized surface area measure) over wave functions compatible with QWPH.

As in the classical case, both QWPH and QSP are vague laws. Given some choices of C-parameters, the macrostate M_0 will correspond to a precise subspace $\mathscr{H}_{PH}$ and the uniform probability distribution will be the surface area measure on the unit sphere in $\mathscr{H}_{PH}$. The actual wave function Ψ_0 will be dynamically central, and it basically screens off the dynamical influence of $\mathscr{H}_{PH}$ on the microscopic histories. So the precision of $\mathscr{H}_{PH}$ becomes untraceable at typical worlds (represented by different wave functions). Hence, a Quantum Strong Past Hypothesis (QSPH) that chooses a particular precise subspace $\mathscr{H}_0$ will be untraceable. Hence, if we stick with these versions of PH, we still have the dilemma between nomic vagueness and untraceable arbitrariness.

[22]It is possible to have additional ontologies such as the Bohmian particles and the GRW mass densities.

4.3 Linking the Macrostate to the Micro-Dynamics

In quantum theory lies a new possibility for avoiding both nomic vagueness and untraceability. Although it is simple and natural, it is easily overlooked. Quantum mechanics in a time-asymmetric universe offers a distinct kind of micro-dynamics that directly connects to the initial macrostate.

First, we observe the following. Given a particular choice of the subspace $\mathscr{H}_{PH}$, we have a simple choice for the initial quantum state: a unique quantum mixed state (density matrix) that is the normalized projection operator onto $\mathscr{H}_{PH}$:

$$W_0 = \frac{\mathbb{I}_{PH}}{\dim \mathscr{H}_{PH}} \tag{2}$$

where $\mathbb{I}_{PH}$ designates the projection operator onto $\mathscr{H}_{PH}$ and dim counts the dimension of that subspace. W_0 is called a density matrix. It is compatible with many different wave functions, and it can be obtained from several probability distributions over wave functions: one is the continuous uniform one on the unit sphere in the subspace, and another is the discrete distribution on the basis vectors of the subspace. Density matrices have traditionally been playing an epistemic role, encoding our ignorances over the underlying wave function.

Second, recent works in the foundations of quantum mechanics suggest that density matrices can directly represent fundamental quantum states.[23] Chen (2018a) calls the view *density matrix realism*. It reverses the traditional relationship between wave functions and density matrices. On the traditional view, the universe is described by a pure state, but we don't know which it is. Our ignorance is represented by a mixed state density matrix. On density matrix realism, the fundamental state of the universe is mixed rather than pure and it has to be described by a density matrix rather than a wave function. There is no longer a fact of the matter about the underlying pure state, as the density matrix is not merely epistemic. Since the density matrix is fundamental, the *fundamental micro-dynamics* needs to be revised (e.g. à la Allori et al. (2013)) to reflect the change: we replace the Schrödinger equation with the von Neumann equation, the Bohmian guidance equation with another that uses the density matrix as an input, the GRW collapse equations with another that stochastically evolves the density matrix, and various definitions of local beables from the wave function with their density-matrix counterparts. Moreover, the density-matrix versions of Bohm, GRW, and Everett are empirically equivalent to the respective wave-function versions.

Third, and most importantly, density matrix realism offers a new possibility for reformulating the Past Hypothesis. We can simply postulate W_0 as the actual initial quantum state and replace the low-entropy initial condition with the following:

Initial Projection Hypothesis (IPH) At one temporal boundary of space-time, the quantum state of the universe is exactly W_0 as described in Equation (2).

[23] See, for example, Dürr et al. (2005), Maroney (2005), Wallace (2011, 2012), Wallace (2016), and Chen (2018a).

This version of PH is exact. The combination of density matrix realism and IPH does the heavy-lifting. The low-entropy initial condition is completely and unambiguously described in W_0. Unlike the SPH or its quantum version QSPH we encountered earlier, W_0 enters directly into the fundamental micro-dynamics. Hence, W_0 will be traceable, from the perspective of two realist interpretations of the quantum state (Chen (2018b)):

1. W_0 is ontological: if the initial density matrix represents something in the fundamental material ontology, IPH is obviously traceable. Any changes to the physical values W_0 will leave a trace in every world compatible with IPH.

2. W_0 is nomological: if the initial density matrix is on a par with the fundamental laws, then W_0 plays the same role as the classical Hamiltonian function or fundamental dynamical constant of nature. It is traceable in the Everettian version with a matter-density ontology as the initial matter-density is obtained from W_0. It is similarly traceable in the GRW version with a matter-density ontology. For the GRW version with a flash ontology, different choices of W_0 will in general lead to different probabilities of the macro-histories. In the Bohmian version, different choices of W_0 will lead to different velocity fields such that for typical initial particle configurations (and hence typical worlds compatible with the theory) they will take on different trajectories.

The traceability of W_0 is due to the fact that we have connected the low-entropy macrostate (now represented by W_0) to the micro-dynamics (in which W_0 occurs). Hence, W_0 is playing a dual role at t_0 (and only at that time): it is both the microstate and the macrostate. In contrast, the untraceability of Γ_0 in the classical-mechanical SPH is due to the fact that classical equations of motion directly involve only the microstate X_0, not Γ_0. Similarly, the $\mathscr{H}_0$ in the standard wave-function version of QSPH is untraceable because the Schrödinger equation directly involves only the wave function, not $\mathscr{H}_0$. There are many changes to Γ_0 and to $\mathscr{H}_0$ that make no changes whatsoever in typical worlds compatible with those postulates.

By linking the macrostate to the microstate through the Initial Projection Hypothesis and by using different microdynamics, we have achieved a kind of unification. It eliminates the source of nomic vagueness without the objectionable arbitrariness, as IPH and the dynamics are traceable. In this class of quantum theories, we have preserved the mathematical expressibility of fundamental laws and kept the close connection between the ontic and the nomic. (These theories have other payoffs, but we omit the discussion here.)

Nomic vagueness is a persistent feature in standard versions of the Past Hypothesis, both classical and quantum. However, by adopting a distinct yet empirically equivalent quantum framework we restore nomic exactness without introducing untraceable arbitrariness. Moreover, we have realist theories of quantum mechanics that do away with the vagueness in the micro-dynamics. Hence, far from the conventional wisdom that quantum theory introduces vagueness, it can help us get rid of it.

5 Conclusion

Nomic vagueness is a new species of vagueness that is (*prima facie*) distinct from semantic vagueness, epistemic vagueness, and ontic vagueness. It is vagueness in the fundamental laws of nature. To find out whether a fundamental law is vague, we can ask whether its domain (set of models) is vague and whether it admits borderline nomologically possible worlds. This account also makes room for vague chances, which will be left to future work.

It is surprising, whether from a Humean or a non-Humean perspective, that actual fundamental laws of nature can fail to be exact. We normally expect all fundamental laws to be completely expressible by precise mathematical equations. That expectation would be mistaken (because of higher-order vagueness) if actual fundamental laws include vague ones such as the Weak Past Hypothesis or the Super Weak Past Hypothesis. One can use the Strong Past Hypothesis to get rid of the vagueness by fiat. However, it introduces *untraceable arbitrariness* that is unseen in any kind of fundamental laws or dynamical constants. Surprisingly, the trade-off between nomic vagueness and untraceability can be avoided when we directly use the initial macrostate to dictate the motion of fundamental material objects, such as when we combine density matrix realism with the Initial Projection Hypothesis.

There may be cases of nomic vagueness that cannot be eliminated in a similar manner and cases of arbitrariness that have a different character. Then one's position on the metaphysics of laws could make a difference about how one should deal with nomic vagueness and arbitrariness. But as the case study shows, the issue is delicate and should not be settled in advance. There may be all sorts of intricate and interesting details concerning metaphysics and physics that can make a difference to how much we should tolerate vagueness in the fundamental laws of nature. Attending to those details may also teach us something new about the nature of laws.

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References

Albert, D. Z. (2000). *Time and chance*. Harvard University Press.

Allori, V., Goldstein, S., Tumulka, R., and Zanghì, N. (2013). Predictions and primitive ontology in quantum foundations: a study of examples. *The British Journal for the Philosophy of Science*, 65(2):323–352.

Armstrong, D. M. (1983). *What Is A Law Of Nature?* Cambridge University Press, Cambridge.

Barnes, E. (2010). Ontic vagueness: A guide for the perplexed. *Noûs*, 44(4):601–627.

Boltzmann, L. (2012). *Lectures on gas theory*. Courier Corporation.

Callender, C. (2004). Measures, explanations and the past: Should 'special' initial conditions be explained? *The British journal for the philosophy of science*, 55(2):195–217.

Callender, C. (2011). Thermodynamic asymmetry in time. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2011 edition.

Calosi, C. and Wilson, J. (2019). Quantum metaphysical indeterminacy. *Philosophical Studies*, 176(10):2599–2627.

Carroll, S. (2010). *From eternity to here: the quest for the ultimate theory of time*. Penguin.

Carroll, S. M. and Chen, J. (2004). Spontaneous inflation and the origin of the arrow of time. *arXiv preprint hep-th/0410270*.

Chen, E. K. (2018a). Quantum mechanics in a time-asymmetric universe: On the nature of the initial quantum state. *The British Journal for the Philosophy of Science*, forthcoming.

Chen, E. K. (2018b). Realism about the wave function. Forthcoming in *Philosophy Compass. PhilSci-archive preprint:15153.*

Cohen, J. and Callender, C. (2009). A better best system account of lawhood. *Philosophical Studies*, 145(1):1–34.

Cowan, C. W. and Tumulka, R. (2016). Epistemology of wave function collapse in quantum physics. *The British Journal for the Philosophy of Science*, 67(2):405–434.

Dürr, D., Goldstein, S., Tumulka, R., and Zanghì, N. (2005). On the role of density matrices in bohmian mechanics. *Foundations of Physics*, 35(3):449–467.

Earman, J. (2006). The "past hypothesis": Not even false. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 37(3):399–430.

Eddon, M. and Meacham, C. J. (2015). No work for a theory of universals. *A Companion to David Lewis*, 57:116.

Elga, A. (2001). Statistical mechanics and the asymmetry of counterfactual dependence. *Philosophy of Science*, 68(S3):S313–S324.

Fenton-Glynn, L. (2019). Imprecise chance and the best system analysis. *Philosophers' Imprint*, 19(23):1–44.

Feynman, R. (2017). *The Character of Physical Law*. MIT press.

Fine, K. (1975). Vagueness, truth and logic. *Synthese*, 30:265–300.

Frege, G. (1973). *Begriffsschrift und andere Aufsätze*. Georg Olms Verlag.

French, S. and Krause, D. (2003). Quantum vagueness. *Erkenntnis*, 59(1):97–124.

Goldstein, S. (2001). Boltzmann's approach to statistical mechanics. In *Chance in physics*, pages 39–54. Springer.

Goldstein, S. (2012). Typicality and notions of probability in physics. In *Probability in physics*, pages 59–71. Springer.

Goldstein, S., Lebowitz, J. L., Tumulka, R., and Zanghi, N. (2019). Gibbs and Boltzmann entropy in classical and quantum mechanics. Valia Allori (ed.). *Statistical Mechanics and Scientific Explanation: Determinism, Indeterminism and Laws of Nature (World Scientific, forthcoming)*.

Hyde, D. and Raffman, D. (2018). Sorites paradox. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2018 edition.

Keefe, R. and Smith, P., editors (1996). *Vagueness: A Reader*. MIT Press, Cambridge, MA.

Lebowitz, J. L. (2008). Time's arrow and boltzmann's entropy. *Scholarpedia*, 3(4):3448.

Leibniz, G. W. F. (1840). *God. Guil. Leibnitii Opera philosophica quae exstant latina, gallica, germanica omnia*, volume 2. sumtibus G. Eichleri.

Lewis, D. (1979). Counterfactual dependence and time's arrow. *Noûs*, 13:455–76.

Lewis, D. (1986). *Philosophical papers II*. Oxford: Oxford University Press.

Loewer, B. (2007a). Counterfactuals and the second law. In Price, H. and Corry, R., editors, *Causation, Physics, and the Constitution of Reality: Russell's Republic Revisited*. Oxford University Press.

Loewer, B. (2007b). Laws and natural properties. *Philosophical Topics*, 35(1/2):313–328.

Loewer, B. (2012). Two accounts of laws and time. *Philosophical studies*, 160(1):115–137.

Loewer, B. (2016). The Mentaculus Vision. In Barry Loewer, Brad Weslake, and Eric Winsberg (eds.) *Time's Arrow and World's Probability Structure*. Harvard University Press, forthcoming.

Lowe, E. (1994). Vague identity and quantum indeterminacy. *Analysis*, 54(2):110–114.

Maroney, O. (2005). The density matrix in the de Broglie–Bohm approach. *Foundations of Physics*, 35(3):493–510.

Maudlin, T. (2007). *The Metaphysics Within Physics*. Oxford University Press, New York.

Miller, M. E. (2019). Fundamental, yet imprecise? manuscript.

Myrvold, W. (2017). Philosophical issues in quantum theory. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2017 edition.

North, J. (2011). Time in thermodynamics. *The oxford handbook of philosophy of time*, pages 312–350.

Penrose, R. (1979). Singularities and time-asymmetry. In Hawking, S. and Israel, W., editors, *General relativity*, pages 581–638.

Raffman, D. (1994). Vagueness without paradox. *The Philosophical Review*, 103(1):41–74.

Rinard, S. (2017). Imprecise probability and higher order vagueness. *Res Philosophica*.

Russell, B. (2009). *The philosophy of logical atomism*. Routledge.

Sorensen, R. (2018). Vagueness. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2018 edition.

Tooley, M. (1977). The nature of laws. *Canadian journal of Philosophy*, 7(4):667–698.

Wallace, D. (2011). The logic of the past hypothesis. manuscript.

Wallace, D. (2012). *The emergent multiverse: Quantum theory according to the Everett interpretation*. Oxford University Press.

Wallace, D. (2016). Probability and irreversibility in modern statistical mechanics: Classical and quantum. *Quantum Foundations of Statistical Mechanics (Oxford University Press, forthcoming)*.

Williamson, T. (1994). *Vagueness*. Routledge, London.

Wilson, J. (2013). A determinable-based account of metaphysical indeterminacy. *Inquiry*, 56(4):359–385.