Multi-field and Bohm's theory

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Abstract

In the recent literature, it has been shown that the wave function in the de Broglie–Bohm theory can be regarded as a new kind of field, i.e., a "multi-field", in three-dimensional space. In this paper, I argue that the natural framework for the multi-field is the original second-order Bohm's theory. In this context, it is possible: i) to construe the multi-field as a real scalar field; ii) to explain the physical interaction between the multi-field and the Bohmian particles; and iii) to clarify the status of the energy-momentum conservation and the dynamics of the theory.

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1 Introduction

Quantum mechanics describes the behavior of microscopic systems via a mathematical function defined on the system's configuration space: the wave function. This is thus the fundamental entity of quantum mechanics, and yet the question "*What is the wave function?*" is still open to debate (the answer varying not only between different interpretations of the theory but also, in some cases, within the same interpretation, as in the case of the de Broglie-Bohm theory). In the standard interpretation, the wave function by itself is not something real: its physical meaning is given by the absolute square, which represents the probability density of finding a particle in a given region of space if we perform a position measurement on the system.[1] In this interpretation, the physical import of the theory is given by the eigenvalues of Hermitian operators, and by the mean values of these operators. All other aspects of the formalism are mostly regarded as formal tools for the computation of these values.

However, the question of the nature of the wave function cannot be ignored in the observer-independent interpretations of quantum mechanics: the Everett, Ghirardi-Rimini-Weber (GRW), and de Broglie-Bohm (dBB) theories. These interpretations, in fact, describe real physical processes for microscopic systems at any time and independently from measurement:[2] the observable's

[1]Note that the absolute square refers to the probability density *of finding the particle in a given region if we perform a measurement on that region*, which is different from the probability density for the particle's *being in a given region of space, independently from the measurement*. Classical statistical mechanics is of the latter type, while quantum mechanics of the former type. Sometimes, this difference is overlooked in physics textbooks, making the conceptual structure of standard quantum mechanics less problematic than it really is.

[2]However, it is important to note that the Everett theory does not explain and describe physical processes at the quantum scale in the same way as the GRW and dBB theories do. In fact, while the latter have a primitive ontology of matter (matter density/flash or point-

eigenvalues are therefore not the "beginning" but the "end" of any physical interaction between two quantum systems, one of which (the apparatus) is composed of many degrees of freedom.

In this paper, I will focus on the status of the wave function in the de Broglie–Bohm theory. There, the wave function plays the role of a "guiding field" for the particles' motion: it is natural then to regard the wave function as a real field guiding the particles through a specific physical interaction, as it happens e.g. in classical electromagnetism. This view was originally proposed by Bohm (1952) and later supported by Valentini (1992), Holland (1993) and Albert (1996, 2013). The wave function indeed has many physical features in common with a classical field: it is the solution of a dynamical equation, it is (generally) time-dependent, it guarantees momentum-energy conservation, and, moreover, it serves as intuitive explanation of typical quantum phenomena (e.g., it explains the quantum interference after the slits in the double-slit experiment). Nevertheless, the wave function is mathematically defined on the configuration space of the system, and this has led some authors (Albert (1996, 2013); Ney (2013)) to consider it literally as a real field "living" in configuration space. However, this interpretation raises major conceptual problems: Is configuration space the fundamental space we live in? If so, why do we think we live in three-dimensional space? What kind of configuration space is fundamental (for the configuration space's dimensions change according to the number of particles considered)? Is it variable or fixed, given that particles, at the fundamental level, are created and annihilated all the time?
Bohm himself came to regard the wave function as a sort of information tool for the particles' motion (Bohm & Hiley (1993)). Building (maybe) on this idea, Dürr, Goldstein and Zanghì (DGZ) (1996) proposed a radical solution to the problem, i.e. to regard the wave function not as a real field but as a law-like or nomological entity, like the Hamiltonian function in classical mechanics.[3] This produced a shift in the discussion about the ontological status of the wave function: if it is a law-like entity, then it can be interpreted according to the different metaphysical stances applicable to the laws of nature, e.g., dispositionalism and Humeanism. Specifically, it can be re-

particles, respectively) and explain measurement outcomes in terms of these elements, a description of the same kind is missing in the Everett theory. In this theory, measurement-like interactions still have a crucial importance.

[3] Dürr, Goldstein and Zanghì do not speak in terms of information. However, there is a common idea between their nomological interpretation and Bohm's informational view, i.e. the idea that the wave function is a mathematical abstract entity that dictates, describes, or encodes the dynamics of the Bohmian particles.

garded as the mathematical representation of a holistic disposition of the particles' configuration (Esfeld *et al.* (2014)), or as the sum of the individual dispositions of the particles (Suarez (2015)), or as part of the best system of the Humean mosaic (Esfeld (2014)).
I will discuss the nomological view in section 2, showing that the wave function, under careful analysis, cannot be regarded as a nomological entity, insofar as we want to adhere to the physics of the theory. On the other hand, how can we solve the problems raised by the realistic field view of the wave function? In Belot (2012); Romano (2016, Ch. 5); Hubert & Romano (2018)), it is shown that the wave function can be interpreted as a new kind of field – a *multi-field* – in three-dimensional space.[4] Whereas a classical field assigns definite values to each point of space, the multi-field, for a general N-particle state, assigns definite values only to N-tuples of points of space: given an actual configuration of particles in three-dimensional space, it generates one complex value for all the points of space where the particles are located. Configuration space is exactly what is needed to describe the non-local connection between the points of an N-tuple, and is the reason why the wave function cannot be always factorized – indeed, it cannot be factorized insofar the position of one particle can influence the value of the multi-field. With the multi-field view, we can project the wave function from mathematical configuration space into physical three-dimensional space, defining a real physical entity in three dimensions.
However, in this original proposal of the multi-field, some problems remain open:

1. The multi-field assigns a complex value to each N-tuple of points, but complex values are generally not physical values. Therefore, the multi-field itself, being a complex-valued field, may be regarded as an artificial or fictitious field.

2. The de Broglie–Bohm theory can be written as a first-order theory (de Broglie's wave mechanics, Bohmian mechanics) or a second-order theory (Bohm's theory). The multi-field interpretation does not specify which one of the two is the privileged dynamics of the theory.

3. What type of interaction is exerted by the multi-field on the Bohmian particles? How should this interaction be understood?

[4]The idea of the multi-field was originally proposed by Forrest (1988) in standard quantum mechanics. However, in that framework, the multi-field notion is extremely unintuitive and ontologically obscure. We can credit Belot (2012) with re-habilitating this option in Bohm's theory.

In this paper, I shall answer the three questions above. Very shortly, I will argue that: i) the wave function can be reduced to two coupled real-valued scalar multi-fields, one for the amplitude and one for the phase of the wave function; ii) this interpretation is naturally based on the original second-order Bohm's theory; iii) each of the two real-valued multi-fields exerts its influence on the Bohmian particles in the same manner as classical gravitational and electromagnetic fields do on classical particles, i.e. through the actions of real potentials and forces in physical space. That is, the classical scheme: field ⇒ potential ⇒ force ⇒ particles' motion is conserved.

2 Is the wave function nomological?

2.1 The nomological view: a brief introduction

The nomological interpretation of the wave function was originally proposed by Dürr, Goldstein & Zanghí (1996),[5] and rapidly became a popular view among philosophers. The idea is to regard the wave function as something law-like, i.e., as part of the law of motion for the Bohmian particles. According to this interpretation, the wave function does not literally "guide" the particles; rather it prescribes, as part of the law of motion, how the particles move in three-dimensional space. Furthermore, these authors distinguish two different kinds of wave function, namely the *universal wave function* (UWF) and the wave function for isolated subsystems, called *effective wave function* (EWF).[6] The former is the wave function of the universe, coming as the solution of the Wheeler-de Witt equation in quantum cosmology; the latter is the usual wave function for closed systems, i.e. for systems that are sufficiently isolated (decoupled from the environment), and it comes as the solution of the Schrödinger equation. According to DGZ, only the UWF has a fully nomological status, since all the other wave functions for subsystems can be mathematically derived from it. The EWF is defined, instead, as "quasi-nomological", for it shares the nomological character of the UWF but is not fundamental. However, the notion of quasi-nomological is not precisely characterized from a metaphysical point of view. This is certainly a problem for the nomological interpretation, since it naturally raises the following questions: has a quasi-nomological entity the same metaphysical status as the nomological one from which it is derived? Or does it have a different status?

[5]A more recent presentation can be found in Goldstein & Zanghì (2013).

[6]Historically, the original definition of the effective wave function in Bohm's theory as the "collapsed" wave function for isolated subsystems is due to Bohm & Hiley (1987, sect. 4).

How should we metaphysically characterize the status of a quasi-nomological entity? In the absence of a clear answer to these questions, the metaphysical status of ordinary (i.e., effective) wave functions in Bohmian mechanics is not well-defined within the nomological view. However, this distinction notwithstanding, I will consider below some objections to the nomological view that remain valid even considering the EWF as fully nomological, on the same token that the UWF.

2.2 The analogy between the wave function and the Hamiltonian function

In order to support this view, DGZ make an analogy between the wave function in the de Broglie–Bohm theory and the Hamiltonian function in classical mechanics. The classical Hamiltonian is a function defined on phase space, and generates a vector field in this space by means of Hamilton's equations:

$$\dot{q} = \frac{\partial H}{\partial p} \quad \dot{p} = -\frac{\partial H}{\partial q} \tag{1}$$

These equations determine, respectively, the time evolution of the position and momentum of the particles in phase space. Similarly, the wave function in the de Broglie–Bohm theory generates a vector field in configuration space via the guiding equation:

$$\dot{q} = \frac{\hbar}{m}\Im(\frac{\nabla\psi}{\psi}) \tag{2}$$

where $\Im$ denotes the imaginary part. DGZ claim that the wave function in eq.(2) plays a similar role to that of the Hamiltonian in eq.(1). However, on careful analysis, this analogy appears to be misleading, and, consequently, the nomological interpretation seems to lose (at least) part of its intuitive support. The next subsections (2.3–2.5) will be devoted to an analysis of this claim.

2.3 Top-down (ψ) *versus* bottom-up (H) functions

The classical Hamiltonian has a bottom-up construction, starting from the positions and velocities of the particles. The general scheme is well known. Consider a classical N-particle system. The state of the system is specified by N positions and N velocities in three-dimensional space. We can define the kinetic energy of the i-th particle as $K_i = \frac{1}{2}m_i v_i^2$ and the total kinetic energy as the sum of the kinetic energy of the individual particles, that is: $K_{tot} = \sum_i K_i$. The kinetic energy is built upon two real properties of the particles: their velocity, and their mass. We can define the potential energy in a similar manner: if the particle with generic position x is located in a region with potential V, we attribute to the particle a potential energy $V(x)$. The total potential energy will be the sum of the potential energy of the individual particles. In this case too, the potential energy refers to a real property of the particle: the position of the particle inside a region characterized by a potential V. Finally, we build the Hamiltonian function as the sum of the total kinetic energy and the total potential energy of the system:[7]

$$H(x, v) = K_{tot}(v_1, \ldots, v_N) + V_{tot}(x_1, \ldots, x_N) \tag{3}$$

The classical Hamiltonian is an abstract mathematical function, for it refers to the physical state (position and velocity) of the particles, through the potential and kinetic energy terms. The distinction between ontology and mathematics is clear in this case: what exist in the world are the positions and velocities of the particles, and the classical (gravitational or electromagnetic) field that produces the corresponding classical potential $V(x)$. So, why the Hamiltonian? The main reason is mathematical convenience: solving Hamilton's equations is usually much simpler than solving the system of differential equations in Newton's theory. In other words, the classical Hamiltonian refers to something external that is in the ontology of the theory, namely particles' positions and velocities. From these, once the classical potential acting on the particles is known, we can mathematically derive the kinetic energy and the potential energy of the system. The Hamiltonian, being the sum of the kinetic and potential energy, is finally just a suitable (i.e., mathematically useful and compact) way to rewrite those properties that eventually refer to the position and velocity of the particles composing the classical system. However, even if useful, this is not a necessary/indispensable description: we can always eliminate the Hamiltonian and continue to do classical Newtonian mechanics in terms of forces acting on the particles.

Now, what about the wave function? Can it be construed in an analogous way? No, the wave function does not have this sort of origin. It cannot be built from below, since it does not refer *prima facie* to objective properties of the Bohmian particles. The wave function, indeed, is derived as a solution of the Schrödinger equation. In particular, it crucially depends on the form of the quantum Hamiltonian operator, and, specifically, on the

[7] The Hamiltonian is defined on all the phase space points of the system, i.e., it takes as variables not just the actual positions and velocities of the particles but all the possible positions and velocities. However, since this will not be relevant for the argument above, I prefer to keep a simple notation.

classical potential V in the Hamiltonian. The positions and velocities of the particles will be relevant – as in the classical case – for the construction of the quantum Hamiltonian, but they will be completely irrelevant for the construction of the wave function. We may call this distinction the "bottom-up *versus* top-down function asymmetry". The former is characteristic of nomological entities, and the classical Hamiltonian is a concrete example. The latter is characteristic of real physical entities, and the wave function in the de Broglie–Bohm theory is a concrete example. The wave function is the solution of a dynamical equation, i.e. the Schrödinger equation, just as the electromagnetic field is the solution of Maxwell's equations. And it is certainly better regarded as a real physical entity than as a nomological one.
Few remarks are in order here. The argument presented above shows that while it is clear that the classical Hamiltonian is a nomological entity (it is just a compact way to rewrite physical properties – kinetic energy and potential energy – of the particles composing the system), it is not clear instead why the wave function should be regarded as a nomological entity. Indeed, the wave function, contrary to the Hamiltonian, is not a compact way to rewrite or describe some properties of the Bohmian particles, but a fundamental, non-eliminable, entity of the theory. While we can always eliminate the Hamiltonian from classical mechanics and still be able to describe the behavior of particles in terms of, e.g., Newtonian forces; the same cannot be done in the de Broglie–Bohm theory, i.e. we cannot describe the dynamics of the Bohmian particles without the wave function. This suggests that the wave-function in the de Broglie–Bohm theory is not like the Hamiltonian in classical mechanics: while the classical Hamiltonian is a useful description of the physical properties of the particles and can be in principle eliminated and substituted by a direct description of particles' properties and forces acting on them, the wave function in the de Broglie-Bohm theory does not refer (and is not just a description of) the properties of the Bohmian particles and cannot be eliminated by the theory. This suggests to regard the wave function in the de Broglie–Bohm theory as something more fundamental than a nomological entity.

However, a possible reply is that, even though the wave function in the de Broglie–Bohm theory is different from the Hamiltonian function in classical mechanics, even though there is an asymmetry between these two mathematical functions, why should this represent a problem for the nomological view? After all – one may argue – the analogy between the wave function and the Hamiltonian, even if suggestive, is not to be taken too much seriously, or, in any case, it is not the only argument in favor of the nomological view. In fact, other important arguments in favor of the nomological view are that it immediately solves two problems: i) the no back-reaction problem, i.e. the lack of mutual influence between the Bohmian particles and the wave-function and ii) the problem of configuration space realism, i.e. the fact that if the wave function is a physical field, then it must be a field defined on configuration space (mathematically, at least). Points (i) and (ii) certainly remain as good points in favor of the nomological view.[8]

Nevertheless, with this argument I wanted to show that the analogy between the classical Hamiltonian and the wave function is not as strong as is usually presented in the literature, and that the way the wave function is introduced in the theory is crucially different from the way a typical nomological entity is. To sum up: I have assumed that a nomological entity is primarily a convenient description of the physical properties of the particles composing the system and, furthermore, that it can always be eliminated from the theory in favor of a more "direct" description, i.e. a description which involves actual properties of the particles and forces acting on them. On this basis, it is possible to note that the wave function is not like the Hamiltonian in a precise sense: i) it is not the sort of entity that is built from the physical properties of the particles and ii) it is a fundamental (not eliminable) entity for the description of the dynamics of the particles. Nevertheless, this is not to be intended as a definitive argument against the nomological view, but rather as an indication that the wave function –after all– does not fall under the box of nomological entities so naturally. In the next two subsections (2.4 and 2.5), I will present further arguments against the nomological view and in favor of a realistic interpretation of the wave function.

2.4 Time-dependence of the wave function

Another problem for the nomological interpretation, already stressed in the literature (see, e.g., Belot (2012)), is that the wave function of isolated systems is generally time-dependent. Instead, genuine nomological entities, like the classical Hamiltonian function, are time-independent for isolated systems. The standard reply to this objection is that, from the ontological point of view, only the universal wave function exists, and this is likely to be static, according to the Wheeler–De Witt equation (see, e.g., Goldstein & Zanghì (2013)). However, there are two problems with this answer:

1. Currently, we do not know what the dynamical equations for realistic cosmological models look like. It seems therefore very speculative to

[8] The multi-field view, however, immediately solves the problem of configuration space by defining the wave function as a (new type of) field in three-dimensional space, while the problem of no-back reaction remains an open issue in this context, and will be discussed in section 6.3.

rely on unknown dynamical features in order to support a metaphysical claim on the nature of the wave function.

2. Even if the wave function, as a solution to the Wheeler-de Witt equation, turns out to be static, its precise mathematical form crucially depends on the boundary conditions of the cosmological model at hand[9] – as, on the same token, the precise form of the wave function of a system depends not only on the Schrödinger equation but also on two boundary conditions: (i) the condition of continuity in regions at different potentials and (ii) the condition of probability normalization. We would expect instead the universal wave function, as a fundamental law of nature, to be not-contingent, i.e. not-dependent on the boundary conditions that we may set for different cosmological models.[10]

2.5 Interaction between V and ψ

Moreover, the nomological interpretation is not well-supported by the physical meaning of the Schrödinger equation:[11]

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla_x^2\psi(x,t) + V(x)\psi(x,t) \tag{4}$$

On the right-hand side, the Schrödinger equation describes the interaction between a classical potential $V(x)$ and the wave-function $\psi(x)$. It is natural to think of this interaction as a physical interaction: the Schrödinger equation is essentially a bare mathematical structure, the concrete solution of which depends on the specific choice of the potential V (apart from the case of free motion). Different classes of potentials V correspond to different classes of solutions of the wave functions: stationary waves, harmonic oscillators, etc. The interplay between $V(x)$ and $\psi(x)$ via the Schrödinger equation is intuitive if we regard the wave function as a field and the interaction between the potential and the wave function as a physical interaction. It is useful to look at a simple example. Let us consider the textbook example of the infinite potential well: the classical potential V is of the form: $V = \infty$ outside the well and $V = 0$ inside the well. Solving the Schrödinger equation with

[9] The Wheeler-deWitt equation indeed has many solutions, and to get to a unique solution one needs to add extra boundary conditions.

[10] One may still argue that the boundary conditions producing static universal wave function should be regarded as nomologically necessary. However, this would be physically odd, since the boundary conditions imposed on the cosmological models are usually set freely, and can vary from one model to another.

[11] I write the equation in the position basis here and, for the sake of simplicity, for a 1-particle system.

these conditions, we find a stationary wave inside the well (corresponding to a coherent superpositions of plane waves). Why does the wave function take this particular form? This is easy to explain if it is regarded as a physical entity: in this case, its particular form is the result of the interaction with the borders of the well. In addition, we can explain why the Bohmian particle is at rest inside the box even if the system has finite energy (see, e.g., Hubert & Romano (2018, sect. 2.2)). On the nomological interpretation, this interplay between the wave function and the potential V is absent: what does the potential V act on? In what manner can the potential V modify the form of the wave function? Of course, we can derive it from the Schrödinger equation, but there is no physical mechanism that connects together the wave function with the physical potential V. In other words: the form of the potential V in the quantum Hamiltonian determines –*via* the Scrödinger equation– the form of the wave function and, thus, the motion of the Bohmian particles. Nevertheless, if the classical potential V is a physical entity and the wave function is a nomological entity, what is the mechanism that ought to explain this physical interplay between the wave function and the classical potential in the Schrödinger equation? A possible reply, within the nomological view, is that this interaction should be also regarded as a nomological interaction, i.e. as a sort of "law constraining another law" mechanism. This is a consistent solution, but comes at a price: since, on this view, the action of the potential V on the wave function is interpreted as a law-like mechanism, it suggests to regard not only the wave function but also the classical (gravitational, electromagnetic) fields as nomological entities, and the resulting potentials and forces as a kind of nomological constraints. From the ontological point of view, this amounts to say that the (Bohmian) particles are the only physical entities existing in the universe, and all fields, potentials and forces have a nomological character. This radical "minimalistic" ontology has been defended, e.g., by Esfeld et al. (2017) and Esfeld & Deckert (2017), but it is not obvious whether it is shared by all the supporters of the nomological view.
Granted, this does not want to be a conclusive argument, but only a (further) indication that the wave function is more naturally interpreted as a physical field: while, indeed, it is certainly possible to regard the potential V(x) as a law-like entity and the $\Psi(x)V(x)$ interaction in the Schrödinger equation as a sort of "law constraining another law" mechanism, it seems more plausible –as long as one endorses a realistic view of classical fields– to interpret the classical potential V as a physical potential and the $\Psi(x)V(x)$ interaction as a typical interaction between a physical potential and a physical field.

From the arguments presented above (subsections 2.3 - 2.5), it is possible to conclude that the wave function in the de Broglie–Bohm theory is more naturally regarded as a physical field than a nomological entity. In particular, as showed in the recent literature by Belot (2012), Chen (2017) and Hubert & Romano (2018), it can be thought of as a new kind of field –a *multi-field*– in three-dimensional space. This interpretation, while keeping a realistic view on the wave function, avoids the problems typically raised by configuration space realism,[12] such as the problem of perception (i.e., explaining the appearance of three-dimensional space at the macroscopic classical level), maintaining three-dimensional space –or four-dimensional space-time, in the relativistic regime– as the fundamental physical space at the quantum and classical level.

In the next sections, I will show that the natural framework of the multi-field interpretation is the original second-order Bohm's theory: in this context, some of the open problems of the multi-field interpretation are easily overcome and the wave function can be literally regarded –as intuitively suggested by the theory– as a physical guiding field for the particles' motion.

3 Bohm's theory and Bohmian mechanics

Bohm's theory and Bohmian mechanics are, from a metaphysical point of view, two different theories. The former is a second-order theory, and explains the particles' motion through the action of a real quantum potential generated by the wave function. The latter is a first-order theory, and explains the particles' motion through the velocity-based guiding equation. In both theories, the complete representation of the state of a system is given by the couple $(\mathcal{Q}, \psi)$, where $\mathcal{Q}$ is the configuration of particles and ψ the wave function of the system. In this section, I will briefly outline these two theories.

3.1 Bohmian mechanics

In Bohmian mechanics (Dürr, Goldstein & Zanghì (2013); Dürr & Teufel (2009)), the time evolution of the particles is given by the guiding equation:

$$\dot{x} = \frac{\hbar}{m} \Im \left(\frac{\nabla \psi}{\psi} \right) \tag{5}$$

where the wave function is the solution of the Schrödinger equation (expressed in the position basis). The theory is thus completely defined by the

[12] See Callender (2015, sect. 2) for a clear exposition of the problems and Albert (2015, ch. 6) for a possible reply.

equations (5) and (4).
Given an ensemble of systems with wave function ψ, the initial configuration of the particles is statistically distributed according to the Born rule, that is:

$$\rho(0) = |\psi(0)|^2 \tag{6}$$

The continuity equation guarantees that the flow of the Born probability distribution stays constant through time:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \Longrightarrow \rho(t) = |\psi(t)|^2 \tag{7}$$

A simple way to see that the Born probability distribution is constant through time is to consider the relation $\rho = |\psi|^2$ and that $|\psi|^2$ follows the continuity equation (7)– therefore $|\psi|^2$ it is a conserved quantity under the Schrödinger evolution.
Bohmian mechanics is a first-order theory, since the particles' motion is fixed by the first-order, velocity based, guiding equation. Historically, this kind of theory was originally proposed by de Broglie.[13] Bohm, instead, recasts the theory as a second-order theory in "pseudo-Newtonian fashion" (see the next subsection). In this respect, Bohmian mechanics should be rather called *de Brogliean mechanics*, since it is more in the spirit of the original de Broglie theory (it has indeed the same dynamics for particles' motion).

3.2 Bohm's theory

Bohm's theory (Bohm (1952); Bohm & Hiley (1993)) is a second-order theory, and can be derived from the following procedure. First, we decompose the wave function in polar form:

$$\psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)} \tag{8}$$

then, inserting (8) into the Schrödinger equation, and separating the real and imaginary part, we obtain two coupled equations for the real fields $R(x,t)$ and $S(x,t)$, respectively the amplitude and the phase of the wave function:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{9}$$

with

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{10}$$

[13]See, e.g., Bacciagaluppi & Valentini (2009, Ch. 2).

and

$$\frac{\partial R^2}{\partial t} + \nabla \cdot \left(R^2 \frac{\nabla S}{m} \right) = 0 \tag{11}$$

Eq.(9) is known as the quantum Hamilton–Jacobi equation, for the structure is the same as the classical Hamilton–Jacobi equation apart from the "quantum potential" term. It defines a real scalar field $S(x,t)$ on configuration space: the particles' trajectories will be the integral curves of $S(x,t)$, and the particles' velocity is given by the formula (mathematically equivalent to eq.(5)):

$$v = \frac{\nabla S}{m} \tag{12}$$

Eq.(9) describes the motion of a particle (or, for an N-particle system, of the particles' configuration) with kinetic energy $K = \frac{(\nabla S)^2}{2m}$ and acted upon by the classical potential V and the quantum potential Q, the latter being generated by the amplitude of the wave function $R(x,t)$.[14] The total energy of the system is then given by the sum of the kinetic energy K, the classical potential V, and quantum potential Q. Equation (11) represents the continuity equation for $R^2 = |\Psi|^2$: it guarantees that the Born probability distribution $|\Psi|^2$ is constant through time, and, consequently, that Bohm's theory (as Bohmian mechanics) is empirically equivalent to standard quantum mechanics (provided that the initial distribution of particles is also $|\psi|^2$–distributed[15]).

Taking the gradient of both sides of eq.(12) and using eq.(9), we obtain the acceleration of the particle:

$$m\ddot{x} = -\nabla V - \nabla Q \tag{13}$$

Defining, in analogy with Newton's theory, the quantum force as (minus) the gradient of the quantum potential: $F_Q = -\nabla Q$, we finally obtain the fundamental equation of Bohm's theory – the *quantum Newton's law*:

$$m\ddot{x} = F_C + F_Q \tag{14}$$

which describes the acceleration of a particle under the influence of a classical and a quantum force.[16]

[14] It is worth noting, however, that the amplitude R and the phase S are not independent terms, for they are dynamically coupled via the continuity equation (11).

[15] Two different approaches have been proposed in the literature to explain why the initial particle configuration of a Bohmian system is distributed according to $|\psi|^2$: the typicality approach by Dürr, Goldstein & Zanghì (1992) and the relaxation dynamical approach by Valentini (1991). A comprehensive and updated review of the two approaches is given by Norsen (2018).

[16] It is worth noting that the amplitude R has a double role in Bohm's theory: on the one hand, it fixes the acceleration of the particles through the quantum potential Q

Bohm's theory is completely described by eqs.(13) or (14), (12), (9) and (11). It describes the motion of particles in three-dimensional space, whose acceleration and velocity are generated, respectively, by the action of the classical and quantum potential and by the phase of the wave function. Originally, Bohm regarded the velocity formula (12) as a boundary condition that could be released in the sub-quantum regime, giving rise to new physics. Contrary to this view, I will regard this formula as a second dynamical equation for the particles' motion. Three reasons for this are as follows:

1. The guiding equation is not incompatible with the quantum Newton's law, since the former determines the velocity and the latter the acceleration of the particles.

2. The velocity formula does not look like a boundary or initial condition, but like a law of motion. Boundary conditions are contingent, whereas the velocity formula is a necessary condition, insofar as the continuity equation is exact. In fact, since the former can be mathematically derived from the latter,[17] the velocity equation will take the exact form: $v = \frac{\nabla S}{m}$ insofar as the continuity equation (11) is valid.

3. The guiding equation should be taken as exact if we want the Born rule to hold in Bohm's theory. Indeed, the continuity equation (11) is equivariant for $|\psi|^2$ insofar as the Bohmian velocity is exactly $v = \frac{\nabla S}{m}$. Approximations of or deviations from the guiding equation would lead to corresponding deviations of the Born rule probability distribution, which has not been observed so far.[18]

4 Why Bohm's theory

Finding the correct (or, at least, a reasonable) ontology of physical theories is a subtle task, especially when theories become so abstract that we do not have immediate empirical support and must rely solely on their formal structures. Mathematical formalism, in fact, can be a dangerous guide to extracting an ontology.

Newtonian mechanics is an example in which we can clearly distinguish the

(*dynamical* role) and, on the other hand, it determines the $|\psi|^2$ probability distribution of the particles' configuration (*probabilistic* role). Why R plays such a double role, and whether these two roles are connected at some other level, are open questions.

[17]For the derivation of the velocity formula from the continuity equation, see, e.g., Sakurai (1994, p. 101-102).

[18]For a technical analysis of this point, see e.g. Goldstein & Struyve (2015).

ontology of the theory from its mathematical formulation. The ontology is clear: what exists in the theory are massive point particles moving under the influence of the classical gravitational field. The field generates a gravitational potential (V) and a gravitational force ($F_C = -\nabla V$) that, in turn, determines the acceleration of particles. The particles' motion is mathematically expressed by Newton's second law:

$$F_C = m\ddot{x} \tag{15}$$

However, we can cast Newtonian mechanics into different and more abstract formulations, such as the Lagrangian, the Hamilton–Jacobi, and the Hamiltonian formulations.

Nevertheless, it is clear that these formulations do not provide an ontology of classical mechanics: we can use, for example, the Hamiltonian function to compute the particles' trajectories, but we still think we live in three-dimensional space and still think that the cause of motion is not the Hamiltonian *per se*, but the gravitational force generated by the classical gravitational field.

Quantum mechanics is a type of Hamiltonian mechanics, for the evolution of systems (mathematically represented by the wave function) is computed through the quantum Hamiltonian function. The Schrödinger equation is, in fact, the natural extension of classical Hamiltonian mechanics in the quantum domain, where the position and velocity variables are replaced by the position and momentum operators. But, *as classical Hamiltonian mechanics is not a guide to the ontology of the classical world, a Schrödinger-based theory of quantum mechanics cannot be a guide to the ontology of the quantum world.* For this reason, the wave function should not be regarded as a physical entity *per se*, but as a mathematical object that unifies the two real multi-fields $R(x, t)$ and $S(x, t)$ (more on this in the next section). The ontology of quantum mechanics should be derived, instead, from a natural extension of classical Newton mechanics to the quantum domain. This theory would represent a natural candidate for the ontology of the quantum world: a theory of this kind is, indeed, Bohm's second-order theory. Bohm's theory describes the particles' motion as a generalization of the classical motion: the particles are acted upon by a classical and a quantum potential. However, in contrast to Newtonian mechanics, the velocity is not a free parameter but is fixed by the gradient of the phase S, which plays the role of an "Aristotelian" potential (Valentini (1992)).[19] So, what does exist in the quantum world according to Bohm's theory? Particles moving in three-dimensional space, classical fields, and two new quantum fields: the R-field, which generates

[19] S is called "Aristotelian" potential because ∇S determines the velocity of the particles.

the quantum potential Q and the quantum force $F_Q = -\nabla Q$, and the S-field, which generates the Aristotelian potential S and the Aristotelian force $F_A = \nabla S$. As in Newton's theory, also in Bohm's theory the potentials are directly connected with the particles' motion: the quantum potential Q and the Aristotelian potential S determine, respectively, the acceleration and the velocity of the particles. These real-valued scalar fields, derived from the polar decomposition of the wave function and mathematically defined on configuration space, can be interpreted as *multi-fields in three-dimensional space*.

5 Multi-field(s) from Bohm's theory

In Hubert & Romano (2018), the main motivation for introducing the multi-field interpretation was to keep a realistic stance on the wave function as a physical field, but existing in ordinary three-dimensional space. There, it is shown that we can project the wave function field values defined on configuration space into multi-field values defined in three-dimensional space. In order to do this, it is sufficient to associate – for a general entangled N-particle system – a definite field value not with single points, but with N-tuples of points of space. That is, the N particles' positions of the system's actual configuration "select" one single value of the multi-field in three-dimensional space. From the mathematical point of view, configuration space is exactly what we need to capture the holistic feature of the multi-field in the assignment of field values. In addition, quantum non-locality is explained very naturally, for the multi-field value depends instantaneously on the total particle configuration. However, this original multi-field proposal encounters two problems:

1. The wave function is a complex-valued scalar function, so the multi-field takes on complex values in physical space, and complex values are not usually intended as physical values. The latter, indeed, are expected to be measurable quantities, i.e. quantities that can be correlated with different possible states of a pointer or, more generally, of a measurement device. While the correlation between pointer states and real numbers is pretty straightforward – e.g., they can be ordered on a line, thus representing magnitudes – it is not obvious that the same can be done with complex numbers – which can only be ordered on a plane, making it difficult to regard them as magnitudes of sort.[20]

[20] It is an interesting question, however, whether pointer states may be possibly correlated not only with real numbers (as it usually happens in a laboratory) but also with

2. The multi-field is a physical re-interpretation of the wave function, but, for the reasons given above (section 4), the wave function as well as the Schrödinger equation should not be taken *prima facie* as expressing ontological features of quantum formalism, even in the de Broglie–Bohm theory.

However, both of these problems are solved if we regard not the wave function but the R-field and the S-field (respectively, the amplitude and the phase of the wave function) as multi-fields.[21] Problem (1) is immediately solved, since these are real-valued scalar functions (eq.(8)). Therefore, the R- and S-field, interpreted as multi-fields, assign a definite real value in physical space to each N-tuple of points, i.e., to the actual location of the Bohmian particles. Problem (2) is also solved, since Bohm's theory naturally selects not the wave function but the R- and S-field as multi-fields. In turn, once the amplitude and the phase of the wave function are interpreted as multi-fields in physical space, this provides a very intuitive understanding of the dynamics of the Bohmian particles. Specifically, the R-field, as a physical field in three-dimensional space, generates a quantum potential (according to eq.(10)) and, consequently, a quantum force: $F_Q = -\nabla Q$. The quantum force, together with the classical force $F_C = -\nabla V$, fixes the acceleration of the Bohmian particles according to (13) or (14). The S-field, instead, generates an Aristotelian potential (mathematically expressed as just $S(x,t)$) and, consequently, an Aristotelian force: $F_A = \nabla S$ that fixes the particles' velocity according to (5).
What is then the role of the wave function? Why is it so important in quantum mechanics? The wave function is a mathematical tool that encodes and compactly expresses relevant information on the R- and S-field and their dynamical coupling. In fact, the amplitude and phase are not independent from each other: they are coupled together via the continuity equation (11) and, through the wave function, via the the Schrödinger equation. In turn, this explains why the wave function is a complex-valued function. An analogy with classical electrical circuits may be helpful here. In the analysis of electrical circuits, a signal is defined by a sinusoidal function:

$$v(t) = v_0 cos(\omega t + \phi) \tag{16}$$

In order to "solve the circuit", i.e., to find the amplitude and the phase of the electrical signal in the circuit, the original sinusoidal function is usually

complex numbers. Thanks to an anonymous referee for this remark.

[21] The idea of splitting the multi-field into two scalar fields is discussed also in Chen (2018).

translated in a complex function by the following relations:

$$v(t) \equiv v_0 cos(\omega t + \phi) = \Re[v_0 e^{i(\omega t + \phi)}] = \Re[v_0 e^{i\phi} e^{i\omega t}] = \Re[V e^{i\omega t}] = \Re[V'] \quad (17)$$

where $V = v_0 e^{i\phi}$ and $V' = V e^{i\omega t}$ are complex functions. The relation between the real function $v(t)$ and the new complex function V' is straightforward: the real part of V' corresponds to the real signal $v(t) : \Re[V'] = v(t)$ and, in particular, the modulus of V' corresponds to the real amplitude of the signal: $|V'| = v_0$. With similar algebraic methods it is also possible to extract the real value of the phase ϕ. In this example, the complex function V' contains all the relevant information on the real signal $v(t)$, and, in particular, we know how to extract from it information about the amplitude and the phase of the signal, which are real measurable quantities. We use the function V' instead of $v(t)$ for it is much easier to calculate the solution of the circuit by making use of the algebra of complex numbers instead of solving systems of differential equations. Nevertheless, the distinction between a notational shorthand for calculation (V') and the real signal ($v(t)$) is clear.
It is the same in Bohm's theory. The analogue of the amplitude v_0 and the phase ϕ of the electrical signal are the amplitude $R(x,t)$ and the phase $S(x,t)$ of the wave function. In order to find the time evolution of these fields, one should solve the quantum Hamilton–Jacobi equation and the continuity equation, which are two coupled differential equations. Since, as in the case of electrical circuits (and Newtonian mechanics), systems of differential equations are practically very difficult to solve, we can compactly express the information of these two real-valued functions into one complex-valued function –the wave function– through the procedure: $\Psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)}$ and compute its time evolution using the Schrödinger equation. The wave function in Bohm's theory (and in quantum mechanics in general) is analogous to the phasors in electrical circuits: as, in the latter case, the phasor (a complex function) encodes all the relevant information regarding the amplitude and the phase of the electrical signal (real physical quantities), the wave function (complex function) in Bohm's theory encodes all the relevant information concerning the multi-fields $R(x,t)$ and $S(x,t)$ and their dynamical coupling.

6 Further considerations

6.1 Why not first order?

One objection is typically raised by supporters of the first-order theory (Bohmian mechanics): since the quantum Newton's law (14) can be mathematically derived from the guiding equation (5), the second-order theory must be regarded as artificial. However, I think there are three main reasons to regard both the R- and the S-field an ontological, and hence Bohm's second-order theory as the natural formulation:

1. *Classical limit*
   Newtonian mechanics is a second-order theory, and emerges/is derived from quantum mechanics in a special regime, i.e., the macroscopic regime. Bohm's theory has the advantage of unifying quantum and classical mechanics as theories of particles in motion. The classical regime can therefore be understood as the regime in which the Bohmian particles behave "classically", i.e., they follow Newton's second law. From an ontological point of view, we could think for example that the Bohmian particles are influenced by the quantum and classical potential and that, when the former is negligible, they will move according to Newtonian trajectories.[22] However, this scheme properly works only in the second-order theory: if Bohm's theory is fundamentally a first-order theory instead, how can a genuine second-order theory emerge from it in a special regime? In the quantum Newtonian equation, the classical potential V and the quantum potential Q are physically on the same level: they are both physical potentials generating real forces acting on the particles. In this framework, the classical limit can be regarded as the specific limit in which the quantum potential and force do not play any role, while the classical potential is still present and acts on the particles. This way, the ontology of Newtonian mechanics as a theory of particles in motion acted upon by forces naturally emerges from the second-order Bohm's theory. In Bohmian mechanics, instead, the motion of particles is not generated by potentials or forces: they just move according to the guiding equation. But how could then we derive, in this framework, a classical ontology of particles acted upon by forces? Even if we were able to derive Newton's laws of dynamics from the guiding equation in a special regime, we would not be able to derive the classical Newtonian ontology, unless we maintain that also in Newtonian mechanics the classical potentials and forces have a sort of nomological status.

   In other words, while a mathematical derivation can always be done, a derivation of the classical ontology of fields and forces from the first-order theory, where these entities do not play any physical role, is conceptually problematic. A similar point is expressed in the following passage by Belousek (2003, p. 149):

[22] See, e.g., Holland (1993, Ch. 6) for an approach of this kind.

> [...]the conceptual coherence of the classical limit in Bohmian mechanics depends upon V and U [the quantum potential] being interpreted in the same terms. It thus seems that, to be consistent, one here should also take U to be a characteristic or property belonging to a physical system that is related to the evolution of the quantum state. Doing so would not only allow for a conceptually coherent classical limit within the guidance view, but would also remove the ambiguity resulting from giving direct physical significance to only the phase of the pilot wave (because the quantum potential depends upon the amplitude).

2. *Symmetry*
   The principal step that allows Bohm's theory to emerge from quantum mechanics is the polar decomposition of the wave function. This procedure extracts from the wave function two real-valued scalar functions, the amplitude $R(x,t)$ and phase $S(x,t)$: why should we take just one of these functions as a real field and the other as a mere mathematical tool? For reasons of pure symmetry, it seems more natural to think of both functions as equally real. In fact, they can both be interpreted as multi-fields.

3. *Guiding equation*
   The guiding equation (5) is valid only for spinless particles. For particles with $\frac{1}{2}$-spin, the velocity is given by the coupling of the spinor field with the external electromagnetic field, which generally involves not only the phase but also the amplitude of the wave function. In fact, whereas for spinless particles the guiding equation is of the form:

$$v \propto S \tag{18}$$

   for particles with spin the guiding equation is generally of the form:[23]

$$v \propto (S, R) \tag{19}$$

   That is, the R-field contributes to the velocity of the particle as the S-field. Consequently, the R-field plays the role of a physical field in the first-order theory too. But, if R influences the velocity of the particles, there is no reason to think that it does not influence the acceleration of the particles through the potential Q.

[23]See e.g. Holland & Philippidis (2003).

Bohm's theory is a second- and first-order theory
However, Bohm's theory is not a genuine second-order theory like Newton's theory. In the latter, there is only one dynamical equation – Newton's second law – that fixes the acceleration of the particles. In the former, instead, there are two dynamical equations (eqs.(5) and (14)) that fix, respectively, the velocity and the acceleration of the particles. While position and velocity are free parameters (initial conditions) in Newton's theory, in Bohm's theory only the position is a free parameter (even though the initial position distribution is constrained by the relation $\rho = |\psi|^2$), while the velocity is fixed by the guiding equation. Nevertheless, the fact that there are two dynamical laws should not be regarded as an inconsistency or an artificial feature of the theory: it naturally follows, in fact, from the consideration that the wave function generates two physical (multi-)fields in three-dimensional space, and both influence the motion of the Bohmian particles.

6.2 Energy and momentum conservation

In classical mechanics, the total energy of an isolated system is conserved. This follows from the time-independence of the classical potential:

$$\frac{\partial V}{\partial t} = 0 \rightarrow E = \frac{1}{2}mv^2 + V = constant \tag{20}$$

In addition, the momentum will be constant (since no external force is acting on it):

$$\nabla V = 0 \rightarrow p = mv = constant \tag{21}$$

In Bohm's theory, the wave function acts as an external field for the particle, for it adds energy to the particle through the quantum potential Q. Therefore, even if a system is classically isolated ($F_C = 0$), it may be not quantum mechanically isolated ($F_Q = 0$), and, in that case, the energy conservation does not hold. The conditions for energy and momentum conservation in Bohm's theory arise as a generalization of the classical conditions, with the quantum potential Q acting as a further potential together with the classical potential V. In particular, it is possible to derive the following relations (Holland 1993, p. 119):

$$\frac{\partial}{\partial t}(V + Q) = \frac{\partial E}{\partial t}, \tag{22}$$

with $E = \frac{(\nabla S)^2}{2m} + V + Q$, and

$$-\nabla(V + Q) = \frac{dp}{dt} \tag{23}$$

with $p = \nabla S$.
Therefore, analogously to the classical case, the energy is conserved when the classical and the quantum potential are time independent:

$$\frac{\partial}{\partial t}(V+Q) = 0 \rightarrow E = constant \tag{24}$$

and the momentum is conserved when no (classical and quantum) force acts on the system:

$$\nabla(V+Q) = 0 \rightarrow p = constant \tag{25}$$

Examples are, respectively, stationary waves (where $V = 0$ and $Q = \frac{P^2}{2m}$), and plane waves (where $\nabla V = 0$ and $\nabla Q = 0$). It is important to note that in these equations the quantum potential plays an analogous role to the classical potential V. For this reason, the contribution of the quantum potential to the total energy of the system remains unexplained insofar as we regard this term as a mere computational tool, as it happens in the nomological interpretation, unless we are keen to regard also the classical potential V as a "nomological entity" of some sort.
The energy conservation law expresses a precise statement: insofar as the system is isolated, the total energy is conserved, and it can be transformed from kinetic to potential energy and the other way round. If the quantum potential is just a computational tool, it becomes hard to explain what the kinetic energy transforms into when it becomes "potential energy". This explanation, instead, is straightforward in the multi-field interpretation proposed here: the R-field is a physical (multi-)field in three-dimensional space, and generates the physical potential Q, which, in turn, contributes to the total energy of the system together with the classical potential V. The conservation of energy is then analogous to the classical case, with a further potential generated by the R-field. The classical energy and momentum conservation laws can thus be regarded as a special case of the energy and momentum conservation laws in Bohm's theory, arising when the quantum potential (Q), the quantum force ($F_Q = -\nabla Q$), and the quantum power ($P_Q = \frac{\partial Q}{\partial t}$) are negligible – conditions that indeed characterize the classical limit.

6.3 A possible objection: the no-back reaction problem

A possible objection to the multi-field view is that, differently from classical (gravitational and electromagnetic) fields, it does not satisfy back reaction. For example, in classical electromagnetism, the electromagnetic field determines the acceleration of charged particles and, at the same time, the latter determine the exact configuration of the field. In Bohm's theory, instead, this is not the case: while the multi-field determines the velocity and acceleration of the Bohmian particles, the latter do not have any influence on the exact configuration of the multi-field. This is certainly a serious objection to the multi-field view: the phenomenon of back reaction (i.e., the mutual influence between the particles and the field) is often taken as an indirect "proof" of the existence of classical fields. Therefore, one may ask: if the multi-field is really a sort of physical field, why do not the Bohmian particles react back on it? While I think there is not yet a conclusive response to this problem, dealing with it can only improve our knowledge of the multi-field as new physical entity. Let us start then making few considerations on this issue.

One answer to the problem (let us call this the *easy* answer) is to define the multi-field as a physical field which is not subject to back reaction. This option has been originally proposed in Hubert & Romano (2018, pp. 523-524):

> A crucial question is to understand which features are essential to fields in general and which are only essential to classical fields. We want to make this distinction starting from a definition of a classical field, and then giving a generalization of it for the multi-field. We can think of a classical field to be defined by the following features: (a) it is an assignment of intrinsic properties to the points of space it is defined on, and (b) it ensures energy and momentum conservation. Now, in the case of the multi-field, we must substitute (a) with (c) it is an assignment of intrinsic properties to particular N-tuples of points of three-dimensional space.
>
> In sum, we suggest that the definition of a multi-field is captured by statements (b)and (c), and that only classical fields are required to obey (a) and (b).

Within this framework, back reaction is simply not viewed as an essential feature of the multi-field. However, I want to propose here some further reflections on this issue which –I think– may be useful for evaluating it within a more general context and may provide, eventually, a more solid justification for it.

A first consideration is that, in physics, the phenomenon of back reaction of a particle on the field is usually regarded as a specific manifestation of a more general principle, i.e. the action-reaction principle, defined by the Third Newton's Law of dynamics. But there is no a priori reason to think that law to hold in quantum mechanics. Newton's laws of dynamics do emerge, in fact, only in the classical limit of quantum theory, and we do not expect them to hold already at the quantum level. This argument has been recently defended by Vera Matarese in her PhD dissertation (2017, p. 87-88):

> Therefore, it seems that the wave-function violates the criterion of back-reaction.[...] A realist of the wave-function may refute our worries by simply stating that the Newtonian laws are not supposed to apply in a quantum theory. After all, given that the first and second Newtonian laws fail to obtain in a Bohmian system, there is no reason to expect the validity of the third one. In particular, it should be noted that the Newtonian principle is based on classical intuitions, which are not supposed to hold in the quantum world. Why should the reaction be equal to the action? Why should the body that is acted upon react back? There is no reason to hold the principle true in quantum mechanics.

Furthermore, the absence of back reaction may be a consequence of the different way a classical field and a multi-field are generated by the particles. While gravitational and electromagnetic fields take origin on the particles themselves (the mass or charge of the particles are the "sources" of the field), the multi-field is not generated by the Bohmian particles but comes independently from the Schrödinger equation (i.e., the Bohmian particles are not sources of the multi-field). It is not implausible to assume that the lack of back reaction in the quantum context arises as a consequence of the different manner the two fields (respectively, classical field and multi-field) are generated. This position has been originally proposed by Riggs (2008, pp. 36-37):

> In the earlier example of a charged particle accelerated by an external electric field between charged plates, there is an obvious action of the external field on the particle but what is the reaction and how is it mediated? [..] A charged particle is surrounded by its own very small electric field which is independent from any external field. Both the particle's field and the external field (each with its own source) are distorted in shape by their mutual interaction. The answer to the above question is, of course, that the particle's electric field exerts a force on the plates equal and opposite to that which it experiences allowing the Third Law to hold. Classical action-reaction holds in cases of contact phenomena and of most mediated field interactions. However, [...] the Schrödinger wave field is not a mediated field[24] and therefore

[24]The term "mediated field" indicates the self-produced field of the charged particle.

> there is no familiar means to carry a classical reaction from the quantum particle to the wave field.

and recently defended by Hubert & Romano (2018, p. 524):

> We think that these two features [no-source and no-back reaction] are intrinsically connected with each other: a particle must act back on a field if it has generated the field itself. In the de Broglie-Bohm theory, the multi-field is not produced by particles, and therefore it is plausible that the action-reaction principle is violated.

However, even if the multi-field does not obey the Third Newton's Law and the Bohmian particles do not act back on the multi-field, still we may ask whether the multi-field as physical field *can be acted upon* by other physical entities in the theory. That is, in order to evaluate the physicality of the multi-field, we may consider not if it is subject to classical laws of dynamics (as the action-reaction principle) but at least to more general principles such as the "principle of reciprocity". Stated in a very general form, this principle can be expressed as follows (Matarese (2017, p. 90)):

> *Leibnizian Principle of Reciprocity*: Any physical element of a theory must be capable of acting and of being acted upon.

An interesting question, therefore, is to ask whether the multi-field as physical field in Bohm's theory satisfies the principle of reciprocity, i.e. if the wave function in Bohm's theory is capable of being acted upon. The answer to this question is affirmative: as shown in section 2.5, the wave function (and, consequently, the multi-fields R and S) is affected by the physical potential V in the Schrödinger equation. Indeed, we already noted in that context that the $\psi(x)V(x)$ interaction can be naturally interpreted as a typical interaction between a physical potential (V) and a physical (multi-)field (ψ).

In conclusion: the multi-field does not satisfy back reaction. However, this should not be seen prima facie as a fatal problem for the existence of the multi-field, for two reasons: i) back reaction in classical mechanics is plausibly a manifestation of a more general principle, the action-reaction principle, defined by the Third Newton's Law, which we do not expect to hold in quantum mechanics; ii) back-reaction may be intrinsically connected to a specific characteristic of classical fields, namely that they are generated by the particles, while multi-fields are not. Furthermore, even if the multi-field is not subject to back reaction, it nevertheless satisfies the principle

of reciprocity, i.e. it can be defined as an entity capable of acting on (it generates the motion of the Bohmian particles) and being acted upon (it is affected by classical potentials in the Schrödinger equation).
Granted, these considerations should not be intended as a definitive response to the problem, but, at least, as an indication that different possible solutions are on the table, and, most of all, that this is still an issue open to debate.

7 Conclusions

I have shown that the wave function in Bohm's theory can be literally interpreted as a guiding field for the particles' motion. More precisely, the real guiding field is not the wave function *per se*, but the amplitude $R(x,t)$ and the phase $S(x,t)$ of the wave function. These fields are naturally interpreted as multi-fields in three-dimensional space, guiding the particles' motion in the same space through the action of physical potentials and forces. The wave function, instead, is a mathematical object that encodes all the relevant information about the multi-fields R and S and their dynamical coupling.

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