Relativity[1]

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The first thing to note about the theory of relativity is that there are two different theories of relativity, the special theory put forward in Albert Einstein's famous 1905 paper, "On the Electrodynamics of Moving Bodies," and the general theory, completed in November 1915 and first presented systematically in a review article published in March 1916.

Special relativity extends the principle of relativity for uniform motion, known in mechanics since the days of Galileo, to all of physics, in particular to electrodynamics, the field out of which the theory grew. Although the key contribution was Einstein's, several other scientists deserve credit for it as well, most importantly the Dutch physicist H. A. Lorentz, the French mathematician Henri Poincaré, and the German mathematician Hermann Minkowski. General relativity, by contrast, was essentially the work of one man. It was the crowning achievement of Einstein's scientific career. Its name, however, is something of a misnomer. The theory does not extend the principle of relativity for uniform motion to non-uniform motion. It retains the notion of absolute acceleration—i.e., acceleration with respect to space-time rather than with respect to other bodies. In this sense, general relativity is no different from Newtonian theory or special relativity. Absolute acceleration, however, is much more palatable in general relativity than in these earlier theories.

From the point of view of modern physics, the question to what extent general relativity fulfilled Einstein's original hopes of relativizing all motion is of secondary importance. What matters most is that general relativity is a powerful new theory of gravity, based on the insight, called the equivalence principle, that the effects of gravity and those of acceleration ought to be described by one and the same structure, curved space-time.

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Special Relativity

Special relativity grew out of problems in electrodynamics. In 1862 James Clerk Maxwell first published the set of equations named after him, providing a comprehensive description of all electric and magnetic phenomena that had been studied up to that point. He also made a new prediction. The equations allow electromagnetic waves propagating at the speed of light. Maxwell famously concluded: "*light consists of the transverse undulations of the same medium which is the cause of electric and magnetic phenomena*" (Niven, I, 500).

As the quotation above illustrates, it was taken for granted in the 19th century that both light waves and electric and magnetic fields need a medium to support them. This medium, which was thought to fill the entire universe, was called the *luminiferous* (= light carrying) *ether*. Most physicists believed that it was completely immobile (for discussion of the reasons for this belief, see Janssen and Stachel). Ordinary matter, they thought, would move through the ether without disturbing it in the least. The earth, for instance, would zip through the ether at a velocity in the order of 30 km/s, the velocity of the earth's motion around the sun. On earth there should therefore be a brisk ether wind blowing in the opposite direction. This *ether drift*, as it was called, could not be felt directly, but ever since the resurgence of the wave theory of light at the beginning of the 19th century, attempts had been made to detect its influence on light from terrestrial and celestial sources. To be sure, such effects were expected to be small. The velocity of light, after all, is ten thousand times greater than the earth's velocity in its orbit around the sun. Yet optical experiments were accurate enough to detect such effects. All attempts to detect the elusive ether drift, however, failed, and optical theory had to be adjusted to account for these failures.

The combination of Maxwell's theory and the concept of an immobile ether likewise faced the problem of how to explain the absence of any detectable ether drift. When Maxwell found that his equations predict electromagnetic waves propagating with the velocity of light, he quite naturally assumed that this would be their velocity *with respect to the ether*. As long as one accepts classical kinematics, as everyone before Einstein tacitly did, it trivially follows that their velocity *with respect to an observer on earth* is the vector sum of the velocity of propagation in the ether and the velocity of the ether drift on earth. This, in turn, meant that Maxwell's equations could only hold in a frame of reference at rest in the ether: in a moving frame, electromagnetic waves would have different velocities in different directions. The frame of reference of the earth is such a moving frame. We are thus driven to the conclusion that Maxwell's equations do not hold in the frame in and for which they were discovered! Experiments with electricity and magnetism were not accurate enough to detect any possible deviations from Maxwell's equations, but experiments in optics were. The failure of such experiments to detect ether drift thus posed a problem for the theory.

In the 1890s Lorentz set out to explain the absence of any signs of ether drift on the basis of Maxwell's theory. Using classical kinematics, Lorentz first determined the laws that electric and magnetic fields obey in a frame moving through the ether given that they obey Maxwell's equations in a frame at rest in the ether. He then replaced the components (E_x, …, B_x, …) of the real electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ by the components (E'_x, …, B'_x, …) of the auxiliary and, as far as Lorentz was concerned, purely fictitious fields $\mathbf{E}'$ and $\mathbf{B}'$. The components of these auxiliary fields mix components of the real electric and magnetic fields (e.g., in a frame moving with velocity v in the x-direction, $E'_y \approx E_y - vB_z$). He did the same with the space and time coordinates. In particular, he replaced the real time t by a fictitious variable t', which he gave the suggestive name 'local time' because it depends on position (in a frame moving with velocity v in the x-direction, $t' \approx t - (v/c^2)x$). Lorentz chose these quantities in such a way that in any frame moving through the ether with some constant velocity $\mathbf{v}$, the fictitious fields $\mathbf{E}'$ and $\mathbf{B}'$ as functions of the fictitious variables $(\mathbf{x}', t')$ satisfy Maxwell's equations, just as the real fields as functions of the real space and time variables in a frame at rest in the ether. Maxwell's equations, in other words, are invariant under the transformation from the real fields $\mathbf{E}$ and $\mathbf{B}$ as functions of the space and time coordinates in a frame at rest in the ether to the fictitious fields $\mathbf{E}'$ and $\mathbf{B}'$ as functions of the fictitious space and time coordinates of a moving frame. This is the essence of what Lorentz called the *theorem of corresponding states* (Janssen 2002, 424). The transformation is an example of what are now called *Lorentz transformations*. Maxwell's equations are invariant under Lorentz transformations— or *Lorentz invariant* for short.

With the help of this mathematical result, Lorentz was able to show in 1895 that, to first order in the small quantity $v/c \approx 10^{-4}$, many phenomena on earth or in any other frame moving through the ether will be indistinguishable from the corresponding phenomena in a frame at rest in the ether. In particular, he could show that, at least to this degree of accuracy, motion through the ether would not affect the pattern of light and shadow obtained in any optical experiment. Since the vast majority of optical experiments eventually boil down to the observation of such patterns, this was a very powerful result.

In 1899—and more systematically in 1904 (Einstein et al., 1954, 9–34)—Lorentz extended his analysis to higher powers of v/c. He now found that there is a tiny difference between the pattern of light and shadow obtained in an experiment performed while moving through the ether and the pattern of light and shadow obtained in the corresponding experiment performed at rest in the ether. Compared to the latter, the former pattern is contracted by a factor $\sqrt{1-v^2/c^2}$ in the direction of motion. Lorentz had come across this contraction factor before. In 1887, the American scientists Albert A. Michelson and Edward W. Morley had tried to detect ether drift in an experiment accurate to order $(v/c)^2$. They had not found any. Independently of one another (in 1889 and 1892, respectively), the Irish physicist George Francis FitzGerald and Lorentz had suggested that this negative result could be accounted for by assuming that material bodies, such as the optical components in the experiment, contract by a factor $\sqrt{1-v^2/c^2}$ in the direction of motion. Lorentz's analysis of 1899 and 1904 showed that this contraction hypothesis, as it came to be known, could be used not just to explain why the Michelson-Morley experiment had not detected any ether drift but to explain much more generally why *no* observation of patterns of light and shadow ever would.

In modern terms, the hypothesis that Lorentz added to his theory in 1899 is that the laws governing matter, like Maxwell's equations, are Lorentz invariant. To this end, Lorentz had to amend the Newtonian laws that had jurisdiction over matter in his theory. As he had shown in the case of Maxwell's equations, it is a direct consequence of the

Lorentz invariance of the laws governing a physical system that the system will undergo the Lorentz-FitzGerald contraction when moving with respect to the ether. From a purely mathematical point of view, Lorentz had thereby arrived at special relativity. To meet the demands of special relativity, all that needs to be done is to make sure that any proposed physical law is Lorentz invariant. Conceptually, however, Lorentz's theory is very different from Einstein's. In Einstein's theory, the Lorentz invariance of all physical laws reflects a new space-time structure. Lorentz retained Newton's conception of space and time, the structure of which is reflected in the invariance of the laws of Newtonian physics under what are now called Galilean transformations. It is an unexplained coincidence in Lorentz's theory that all laws are invariant under Lorentz transformations, which have nothing to do with the Newtonian space-time structure posited by the theory.

This mismatch between the Newtonian concepts of space and time (and the classical Galilean kinematics associated with it) and the Lorentz invariance of the laws governing matter and fields in space-time manifests itself in many other ways. Einstein hit upon a particular telling example of this kind and used it in the very first paragraph of his 1905 paper. The example is illustrated in Figure 1.

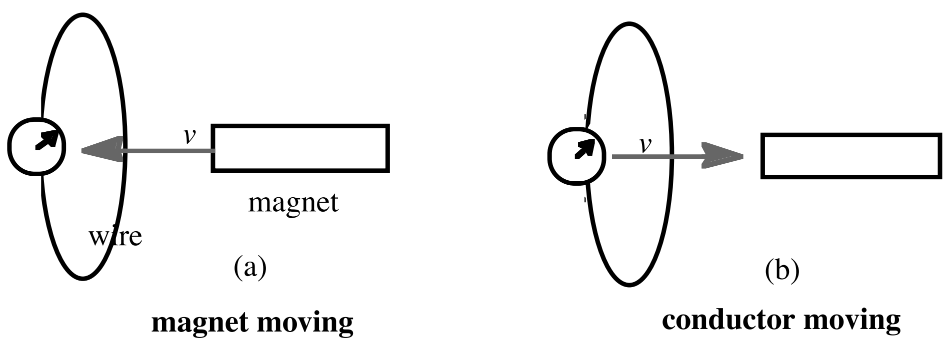

Fig. 1. Einstein's magnet-conductor experiment.

Consider a bar magnet and a conductor—a piece of wire hooked up to an ammeter—moving with respect to one another at relative velocity v. In Lorentz's theory we need to distinguish two cases, (a) with the conductor and (b) with the magnet at rest in the ether. In case (*a*) the approaching magnet causes the magnetic field at the location of the wire to grow. According to Faraday's law of induction, this change in magnetic field induces an electric field, producing a current in the wire, which is registered by the ammeter. In case (*b*) the magnetic field is not changing and there is no induced electric field. The ammeter, however, still registers a current. This is because the electrons in the wire are moving in the magnetic field and experience a Lorentz force that makes them go around the wire. It turns out that the currents in cases (a) and (b) are exactly the same. Yet, Lorentz's theoretical account of what produces these currents is very different for the two cases. We have here, in Einstein's words, an example of theoretical "asymmetries that do not appear to be inherent in the phenomena" (Einstein et al., 1954, 37). Einstein proposed to remove the asymmetry by insisting that cases (a) and (b) are just one and the same situation looked at from different perspectives. Even though this meant that he had to jettison the ether, Einstein took the relativity principle for uniform motion from mechanics and applied it to this situation in electrodynamics. He then proposed to extend the principle to all of physics.

In 1919, in an article intended for *Nature* but never actually submitted, Einstein explained the importance of the example of the magnet and the conductor for the genesis of special relativity: "The idea that we would be dealing here with two fundamentally different situations was unbearable to me [...] The existence of the electric field was therefore a relative one, dependent on the coordinate system used, and only the electric and magnetic field taken *together* could be ascribed some kind of objective reality. This phenomenon of electromagnetic induction forced me to postulate the [...] relativity principle" (Stachel et al., Vol. 7, 264–265; Janssen 2002, 504). The lack of documentary evidence for the period leading up to his creative outburst in his miracle year 1905 makes it very hard to reconstruct Einstein's path to special relativity (for a valiant attempt see Rynasiewicz). It seems clear, however, that Einstein hit upon the idea of 'the relativity of electric and magnetic fields' expressed in the quotation above before he hit upon the new ideas about space and time for which special relativity is most famous. His reading of works of Lorentz and Poincaré probably helped him connect the dots from one to the other.

Once again consider the example of the magnet and the conductor in Fig. 1. From the point of view of the magnet (b), the electromagnetic field only has a magnetic component. From the point of view of the conductor (a), this same field has both a magnetic and an electric component. Lorentz's work provides the mathematics needed to describe this state of affairs. Einstein came to recognize that the fictitious fields of Lorentz's theorem of corresponding states are in fact the fields measured by a moving observer. (He also recognized that the roles of Lorentz's two observers, one at rest and one moving in the ether, are completely interchangeable.) If the observer at rest with respect to the magnet measures a magnetic field with z-component B_z, then the observer at rest with respect to the conductor will not only measure a magnetic field but also an electric field with y-component E'_y. This is captured in Lorentz's formula $E'_y \approx E_y - vB_z$ for one of the components of his fictitious fields.

Maxwell's theory is compatible with the relativity principle if it can be shown that the observer measuring the Lorentz-transformed electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ also measures the Lorentz-transformed space and time coordinates $(\mathbf{x}',t')$. Carefully analyzing how an observer moving through the ether would synchronize her clocks, Poincaré had already shown to first order in v/c that such clocks register Lorentz's local time (Janssen, 428). A direct consequence of this is that observers in relative motion to one another will disagree about whether two events occurring in different places happened simultaneously or not. Distant simultaneity is not absolute but depends upon the state of motion of the observer making the call. This insight made everything fall into place for Einstein about six weeks before he published his famous 1905 paper.

Einstein modeled the presentation of his theory on thermodynamics (as he explained, for instance, in an article for *The London Times* in 1919; see Einstein 1954, 228). He started from two postulates, the analogues of the two laws of thermodynamics. The first is the relativity postulate, which extends the principle of the relativity of uniform motion from mechanics to all of physics; the other, known as the light postulate, is the key prediction that Einstein needed from electrodynamics to develop his theory: "light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body" (Einstein et al., 1954, 37). The combination of these two postulates appears to lead to a contradiction: two observers in relative motion will both claim that one and the same light beam has velocity c with respect to them! Einstein reassured the reader that the two postulates are "only apparently irreconcilable" (ibid.). Reconciling the two, however, requires giving up several common-sense notions about space and time and replacing them with unfamiliar new ones, such as the relativity of simultaneity, length contraction (moving objects are shorter than identical objects at rest), and time dilation (processes in moving systems take longer than identical processes in systems at rest). Einstein derived these effects from his two postulates and the plausible assumption that space and time are still homogeneous and isotropic in his new theory. Following Poincaré's lead but without neglecting terms smaller than of order v/c, Einstein showed that the time and space coordinates of two observers in uniform relative motion are related to one another through a Lorentz transformation. Einstein thus introduced a new kinematics. In the second part of his paper, he showed that this new kinematics removes the incompatibility of Maxwell's equations with the relativity principle. He did this simply by proving that Maxwell's equations are Lorentz invariant. Since he was familiar with an early version of Lorentz's theorem of corresponding states (valid to order v/c), he would have had no trouble with this proof.

In 1908, Minkowski supplied the geometry of the space-time to which Einstein's new kinematics applies. The geometry of this Minkowski space-time is similar to Euclidean geometry. Frames of reference in different states of motion resemble Cartesian coordinate systems with different orientations of their axes. Lorentz transformations in Minkowski space-time are akin to rotations in Euclidean space. The demand that physical laws be Lorentz invariant thus acquired the same status as the demand that physical laws should be independent of the orientation of the axes of the coordinate system used to formulate them.

In a passage that echoes Einstein's comments about electric and magnetic fields quoted above, Minkowski wrote: "space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (Einstein et al., 1954, 73). Different observers in Minkowski space-time will agree on the space-time distance between any two events, but disagree on how to break down that spatio-temporal distance into a spatial and a temporal component. It is this disagreement that lies behind relativity of simultaneity, length contraction, and time dilation, the effects Einstein had derived from his two postulates (Janssen 2002, 429–430). In light of this analysis, Minkowski pointed out, the phrase 'relativity postulate' seemed ill-chosen: "Since the postulate comes to mean that only the four-dimensional world in space and time is given […] but that the projection in space and in time may still be undertaken with a certain degree of freedom, I prefer to call it the *postulate of the absolute world*" (Einstein et al., 1954, 83).

Einstein agreed with Minkowski on this point but felt that it was already too late to change the theory's name. Minkowski's geometrical reformulation of the theory, however, initially met with resistance on Einstein's part, who dismissed it as "superfluous learnedness" (Pais, 152). He only came to value Minkowski's contribution in his work on general relativity.

General Relativity

Einstein was not satisfied with special relativity for very long. He felt strongly that the principle of relativity for uniform motion ought to be generalized to arbitrary motion. In his popular book on relativity of 1917 (Einstein 1961, 72, ch. 21), he used a charming analogy to explain what he found so objectionable about absolute motion. Consider two identical tea kettles sitting on a stove. One is giving off steam, but the other is not. You are puzzled by this until you see that the burner under the first kettle is turned on, but that the burner under the second is not. Compare this situation to that of two identical globes rotating with respect to one another around the line connecting their centers (Einstein et al., 112–115, sec. 2). This situation, like that of the two tea kettles, at first seems to be completely symmetric. Observers on both globes will see the other globe rotating. Yet one globe bulges out at the equator, while the other does not. What is responsible for this difference? The Newtonian answer is: absolute space. What makes a globe bulge out is not its rotation with respect to the other globe, but its rotation with respect to absolute space. The special-relativistic answer is the same except that Newton's absolute space needs to be replaced by Minkowski's absolute space-time. Einstein found this answer unsatisfactory, because neither Newton's absolute space nor Minkowski's absolute space-time can be directly observed. And without an observable cause for the difference between the two globes, we would have a violation of Leibniz's Principle of Sufficient Reason. It would be as if the burners under both tea kettles were turned on, but only one of them gave off steam.

There is a crucial difference, however, between the Newtonian and the special-relativistic answer (Dorling). In special relativity, the situation of the two globes is not even symmetric at the kinematical level. Because of time dilation, one revolution of the other globe takes less time for an observer on the globe rotating in Minkowski space-time than it does for an observer on the other one. It therefore need not surprise us that the situation is not symmetric at the dynamical level either, i.e., that only one globe bulges out. In terms of Einstein's analogy: if the two tea kettles do not *look* the same, it need not surprise us that they do not *behave* the same. Absolute acceleration in special relativity thus does not violate the Principle of Sufficient Reason. Einstein failed to appreciate that special relativity had already solved what he himself identified as the problem of absolute motion.

Einstein set out on the road that would lead to general relativity in 1907. In the 1919 article quoted in the preceding section, he vividly recalls the initial flash of insight. Immediately following the passage quoted earlier, he writes: "Then came to me the best idea [*die glücklichste Gedanke*] of my life […] Like the electric field generated by electromagnetic induction, the gravitational field only has a relative existence. *Because, for an observer freely falling from the roof of a house, no gravitational field exists while he is falling*. The experimental fact that the acceleration due to gravity does not depend on the material is thus a powerful argument for extending the relativity postulate to systems in non-uniform relative motion" (Pais, 178; Janssen 2002, 507).

This extended relativity postulate, it turns out, is highly problematic. What it boils down to is that two observers accelerating with respect to one another can both claim to be at rest if they agree to disagree about whether a gravitational field is present or not. This curious principle is best illustrated with a couple of examples.

First, consider the unfortunate observer falling from the roof in Einstein's example. For a fleeting moment this person will feel like a modern astronaut orbiting the earth in a space-shuttle. Einstein, watching his observer accelerate downwards from the safety of his room in the Berne Patent Office where he was working at the time, is at rest in the gravitational field of the earth. For the falling person, however, there seems to be no gravitational field. He can, if he wishes, maintain that he is at rest and that Einstein and the Patent Office are accelerating upwards.

As a second example, imagine two astronauts in rocket ships hovering side-by-side in somewhere in outer space where the effects of gravity are negligible. One of the astronauts fires up the engines of her rocket ship. According to the other astronaut she accelerates, but she can, if she were so inclined, maintain that she is at rest in the gravitational field that suddenly came into being when her engines were switched on and that the other rocket ship and its crew are in free fall in this gravitational field. Einstein produced an account of the twin paradox along these lines as late as 1919 (Stachel et al., Vol. 7, Doc. 13).

Notice that the 'relativity of acceleration' in these two examples is very different from the relativity of uniform motion in special relativity. Two observers in uniform relative motion are physically fully equivalent to one another; two observers in non-uniform relative motion are not. This is clear in both examples. **(1)** Free fall in a gravitational field **(a)** *feels different* from resisting the pull of gravity **(b)**. **(2)** Hovering in outer space **(a)** *feels different* from accelerating in outer space **(b)**. In both cases, the 'relativity of acceleration' is relativity in name only. In fact, the pair (1a)–(2a) and the pair (1b)–(2b) feel the same.

Einstein's excitement about "the best idea of his life" was nonetheless fully justified. Galileo's principle that all matter falls with the same acceleration in a given gravitational field cries out for an explanation. Newton incorporated the principle by giving two very different concepts of mass the same numerical value. He set inertial mass, a measure of a body's resistance to acceleration, equal to gravitational mass, a measure of a body's susceptibility to gravity. In Newton's theory this is an unexplained coincidence. Einstein correctly surmised that this equality points to an intimate connection between acceleration and gravity. He called this connection the equivalence principle. It was not until after he finished his general theory of relativity, however, that he was able to articulate what exactly the connection is.

That did not stop him from relying heavily on the equivalence principle in constructing his theory. Since accelerating in Minkowski space-time feels the same as resisting the pull of gravity, Einstein was able to glean some features of gravitational fields in general by studying acceleration in Minkowski space-time. In particular, he examined the situation of an observer on a rotating disk or a merry-go-round in

Minkowski space-time. Appealing to the embryonic equivalence principle, the man on the disk can claim to be at rest and attribute the centrifugal forces due to his centripetal acceleration to a centrifugal gravitational field. Suppose the man on the disk and a woman standing next to it are both asked to determine the ratio of the disk's circumference to its radius. The woman will find 2π, the answer given by Euclidean geometry. Because of length contraction, which affects the measuring rods placed along the circumference but not the ones along the radius which move perpendicularly to their length, the man on the disk will find a value greater than 2π. This means that the spatial geometry for an observer rotating in Minkowski space-time is non-Euclidean. The equivalence principle says that the spatial geometry for an observer in a gravitational field will then, in general, also be non-Euclidean. This simple consideration, in all likelihood, gave Einstein the idea to represent gravity by the curvature of space-time (Howard and Stachel, 48–62).

Recognizing that gravity is part of the fabric of space-time made it possible to give a more precise formulation of the equivalence principle. The mature form of the equivalence principle is brought out very nicely by the analogy that Einstein sets up but never quite finishes in the 1919 passage quoted above. Special relativity made it clear that electric and magnetic fields are part of one entity, the electromagnetic field, which breaks down differently into electric and magnetic components for different observers. General relativity similarly made it clear that the inertial structure of space-time and the gravitational field are not two separate entities but components of one entity, now called the *inertio-gravitational field*. Inertial structure determines the trajectories of free particles. Gravity makes all particles deviate from these trajectories in identical fashion, regardless of their mass. These are the only marching orders that *all* particles have to obey. In general relativity, they are all issued by one and the same authority, the inertio-gravitational field, represented by curved space-time. Which marching orders are credited to inertial structure and which ones to gravity depends on the state of motion of the observer. The connection between acceleration (or, equivalently, inertia) and gravity is thus one of unification rather than one of reduction, as with the nominal 'relativity of acceleration' discussed above in which acceleration was being reduced to gravity.

How do the examples of non-uniform motion discussed above fit into the new scheme of things? Free fall in a gravitational field (1a) and hovering in outer space (2a) are both represented as motion along the straightest possible lines in what in general will be a curved space-time. Such lines are called *geodesics*. Resisting the pull of gravity (1b) and accelerating in outer space (2b) are both represented as motion along crooked lines or non-geodesics. Since no change of perspective will transform a geodesic into a non-geodesic or vice versa, there is an absolute difference between (1a) and (1b) as well as between (2a) and (2b). Absolute acceleration survives in general relativity, as in special relativity, in the guise of an absolute distinction between geodesic and non-geodesic motion. This does not violate the Principle of Sufficient Reason since geodesics and non-geodesics are already different at a purely kinematical or geometrical level.

Einstein did not give up his crusade against absolute motion so easily. Once he had realized that gravity is space-time curvature, he quickly came up with a new (though once again flawed) strategy for extending the principle of relativity from uniform to arbitrary motion. To describe curved space-time, Einstein had turned to the theory of curved surfaces of the great 19^{th}-century German mathematician Carl-Friedrich Gauss. To describe such surfaces (think of the surface of the earth for instance) one needs a map, a grid that assigns unique coordinates to every point of the surface, and sets of numbers to convert coordinate distances (i.e., distances on a map) to real distances (i.e., distance on the actual surface). These sets of numbers are called the components of the *metric tensor*. In general they are different for different points. The conversion from coordinate distances to real distances is thus given by a field, called the *metric field*, which assigns the appropriate metric tensor to every point.

A simple example may help to better understand the concept of a metric tensor field. On a standard map of the earth, countries close to the equator look smaller than countries close to the poles. The conversion factors from coordinate distances to real distances are therefore larger near the equator than they are near the poles. The metric tensor field thus varies from point to point, just like an electromagnetic field. Furthermore, at one and the same point, the conversion factor for north-south distances may differ from the conversion factor for east-west distances. The metric tensor at one point thus has different components for different directions.

Gauss' theory of curved surfaces was generalized to spaces of higher dimension by another German mathematician, Bernhard Riemann. This Riemannian geometry can handle curved space-time as well. In the case of four-dimensional space(-time), the metric tensor has ten independent components. In Einstein's theory, the metric tensor field does double duty: it describes both the geometry of space-time and the gravitational field. Mass—or its equivalent, energy—is the source of gravitational fields. Which field is produced by a given source is determined by so-called *field equations*. To complete his theory Einstein thus had to find field equations for the metric field.

Einstein hoped to find field equations that would retain their form under arbitrary coordinate transformations. This property is called *general covariance*. The description of curved space-time outlined in the preceding paragraph clearly is generally covariant. One can choose any grid to assign coordinates to the points of space-time. Each choice will come with its own sets of conversion factors. In other words, the metric field encoding the geometry of the space-time will be represented by different mathematical functions depending on which coordinates are used. Riemannian geometry is formulated in such a way that it works in arbitrary coordinates. It provides standard techniques for transforming the metric field from one coordinate system to another. If only Einstein could find field equations for the metric field that retain their form under arbitrary transformations, his whole theory would be generally covariant. In special relativity, Lorentz invariance expresses the relativity of uniform motion. Einstein—understandably perhaps but mistakenly—thought that extending Lorentz invariance to invariance under arbitrary transformations would automatically extend the principle of relativity from uniform to arbitrary motion.

This line of thinking, however, conflates two completely different traditions in 19^{th}-century geometry (Norton). Minkowski's work with special relativity is in the tradition of projective geometry, associated with the so-called *Erlangen Program* of Felix Klein. Einstein's work with general relativity is in the tradition of differential geometry of Gauss and Riemann. The approaches of Klein and Riemann can be characterized as 'subtractive' and 'additive', respectively (Norton).

In the subtractive approach one starts from an exhaustive description of space-time with all bells and whistles and then strips down this description to its bare essentials. The recipe for doing that is to assign reality only to elements that are invariant under the group of transformations that relate different perspectives on the space-time. This group of transformations is thus directly related to some relativity principle. The most famous application of this strategy in physics is Minkowski's geometrical formulation of special relativity. The group of transformations in this case is the group of Lorentz transformations.

In the additive approach one starts with the set of space-time points stripped of all their properties and then adds the minimum geometrical structure needed to define straight(est) lines and distances in space-time. To guarantee that the added structure describes only intrinsic features of space-time, the demand is made that the description be generally covariant, i.e., that it does not depend on the coordinates used. This procedure can obviously be applied to any space-time. Only in certain special cases, however, will there be symmetries such as Lorentz invariance in Minkowski space-time reflecting the physical equivalence of different frames of reference and thereby some relativity principle. In the generic case there will be no symmetries whatsoever and hence no principle of relativity at all. This shows that general covariance has nothing to do with general relativity. Comparing Lorentz invariance in special relativity and general covariance in general relativity is like comparing apples and oranges.

It was not until 1918 that a German high school teacher by the name of Erich Kretschmann set Einstein straight on this score (Stachel et al., Vol. 7, Doc. 4; Goenner et al., 431–462). This was a few years after Einstein had finally found generally-covariant field equations. These equations, first published in November 1915, formed the capstone of his general theory of relativity. For more than two years prior to that, Einstein had used field equations that are not generally covariant. He had even found a fallacious but ultimately profound argument purporting to show that the field equations for the metric field cannot be generally covariant. For reasons that need not concern us here the argument is known as the "hole argument" (Howard and Stachel, 63–100; Earman, ch. 9). The problem with generally-covariant field equations, according to the hole argument, is that they allow one and the same source to produce what look like different metric fields, whereas the job of field equations is to determine uniquely what field is produced by a given source.

The escape from the hole argument is that on closer examination the different fields compatible with the same source turn out to be identical. The hole argument rests on the assumption that space-time points can be individuated and identified before any of their spatio-temporal properties are specified. Reject this assumption and the argument loses its force. The allegedly different metric fields only differ in that different featureless points take on the identity of the same space-time points. If space-time points cannot be individuated and identified independently of their spatio-temporal properties, this is no difference at all.

This comeback to the hole argument—a popular gloss on Einstein's so-called "point-coincidence argument" (Goenner et al., 463–500)—amounts to a strong argument against the view that space-time is a substance, a container for the contents of space-time. The comeback shows that there are many ways to map spatio-temporal properties onto featureless points, all indistinguishable from one another. According to Leibniz's Principle of the Identity of Indiscernibles all these indistinguishable ways must be physically identical. But then these points cannot be physically real for that would make the indistinguishable ways of ascribing properties to them physically distinct.

The combination of the hole argument and the point coincidence argument is thus seen to be a variant of an argument due to Leibniz himself against the Newtonian view that space is a substance (Alexander). If space were a container, one version of the argument goes, God could have placed its contents a few feet to the left of where He actually placed it. But according to the Principle of the Identity of Indiscernibles these two possible universes are identical and that leaves no room for the container, which could serve to distinguish the two. Einstein's fallacious argument against general covariance thus turned into an argument in support of a Leibnizian relational ontology of space-time.

During the period that Einstein accepted that the field equations of his theory were not generally covariant, he explored yet another strategy for eliminating absolute motion. This strategy was inspired by his reading of Mach's response to Newton's famous bucket experiment. Set a bucket filled with water spinning. It will take the water some time to catch up with the rotation of the bucket. Just after the bucket starts rotating, the water will still be at rest and its surface will be flat. Once the water starts rotating, the water will climb up the sides of the bucket and its surface will become concave. Newton pointed out that this effect cannot be due to the relative rotation of the water with respect to the bucket. After all, the effect increases as the relative rotation decreases and is at its maximum when there is no relative motion at all because the water is rotating with the same angular velocity as the bucket. Newton concluded that the water surface becomes concave because of the water's rotation with respect to absolute space. Mach pointed out that there is another possibility: the effect could be due to the relative rotation of the water with respect to all other matter in the universe. Picture the earth, the bucket, and the water at the center of a giant spherical shell representing all other matter in the universe. If Mach were right, it would make no difference whether the bucket or the shell is set rotating: in both cases the water surface should become concave. According to Newton's theory, however, the rotating shell will have no effect whatsoever on the shape of the water surface.

In 1913–1914, Einstein was convinced for a while that this was a problem not for Mach's analysis but for Newton's theory and that his own theory vindicated Mach's account of the bucket experiment. It only takes a cursory look at Einstein's calculations in support of this claim to see that this attempt to relativize rotation is a non-starter. When Einstein calculated the metric field of a rotating shell at its center, he considered a shell rotating in Minkowski space-time. The rotating shell does produce a tiny deviation from the metric field of Minkowski space-time, but nothing on the order needed to make the water surface concave. What Einstein would have had to show to vindicate Mach is that the metric field produced by the rotating shell near its center mimics Minkowski space-time as seen from a rotating frame of reference. In that case the situation of the bucket at rest in this metric field would have been identical to that of the bucket rotating in the opposite direction in Minkowski space-time. But in order to calculate the metric field of a rotating shell, one needs to make some assumption about boundary conditions, i.e., the values of the metric field as we go to spatial infinity. Rotation with respect to space-time rather than other matter thus creeps back in.

Einstein's flawed Machian account of Newton's bucket experiment receded into the background when he finally found generally-covariant field equations for the metric field in November 1915. As is clear from Einstein's first systematic exposition of the theory in

1916 (Einstein et al., 109–164), he still believed at this point that general covariance guarantees the relativity of arbitrary motion. The Dutch astronomer Willem de Sitter disabused him of this illusion in the fall of 1916 (Stachel et al., Vol. 8, 351–357). De Sitter pointed out that Einstein used Minkowskian boundary conditions in his calculations of metric fields produced by various sources (such as the rotating shell discussed above) and thereby retained a remnant of absolute space-time. By early 1917, Einstein had worked out his response to De Sitter (Einstein et al., 175–188). He eliminated the need for boundary conditions at infinity simply by eliminating infinity! He proposed a model for the universe that is spatially closed. He chose the simplest model of this kind, which is static in addition to being closed. Such a static universe would collapse as a result of the gravitational attraction between its parts. Einstein therefore needed to add a term to his field equations that would provide the gravitational repulsion to neutralize this attraction. This term involved what has become known as the *cosmological constant*. In the late 1920s it became clear that the universe is expanding, in which case the gravitational attraction can be allowed to slow down the expansion and does not need to be compensated by a gravitational repulsion. In the wake of these developments, Einstein allegedly called the cosmological constant the biggest blunder of his life. In 1917, however, he felt he needed it to get rid of boundary conditions.

De Sitter quickly produced an alternative cosmological model that was allowed by Einstein's new field equations with cosmological term. In this De Sitter world there is no matter at all. Absolute space-time thus returned with a vengeance. In reaction to De Sitter's model, Einstein formulated what would come to be known as *Mach's principle* (Stachel et al., Vol. 7, Doc. 4): the metric field is fully determined by matter and cannot exist without it. Einstein was convinced at this point that the addition of the cosmological term guaranteed that general relativity satisfies this principle, despite the apparent counter-example provided by the De Sitter solution. Early in 1918, Einstein argued that the De Sitter world was not empty after all, but that hidden from view a vast amount of matter was tucked away in it. He concluded that general relativity satisfies Mach's principle and that this finally established complete relativity of arbitrary motion. All motion in general relativity is motion with respect to the metric field. But if the metric field can be reduced to matter, talk about such motion can be reinterpreted as a *façon de* *parler* about motion with respect to the matter generating the metric field. This certainly was a clever idea on Einstein's part, but by June 1918 it had become clear that the De Sitter world does not contain any hidden masses and is thus a genuine counter-example to Mach's principle. Another one of Einstein's attempts to relativize all motion had failed.

Einstein thereupon lost his enthusiasm for Mach's principle. He accepted that motion with respect to the metric field can not always be translated into motion with respect to other matter. He also realized that motion with respect to the metric field or curved space-time is much more palatable than motion with respect to Newton's absolute space or Minkowski's absolute space-time anyway. The curved space-time of general relativity, unlike absolute space(-time), is a bona fide physical entity. It not only acts upon matter, like absolute space(-time), by telling matter how to move, but is also acted upon, as matter tells it how to curve (to borrow two slogans from Misner et al., 5). In his lectures in Princeton in May 1921, Einstein reformulated his objection against absolute space(-time) accordingly: it is something that acts but is not acted upon (Einstein 1956, 99–108).

Einstein had a deeper reason to abandon Mach's principle. It was predicated on an antiquated 19th-century billiard-ball ontology. Einstein thought of matter as consisting of electromagnetic fields, in combination perhaps with gravitational fields (Einstein et al., 190–198). Mach's principle would thus amount to reducing one field to another. As can be inferred from a lecture delivered in Leyden in October 1920 (Einstein 1983, 1–24), Einstein came to accept that the metric field exists on a par with the electromagnetic field. Just as he had unified the electric and the magnetic field in special relativity and space-time and gravity in general relativity, he now embarked on the quest for a theory unifying the electromagnetic and the inertio-gravitational field. He would spend the rest of his life looking for such a theory.

Einstein's struggle to relativize all motion, uniform and non-uniform, illustrates the old travelers saying that the journey is more important than the destination. Although Einstein never reached the destination he originally had in mind, he found many valuable results along the way. For starters, he fulfilled many of his philosophical hopes, albeit in ways very different from what he originally envisioned. Absolute motion survives in general relativity since there is an absolute difference between moving on a geodesics and moving on a non-geodesic. But motion with respect to curved space-time with a geometry described by a field interacting with matter (itself described by other fields) is a much more agreeable proposition than motion with respect to the absolute space(-time) of Newtonian theory and special relativity. The combination of the hole argument and the point-coincidence argument, moreover, had provided a strong argument against a Newtonian substantival view of space-time and strong support for the rival Leibnizian relational view.

More importantly, Einstein had found a new theory of gravity, which does away with the artificial split between space-time and gravity of Newtonian theory. This theory opened up such exciting research areas as modern cosmology, black holes, singularities, gravitational waves, and gravitational lensing. Even some of the dead ends in Einstein's crusade against absolute motion led to interesting physics. As these lines are written NASA's *Gravity Probe B* is trying to detect frame dragging, a phenomenon first investigated in the context of Einstein's misguided attempt to vindicate Mach's account of Newton's bucket experiment. The cosmological constant, originally introduced in the context of Einstein's ill-fated attempt to make general relativity satisfy Mach's principle, has made a spectacular comeback in modern cosmology as a straightforward phenomenological description of the repulsion driving the acceleration of the expansion of our universe discovered in recent years through Type Ia Supernovae observations. Einstein's quest for general relativity was a very rewarding journey indeed.

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