Local symmetries with direct empirical status, gauge symmetries, and the empirical approach*

Valeriya CHASOVA
Centre de philosophie des sciences et sociétés
Institut supérieur de philosophie
Université catholique de Louvain

I use the empirical approach to the direct empirical status (DES) to clarify the question of which theoretical symmetries have DES. I prove in particular that if a global symmetry has the identifiable observational DES by virtue of representing observable features of an identifiable empirical symmetry, then using gauge symmetries a local symmetry with the same DES can always be constructed. I explain why this demonstrates that currently we should take local symmetries to be at least as much relevant for the ontology as global symmetries are.

1. Introduction

Symmetries are widespread in physical theories, so the question of their physical content naturally arises. Traditional responses, appealing for example to the hole argument or to the gauge argument, tend to interpret theoretical symmetries as a mathematical surplus or a heuristic tool, or at best as accounting for points of view of different observers. *Direct empirical status* (DES) provides a new look on the question. DES is a status that a theoretical symmetry has in virtue of its correspondence to an empirical symmetry. In this context the theoretical symmetry transformation and the theoretical elements which vary under it have a correspondence in the world. But does this correspondence concern the observable features alone ('the observational DES') or the unobservable features as well ('the ontological DES')? Responding to this question would show to what extent theoretical symmetries can be ontologically significant, and the present article aims at helping to determine the answer.

So far DES has been addressed (to my knowledge) in Kosso ([2000]), Brading and Brown ([2004]), Healey ([2009]), Greaves and Wallace ([2014]), Friederich ([2015]), Teh ([2016]), and Ladyman and Presnell ([in submission])[1]. Despite the progress brought about by these articles, they contain a number of drawbacks: empirical symmetries are only given through examples; the observational and the ontological DES are often conflated; it is mostly held that only global but not local theoretical symmetries have the ontological DES; the empirical approach to DES is

* This is a de-anonymised version of the text which got a 2nd place in the 2018 competition for the Young Researchers Prize of the Société de Philosophie des Sciences (SPS). Submitted for the competition on 1 March 2018.

1 The notions of direct empirical significance and direct empirical consequences have also been used in this literature. I concentrate on the more general and less charged notion of direct empirical status whose name derives from Kosso's ([2000]) original vocabulary and whose varieties I define below.

underestimated; gauge symmetries are deemed irrelevant to the analysis of DES. The present article, by contrast, presents a generalised notion of empirical symmetry, consistently distinguishes the two DESs, shows that local symmetries are able to have the ontological DES, and illustrates the usefulness of gauge symmetries and of the empirical approach to the analysis of DES.

Section 2 explains what empirical symmetries are, and Section 3 when they are identifiable. Section 3 also defines the identifiable observational DES and the identifiable ontological DES and explains why gauge symmetries do not have them. Section 4 explains which theoretical symmetries have the identifiable observational DES. Sections 5-6 discuss global and local symmetries and motivations for thinking that only global symmetries have the identifiable ontological DES. Section 7 exhibits local symmetries with the identifiable observational DES, explains why these symmetries can also have the identifiable ontological DES, and shows how gauge symmetries are useful in this context. Section 8 is a conclusion.

2. Empirical symmetries

For Kosso ([2000]) an empirical symmetry is a phenomenon exhibiting some observable difference and some observable invariance under a transformation. For instance, in Galileo's ship empirical symmetry mechanical experiments performed within a ship look the same from within a ship which stands still with respect to the shore and from within a ship which moves uniformly with respect to it. However, the four recognised empirical symmetries found so far have much more in common than what Kosso was highlighting. I will encompass this in the notion of generic empirical symmetry below.

The following vocabulary will be used. *A physical feature* is any monadic or relational generic physical property (which can be instantiated by several systems or is an abstraction of such instantiations). *A physical system* is a part of the world sufficiently autonomous for our purposes. *A(n extended) physical state* of a physical system relative to a period of time is a collection of physical features which the system or its parts possess at one, some or all moments within this period of time. *A (conservative) physical transformation* is a transformation between physical states which preserves the identity of a physical system. *A dynamics* is a change in time of physical features of a system or its parts which either originates in the system or is induced but not immediately caused by an external intervention (e.g. as when the intervention only consists in setting the initial conditions).

In this framework I propose to describe *a generic empirical symmetry* as constituted by two physical states of a physical system such that one (*the initial state*) can be physically transformed into the other (*the final state*) and such that these states exhibit *the (dynamical) constitutive features*. The latter are firstly *the identical features*, namely the dynamics (for macroscopic systems) or the results of dynamics (for microscopic systems) identical for the two states and such that the dynamics is interior to the system in question, and secondly *the constitutive differences,* i.e. features which cannot be instantiated by the same physical system simultaneously, are different for the two states, and consist in or induce some dynamics between that system and exterior objects which form a larger system together. Additionally, each of the constitutive differences should persist throughout the time during which the interior dynamics unfolds (*the relevance condition*) and there should exist constitutive differences which are similar to those constituting the empirical symmetry but which cannot be instantiated with the same identical features (*the non-triviality condition*).

A two-state empirical symmetry just described corresponds to a particular realisation of a recognised empirical symmetry (e.g. a particular pace of relative displacement between two Galileo's ships and a particular mechanical experiment within each ship). A recognised empirical symmetry itself is a class of such realisations defined by the fact for their constitutive features to be of specific kinds, such as those shown in Table 1. I will be concentrating on particular two-state empirical symmetries rather than on their classes in what follows.

Recognised empirical symmetry	Galileo's ship	Einstein's elevator	Faraday's cage	't Hooft's beam-splitter
Scale	Macroscopic			Microscopic or macroscopic[2]
System	Ship	Elevator	Cage	Beam[3]
Identical features	Interior dynamics[4]			Interference pattern resulting from a dynamics
	mechanical	inertial	electromagnetic	
Constitutive differences				
compatible with the invariance	Some / no[5] uniform rectilinear displacement Some / no friction	A neighbouring mass, no working engine / No neighbouring mass, a working engine	Some / no sparkles on the outer boundaries	Two / no half-wave plates A half-wave plate and a solenoid with a current inside / None
incompatible with the invariance	Some / no accelerated displacement Some / no curvilinear displacement	A neighbouring mass, no working engine / A neighbouring mass, a working engine	To be determined	Two half-wave plates with / without a solenoid with a current inside

The condition above that it be possible to link two states constituting a given (realisation of an) empirical symmetry by a physical transformation is compatible with the states not being linked by a physical transformation in some instances of this empirical symmetry. Thus in different instances of an empirical symmetry the two states can belong to the same or to distinct system(s), which constitutes respectively *the first* and *the second way of instantiating empirical symmetries*. The equivalence of these ways was stated but not motivated by Brading and Brown ([2004], p. 647, n. 5). One simple motivation I can propose is that distinct systems can instantiate an empirical symmetry via the first way, so the identity of systems seems irrelevant to instantiating any of the states constituting an empirical symmetry (unless when the constitutive differences are essential) and thus the second way seems as much acceptable as the first.

3. Identifiability

DES can be established in two ways. In the *theoretical approach*, one starts with a theoretical symmetry and asks which empirical symmetry adequately instantiates it (e.g. Greaves and Wallace [2014]). In the *empirical approach*, one starts with an empirical symmetry and asks which theoretical symmetry adequately represents it (e.g. Healey [2009]). If the matching empirical or theoretical symmetry is found, the relevant theoretical symmetry is said to have DES.

At least in the empirical approach a pre-requisite for establishing a DES is that an empirical symmetry be *identifiable*, i.e. that it be possible to detect it in the world. Arguably empirical symmetries can be detected most straightforwardly if their constitutive features are observable. If they are not, one could try to claim that a phenomenon is an empirical symmetry (or that an empirical symmetry exists) because a theory says so. But this requires presupposing that some

2 Respectively in ('t Hooft [1980]) and (Naeij and Shafiee [2016]). Note that the name and page numbers of the 't Hooft's article do not seem to be cited correctly in Brading and Brown ([2004]) and Greaves and Wallace ([2014]).

3 Not one of the two parts the beam is split into as in Greaves and Wallace ([2014]). This takes into account objections to the latter position (Greaves and Wallace [2014], p. 83, n. 24).

4 E.g. falling balls, bending light, burning candles; for other examples for Galileo's ship and Faraday's cage, see quotes in Healey [2009], pp. 698-699.

5 Or two qualitatively different realisations (note also valid for other 'some / no' cases).

theoretical symmetry has the ontological DES with respect to it, and so will beg the question of which theoretical symmetry has DES. But this is the question we want to answer after having identified what counts as an empirical symmetry, not before. Therefore not only should it be possible to identify an empirical symmetry without any reference to theories (Kosso [2000], p. 85; Healey [2009], p. 699), but it is necessary to do so in our context. If no other means of access to unobservable features remains, it follows that identifiable empirical symmetries are precisely empirical symmetries where both the identical features and the constitutive differences are observable. All the recognised empirical symmetries are of this kind.

This rules out in particular empirical symmetries where only the identical features are observable (excluded by Kosso [2000] but admitted by Ladyman and Presnell [in submission]). It might seem that we could establish the unobservable constitutive differences through the observability of the physical transformation, like in Ladyman and Presnell's example of a square tile rotated by 90 degrees. There the process of performing the transformation is indeed observable, but this is because the intermediary states by which it passes are observationally distinguishable from the initial and the final states constituting this empirical symmetry. But then if we match an intermediary state with the initial or the final state, the constitutive differences will be observable contrary to our presupposition; and if we match an intermediary state with its 90-degree rotated state, the constitutive differences will again be unobservable and this will be of no help[6]; and if we consider the initial and the final state together with an intermediary state, then this may help for the first way of realising empirical symmetries but not for the second way because there are no intermediary states there. Moreover, this will not help in cases like Ladyman and Presnell's example of the Michelson-Morley experiment where for any couple of states the presumed constitutive differences of position with respect to the aether are unobservable[7].

If we cannot identify empirical symmetries whose constitutive differences are unobservable, we cannot establish a DES of those theoretical symmetries which would be best instantiated by, or would best represent, such empirical symmetries. Arguably the latter theoretical symmetries are those whose transformations do not induce any (non-trivial) observational consequences. But then these are just *gauge symmetries* by one of the understandings of this term[8]. Hence *no DES of such gauge symmetries can be established*, and this independently of whether their transformations are local or global, external (i.e. spatiotemporal) or internal (i.e. non-spatiotemporal), independently of whether gauge symmetries do have DES, and despite the possibility for us to put the identical observational consequences of a gauge symmetry in correspondence with the observable identical features of an empirical symmetry.

Thus DESs we have a hope to establish (*identifiable DESs*) should be those of non-gauge symmetries. Moreover, given the argument against appeal to theories above, these DESs should only rely on physical features and theoretical elements related to the level of observation. This motivates the following definitions (which both do not make appeal to features and elements unrelated to observation): A theoretical symmetry has *the identifiable observational DES* if it provides an adequate representation of the observable constitutive features of an identifiable empirical symmetry in the empirical approach or if the observable constitutive features of an identifiable empirical symmetry provide an adequate instantiation of the theoretical symmetry's observational consequences in the theoretical approach. And a theoretical symmetry has *the identifiable ontological DES* if moreover the same relationship holds between the unobservable

6 This shows that identifiability of an empirical symmetry depends on two states or on a state and a transformation, not on a single state.

7 Ladyman and Presnell claim that this is not an empirical symmetry because theories say it is not, but if the argument against appeal to theories above also holds for establishing what should not count as an empirical symmetry, we should instead remain agnostic on whether this experiment is an empirical symmetry.

8 This is the understanding we adopt throughout the article. On another often used understanding, gauge symmetries are local symmetries whose empirical status is unclear. These are simply called local symmetries below, and their status is discussed in Section 7.

underpinnings of these observable constitutive features and the theoretical underpinnings of these observational consequences. This corresponds to Ladyman and Presnell's distinction between weaker 'logical' and stronger 'physical' consequences of theoretical symmetries, but is free of some of its drawbacks[9]. We will be analysing these identifiable DESs of non-gauge symmetries until we see how gauge symmetries can be useful in that context.

4. Theoretical symmetries with the identifiable observational DES

Even the most comprehensive analysis of DES so far by Greaves and Wallace ([2014]) does not tell which theoretical symmetries have the (identifiable) observational DES[10]. They proceed within the theoretical approach, by considering which theoretical symmetries are instantiated by empirical symmetries, and obtain a list of theoretical symmetries corresponding to the recognised empirical symmetries as a response (ibid., p. 87)[11]. Beforehand they divide theoretical symmetries into boundary-preserving and non-boundary-preserving ones, as well as into interior and non-interior ones, so one expects their list to follow from their framework. But the first distinction is orthogonal to the question of which symmetry has the observational DES, for both boundary-preserving and non-boundary-preserving symmetries figure in their list. Meanwhile, classifying a symmetry as interior trivially implies the answer to that question, for by their definition interior symmetries are those where both theoretical states describe the same possible world, and so they cannot correspond to any empirical symmetry (ibid., p. 71); while classifying a symmetry as non-interior does not generate the answer precisely enough, for some non-interior symmetries may fail to correspond to identifiable empirical symmetries or even to generalised not necessarily identifiable empirical symmetries from Section 2.

I will show how to determine which theoretical symmetries have the identifiable observational DES using the empirical approach, a general knowledge about current physics and a characterisation of a generic theoretical symmetry[12].

We start from the empirical approach by asking what it takes to represent an identifiable empirical symmetry. A general knowledge of physics tells us that observable features static within a state can be represented either by static values of variables or by evolving values whose observational consequences are static, while observable dynamical features can be represented by evolving values whose observational consequences evolve. To obtain the latter, we can first represent observable features of an initial *momentary physical state*, i.e. the fraction of a physical state corresponding to the moment when the dynamics begins, using an initial *momentary theoretical state*, i.e. a set (or distribution) of certain values of suitable variables indexed by the same value of a time variable, next generate a history (i.e. a sequence of momentary theoretical states for all other times) using differential equations of motion or field equations, and then restrict that history to the period of time during which a non-trivial observable identical dynamics or a non-trivial dynamics leading to identical observable static features occurs. Alternatively, besides specifying an initial momentary theoretical state we can also specify a final momentary theoretical state corresponding to the last moment of the non-trivial dynamics in question, then determine the variation along which path (i.e. a restricted sequence of momentary states) between the two momentary theoretical states extremises an action, and choose either that path in the classical case,

9 Namely, here both approaches rather than the theoretical approach alone are clearly taken into account, and the theoretical and the physical level are clearly distinguished while Ladyman and Presnell's distinction suggests they are mixed if the idea that theoretical symmetries have physical consequences is taken seriously.

10 I take them to examine this question because Ladyman and Presnell ([in submission]) say Wallace confirms dealing with weaker 'logical' rather than stronger 'physical' consequences in that article.

11 To which for completeness global symmetries corresponding to 't Hooft's beam-splitter must be added from Brading and Brown ([2004]).

12 This is a theoretical approach in that a knowledge of physics is used, but not the theoretical approach as defined above.

or a combination of paths contributing proportionally to their proximity to the classical path in the quantum case. Thus a representation of a physical state, which we will name *a(n extended) theoretical state*, will be a sequence of momentary theoretical states comprising a theoretically static or dynamical representation of observably static features and a theoretically dynamical representation of observable dynamical features.

As to the physical transformation, its occurrence is accidental to the way of instantiating empirical symmetries, and the details of bringing about this transformation are accidental to the particular instantiations within the first way. Moreover, representing e.g. how one decided to physically boost a ship, if possible at all, may require going beyond physics. So we only need to represent a possibility of a physical transformation, and this can be done using a non-dynamical (i.e. without intermediary stages) theoretical transformation which resumes or brings about all the differences between the initial and the final theoretical states (representing respectively the initial and the final physical states), and which we will name *the main transformation* to distinguish it from the infinitesimal dynamical theoretical transformations which can be used to generate a single (momentary or extended) theoretical state.

A particular representation of a realisation of an identifiable empirical symmetry will thus consist of two particular theoretical states linked by a particular main transformation. In virtue of representing the physical states of an identifiable empirical symmetry these theoretical states will have to share some but not all of their observational consequences besides possibly sharing some theoretical elements as well. Now *a generic theoretical symmetry* consists of two partially identical theoretical constructions and a theoretical transformation between them[13]. Therefore, under a natural identification of the theoretical states with 'constructions' and of the main transformation with the transformation linking those constructions, a representation of an identifiable empirical symmetry we have built is precisely *a non-trivial theoretical symmetry with the identifiable observational DES in the empirical approach*.

5. Global and local transformations and states

The literature on DES has concentrated on whether a DES of a theoretical symmetry depends on the global or local character of its (restricted) main transformation. But as explained above the main transformation only has to resume the differences between the theoretical states because of the accidental character of much there is in the physical transformation. So most of the identifiable observational DES has to be ensured by the theoretical states. To keep the possibility for the global/local distinction to be important in this context, I am therefore going to extend it to states by analysing how it applies to transformations, supposing both states and transformations are already specified.

There are three usual ways of dividing given transformations into global and local. Firstly, a continuous group of transformations is global or local according to whether it depends on a finite number of parameters or functions respectively. Secondly, a transformation is global or local according to whether it is specified using a finite number of parameters or functions respectively. Thirdly, a transformation is global or local according to whether its prescriptions of change in value vary within its domain of application.

From these the first definition is mostly used in mathematics (from which the notion of group comes as well) and the other two are used as equivalent in the literature on DES. I will not use the second definition, because if it is an attempt to particularise the first in view of applying it to physics, the justification in our context is not self-evident, and it is not equivalent to the third either. Indeed, a transformation global by the third definition can be global or local by the second definition, i.e. described using a parameter or a function (which can be constant or not on the

13 A transformation alone is not enough because there should also be something it preserves (for this to be a symmetry) and something it changes (for the symmetry to be non-trivial).

relevant domain). Conversely, a transformation local by the second definition can be global or local by the third definition. Both issues arise because variables concerned by the two definitions are different. E.g. in the (restricted) theoretical symmetry $\psi_0 \rightarrow \psi_1 = \psi_0\exp(iq\theta)$, where the phase transformation $\cdot\exp(iq\theta)$ can represent the change in the constitutive differences of 't Hooft's beam-splitter empirical symmetry, the second definition concerns θ and the third $\cdot\exp(iq\theta)$, i.e. the change in the phase ψ. But it is the latter quantity that represents the change in the constitutive differences. So what matters primarily is whether the change in ψ varies within the domain, not whether the change in θ varies within the domain. Therefore the third definition will be used.

Thus *a given theoretical transformation* is *global* or *local* with respect to a variable depending on whether the prescriptions of change in value of that variable are the same for all items or different for at least two items within the initial domain. Therefore, the most straightforward extension to states is to say that *a given theoretical state* is *global* or *local* with respect to a variable depending on whether the values of that variable are same for all items or different for at least two items within the initial domain.

Arguments in favour of this way of defining the extension to states are as follows.

Firstly, in this case the only difference between the distinctions, and perfectly natural, is that the distinction for transformations is about prescriptions of change in value while the distinction for states is about values.

Secondly, the latter distinction needs the same precondition for being well-defined (i.e. allowing either option to be realised) as the former, namely that the domain be always composed of several items. Indeed, if the domain is composed of a single item, a state or transformation will always be global, either trivially or after we will have reinterpreted the domain as composed of several parts of the original item to each of which the same value or change in value is attributed. E.g. the (restricted) theoretical symmetry $v_0 \rightarrow v_1 = v_0 + v'$ capable of representing the constitutive differences of Galileo's ship empirical symmetry, usually considered as involving two values of velocity v_0 and v_1 assigned each to a whole Galileo's ship as well as a single change in value $+v'$ between them, thus comes out either way as global in the main transformation and in the states with respect to velocity.

Thirdly, the global or local character of given states and transformations can be determined using the same tool, namely a function which evaluates how values or changes in value differ for two items[14]. The conditions for having a global or a local character according to this tool will again be similar for states and transformations, namely, given the adopted definitions of the global/local distinction for states and transformations, *a theoretical state or transformation* will be *global* or *local* according to whether the difference in value or in change in value as evaluated by the function respectively is or is not zero for respectively any or at least some two items within the domain. Also if the function fails to be well-determined, this will affect the characterisation of states and transformations alike. If it only fails to give a unique evaluation for two distant items from the same domain, as is common in particular to many general-relativistic topologies, restricting the function to comparisons to neighbouring items, i.e. making it a kind of derivative on the domain, can help to determine the character of both states and transformations.

An already specified main theoretical transformation can be interpreted *more actively* (as representing a production of physical change of a physical state) or *more passively* (as resuming a difference between two physical states) depending on whether we match it with instantiations of an empirical symmetry where physical transformations are present or those where they are absent[15]. Above I only took the more active sense, but the same discussion could be made for the more passive sense, with global and local defined as above except that items go into couples of items, the initial domain into the initial and the final domain, and prescriptions of change in value into differences between values.

14 While the function in the second definition above concerned variables like θ, this function concerns variables like ψ.
15 Both these senses are active with respect to the passive sense from Section 7.

6. Properly global and properly local theoretical symmetries

To establish a correspondence between the identical features and suitable theoretical elements we could use a single physical state and so no DES would arise. It is for establishing a correspondence between (the change in) the constitutive differences and other suitable theoretical elements that we need an empirical symmetry and a theoretical symmetry, whence a DES[16]. Thus a DES of a theoretical symmetry is primarily a property of its *restricted theoretical symmetry* constituted by those theoretical elements corresponding to the observable constitutive differences, together with that part of the main transformation which corresponds to the difference between these elements in the two theoretical states. We will therefore be concentrating on the DES of restricted theoretical symmetries for the rest of the article[17].

Examples of restricted theoretical symmetries mentioned in Section 5 are $v_0 \rightarrow v_1 = v_0 + v'$ and $\psi_0 \rightarrow \psi_1 = \psi_0\exp(iq\theta)$. Here v_0 and ψ_0 are restricted initial theoretical states, $+v'$ and $\cdot\exp(iq\theta)$ restricted main transformations, v_1 and ψ_1 restricted final theoretical states. These symmetries involve velocity (v) and phase (ψ) variables respectively, and can represent (be instantiated by) the constitutive differences and the change in them respectively in Galileo's ship and 't Hooft's beam-splitter empirical symmetries (the latter usually when supplemented with a restricted symmetry involving the electromagnetic potential variable). For Faraday's cage, restricted symmetries involving electrostatic or electromagnetic potentials have been evoked in the literature on DES, and for Einstein's elevator, restricted symmetries involving acceleration and gravitational potentials.

In our context *a theoretical symmetry* is qualified as *global* or *local* according to whether its restricted main transformation is global or local. As many authors as Kosso ([2000]), Brading and Brown ([2004]), Healey ([2009]), Friederich ([2015]), and Ladyman and Presnell ([in submission]) can be taken to claim that only symmetries global in that sense have the ontological DES. But given the importance of the states explained above, admitting that a symmetry with the ontological DES global in the restricted main transformation can be local in the restricted states seems to significantly weaken this view. Therefore, if we say that *theoretical symmetries* are *properly global* or *properly local* according to whether both the restricted main transformation and the restricted theoretical states are respectively global or local, the strongest version of the common view will be the claim that only properly global symmetries have the ontological DES.

I think that within the empirical approach the strengthened common view can be supported by the following pragmatic motivation. It is a fact that any realisation of a recognised empirical symmetry yields the observational DES to an infinity of theoretical symmetries (call them *DES-equivalent*). E.g. two Galileo's ships uniformly moving at a definite observable pace with respect to each other can be represented by an infinity of pairs (v_0,v_1) such that velocities in each pair differ by the same v'. Usually DES-equivalent theoretical symmetries have the same observational consequences, and so empirical adequacy, and do not differ significantly in theoretical virtues such as simplicity. A common strategy for extracting ontology in such cases is to hold as ontologically faithful what all the theoretical descriptions agree on. But this gives a very poor ontology if applied to only local or to both global and local DES-equivalent symmetries because these do not have much in common. For example, properly global and properly local symmetries obviously disagree on the global or local character of states and transformations, let alone on particular values and their particular changes and/or differences, while local DES-equivalent symmetries disagree for instance on whether Faraday's cage or its exterior alone is transformed (Healey [2009], appendix B, gives an example of the latter representation). Meanwhile, properly global DES-equivalent symmetries disagree on the absolute values of the variable(s) subject to transformation, but agree numerically

16 More precisely, DES also involves a correspondence between the compatibility of the identical features with either of the constitutive differences and the compatibility between the theoretical elements put in correspondence to them.

17 Even when we will be considering unrestricted symmetries in Section 7.

on the transformation besides agreeing on the global character of both the transformation and the states. Abstracting away the absolute values from DES-equivalent properly global symmetries is thus sufficient to get a rather rich ontology with respect to the other options[18]. This pragmatically motivates the attribution of the ontological DES to properly global symmetries alone, as well as the ontological importance of the main transformation over the theoretical states it links.

However, if ontology is to be understood in objective rather than pragmatic way, additional motivation independent of pragmatic considerations has to be provided. But motivations from the early literature on DES are often elliptic or unclear, need to be updated in light of recent articles (such as Greaves and Wallace [2014] and Teh [2016]) and concern particular cases, by which reason they require going into technical and other details and are not easily generalisable. So instead of discussing them, I will concentrate on one general and easy-to-understand non-pragmatic motivation which has persisted throughout the debate. This is the idea, also arising within the empirical approach, that identifiable empirical symmetries often yield the identifiable observational DES to (properly) global symmetries alone.

Already Kosso's main claim is that “global symmetries … are, in an important sense, directly observed, while local symmetries … are not” ([2000], p. 81), where the relevant sense is a correspondence between an identifiable empirical ('physical') symmetry, possibly established prior to any theories, and a theoretical ('analytic') symmetry (ibid., p. 85)[19]. However, later on Brading and Brown ([2004]) and Healey ([2009]) show respectively of 't Hooft's beam-splitter and of Faraday's cage empirical symmetries that they yield the identifiable observational DES to both global and local symmetries. Still, when Greaves and Wallace introduce Einstein's elevator empirical symmetry into discussion, they only exhibit global (although not properly global) symmetries capable of representing its observable constitutive differences ([2014], eq. 11). Furthermore, despite suggestions by Greaves and Wallace ([2014], p. 63, n. 4) and Friederich ([2015], p. 556) no-one has exhibited local or properly local symmetries capable of representing the observable constitutive differences of Galileo's ship empirical symmetry yet, so it continues to be represented by restricted symmetries like the one involving velocity above, which is properly global as argued in Section 5. The upshot is that so far two out of four recognised empirical symmetries still bestow the identifiable observational DES on (properly) global symmetries alone.

An argument for the view that properly local symmetries do not have the ontological DES can then be easily constructed as follows. It is plausible (given the current situation, and perhaps by other reasons as well) that while some empirical symmetries bestow the identifiable observational DES on both global and local symmetries, other empirical symmetries bestow this DES on global symmetries alone in the sense that their observable constitutive differences can only be represented by properly global symmetries or at least by global symmetries, but not by properly local symmetries. Then in the case of the latter empirical symmetries only (properly) global symmetries will have the identifiable ontological DES because there will be no properly local symmetries to consider. But to remain consistent, we have to build our ontology in the same way whatever the empirical symmetry. Hence, the argument would go, likewise in the case of the former empirical symmetries properly local symmetries cannot have the identifiable ontological DES. In short, (properly) global but not properly local symmetries would have to always get the identifiable ontological DES because of them having the identifiable observational DES with respect to a greater number of empirical symmetries.

To address this argument, exhibiting properly local symmetries representing the observable constitutive differences of the two recognised empirical symmetries above would be useful not be enough, for the worry would still remain of discovering new empirical symmetries which bestow the identifiable observational DES on (properly) global symmetries alone. Instead, its crucial

18 Obviously allowing global symmetries not to be properly global removes the agreement on the character of the states and makes the ontology poorer.

19 Whether he is concerned with the observational or the ontological DES is unclear.

assumption that some empirical symmetries yield the identifiable observational DES to (properly) global symmetries alone will be challenged in the next section in a way which does not depend on details of particular theories or identifiable empirical symmetries.

7. The proof

I am going to prove that from an unrestricted properly global theoretical symmetry which has the identifiable observational DES in virtue of adequately representing the observable constitutive features of a realisation of an identifiable empirical symmetry, an unrestricted properly local theoretical symmetry with the same status with respect to the same realisation of the identifiable empirical symmetry can always be constructed using suitable gauge symmetries, i.e. those theoretical symmetries which do not induce any change in observational consequences.

The recipe is as follows:

(1) construct new theoretical states and ensure they are observationally equivalent to the original states;

(2) ensure the new states are local in all the variables relevant for the representation of the observable constitutive differences;

(3) construct a theoretical transformation between these local states;

(4) ensure this transformation is local in all the variables relevant for the representation of the observable constitutive differences.

All this will be done using requirements on gauge transformations (and using the I defined below).

The notation in the proof is as follows. In the original theoretical symmetry the initial theoretical state is labelled by S_0, the main transformation by T, the final theoretical state by S_1. The common part of the two states is denoted by S. In the resulting theoretical symmetry the same labels with a prime are used. The (gauge) transformation linking S_0 and S_0' is labelled by T_0, the (gauge) transformation linking S_1 and S_1' is labelled by T_1. After all these labels, variables acted on by a transformation or involved in a state can be indicated in brackets. Variables (or a single variable) are denoted by A if they are acted on by the restricted main transformation, by B if they are acted on by the unrestricted but not restricted main transformation, by C if they are involved in the states but are not acted on by the main transformation, and by N if they are not involved neither in the main transformation nor in the states prior to introducing the gauge transformations. These letters are unprimed or primed depending of whether the involvement is with respect to the states and the transformation of the original or the resulting symmetry. Unspecified variables are denoted by X, while I stands for values of variables ensuring the individuation of the items for the state of which these values make part.

In this notation our task reads as follows: starting with a theoretical symmetry with the identifiable observational DES whose restricted symmetry is constituted by $S_0(A)$, $T(A)$, $S_1(A)$ all global in A, we should arrive at another theoretical symmetry with the identifiable observational DES whose restricted symmetry is constituted by $S_0'(X)$, $T'(X)$, $S_1'(X)$ all local in X. I am going to obtain this using gauge transformations T_0, T_1 as depicted in Figure 1.

$$\begin{tikzcd} S_1 \arrow[rr, "T_1"] & & S_1' \\ \\ S_0 \arrow[uu, "T"] \arrow[rr, "T_0"'] & & S_0' \arrow[uu, dashed, "T'"'] \end{tikzcd}$$

Figure 1

(1) We firstly need gauge transformations T_0, T_1. One option is to construct T_0 and T_1 using *the first approach* to constructing transformations, which consists in attaching prescriptions of change in value of some variable to the items which belong to the initial domain, i.e. to the domain of S_0 for T_0 and of S_1 for T_1. That T_0, T_1 are not given in advance implies in particular that a theory used to build to the original symmetry does not tell us anything about the observational impact of T_0, T_1. Therefore we can always impose as a requirement that these transformations do not induce any change in observational consequences when applied to respectively S_0 and S_1.

Alternatively, we can use gauge transformations already given by a theory used to build the original symmetry or by another theory. This option may seem to imply that a properly local symmetry that we wish to construct is already given too. However, if T_0, T_1 are given as prescriptions of change on the domains of S_0, S_1, then S_0', S_1' are still not given until we apply T_0, T_1 to S_0, S_1, i.e. until we apply to a value associated by a state to a given item a prescription of change in value associated by a transformation to that item. Moreover, neither giving T_0, T_1 nor applying them to S_0, S_1 may be sufficient to define T'. For even though we will presuppose that the domains of S_0, S_1, S_0', S_1' have the same cardinality, a way of defining T' can be non-unique (see (3) below) and then T' will not be defined until a choice is made. Besides, not all T_0, T_1 lead to properly local symmetries, otherwise we would not need all the requirements on them specified below.

Next, as a given or constructed T_0 is gauge when applied to S_0, the resulting state S_0' is observationally equivalent to S_0, and likewise the state S_1' resulting from the application of T_1 to S_1 is observationally equivalent to S_1. Thus S_0' represents the initial physical state as good as S_0 does, and likewise for S_1' as compared to S_1. Moreover, S_0 and S_1 are distinct observationally inequivalent states, for otherwise the same theoretical state would be allowed to represent two observationally distinct physical states, and this would contradict the assumption (implied by the identifiable observational DES) that the observable constitutive differences are adequately represented. Therefore S_0' and S_1' are distinct observationally inequivalent states, and if they can be linked by a main transformation, it will be non-trivial and will have to bring the difference in the observational consequences of these states, which will be the same as the difference in the observational consequences brought by T.

(2) The variables relevant to the representation of the observable constitutive differences in S_0' and/or S_1' are those the action of T_0 or T_1 on which in S_0 and S_1, coupled to a non-trivial action of T_0 or T_1 on A, is sufficient to make S_0' and S_1' observationally equivalent to respectively S_0 and S_1 as far as the representation of the observable constitutive differences is concerned. Both S_0' and S_1' should be local in all the relevant variables including A (which can comprise A alone or more variables), or both should be both local in all the relevant variables and zero in A (which then is not a relevant variable strictly speaking). E.g. quantum mechanics says that passing from a state global in phase to a state local in phase changes the prediction of the interference pattern in 't Hooft's beam-splitter, but a theory coupling quantum particles with classical electromagnetic fields gives gauge transformations allowing to restore the original prediction by making the state also local in electromagnetic potential (Brading and Brown [2004], p. 654). Here the phase corresponds to A and the potential to N. To act on a new variable N one presupposes that it characterised the initial state but was zero across it. To ensure that the DES of the original symmetry does not derive from (changes in) values of N, we can thus impose that if N are acted on by at least one of T_0 and T_1, N have value zero across both S_0 and S_1 and are unaffected by T. For completeness we will suppose that the relevant variables can be among any of A, B, C and N.

If the action of T_0, T_1 on the relevant variables makes S_0', S_1' incompatible with the way other features are represented in S_0, S_1, then T_0, T_1 can additionally act on the theoretical elements accounting for the latter features to make S_0' and S_1' observationally equivalent to respectively S_0 and S_1 as far as the representation of these features is concerned. However, S_0 and S_1 need not be local in these theoretical elements. E.g. in Brading and Brown's example introducing the potential requires changing the Lagrangian or the equations for generating the dynamics ([2004], eq. 1.7 instead of eq. 1.1)[20].

To make S_0', S_1' local in the relevant variables, recall that being global means attaching the same (change in) value to each item within the domain of application and being local means attaching a different (change in) value to different items within the domain. It follows that, within the domain of application of the transformation in the first approach and with respect to the variable(s) acted on by it, /1/ a global transformation applied to a global state always yields a global state; /2/ a global transformation applied to a local state always yields a local state; /3/ a local transformation applied to a local state can yield a local or a global state; and /4/ a local transformation applied to a global state always yields a local state.

Therefore, /1/ allows to put values of A to zero by requiring that $T_0(A)$ and $T_1(A)$ be certain global transformations. Meanwhile, for making S_0', S_1' local in B and C in which S_0, S_1 are local, either we should require using /2/ that T_0, T_1 be global, or using /3/ we should require that T_0, T_1 be local and exclude the possibility in this context for S_0', S_1' to be global. For this, as T_0, T_1 are given or have been already constructed, we can simply check different S_0', S_1' obtained using different local T_0, T_1 and only retain those local T_0, T_1 which yield local S_0', S_1'. Or else we can require for this that T_0, T_1 be identity on some subset of items of S_0, S_1. Finally, /4/ allows to make S_0', S_1' local in A and N, as well as in B and C in which S_0, S_1 are global, by requiring that T_0, T_1 be local in these variables.

For T_0 or T_1 to be local, this transformation should associate different amounts of change to different items characterised by S_0 or S_1, and this means that these items should be distinguishable beforehand. To ensure this we will presuppose that S_0 and S_1 include or are supplemented with the individuating distributions of values I allowing to distinguish between the items. E.g. values of spatiotemporal variables can be used because within any given coordinate frame any two items always get associated to them different sets of values of these variables. Of course, associations between sets of values and items vary with coordinate frames, so distinct coordinate frames can associate the same set of values to distinct items. Thus S_0 and S_1 can share the same I (as part of C) e.g. because they represent objects participating in the interior dynamics within two systems (e.g. Galileo's ships or 't Hooft's beam-splitters) as having the same locations in the coordinate frames defined by the observers of these systems.

(3) As by now S_0' and S_1' have already been obtained, we can use *the second approach* to constructing transformations and define T' as a difference, for any variable by values of which S_0' and S_1' differ, between a given initial value and a given final value of that variable. Which variables T' acts upon is determined by a *strong key fact* by which T' should be equivalent to the combination of the reverse of T_0 followed by T and T_1 in case T_0 is reversible, given that both T' and the combination just mentioned should lead from the same S_0' to the same S_1', or at least by a *weak key fact* by which the combination of T_0 and T' should be equivalent to the combination of T and T_1 given that both combinations should lead from the same S_0 to the same S_1'. Either fact means T' should only act on those variables which are needed to make the square commute. E.g. if $T_0(N)$ and $T_1(N)$ are identical, T'(N) is identity, and if neither of T, T_0 and T_1 act on C, nor can T'. Thus variables on which T' does not act (denoted by C') and variables on which T' acts (denoted by A'&B') are those whose change by T' is respectively incompatible or compatible with one of the key facts.

We need not have I in S_0' and S_1' to define T', but their presence together with the key facts determines whether T' is defined uniquely. Firstly, if there are I among the values of C' (e.g. if the same I are incorporated in S_0 and S_1 (as part of C) and if T_0, T_1 preserve these I), T' is unique. Secondly, if C' do not provide I and A'&B' do but only for one of S_0' and S_1', then some items characterised by the same values of C' will be distinguishable in one state because of having different values of A'&B' and indistinguishable in another state because of having the same values

20 More exactly one would need to change them in the original symmetry too similarly to how one introduced N there.

of A'&B'. In this case T' preserving C' is seemingly underdetermined as to which of the distinguishable items should be matched with which of the indistinguishable items. But due to the indistinguishability of the items the matching options are indistinguishable too, hence no real underdetermination arises. Thirdly, the same holds if neither C' nor A'&B' provide I for either of S_0' and S_1', i.e. when indistinguishable items are matched. Fourthly, if C' does not provide I and A'&B' do for either of S_0' and S_1', then T' is indeed underdetermined because there are several distinguishable ways of matching the distinguishable items of the two domains (or, more precisely, of matching values of A'&B' associated to these items) even if C' is required to be preserved given the key facts. This underdetermination is however not a problem provided among the ways of defining T' we can single out those which make it local in the relevant variables.

(4) We deduce from /1/-/4/ that /1'/ two global states can be linked by an identity or a global transformation, /2'/ two local states can be linked by an identity or a global or a local transformation, /3'/ a global state and a local state can only be linked by a local transformation. So by /2'/ T' is not automatically local in all the relevant variables in which S_0' and S_1' are local. Moreover, in the third case above T' limited to the matchings of indistinguishable items (states global in C' and in A'&B') cannot be local by /1'/, although the whole T' can still be local. On the other hand, in the second case above T' limited to the matchings of indistinguishable items with distinguishable items (the latter state global in C' and local in A'&B') is necessarily local by /3'/, and hence the whole T' is necessarily local. So one way of making T' local is to ensure that the second case obtains. E.g. if S_0, S_1 have the same I (as part of C), we can make T_0, T_1 both act on C to alter the same value (or the same set of values) of I into another value among those used in I, while we make say T_0(A,N) identity and T_1(A,N) local on the items whose values are concerned, and both T_0(A,N) and T_1(A,N) local on the other items. (That local transformations act on I needed to define them is not a problem.) However, this hardly suits for I such as values of spatiotemporal variables, so I will also describe another way of ensuring T' is local which it suitable for them too.

Namely, it follows from our definition of global and local transformations above that if one transformation is equivalent to the combination of a second and a third transformation, then /1''/ if from first and the second transformations, or from the second and the third, both are global, respectively the third or the first is either identity or another global transformation; /2''/ if from the first and the second transformations, or from the second and the third, one is global and the other local, respectively the third or the first is another local transformation; /3''/ if the first and the second transformations are local, the third is identity or global or local. Therefore we can make a transformation global or local by considering it as part of a triangle of transformations where the global or local character of the other transformations is suitably chosen.

A triangle is available directly for variables not acted on by T, i.e. C and N, for which by a *reduced weak key fact* T_1 is equivalent to the combination of T_0 and T' (Figure 2). For A and B we can transform our square into two triangles. For this we define T'': $S_0 \rightarrow S_1$' using the second approach and determine the variables T'' acts upon using another reduced weak key fact by which T'' is equivalent to the combination of T and T_1. T'' also gets involved in a third reduced weak key fact by which T'' is equivalent to the combination of T_0 and T' (Figure 3).

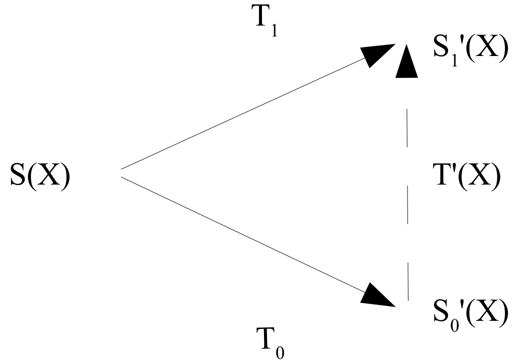

Figure 2

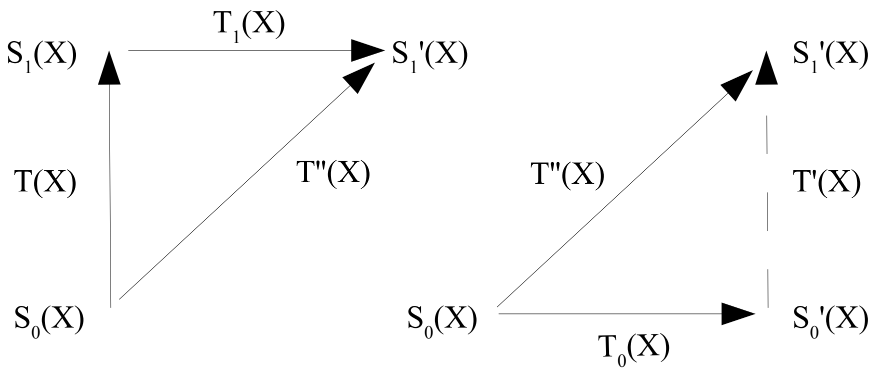

Figure 3

The character of T' can then be determined given that in both triangles where it is present, it is the third transformation from the rules /1"/-/3"/ above. Thus in general T' can be made local via /2"/, or via /3"/ e.g. if the other two transformations are required to be non-identical on their domain (which precludes the third transformation from being identical) but identical on some restriction of their domain (which precludes the third transformation from being global). However, not all solutions suit for particular variables.

T'(A) cannot be made local via /2"/, for if T_0(A) is global and T"(A) local, by /2"/ T_1(A) should be local and so S_0' will be zero in A while S_1' local in A contrary to (2); and if T_0(A) is local and T"(A) is global, by /1"/ T_1(A) should be global and so S_0' will be local in A while S_1' zero in A contrary to (2). Thus T'(A) should be made local by choosing a local T_1(A), which by /2"/ makes T"(A) local, and a local T_0(A), which allows to use /3"/ provided the requirements for /3"/ above are satisfied, i.e. provided we require that T_0(A) be different from T"(A) except for some subdomain of S_0.

Similarly, T'(N) or T'(C) with global S(C) cannot be made local via /2"/, for if one of T_0 and T_1 is global in C or N and another local in C or N, by /1/ one of S_0' and S_1' will be global in C or N and by /4/ the other of S_0' and S_1' will be local in C or N contrary to (2). Thus T'(N) or T'(C) with global S(C) should be made local via /3"/ by choosing both T_0 and T_1 local in N or C and by imposing on them the requirements for /3"/ above.

Also T'(C) with local S(C) can be made local via /3"/ as just described provided we ensure that S_0'(C) and S_1'(C) are local. It is fortunate that one of the ways of ensuring this proposed in (2), namely to require that T_0(C) and T_1(C) be identity on a subdomain of S, implies (if for T_0(C) and T_1(C) the same subdomain is concerned) that these transformations are identical on this subdomain and so allows to satisfy the requirements for /3"/ at the same time. Besides, T'(C) with local S(C) can be made local via /2"/ by choosing one of T_0(C) and T_1(C) to be global and another local while ensuring, e.g. as proposed in (2), that the state resulting from the action of the latter transformation is local in C.

Finally, if S_0(B), T(B), S_1(B) are all global, T'(B) can be made local exactly as it was done with T'(A). While if S_0(B) is global and S_1(B) is local, and hence T(B) local by /3'/, we can choose T_1(B) to be global, which makes T"(B) local by /2"/, then choose T_1(B)** to be local and use /3"/ and the requirements above, i.e. require that T_0(B) be different from T"(B) except for some subdomain of S_0. Meanwhile, if S_0(B) is local and S_1(B) is global, and hence T(B) local by /3'/, we can choose T_1(B) to be local and use /3"/ and the requirements above, i.e. require that T_1(B) be different from T"(B) except for some subdomain of S_1, then choose T_1(B)*** to be global and use /2"/. Finally, if S_0(B), T(B), S_1(B) are all local, we can choose T_1(B) global, which makes T"(B) local by /2"/, and choose T_0(B) global, which makes T'(B) local by /2"/.

This concludes the proof.

** This should have been "T_0(B)".

*** This should have been "except for some subdomain of S_1 and some subdomain of S_0 which share the same I, then choose T_0(B)".

Example (adapted from Healey [2009], Appendix B): The observable constitutive differences of Faraday's cage empirical symmetry are represented by a properly global symmetry $\varphi_0 = 0 \rightarrow \varphi_1 = \varphi_0 + \varphi' \neq 0$ with φ the electrostatic potential. T_1 sets φ_1 to 0 and acts on the magnetic potential Ax as follows: $Ax_0 = 0 \rightarrow Ax_1 = (\varphi_0 / a)t$ where *a* is a distance from the cage's exterior borders at which sparkles are no longer observed. In our notation φ is A and Ax is N. T_1 and S_1' are local in Ax. T'' is identical to $T_1(Ax)$. We expect a properly local symmetry to arise if T_0 is local in our sense. Perhaps this T_0 is the usual gauge transformation (Healey [2009], p. 718, eq. 2) with a suitable choice of function Λ.

Some consequences of the proof are as follows. Firstly, the same theoretical state (S_0) can give rise to a theoretical symmetry with an identifiable DES (S_0, T, S_1) and to a gauge symmetry (S_0, T_0, S_0'). Thus which of these kinds a theoretical symmetry belongs to is determined by a couple of states or by a state and a transformation rather than a single state. This parallels the way an empirical symmetry happens to be identifiable or non-identifiable (Section 3 n. 6). Secondly, gauge symmetries (including local ones) are not irrelevant to DES, contrary to the usual belief supported even by those who attribute a DES to some local symmetries (Greaves and Wallace [2014], p. 87; Teh [2016], p. 116). For gauge symmetries can produce properly local (S_0', T', S_1') and mixed (S_0, T'', S_1') symmetries with the identifiable observational DES from properly global ones, and we can expect them to link other combinations of symmetries too. Thirdly, the ontological status of gauge symmetries can affect the status of symmetries with the identifiable observational DES. In particular, if each of the gauge symmetries (S_0, T_0, S_0' and S_1, T_1, S_1') is interpreted *passively*, i.e. with the two theoretical states redescribing the same physical state and the theoretical transformation describing a passage between these redescriptions, then the three theoretical symmetries (S_0, T, S_1 and S_0', T', S_1' and S_0, T'', S_1') have the identifiable observational DES with respect to the same realisation of the same empirical symmetry and cannot all have the ontological DES with respect to it. Meanwhile, if each of the gauge symmetries is interpreted *actively*, i.e. with each of the two theoretical states describing a different physical state (such that the two physical states are observationally indistinguishable) and the theoretical transformation describing a passage between the two physical states (whether interpreted more actively or more passively in the sense of Section 5), then the three theoretical symmetries have the identifiable observational DES with respect to either of the physically distinct observationally indistinguishable realisations of the empirical symmetry so obtained and more than one of these theoretical symmetries can have the ontological DES with respect to one (but normally not the same) of these realisations of the empirical symmetry. Finally, if only one of the gauge symmetries (e.g. T_1) is interpreted actively, the two realisations of the empirical symmetry share one of the physical states, and so at most one of the theoretical states (S_0 and S_0') should be able to represent the unobservable underpinnings of that state, but again more than one theoretical symmetry can have the ontological DES.

8. Conclusion

A number of new interrelated results have been presented: a general account of empirical symmetries, a discussion of their identifiability, a demonstration of why gauge symmetries do not have an identifiable DES, two notions of identifiable DES, a description of a theoretical symmetry with the identifiable observational DES, a demonstration in this context of how the empirical approach is more useful than the current version of the theoretical approach, an extension of the global/local distinction to theoretical states, a motivation for concentrating on reduced theoretical symmetries, a pragmatic and a non-pragmatic argument within the empirical approach on why only properly global symmetries have the identifiable ontological DES, a refutation of the non-pragmatic argument by the proof that from a properly global symmetry with the identifiable observational DES a properly local symmetry with the same DES can always be constructed, and a demonstration via the same proof of how gauge symmetries are useful for the analysis of DES. The upshot is that so far properly local symmetries should be considered as having at least as much chances for the identifiable ontological DES as properly global symmetries do.

9. Acknowledgements

All the new content of this article, including the proof, has been elaborated by myself. Nevertheless, I would like to thank some persons and organisations as follows.

I thank Guido Bacciagaluppi, Steven French and Jean-Pierre Marquis for a discussion of my early thoughts on the topic, in particular concerning the formalisaiton of the notion of empirical symmetry.

I also thank the participants of the conference “Direct empirical status and the ontology of symmetries in physics”, including the invited speakers Gordon Belot, Richard Healey, James Ladyman and David Wallace, for their participation and their interest in the topic.

Besides, I thank particularly Alexandre Guay for helping me to organise the conference, obtaining a funding for my PhD studies, bringing the topic of DES to my attention and having been my research director for seven years.

I am also grateful to the Fonds de la Recherche Scientifique – FNRS (F.R.S.-FNRS) for a mobility funding and to the Fonds Spéciaux de Recherche (Université catholique de Louvain) for sponsoring my PhD studies.

Finally, I am grateful to the Société de Philosophie des Sciences (SPS) for giving this text (before the de-anonymising) a 2nd place in the 2018 competition for the Young Researchers Prize.

10. References

Brading K., and H. Brown [2004], «Are Gauge Symmetry Transformations Observable?», *The British Journal for the Philosophy of Science*, Vol. 55, pp. 645-665.

Friederich S. [2015], «Symmetry, Empirical Equivalence, and Identity», *The British Journal for the Philosophy of Science*, Vol. 66, pp. 537-559.

Greaves H., and D. Wallace [2014], «Empirical Consequences of Symmetries», *The British Journal for the Philosophy of Science*, Vol. 65, pp. 59-89.

Healey R. [2009], «Perfect Symmetries», *The British Journal for the Philosophy of Science*, Vol. 60, pp. 697-720.

Kosso P. [2000], «The Empirical Status of Symmetries in Physics», *The British Journal for the Philosophy of Science*, Vol. 51, pp. 81–98.

Ladyman J., and S. Presnell [in submission], «Representation and Symmetry in Physics».

Naeij H.R., and A. Shafiee [2016], «Double-Slit Interference Pattern for a Macroscopic Quantum System», *Foundations of Physics*, DOI 10.1007/s10701-016-0034-7.

Teh N. [2016], «Galileo's Gauge: Understanding the Empirical Significance of Gauge Symmetry», *Philosophy of Science*, Vol. 83, pp. 93-118.

't Hooft G. [1980], «Gauge Theories of the Forces between Elementary Particles», *Scientific American*, Vol. 242, pp. 104-138.