Jean van Heijenoort. *From Frege to Gödel. A source book in mathematical logic, 1879–1931*. Cambridge, Mass., Harvard University Press, 1967. New paperback edn., 2002. 664 pages, 1 halftone. ISBN: 0-674-32449-8. Price: $27.95.

On April 1, 1967, a bibliographic note dealing with van Heijenoort's Source Book, in the *Library Journal*, ended with the following words: "this book will long remain a standard work, essential to the study of symbolic logic". This was obviously an overstatement, but easy to correct by inserting three words – *From Frege to Gödel* has been and remains an essential tool in the study of *the history of* mathematical logic, and even more, for the history and philosophy of logic and foundational research. As Quine said:

> [Van Heijenoort] made a contribution to mathematical logic that was second only to what Alonzo Church had contributed in molding the *Journal of Symbolic Logic* itself. … For us it was a boon just to have these papers brought together, to have xeroxes in our hands of the original German. But then, he did these painstaking translations and painstaking commentaries … a collaborative job, but very largely Van's own work.[1]

Thus, it should not come as a surprise that this source book is reissued, after 35 years! There are many reasons to welcome the new paperback edition, not least the low price, making it accesible to interested students and to libraries throughout the world.

I believe every single person who cares about the topics mentioned above does know and value highly van Heijenoort's achievement. The book not only collected and made available to the English-speaking public an impressively wide range of key works, in translations of very high quality. *From Frege to Gödel* contributed to establishing higher standards for editorial work in collections on logic and mathematics, as witnessed e.g. by the *Collected Works* of Kurt Gödel, explicitly modeled upon its example.[2] Particularly noticeable is the quality of the editorial notes that introduce each paper, always clear, deep, and yet concise. These were written by leading logicians and philosophers like Quine, H. Wang, Dreben, and Ch. Parsons, besides the editor himself.

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[1] Quoted in [2], 281.

[2] [3], vol. I, p. iv (preface).

The person responsible for producing this huge, admirable work was a professor of philosophy at Brandeis University. There was more than met the eye behind his modest appearance, because Jean van Heijenoort (1912–1986) had been intensely involved not only in logic, but also – to speak with his biographer – in politics and love.[3] For seven years he was personal secretary and bodyguard to Leon Trotsky, following him from France to Mexico through Turkey. In 1939 van Heijenoort settled in the USA, and after Trotsky's assassination and the war he went back to his first love, mathematics, doing graduate studies and becoming especially interested in mathematical logic. Love came in again at the time of his death, shot in the head by his fifth wife, near Cuernavaca in Mexico, where she was from.

Van Heijenoort had strong ties to the influential school of logicians-philosophers at Harvard University. It was through them that he was appointed editor of the Source Book, they assisted him in its production, and it is a safe assumption that they influenced heavily his conception of logic and his selection of material. The plan to publish a logic anthology in the Harvard series *Source Books in the History of Science* began with Quine and Dreben in 1959. Dreben happened to know van Heijenoort and was aware of his deep interest in the development of logic. By then, van Heijenoort had independently come to think about editing a collection of seminal logic papers, and his passion for exactitude and knowledge of languages were obvious extra bonuses. After meeting Quine and Harvard University Press, still in 1959, he was recruited as editor.

Obviously any selection must have its biases, and also its history. Let us examine them in turn, beginning with a few words on the figures just mentioned. There is little need to introduce W. V. O. Quine (1908–2000), the extremely influential analytical philosopher who began his career working with Whitehead and learning from Carnap. Quine's initial work was in mathematical logic, developing two original systems that offered an alternative to Russell's theory of types (and therefore to set theory), and later championing first-order logic as "the one" system of elementary logic.[4] He was the senior man in van Heijenoort's advisory board. Burton Dreben (1927–1999) is well

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[3] See the delightful biography by A. Feferman [2].

[4] See [4], which analyzes Quine's path from logicism to first-order logic.

known for Quineans, if only because his name is present in the acknowledgements of almost all of Quine's books and papers. A member of the Society of Fellows at Harvard in 1952–1955 (as Quine was in the 1930s), he taught at the University thereafter. He wrote little, being mainly interested in proof theory and very especially in the work of the French logician Herbrand. But Dreben kept a deep interest in the history of foundations, logic, and analytical philosophy, being quite influential in these three fields indirectly, through personal contacts. There are indications that he may have been very intensely involved in the Source Book project. Dreben not only enlisted van Heijenoort, but he collaborated closely in making the selections and polishing translations and introductions.[5]

Van Heijenoort's preface made it clear that the Source Book was targeted at three main foci, one being modern symbolic logic, the other, two fields which "emerged on the borders of logic, mathematics and philosophy", namely set theory and foundations of mathematics.[6] In this triangle, logic formed the longer side, and it conditioned the dimensions and aspect of the remaining two. Van Heijenoort's selection highlighted those aspects of foundations and set theory which are intimately linked with formal logic, while it clearly downplayed mathematical and philosophical aspects. In the case of set theory, the papers included deal only with basic principles and the main axioms systems, while properly set-theoretical developments are conspicuously absent. This one-sided approach was a sensible one, given limitations of space and the main focus of the volume. With that feature in mind, one can easily understand some differences between the book under review and the more recent anthologies *From Kant to Hilbert* (1996) and *From Brouwer to Hilbert* (1998).[7]

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[5] G. Hellman reminisces that, while lecturing, van Heijenoort once referred to the Source Book as a joint project with Dreben (personal communication). According to W. Goldfarb (personal communication), Dreben's files contain typescript copies of most of the translations, some in several versions. Goldfarb seems to believe that Dreben was an uncredited co-editor (for the straight reason that he preferred to remain in the dark, free from the drudgery of editing).

[6] P. vi; notice the very Quinean characterization of set theory.

[7] Ewald, ed. [1]; Mancosu, ed. [10]. The titles make it clear that the souce book under review has played an exemplary role.

The original plan was two volumes, the first to span the period from Frege's *Begriffsschrift* to Gödel's celebrated paper, the second from thence "to the present".[8] Van Heijenoort optimistically thought that the editorial work would be done in six months, but he ended up working "terribly hard, under heavy pressure, for seven years".[9] He enlisted three collaborators for the task of translating, including Stefan Bauer-Mengelberg, who translated 28 out of the total of 46 papers, working on them "almost full time" for six years. The care with which they discussed any imaginable detail in the translations, dug up additional sources, and analyzed them all to produce high-quality introductions and explanatory footnotes – this was the reason for the heavy pressure. The outcome was the impressive first volume, but the projected second never came into being.

To some extent, the very success of *From Frege to Gödel* has been the source of criticism directed against it. Van Heijenoort's volume never intended to represent the complex historical development of logic from 1879 to 1931. "Encyclopedic completeness" was precluded if only because the "main constraint" was that the outcome had to be a single volume (p. vi). Yet it was so representative of this history, that many of us have wondered why the book excludes some chapters.[10] It must be acknowledged that such a criticism is unfair, but at the same time one must warn readers of this anthology that, if they are looking for a complete picture of the historical emergence of mathematical logic, they must complement *From Frege to Gödel* with other anthologies, original treatises, textbooks, and secondary sources.[11]

One obvious bias, that can only be expected in such an anthology, is that the vista displayed before us is clearly whiggish. This could only be a selection of successes, excluding programmatic work, dead ends, and influential confusions.[12] It may be for

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[8] See van Heijenoort's letters to Gödel in the latter's [3], vol. IV (forthcoming), and the introduction by Goldfarb (who kindly made it available to me).

[9] Quoted in [2], 275. See also 274–282.

[10] See the reviews by Mostowski [14] and above all by Moore in this journal [11].

[11] Among the latter, see Goldfarb [6] and Moore [12] & [13]. For complements, Peckhaus [15] and Grattan-Guinness [7].

[12] A relevant example of the last category is Wittgenstein's famous *Tractatus*, whose conception of logic is remote from modern mathematical logic.

these reasons that important authors like Frank Ramsey and Leon Chwistek are absent (both contributed very much to simplifying type theory). Also absent are all members of the very important Polish school in logic, counting Sierpinski, Lukasiewicz, Tarski, Lindenbaum and others among its members, and Hermann Weyl's seminal work in predicative foundations (*Das Kontinuum*, 1918). The reason for this last omission is likely to be that the interest of predicative approaches to foundations was lost of sight for many years, and that the editors ignored the historical significance of Weyl's work.[13]

A very important source for probing the state of development of logic, both in its main body and, particularly, in the changing images that researchers project of the enterprise, can be found in comprehensive treatises and textbooks. Three key examples, which complement van Heijenoort's source book informatively, are: Schröder's *Vorlesungen über die Algebra der Logik* (3 vols., 1890–95), Whitehead & Russell's *Principia Mathematica* (3 vols., 1910–13), and Hilbert & Ackermann's *Grundzüge der theoretischen Logik* (1928). In spite of the legendary stature of *Principia Mathematica*, only Hilbert & Ackermann's *Grundzüge* can be regarded as the first modern presentation of mathematical logic, for only here one finds a stern formal approach to logical systems plus the all-important focus on metatheoretical questions.

At the time of Ernst Schröder's lectures, most of mathematical logic was in the algebraic tradition of Boole and Peirce, but this is absent from the Source Book. A conspicuous characteristic of logical systems around that time is that they routinely include set theory as a core part of elementary logic. This would change radically with the discovery of the paradoxes. Contemporaneous with Schröder were the forward-looking contributions of Frege and Peano. Frege introduced the very idea of a formal system, and many years later Gödel would rightly emphasize that Whitehead & Russell's treatment of their system represents "a considerable step backwards as compared with Frege" ([3], vol. II, 120). Note that Frege's key works appeared 30 and 20 years before *Principia Mathematica*!

Nevertheless, *Principia* marked an epoch in the development of logical theory and left strong traces in many authors. It was an impresively detailed treatise, which developed

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[13] See the reviewer's [5], chapter X.

much of mathematics in the symbolic language of logic, starting with a plausible solution to the paradoxes that for some time seemed sufficient to rescue the logicist project. During the 1920s, Russell's type theory was generally regarded as the "natural" system of logic.[14] But neither Russell nor Frege encouraged the metatheoretical study of logical systems, a fact reflected upon by van Heijenoort in a famous short paper [8]. Interest in metalogic came from the algebraists in Schröder's tradition (Löwenheim in particular, whose work the Source Book did much to call attention to) and from the Hilbert tradition in axiomatics. All of these crucial threads were tied and further developed in the work of Hilbert and his school during the 1920s, without which the decisive contributions of Gödel, Tarski and Turing would have been impossible.

To the list of absences above, we may add that the work of Hilbert's main collaborator Paul Bernays is underrepresented, probably because his original papers of the 1920s are too philosophical. And one further omission appears hard to explain, even taking into account the already mentioned bias in the treatment of set theory: Zermelo's paper 'Über Grenzzahlen und Mengenbereiche'.[15] This presents today's ZF system, including the Axiom of Foundation, and offers an extremely interesting study of the cumulative hierarchy, inspired in earlier work by von Neumann. We might try to explain its omission by the antagonism between Zermelo's platonistic attitude toward the higher infinite, and the anti-realist leanings of both Quine and Dreben. That led Zermelo to offer a second-order axiomatization of ZF, which again conflicted with the opinion of Quine and Dreben that first-order is the only logic worthy of this name. (However, the true reason might have been simply that Zermelo's paper, in the Polish journal *Fundamenta Mathematicae*, escaped the attention of van Heijenoort and collaborators.)

From Frege to Gödel deals with the period 1879–1931. Today we might prefer a broader delimination of the formative period of mathematical logic, say, from 1847 (Boole's *Mathematical analysis of logic*) to 1936. Van Heijenoort justified his exclusion of Boole and the whole trend of the algebra of logic saying that it was important developments, but not "a great epoch". And we may concede that, faced with the

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[14] See [5], chapter X.

[15] *Fundamenta Mathematicae* **16** (1930), 29–47. See the translation in Ewald [1] and the introductory comments by M. Hallett.

necessity to cut somewhere, it is not bad to start with Frege's *Begriffsschrift*, which in retrospect emerges as impresively clear and precise, truly epoch-making. Of course, this decision leads to historical injustice in case readers are misled into thinking that the contributions of authors like Boole, De Morgan, Peirce and Schröder were not very relevant.

As regards the final date, one might prefer to stretch the interval so as to include seminal papers in formal semantics by Tarski and the crucial contributions of 1936, by Turing and Church, on computability and the *Entscheidungsproblem*. Again, one might justify van Heijenoort's choice by pointing to the large extra space that the inclusion of such papers would call for. Moreover, there was the initial idea of a second volume that would probably have covered all of this. And, as regards Alfred Tarski, – probably the name whose omission is most striking, being the founder of the Berkeley school, usually regarded as second only to Gödel among 20th century logicians – it is also true that the well-known collection *Logic, semantics, metamathematics* was available since 1956.

But I suspect that both the exclusion of Tarski and the decision to begin with Frege were related to the conception of logic emanating from Harvard. The viewpoint behind van Heijenoort's selection is, quite clearly, a strictly formal one, by which I mean one that is focused on a *syntactic* presentation and investigation of classical logic. By beginning with Frege and closing with Gödel's investigations in proof theory, that conception was enhanced. (The role of Frege as a founding figure had been repeatedly emphasized by both Russell and Quine, and thus to start with him was very much in line with Harvard views.) If it is true that Dreben was intensely involved in the project, one would have even more reason to expect a bias toward the syntactic approach to logic. Those who knew him report that he disliked Tarskian set-theoretic semantics and model theory. In general, he rejected all kinds of speculative trends[16] – and model theory, by depending on the strong "philosophical" assumptions of set theory, was in his eyes a speculative, risky tendency within mathematical logic. This again was in line with

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[16] Although very interested in the history of analytical philosophy, Dreben is reported to have frequently said that all philosophy is nonsense, "garbage," although "the history of garbage – that's sholarship!" (personal communications, G. Hellman and A. Kanamori).

Quine's views concerning logical matters. (Needless to say, Dreben's hand led to interesting additions too, for instance the extremely adequate inclusion of a good number of Skolem's papers.)

Another feature that is strongly in line with the Harvard perspective, as represented by Quine, is the strong thesis that "there are no two logics" (p. vii), reinforced in *From Frege to Gödel* by the exclusion of alternative logics, modal logic, and the like. It may be surprising to find such a forceful statement in a book that gives much space to intuitionism, but this becomes less so when one realizes that van Heijenoort did not include contributions like Heyting's 1930 formalization of intuitionistic logic.[17]

It is interesting to reflect on the fact that, even though he was guided by a stern, purely formal, and seemingly ahistoric conception of logic, van Heijenoort's selection still bears the mark of historical developments. The decision to include papers in set theory and foundations could only be justified historically, in terms of the great contribution those fields made to the reform and delimitation of mathematical logic, as well as to its philosophy. For a long time, from Frege and Dedekind to Carnap, set theory was conceived to be merely a part of elementary logic, and the evolution of modern logic was intimately entangled with debates on the principles of set theory. Subsequently, in the 1920s, most of the novelties introduced into logical theory and the conception of logic itself were closely linked with the foundational debate. I believe it is for these reasons that an anthology of logic in the first third of the 20th century was felt to require inclusion of material in set theory and the foundations of mathematics.[18]

As we see, an analysis of the contents and origins of *From Frege to Gödel* offers quite an interesting overview of important chapters in the history of logic during the 20th century. Philosophically, this would appear as an important instantiation of a far-reaching general idea: that historical factors, the historical situation (including its immediate or even its remote past), are present whenever we offer an evaluation in any subject matter, however abstract and universal it may seem. This principle had already

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[17] For this, see Mancosu [10].

[18] This way of presenting the matter had already appeared in S. C. Kleene's famous *Introduction to Metamathematics* [9], which may well have served as a model for van Heijenoort.

been established for scientific methodology, and it is instructive to find it confirmed in the very abstract realm of logic itself.

But if we come down to more practical matters, one thing can be regarded as certain. The new paperback edition of van Heijenoort's famous source book must be welcome by all those who have an active interest in the history of mathematical logic and the foundations of mathematics. Which, of course, includes anybody who is truly interested in the history of 20th century mathematics.

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[3] S. Feferman *et al.*, eds., *Kurt Gödel. Collected Works*, New York, Oxford University Press, 1986 (vol. I), 1990 (vol. II).

[4] J. Ferreirós, Notes on types, sets and logicism, 1930–1950, *Theoria* (San Sebastián) **12** (1997), 91–124.

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[6] W. Goldfarb, Logic in the Twenties: The nature of the quantifier, *Journal of Symbolic Logic* **44** (1979), 351–68.

[7] I. Grattan-Guinness, *The Search for Mathematical Roots, 1870–1940. Logics, set theories and the foundations of mathematics from Cantor through Russell to Gödel*, Princeton University Press, 2000.

[8] J. van Heijenoort, Logic as calculus and logic as language, *Boston studies in the philosophy of science* **3** (1967), 440–46.

[9] S. C. Kleene, *Introduction to Metamathematics*, New York, Van Nostrand, 1952.

[10] P. Mancosu, ed., *From Brouwer to Hilbert. The debate on the foundations of mathematics in the 1920s*, Oxford University Press, 1998.

[11] G. H. Moore, Review of J. van Heijenoort, *From Frege to Gödel*, *Historia Mathematica* **4** (1977), 468–471.

[12] G. H. Moore, The Emergence of First-Order Logic, in W. Aspray & P. Kitcher, *History and Philosophy of Modern Mathematics*, University of Minnesota Press, 1988.

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[14] A. Mostowski, Review of J. van Heijenoort, *From Frege to Gödel*, *Synthese* **18** (1968), 302–305.

[15] V. Peckhaus, *Mathesis universalis. Leibniz und die Wiederentdeckung der formalen Logik im 19. Jahrhundert*. Berlin, Akademie-Verlag, 1998.

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