Perverted space-time geodesy in Einstein's views on geometry

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Résumé

Une géodésie spatio-temporelle pervertie résulte des notions de règles et d'horloges variables, qui sont prises pour avoir leur longueur et leur rythme affectés par le champ gravitationnel. D'autre part ce que nous pourrions appeler une géodésie concrète repose sur les notions de règles et d'horloges invariables de mesure d'unité. En fait, il s'agit d'une hypothèse de base de la relativité générale. Les règles et les horloges variables conduisent à une géodésie pervertie dans le sens où un espace-temps courbe pourrait être vu comme provenant du départ de l'espace-temps Minkowskien comme un effet du champ gravitationnel sur le rythme d'horloges et la longueur des règles. Dans le cas d'une géodésie concrète, nous avons «directement» un espace-temps courbe dont la courbure peut être déterminée en utilisant des règles et des horloges de mesure d'unité (invariables). Dans cet article, nous défendrons la plausibilité que le point de vue d'Einstein sur la géométrie par rapport à la relativité générale est imprégné par une géodésie pervertie.

Abstract

A perverted space-time geodesy results from the notions of variable rods and clocks, which are taken to have their length and rates affected by the gravitational field. On the other hand, what we might call a concrete geodesy relies on the notions of invariable unit-measuring rods and clocks. In fact, this is a basic assumption of general relativity. Variable rods and clocks lead to a perverted geodesy in the sense that a curved space-time might be seen as arising from the departure from the Minkowskian space-time as an effect of the gravitational field on the rate of clocks and the length of rods. In the case of a concrete geodesy we have “directly” a curved space-time whose curvature can be determined using (invariable) unit-measuring rods and clocks. In this paper, we will make the case for the plausibility that Einstein's views on geometry in relation to general relativity are permeated by a perverted geodesy.

1. Introduction

In his detailed analysis of Einstein's 1916 review paper on general relativity, Darrigol noticed that Einstein when making the so-called Newtonian approximation, or in his gravitational redshift derivation, seems to start with a "fictitious" Minkowskian space-time and arrive at a curved space-time by considering the effect of the gravitational field on rods and clocks [Darrigol 2015, 172]. According to Darrigol, "in this view, space is originally Euclidean but it appears to be non- Euclidean when the measuring rods are affected by a gravitational field" [Darrigol 2015, 172], with an equivalent situation holding for clocks. Darrigol calls this view a "perverted geodesy". Darrigol notices, en passant, in a footnote, that Einstein reliance on this view is confirmed in his 1917 book, "where he compares the non-Euclidean geometry of general relativity with the apparent geometry of a table top of uneven temperature when gauged with dilatable rods" [Darrigol 2015, 172]. This paper starts from this "seed". Our intention is to make an historical reconstruction of Einstein's views on geometry and determine to what extent his views are permeated by a perverted geodesy. We will make the case for the plausibility of Darrigol's view that Einstein relied on a perverted geodesy in relation to the space-time of general relativity.[1] While we will mention them, we will not focus in detail on the inconsistencies that would be brought by this situation.

The work is organized as follows. Section 2 is a preliminary section, in which we review some basic elements of the theory – the meaning of Gaussian coordinates and the physical assumption of the "independence from past history" of unit-measuring rods and clocks –, and we consider a gravitational redshift derivation made by applying the equivalence principle[2] in the case of clocks in a rotating disk. We will see that this heuristic derivation is based on the "dynamical-like" view that the gravitational field (the rotation of the disk) affects the rate of clocks. This result goes hand in hand with the adoption of a perverted geodesy in this case. The bulk of this work is section 3, in which we consider the development of Einstein's views on geometry and see how in several places Einstein adopts (implicitly) a perverted geodesy: 1) In his rotating disk argument for the adoption of a non-Euclidean geometry due to the presence of a gravitational field (using the equivalence principle, the rotation of the disk is reinterpreted as a stationary gravitational field "existing" in relation to the rotating reference frame – the disk), 2) In the analogy with a heated marble slab in which "little rods" are differently deformed due to the temperature gradient in the table (which is an analogy to the curved space-time of general relativity), 3) In his reference to the Gaussian coordinates of a two dimensional surface as "nothing other than an arbitrary deformed and stretched planar Cartesian coordinate system" [Einstein 1920, 145], which are straightforwardly applied to the case of general relativity as the Gaussian coordinates of a four-dimensional space-time, 4) In the persistence, in the context of general relativity, of the reference to the effect of the gravitational field on rods and clocks, e.g. in redshift derivations, which, in cases 1 and 2, clearly gives rise to a perverted geodesy.

Section 4 is a complementary section in which we address some further elements pointing to the presence of a perverted geodesy in Einstein's work. We mention briefly the evidence for the presence of a perverted geodesy in Einstein's early metric theory of gravitation (the Entwurf theory) and in Einstein's work on Nordström's scalar theory of gravitation. In the final section, we reiterate once

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1 We will consider in detail Einstein's views on geometry up until 1921, which is the year in which was published "geometry and experience" where he presented his view on geometry called "practical geometry", and when he wrote his Princeton lectures on special and general relativity (published in 1922). Together with the 1916 review paper and the 1917 "popular" book, these constitute the most detailed and influential writings by Einstein on general relativity and geometry.

2 In its earlier formulation, the equivalence principle entails that an inertial reference frame in a homogeneous gravitational field is physically equivalence to a uniformly accelerated reference frame in a space free of a gravitational field [Einstein 1907, 302], [Einstein 1911], [Einstein 1913, 208]. Einstein made an alternative formulation of the principle of equivalence in which we consider a four-dimensional space-time region in which special relativity is valid (i.e. in which we can adopt an inertial reference frame K); we then consider a second reference system K' uniformly accelerated with respect to K. According to Einstein "nothing prevents us from considering [the] system K' as at rest, provided we assume a gravitational field (homogeneous in first approximation) relative to K'" [Einstein 1916b, 237-8]; see also [Einstein 1916a, 150], Einstein 1917, 318-9], [Einstein 1922, 317], [Norton 1985, 205-6].

again the thesis of this work and address some possible criticism and alternatives to it.

2. Perverted geodesy in the rotating disk gravitational redshift derivation

The physical meaning of coordinates in general relativity is not as direct as in special relativity. In the latter case, we identify the reading of unit-measuring rods and clocks with the differential dx (dy, dz) and dt of the line element $\mathrm{ds}^2 = - \mathrm{dx}^2 - \mathrm{dy}^2 - \mathrm{dz}^2 + \mathrm{c}^2\mathrm{dt}^2$. In the case of general relativity this expression is only valid locally and in general $\mathrm{ds}^2 = \sum_{\mu\nu} \mathrm{g}_{\mu\nu}\mathrm{dx}_\mu\mathrm{dx}_\nu$. The Gaussian coordinates x_μ (x_ν) are in this expression related to local measurements made with rods and clocks through the metric $g_{\mu\nu}$. This was already clear to Einstein in his early effort on a metric field theory of gravity based on Riemann geometry and tensor calculus – the Entwurf theory.[3] According to Einstein:

The relation between coordinate differentials on the one hand and measuring lengths and times on the other hand is given in this way; since the quantities $g_{\mu\nu}$ enter into this relationship, the coordinates by themselves have no physical meaning. [Einstein 1913, 211]

Einstein gives a detailed introduction to Gaussian coordinates in his early 1917 book on special and general relativity. We begin with the original context in which Gauss developed this type of coordinates – its application to curved surfaces. Einstein asks us to imagine "a system of arbitrary curves drawn on the surface of a table" [Einstein 1917, 340]. We have a mesh of "horizontal" and "vertical" curves *u* and *v*, and each point of the surface is identified by the intersection of two of these curves. It follows that "a value of *u* and a value of *v* belong to every point on the surface" [Einstein 1917, 341]. These numbers are our Gaussian coordinates. Accordingly, "two neighbouring points P and P' on the surface then correspond to the coordinates P: *u*, *v* [and] P': *u* +d*u*, *v* + d*v*, where d*u* and d*v* signify very small numbers" [Einstein 1917, 341]. Here we do not have any notion of length or distance between P and P'. For that purpose, we can do as follows:

We may indicate the distance (line-interval) between P and P', as measured with a little rod, by means of the very small number ds. Then according to Gauss we have In the latter case $\mathrm{ds}^2 = \mathrm{g}_{11}\mathrm{du}^2 + 2\mathrm{g}_{12}\mathrm{dudv} + \mathrm{g}_{22}\mathrm{dv}^2$. [Einstein 1917, 341]

We see that the Gaussian coordinates do not have a direct metrical significance. As Einstein remarks, "the Gaussian method can be applied also to a continuum of three, four or more dimensions" [Einstein 1917, 342]. In this way:

We refer the four-dimensional space-time continuum in an arbitrary manner to Gauss coordinates. We assign to every point of the continuum (event) four numbers x_1, x_2, x_3, x_4 (coordinates), which have not the least direct physical significance. [Einstein 1917, 349]

For this approach to be meaningful it is crucial that our measuring rods (our "little rods") and clocks correspond to standards of length and time that do not change depending on where and when the measurements are being made. Accordingly, it is an assumption of the theory that the rate (length) of clocks (rods) is independent from their path of transfer (see, e.g., [Einstein 1921b, 225]; see also, e.g.,

3 Einstein worked on a tentative non-convariant metric field theory, the so-called Entwurf theory, from mid-1912 to late 1915 when Einstein found the field equations of general relativity (see, e.g., [Norton 2005]).

[Synge 1960, 106], [Geroch 1972, 8]).[4, 5] According to Einstein, it is this assumption that enables an invariant ds, as we can see, e.g., in a letter Einstein wrote to Weyl in 1918:

If light rays were the only means of establishing empirically the metric conditions in the vicinity of a space-time point, a factor would indeed remain undefined in the distance ds (as well as in the $g_{\mu\nu}$'s). This indefiniteness would not exist, however, if the measurement results gained from (infinitesimal) rigid bodies (measuring rods) and clocks are used in the definition of ds. A timelike ds can then be measured directly through a standard clock whose world line contains ds.

Such a definition for the elementary distance ds would only become illusory if the concepts "standard measuring rod" and "standard clock" were based on a principally false assumption; this would be the case if the length of a standard measuring rod (or the rate of a standard clock) depended on its prehistory. If this really were the case in nature, then no chemical elements with spectral lines of a specific frequency could exist, but rather the relative frequencies of two (spatially adjacent) atoms of the same sort would, in general, have to differ. [CPAE Vol 8, 533]

An immediate consequence of this is that the standard (or unit-measuring) rods and the standard (or unit-measuring) clocks cannot be considered to be affected by the gravitational field. If we have a unit clock in a particular region of the gravitational field (i.e. of space-time) and move it to another region it is still a unit clock, i.e. its rate is not affected by the path of transfer *neither* by its new location in space-time.[6]

In his detailed analysis of Einstein's review paper on general relativity from 1916, Darrigol noticed its "lack of a clear distinction between heuristic and deductive arguments" [Darrigol 2015, 163]. This is noticed, e.g., in Einstein's derivation of the gravitational redshift. In his 1916 paper, Einstein made a (heuristic) derivation of the redshift in which a clock was taken to be affected by the gravitational field, and the "temporal" Gaussian coordinate was given a direct metrical significance as corresponding to the measured time interval of the clock "at rest" in the gravitational field [Einstein 1916a, 197-8], [Darrigol 2015, 172-3].[7] Darrigol considered two possible explanations for this state of affairs, one of them being that "Einstein seems to be starting from a fictitious Minkowskian space-time and to be treating the departure from Minkowskian geometry as an effect of the gravitational field on the rate of clocks" [Darrigol 2015, 172].[8] Darrigol called this view "perverted geodesy".

4 While Einstein considers rods and clocks at an equal footing, in, e.g., Malament, Synge or Geroch's approaches only clocks have a special role; light rays "complement" clocks to provide a measurement of spatial lengths [Malament 2012, 118], [Synge 1960, 112-3], [Geroch 1972, 11-2]. To make easier the presentation and analysis of Einstein's ideas we will adopt his original views and mention rods and clocks. However, we can define a practically-rigid rod by what Darrigol called "chrono-optical control of rigidity" [Darrigol 2015, 167]. This implies that from a modern perspective we can justify the adoption at an equal footing of rods and clocks. As Darrigol mentioned, "what is truly incompatible with general relativity is the existence of extended rigid bodies" [Darrigol 2015, 167].

5 There is also a more "technical" detail at play. The local Minkowskian space-time corresponds to a local inertial reference frame that we can regard as being in free fall in the gravitational field (see, e.g., [Einstein 1922, 322-3]; [Einstein 1923, 78]). If we consider, e.g., a clock "at rest" in a gravitational field we must check that its rate is not affected by the non-gravitational forces that "pull" the clock away from its free fall along a geodesic. We will consider that when mentioning measurements made by rods and clocks these are part of a free falling local inertial frame or that these measurements would be identical to that of rods and clocks in free fall momentarily at rest in relation to the rods and clocks being used to make the measurements.

6 In this presentation of the "standard interpretation" of general relativity in terms of a concrete geodesy we make reference to several writings by Einstein. However, as we will see, he did not make any consistent presentation of this view in his writings (the references we found are: [Einstein 1921a], [Einstein 1921b], [Einstein 1922, 323], and some correspondence like [CPAE Vol 8, 529] and [CPAE Vol 8, 533]). In fact, we mostly see elements that can be interpreted as referring to a perverted geodesy.

7 This general relativistic derivation is basically the same as the derivations made using the Entwurf theory [Einstein 1913, 216-8], [Einstein 1914, 82].

8 Regarding this derivation, Earman and Glymour had already noticed that "the derivation actually relied on the same ideas as the 1907 and 1911 heuristic derivations, especially the idea that the rate of clocks is affected by the gravitational field" [Earman & Glymour 1980, 182]. Overall, regarding Einstein's and others heuristic derivations, Earman and Glymour position is that "all heuristic derivations of the red shift can be faulted on various technical arguments. But to raise such objections is to miss the purpose of heuristic arguments, which is not to provide logically seamless proofs but

For the purpose of this paper it will be useful to consider the redshift derivation made by Einstein in his 1917 book, which relied on the use of the equivalence principle and clocks located in a rotating disk.[9] Einstein considers a disk rotating with a constant angular velocity ω (relative to an inertial reference frame K), which constitutes an accelerated reference frame K'. Einstein considers a clock located at a distance γ from the center of the disk. According to special relativity the frequency of the clock (number of ticks of the clock per unit time) is $\nu = \nu_0(1 - \omega^2\gamma^2/2c^2)$, where ν_0 is the frequency of an identical clock at rest at the origin. By resort to the equivalence principle, judged from K' the clock "is in a gravitational field of potential Φ" [Einstein 1917, 389], where $\Phi = -\omega^2\gamma^2/2$. From this result – taken to "hold quite generally" [Einstein 1917, 389], and regarding "an atom which is emitting spectral lines as a clock" [Einstein 1917, 389] –, Einstein concludes that:

An atom absorbs or emits light of a frequency which is dependent on the potential of the gravitational field in which it is situated.

The frequency of an atom situated on the surface of a heavenly body will be somewhat less than the frequency of an atom of the same element which is situated in free space (or on the surface of a smaller celestial body) … thus a displacement towards the red ought to take place for spectral lines produced at the surface of stars as compared with the spectral lines of the same element produced at the surface of the earth. [Einstein 1917, 389-90]

In this derivation of the gravitational redshift we have an "underlying physical mechanism" that leads to the gravitational redshift and that is the effect of the gravitational field on clocks. According to this derivation the clocks go slower in the vicinity of a gravitational source (in this case the "gravitational field" is stronger the further away the clock is from the center of the disk).[10] This view is in complete contradiction with the physical assumption of the "independence from past history" of unit-measuring rods and clocks that sustains and gives physical meaning to the invariant line element ds.

Looking more closely into this derivation we can also see a perverted geodesy at work. In fact, the perverted geodesy goes hand in hand with the idea that clocks and rods are affected by the gravitational field. The derivation shows us that the frequency of a clock in a rotating disk changes according to the clock's distance to the center, according to the expression $\nu = \nu_0(1 - \omega^2\gamma^2/c^2)$. We might say that we have a curved time (we will see in the next section that there is an equivalent "effect" regarding rods in a rotating disk so that instead of a Euclidean space we have a non-Euclidean space. Overall, we have a curved space-time). As Einstein recognizes for the similar case of rods in a rotating disk:

Throughout this [derivation] we have to use the Galilean (non-rotating) system K as reference-body, since we may only assume the validity of the results of the special theory of relativity relative to K (relative to K' a gravitational field prevails). [Einstein 1917, 334]

The mathematical expression $\nu = \nu_0(1 - \omega^2\gamma^2/c^2)$ is derived in relation to K, i.e. in relation to an

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rather to give a feel for the underlying physical mechanism. It is precisely here that most of the heuristic red shift fail – they are not good heuristics" [Earman & Glymour 1980, 181].

9 Even if this "derivation" appeared in a "popular" exposition of the theory from 1917, we must take into account that Einstein did not considered it as less valuable than his heuristic field theory derivations. We can notice this from Einstein reliance on this derivation on a 1920 unpublished manuscript that was intended to be published in Nature [Einstein 1920, 141-2], and in a letter to Eddington from the previous year, in which Einstein considered that this derivation showed that the gravitational redshift "is an absolutely compelling consequence of relativity theory" [CPAE Vol 9, 184]. It is clear that Einstein was confident on this derivation. Importantly, in this letter Einstein also mentions that "measuring rods and clocks exhibit a behavior independent of their prehistories" [CPAE Vol 9, 184]. It seems that Einstein did not notice the incompatibility between the presuppositions and consequences implied in the derivation and the physical assumption of "independence from past history" regarding rods and clocks.

10 This "underlying physical mechanism" is already present in Einstein's initial derivations in terms of the application of the equivalence principle in the case of a uniformly accelerated reference frame [Einstein 1907, 302-7], [Einstein 1911, 384-5].

underlying non-curved time. The special relativistic time dilation, as determined in K, implies in K that the clock appears to run slower the more distant it is from the center. According to Einstein this result also holds in K', being reinterpreted, applying the equivalence principle, in terms of the presence of a gravitational field, i.e. we have $\nu = \nu_0(1 - \Phi/c^2)$, where $\Phi = -\omega^2\gamma^2/2$ [Einstein 1917, 389]. In this way, it is the departure of the clock's rate from its underlying non-curved/uniform time due to the gravitational field (the rotation of the disk) that curves time. We can say, using a phrasing similar to Darrigol's, that time is originally non-curved but it appears to be curved when clocks are affected by a gravitational field: we start with the non-curved time of a Minkowskian space-time, as described in an inertial reference frame K, and we treat the departure from the Minkowskian non-curved time as an effect of the gravitational field on the rate of clocks. We have a case of a perverted geodesy.

3. Perverted geodesy in Einstein’s practical geometry

While Einstein mature views on geometry appeared on the well-known “geometry and experience” from 1921, some of his views had already “consolidated” and appeared on a paper on the Entwurf theory from late 1914. We start with the idea of geometry as “pure” mathematics: “[Euclidean] geometry means originally only the essence of conclusions from geometric axioms; in this regard it has no physical content” [Einstein 1914, 78]. Then comes the view that geometry can become a physical science (by 1921 this view is elaborated in a way that only geometry as a physical science is meaningful in the present stage of development of physics): “[Euclidean] geometry becomes a physical science by adding the statement that two points of a “rigid” body shall have a distinct distance from each other that is independent of the position of the body” [Einstein 1914, 78].[11] This leads to a view that Einstein will maintain throughout: “After this amendment, the theorems of this amended [Euclidean] geometry are (in a physical sense) either factually true or not true” [Einstein 1914, 78]. That is, we can experimentally determine if the adopted geometry is in agreement with observation and/or experimental results. Regarding Euclidean geometry, Einstein's position is then that: “Euclidean geometry too – as it is used in physics – consists of physical theorems that, from a physical aspect, are on an equal footing with the integral laws of Newtonian mechanics” [Einstein 1914, 78].

Einstein mentions again Euclidean geometry as a physical science briefly in his review paper on general relativity from 1916. Here, Einstein is more explicit regarding what becomes of Euclidean geometry as a physical science: “the laws of [Euclidean] geometry, even according to the special theory of relativity, are to be interpreted directly as laws relating to the possible relative positions of solid bodies at rest” [Einstein 1916a, 148]. Here we find expressed (implicitly) the view that the amended or physical Euclidean geometry only applies for solid bodies in inertial motion. In fact, a few pages later Einstein considers a coordinate system K' in uniform rotation around the Z-axis of an inertial (or Galilean) system of reference K having both the same origin. Einstein concludes that “Euclidean geometry does not apply to K'” [Einstein 1916a, 152]. To arrive at this conclusion Einstein considers “a circle around the origin in the X, Y plane of K [which] may at the same time be regarded as a circle in the X', Y' plane of K'” [Einstein 1916a, 152]. If we measure the circumference and diameter of this circle using unit-measuring rods at rest relative to K the quotient is π. However, if we make the same measurements with unit-measuring rods at rest relatively to K' the quotient is greater than π. According to Einstein:

This is readily understood if we envisage the whole process of measuring from the “stationary” system K, and take into consideration that the measuring-rod applied to the periphery undergoes a Lorentzian contraction, while the one applied along the radius does not. Hence Euclidean geometry

11 Already in 1912 Einstein mentioned that “the propositions of Euclidean geometry acquire a physical content through our assumption that there exist objects that possess the properties of the basic structures of Euclidean geometry” [Einstein 1912, 28].

does not apply to K'. [Einstein 1916a, 152]

Like in the case of the redshift derivation considered in the previous section we have a case of a perverted geodesy. Applying the equivalence principle, we might consider that the gravitational field "existing" in K' affects the length of the unit-measuring rods located on the periphery of the circle in a way similar to its effect on clocks. Again, we start with a description in a Minkowskian space-time and by calculations made in relation to the inertial reference frame K we determine the "distortion" suffered by unit-measuring rods, which leads to conclude from "measurements" made with the whole set of rods (the ones along the radius and the ones along the periphery) that space is non-Euclidean.

After considering the issue of spatial geometry, Einstein considers what occurs to clocks rotating with K'. We have a situation similar to that of the unit-measuring rods along the periphery. Due to the time dilation, a "clock at the circumference – judged from K – goes more slowly than [a clock at the origin]" [Einstein 1916a, 152]. In this case he does not mention the gravitational redshift, which, however, is a direct consequence. As mentioned in the previous section we have here also a case of a perverted geodesy.

Taken together the results for rods and clocks in a rotating coordinate system (which we associate with a rotating reference frame – the disk), we see that, in this case, more than "starting from a fictitious Minkowskian space-time" [Darrigol, 2015, 172], we start from what we might call a "background" Minkowskian space-time (which is always "there" for the coordinate system K), and due to the "influence" of the gravitational field (i.e. the rotation) we have in K' an "effective" curved space-time.

Einstein gives a new presentation of his views on geometry in his early 1917 book. Again, Einstein starts with "pure" geometry. As such, "geometry, however, is not concerned with the relation of the ideas involved in it [e.g. point, straight line] to objects of experience, but only with the logical connection of these ideas among themselves" [Einstein 1917, 250]. Again, like in 1914, we can consider geometry to be amended so that we can use it in physics. At this point Einstein still treats geometry at two levels not being clear that the amended geometry is necessary beyond practical reasons:

We now supplement the propositions of Euclidean geometry by the single proposition that two points on a practically rigid body always correspond to the same distance (line-interval), independently of any changes in position to which we may subject the body. [Einstein 1917, 251]

This is with a different wording the statement already made in late 1914. Like in early 1916, Einstein stresses that "the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies" [Einstein 1917, 251]. After this sentence Einstein presents two of the key ideas we already found in 1914:

Geometry which has been supplemented in this way is then to be treated as a branch of physics. We can now legitimately ask as to the "truth" of geometrical propositions interpreted in this way, since we are justified in asking whether these propositions are satisfied for those real things we have associated with the geometrical ideas. [Einstein 1917, 251]

Afterwards, Einstein addresses, similarly to the 1916 paper, the case of a reference frame K' rotating uniformly in relation to an inertial reference frame K. Our rotating frame consists in a plane circular disk rotating about its center. Let us consider two identical clocks, one at the center of the disk and the other at its periphery. Applying the results of special relativity to the clock located at the periphery with a constant angular velocity (in K), we find that this clock "goes at a rate permanently slower than that of the clock at the center of the circular disk" [Einstein 1917, 333]. Einstein extends this result and considers that "in every gravitational field, a clock will go more quickly or less quickly, according to the position in which the clock is situated (at rest)" [Einstein 1917, 333]. Considering the case of spatial geometry, we imagine that an "observer applies his standard measuring-rod …

tangentially to the edge of the disk” [Einstein 1917, 333]. Again, applying special relativity Einstein concludes that:

As judged from the Galilean system, the length of this rod will be less than 1 … on the other hand, the measuring rod will not experience a shortening in length, as judged from K, if it is applied to the disk in the direction of the radius. If then, the observer first measures the circumference of the disk with his measuring-rod and then the diameter of the disk, on dividing the one by the other, he will not obtain as quotient the familiar number $\pi = 3.14...$, but a larger number, whereas of course, for a disk which is at rest with respect to K, this operation would yield π exactly. [Einstein 1917, 334]

From this result Einstein concludes that “the propositions of Euclidean geometry cannot hold exactly on the rotating disk, nor in general in a gravitational field, at least if we attribute the length 1 to the rod in all positions and in every orientation” [Einstein 1917, 334]. It is important to notice that Einstein's conclusion is dependent on accepting that the measuring rod is always a unit rod. Here Einstein links the experimental determination of what geometry we have to a previous stipulation of what is the length of our measuring rod (and its independence from orientation). If an observer in K' could choose the rods at the periphery to have a length different from one, taking into account the Lorentz contraction, then these rods could be considered in K as having the length 1 and we could retain a Euclidean geometry; however, these rods would have, in K', a length different from that of the rods positioned along the radius (i.e. the lengths of the rods in K' would be taken to depend on the orientation).

The next chapter of the book is a preparation for the subsequent introduction of the notion of Gaussian coordinates. This chapter is informed by Einstein's analysis of the rotating disk. Einstein ask us to imagine the disposition (placement) of little rods of equal length (i.e. standard/unit measuring rods) on the surface of a marble table. We lay these rods forming quadrilateral figures, constituting in this way a network of squares on the surface of the marble table. According to Einstein, “if everything has really gone smoothly, then I say that the points of the marble slab constitute a Euclidean continuum with respect to the little rod, which has been used as a “distance” (line-element)” [Einstein 1917, 337]. Again, it is essential to take into account what is the measurement standard in relation to which we are determining the geometry.

Now we imagine that “we heat the central part of the marble slab, but not the periphery” [Einstein 1917, 338]. Here we can make an analogy to the case of the rotating disk in which the central part is at rest and as we go to the periphery the angular velocity is bigger (in this case the temperature is falling). According to Einstein, in this case “two of our little rods can still be brought into coincidence at every position on the table” [Einstein 1917, 338]. This means that the rods are congruent independently of the position at the table (i.e. independently of the temperature). They are all affected in the same way by the temperature (like the rods at the periphery of the rotating disk that all suffer the same Lorentz contraction as determined in the inertial reference frame). What happens is that “our construction of squares must necessarily come into disorder during the heating, because the little rods on the central region of the table expand, whereas those on the outer part do not” [Einstein 1917, 338]. As such, “with reference to our little rods – defined as unit lengths – the marble slab is no longer a Euclidean continuum” [Einstein 1917, 338]. This is exactly the same situation as with the rotating disk: if we take the rods at the periphery to be unit rods then we must conclude that the disk does not form a Euclidean continuum. Einstein gives an example in the case of the marble slab of how we could avoid the effect of the temperature by using different types of rods that are not affected by the temperature of the marble slab (or at least not affected in the same way). In this case “it is possible quite naturally to maintain the point of view that the marble slab is a “Euclidean continuum”” [Einstein 1917, 338]. But let us imagine a situation in which “rods of every kind (i.e. of every material) were to behave in the same way as regards the influence of temperature” [Einstein 1917, 338]. In this situation, we cannot detect experimentally the effect of the temperature on measuring rods. The same occurs with gravity (or with the case of the rotating disk when applying the equivalence principle). According to Einstein, in this case, “our best plan would be to assign the distance one to two points on the slab, provided that the ends of one of our rods could be made to coincide with these two points" [Einstein 1917, 338-9].

In this example, like in the rotating disk case, we have a perverted geodesy. We have the "background" Euclidean geometry of the marble slab. However, since "we heat the central part of the marble slab, but not the periphery … the little rods on the central region of the table expand, whereas those on the outer part do not" [Einstein 1917, 338]. If an "observer" does not have independent means to determine the length expansion of the little rods, this means that he/she cannot determine experimentally their "real" length. By taking the rods to be unit rods (even if we consider that they are "influenced" by the temperature of the marble slab), we must conclude that the geometry of the marble slab as determined using the rods is non-Euclidean.

Returning to the "seed" of this work – Darrigol's footnote, it is important to notice that Darrigol goes beyond noticing that, in his 1917 book, Einstein adopts a perverted geodesy in the case of a marble slab with a temperature gradient. Darrigol considers that Einstein relies on a perverted geodesy in this work, where, according to Darrigol, "he compares the non-Euclidean geometry of general relativity with the apparent geometry of a table top of uneven temperature when gauged with dilatable rods" [Darrigol 2015, 172]. That is, Darrigol is saying that Einstein relies on a perverted geodesy in case of the non-Euclidean geometry of general relativity, i.e. in relation to the curved space-time.

Einstein does not explicitly make this comparison, but when he considers the consequences arising from considering the possibility that "rods of every kind (i.e. of every material) ... behave in the same way as regards the influence of temperature" [Einstein 1917, 338], this is a clear analogy with the universal "effect" of gravity. Also, like in the case of the marble slab in which the measuring rods are heated differently depending of their location, i.e. have their "behaviour" influenced by the temperature "field" of the marble slab, Einstein speaks, in the context of the four-dimensional space-time of general relativity, of the (physical) "behaviour" of measuring rods and clocks "under the influence of the gravitational field" [Einstein 1917, 356]. Darrigol's view that Einstein maintained a perverted geodesy also in the case of general relativity seem plausible. Below we will consider some other "hints" reinforcing this plausibility.

Einstein next paper presenting his views on geometry was an unpublished manuscript from 1920. By this time Einstein had already developed key ideas that will consolidate views he had at least since 1914; these however are not included in his treatment of geometry in the 1920 paper. During 1918 Einstein felt necessary to make explicit that it is an assumption of the theory that the length of a standard measuring rod or the rate of a standard clock do not depend on their past history (see, e.g., [Giovanelli 2014], [Ryckman 2005, 85-94]). This assumption is strengthened by taking into account that atoms as clocks (that also enable a definition of length) are independent from their past history. According to Einstein, if this assumption was incorrect, "then no chemical elements with spectral lines of a specific frequency could exist" [CPAE Vol 8, 533].

In the 1920 paper Einstein refers to the independence of rods and clocks from their past history just in the context of special relativity, substituting his earlier reference to the more particular case of the boostability assumption [Einstein 1920, 127].[12] Einstein also presents again his rotating disk argument for the adoption of a non-Euclidean geometry (and the gravitational redshift derivation), along the lines of the previous presentations of 1916 and 1917 [Einstein 1920, 141-3]. Regarding his views on geometry we are still mostly with the views presented in 1914 ([Einstein 1920, 144]), 1916 ([Einstein 1920, 146]), and 1917 ([Einstein 1920, 145]). However, there is an analogy made in this paper, which presents the Gaussian coordinates of a curved surface in terms of a perverted geodesy. Writing about a Gaussian coordinate system defined over a surface, Einstein mentions that:

[A Gaussian coordinate system] is – speaking graphically – nothing other than an arbitrary deformed and stretched planar Cartesian coordinate system. It is by virtue of this bending that Gaussian coordinates no longer have any physical meaning whatsoever. [Einstein 1920, 145]

12 According to Einstein it is an assumption of special relativity that measuring rods and clocks when boosted from a state on inertial motion into another maintain their length and rates ([Einstein 1907, 260], [Einstein 1910, 130]; see also [Bacelar Valente 2016]).

We must take into account that in the heated marble slab example or the rotating disk case the rods are so to speak “deformed and stretched” by the effect of temperature or Lorentz contraction (a similar effect occurring with the clocks in the rotating disk due to the time dilation). If we think of the space and time measurements as made by rods and clocks spread over a material reference frame (the marble slab or the rotating disk), it is their “deformation” in comparison to the “background” flat Minkowskian space-time that leads to a view in terms of a non-Euclidean space-time. While Einstein applies the analogy in the case of a two-dimensional surface, not yet in relation to the four-dimensional space-time of general relativity, we have to bear in mind that the extension of Gaussian coordinates to this later case is straightforward (as we have mentioned in the previous section; see, e.g., [Einstein 1917, 340-4]). This gives us a further indication for the plausibility that Einstein relies on a perverted geodesy in relation to the space-time geometry of general relativity.

It was only in a 1921 paper that Einstein's views on geometry previous to 1918 were “integrated” with the views Einstein developed regarding rods and clocks in 1918. In “geometry and experience” Einstein starts, like in the brief paragraphs in the 1914 paper, with axiomatic geometry, which as such is not able to make “assertions as to the behavior of real objects” [Einstein 1921a, 210]. According to Einstein:

To be able to make such assertions, geometry must be stripped of its merely logical-formal character by the coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry … Then the propositions of Euclid contain affirmations as to the behavior of practically-rigid bodies. [Einstein 1921a, 210-1]

With this amendment of axiomatic geometry Einstein concludes, similarly to 1914, that “geometry thus completed is evidently a natural science … We will call this completed geometry “practical geometry”” [Einstein 1921a, 211]. There is nevertheless a difference with previous remarks on geometry. Now we must consider that previous to the coordination with practically-rigid bodies geometry is still not completed. Again, like in 1914, the completed geometry as a physical science is submittable to experimentation: “The question whether the practical geometry of the universe is Euclidean or not has a clear meaning, and its answer can only be furnished by experience” [Einstein 1921a, 211].

Einstein then considers two different possible points of view on geometry, something that was not present in his previous remarks on geometry. In the first “we reject the relation between the practically-rigid body and geometry” [Einstein 1921a, 212]. It follows according to Einstein that:

Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification. Thus (G) may be chosen arbitrarily, and also parts of (P); all these laws are conventions. All that is necessary to avoid contradictions is to choose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience. [Einstein 1921a, 212]

The second point of view is that of practical geometry. Why are we entitled to choose this point of view? According to Einstein part of the answer is that:

In the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. [Einstein 1921a, 213]

To put it simply, the first option is not viable because we cannot construct rods and clocks from (G) + (P), they are irreducible conceptual elements that we coordinate directly with (G). This also means that, in the present stage of development of physics, geometry must be completed not simply amended or supplemented, like Einstein had written before. But is it actually viable the option of practical geometry? We can find an answer to this question already in a letter of Einstein to Weyl from 1918, which we already considered in the previous section:

The measurement results gained from (infinitesimal) rigid bodies (measuring rods) and clocks are used in the definition of ds. A timelike ds can then be measured directly through a standard clock whose world line contains ds.

Such a definition for the elementary distance ds would only become illusory if the concepts "standard measuring rod" and "standard clock" were based on a principally false assumption; this would be the case if the length of a standard measuring rod (or the rate of a standard clock) depended on its prehistory. If this really were the case in nature, then no chemical elements with spectral lines of a specific frequency could exist, but rather the relative frequencies of two (spatially adjacent) atoms of the same sort would, in general, have to differ. [CPAE Vol 8, 533]

Einstein presents a similar reasoning in "geometry and experience" in a part of the text addressing the adoption of practical geometry [Einstein 1921a, 213-4]. From this it should be evident that the unit-measuring rods and clocks are not "influenced" by the gravitational field. This is in strict contradiction with the presupposition leading to a perverted geodesy that rods and clocks are affected by the gravitational field.

Einstein made further remarks on geometry that are relevant for this work in his lectures on special and general relativity held at Princeton University in 1921, which were published in the form of a book in the following year. In this text Einstein considers again the case of a coordinate system K' rotating uniformly relative to an inertial system K [Einstein 1922, 319-20].[13] Einstein arrives at the conclusion that:

The gravitational field influences and even determines the metrical laws of the space-time continuum. If the laws of configuration of ideal rigid bodies are to be expressed geometrically, then in the presence of a gravitational field the geometry is not Euclidean. [Einstein 1922, 321]

With an almost implicitly reference to what Darrigol called "perverted geodesy" [Darrigol 2015, 172], Einstein is saying that if the (physical) laws of chronometry and spacial geometry as determined by the time dilation and Lorentz contraction are expressed geometrically then we arrive at a curved space-time, which means, taking into account previously published remarks, that implicitly we take the rods and clocks to be unit rods and unit clocks. Symbolically we would go from $(G_{\text{Euclidean}}) + (P_{\text{Lorentz contracted rods + clocks with time dilation}})$ to $(G_{\text{non-Euclidean}}) + (P_{\text{unit rods + unit clocks}})$. This means that in the back of Einstein's mind there seems to be the idea that non-Euclidean geometry (i.e. curved space-time) "expresses geometrically" the effect of the gravitational field on rods and clocks, which are now treated as unit rods and unit clocks. In fact, in these lectures Einstein maintains, in the context of general relativity, the view that rods and clocks are affected by the gravitational field [Einstein 1922, 351-2]. This is particularly clear in the case of clocks (giving rise to the gravitational redshift), in relation to which Einstein considers that:

The interval between two beats of the unit clock (dT = 1) corresponds to the "time" 1 +

13 Let us imagine that we have identical measuring rods "laid in series along the periphery and the diameter of the circle, at rest relative to K'" [Einstein 1922, 319]. Being U the number of rods along the periphery and D the number of rods along the diameter, if K' does not rotate relative to K then U/D = π. As we have already seen previously, if K' is rotating then the rods along the periphery experience a Lorentz contraction but not the rods along the diameter. In this case U/D > π. According to Einstein, "it therefore follows that the laws of configuration of rigid bodies with respect to K' do not agree with the laws of configuration of rigid bodies that are in accordance with Euclidean geometry" [Einstein 1922, 320]. Also, as we have seen, when placing two identical clocks one at the center of the circle and the other at the periphery (rotating with K'), due to the time dilation "the clock on the periphery will go slower than the clock at the center" [Einstein 1922, 320]. Taking into account the principle of equivalence, "K' may also be considered as a system at rest, with respect to which there is a gravitational field" [Einstein 1922, 320].

$\kappa/8\pi \int \sigma dV_0 / r$ in the unit used in our system of coordinates. The rate of a clock is accordingly slower the greater is the mass of the ponderable matter in its neighbourhood. [Einstein 1922, 352]

These seem to us as further elements making plausible that Einstein's views on geometry (in the context of general relativity) were permeated by a perverted geodesy, as Darrigol calls the attention to.[14, 15]

4. Further evidence of perverted geodesy in Einstein's work

As we have seen, Einstein's views on geometry since the development of the Entwurf theory seem to be permeated by a perverted geodesy. But why would the adoption of Riemann geometry and tensor calculus be accompanied by this view? An explanation for this might be in the crucial role that the rotating disk argument might have had in the adoption of the new mathematics. According to Stachel, the rotating disk example seems to be “a 'missing link' in the chain of reasoning that led [Einstein] to the crucial idea that a nonflat metric was needed for a relativistic treatment of the gravitational field” [Stachel 1986, 245]. As Stachel found, in his private correspondence Einstein wrote that the argument for a non-Euclidean geometry in the case of a rotating disk was of decisive importance to adopt Riemann geometry in his quest for a theory of gravitation; however, in his published writings Einstein did not mention this important point [Stachel 1986, 252-3].

Einstein set forward a few times a compact argument for the adoption of the general expression for the line element and a generalization of the flat space-time equations $R^{\mu}_{\nu\sigma\tau} = 0$. This argument is based on the equivalence principle and we can see a “trace” of the rotating disk argument in it. It goes as follows: since the interval of the Minkowski space-time element $ds^2 = - dx^2 - dy^2 - dz^2 + c^2dt^2$ can be given by $ds^2 = \sum_{\mu\nu} g_{\mu\nu}dx_{\mu}dx_{\nu}$ in an arbitrary coordinate system, and due to the equivalence principle it describes a gravitational field, then the “free-field” Minkowski space-time can be interpreted as a “special case” satisfying the “Riemann condition” $R^{\mu}_{\nu\sigma\tau} = 0$ to be generalized for an arbitrary gravitational field [Einstein 1936, 308-9], [Einstein 1954, 372-5]. This more abstract compact argument can be related very directly with the heuristic rotating disk argument. The “free-field” Minkowski space-time line element $ds^2 = - dx^2 - dy^2 - dz^2 + c^2dt^2$ can be described in terms of a rotating frame of reference, e.g., as $ds^2 = - dr'^2 - r'^2 d\varphi'^2 - dz'^2 - 2\omega r' d\varphi' dt' + (c^2 - r'^2\omega^2)dt'^2$ (see, e.g., [Dieks 2004, 31-2]). This is a particular case of the general expression $ds^2 = \sum_{\mu\nu} g_{\mu\nu}dx_{\mu}dx_{\nu}$ in an arbitrary coordinate system. Applying the equivalence principle, we can reinterpret these expressions as resulting from, and expressing the presence of, a gravitational

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14 By this time Einstein had already the view that the unit/standard clocks in free fall give a “natural measure” of time [Einstein 1922, 322-3]. This is in strict contradiction with a direct metrical interpretation of the time coordinate. In fact, in this part of the text Einstein refers to the independence from past history assumption, in what seems to be a reference to a concrete geodesy [Einstein 1922, 323]. The reason we adopt the word “seems” is that Einstein adopted different presentations of the assumption. The rigorous one states that the length of a rod or the rate of a clock is independent from its past history (see, e.g., [Einstein 1921b, 225], [CPAE Vol 8, 533]). The more “loose” one can be expressed as follows: “if two ideal clocks are going at the same rate at any time and at any place (being then in immediate proximity to each other), they will always go at the same rate, no matter where and when they are again compared with each other at one place. If this law were not valid for natural clocks, the proper frequencies for the separate atoms of the same chemical element would not be in such exact agreement as experience demonstrates” [Einstein 1921a, 213-4]; see also [Einstein 1922, 323], [CPAE Vol 8, 529].This second “loose” version of the assumption might seem to be compatible with a perverted geodesy, since, e.g., in a gravitational field all clocks (located “side-by-side”) are supposed to be “affected” in the same way, i.e. they all go slower or faster in a way that their relative rate is constant. This might explain in part the persistence of references that seem to imply a perverted geodesy at a stage in which it was clear to Einstein the need for the independence from past history assumption.

15 Even by 1938, Einstein relies on the rotating disk argument to make the case for the need of a non-Euclidean geometry when we have a gravitational field. We still have the “underlying physical mechanism” of the effect of the gravitational field on rods and clocks, which would explain the gravitational redshift. The presence of this “mechanism” leads to a perverted geodesy, at least in this case. Einstein also returns to the example of the marble slab, again an example of a perverted geodesy [Einstein & Infeld 1938, 238-55].

field; by loosening its application beyond the case of the flat Minkoswki space-time we consider more general kinds of space-times/gravitational fields, i.e. we go from $R^{\mu}_{\upsilon\sigma\tau} = 0$ to $R_{\upsilon\sigma} = R^{\mu}_{\upsilon\sigma\mu}$ (see, e.g., [Norton 1985, 230-2]).

In this way, the perverted geodesy present in the heuristic rotating disk argument might be seen to be compatible with the new mathematical formalism, in particular with the curved space-time. That this seems to be the case can be seen by considering some of Einstein's work on the Entwurf theory and in relation to Nordström's gravitation theory.

In a paper where he compares both theories, Einstein calculates the Newtonian approximation in his Entwurf theory [Einstein 1913, 216-19]. This approximation, of which we can find another example, in the context of the Entwurf theory, in [Einstein 1914], is basically the one made by Einstein in his 1916 review paper on general relativity. As pointed out by Darrigol, this procedure "has the defect of presuming a partial metric interpretation of the coordinates before the metric is given" [Darrigol 2015, 171]. This, as Darrigol notices, might be due to Einstein's reliance on a perverted geodesy [Darrigol 2015, 172].[16] That this seems quite plausible can be seen in the results obtained by Einstein when adopting the Newtonian approximation. According to Einstein "the rate of a clock depends on the gravitational potential … Thus, the greater the masses arrayed in its vicinity, the slower the clock runs" [Einstein 1913, 218]. In this way, the change in the time coordinate in relation to the Minkowskian case arises due to the effect of the gravitational field – we have a perverted geodesy.[17]

Darrigol speaks of Einstein's use of a fictitious flat space-time [Darrigol 2015, 172]. However, considering the influence that Einstein's work on Nordström's theory might have had on him, it might well be the case that Einstein, at some point, was thinking in terms of an "actual" background flat space-time (as it seems to be the case in some of his comments on geometry, as we have seen in the previous section). To make a long story short, Nordström proposed a scalar theory of gravitation which was Lorentz invariant; i.e. a theory formulated in a Minkowski space-time [Nordström 1912], [Norton 1992, 35-9]. After an important debate and contributions by Einstein and Laue, Nordström formulated his "second" theory [Nordström 1913], [Norton 1992, 61-76]. In this theory, the length of rods and the rate of clocks was affected by the gravitational field in a way similar to what Einstein considered to be the case in the Entwurf theory (see, e.g., [Einstein 1913], [Norton 1992, 65-9]). This means that while the theory is formulated with an "actual" background Minkowski space-time, the effect of the gravitational field on rods and clocks makes it an effective curved space-time. This is made particularly clear in a work in which Einstein and Fokker developed Nordström's theory directly in terms of Riemann geometry and tensor calculus, showing that it could be seen as a more restricted case of the Entwurf theory [Einstein & Fokker 1914], Pais [1982, 232-7], [Norton 1992, 76-9]. As Norton remarked:

Under continued pressure from Einstein, Nordström made his theory compatible with the equality of inertial and gravitational mass by assuming that rods altered their length and clocks their rate upon falling into a gravitational field so that the background Minkowski space-time had become inaccessible to direct measurement. As Einstein and Fokker showed in early 1914 [Einstein & Fokker 1914], the space-time actually revealed by direct clock and rod measurement had become curved, much like the space-times of Einstein's own theory. ([Norton 1993, 5]; see also [Deruelle & Uzan

16 In fact, even in the thirties, Einstein adopted a post-Newtonian approximation in his work on the problem of motion in general relativity ([Einstein, Infeld, & Hoffmann 1938]; see also [Deruelle & Uzan 2014, 530-2]). Already in 1915, Einstein adopted a post-Newtonian approximation to calculate the anomaly in the advance of mercury's perihelion ([Einstein 1915]; see also [Eisenstaedt 2002, 258-9]). This reliance, in practice, on a sort of background Minkowski space-time in which the coordinates, in Darrigol's words, had a "partial metric interpretation" [Darrigol 2015, 171] might be an indication of the presence of a perverted geodesy still in the thirties (see also footnote 15). We must notice that this kind of approach is not an "exotic" early scheme adopted by Einstein (see, e.g., [Deruelle & Uzan 2014, 526-38], [Poisson & Will 2014, 290-619]). In this work, we will not treat in any detail the evidence for perverted geodesy arising from Einstein's perturbative calculations. This subject needs a full-length paper.

17 In a similar calculation made within the Entwurf theory using the Newtonian approximation, Einstein concluded again that "the clock rate … increases with the gravitational potential" [Einstein 1914, 82].

2014, 275])

This result can be seen as reinforcing the view that the curved space-time was a result of the effect of gravity on rods and clocks "existing" in a flat space-time; i.e. it reinforces a perverted geodesy view. In this way, the early development of general relativity was made, very plausibly, in the context of a perverted geodesy.[18] We must recall (see previous section) that only in 1918 Einstein made explicit (in some of his correspondence) the assumption of the independence of rods and clocks from their past history (for details see [Giovanelli 2014]) – which is related to the adoption of a concrete geodesy; but this had little impact on his subsequent texts on general relativity, since, as we have seen, he did not "retract" from perverted geodesy views.

In relation to this we must also notice that until later Einstein did not take into account more consistently the state of motion of the local inertial reference frame, in relation to which special relativity is valid ([Einstein 1922, 322-3 & 350]; see also [Einstein 1923, 78]). This might be a factor preventing Einstein to associate a natural measure (in terms of the proper time) to clocks in a gravitational field, since, e.g., in his calculations of the redshift in the Entwurf theory, Einstein considered clocks at rest in the gravitational field (see [Einstein 1913, 218], [Einstein 1914, 82]).[19] However, even when explicitly taking this into account in his Princeton lectures, where he also mentions the independence from past history assumption, Einstein still considers that "the rate of a clock is accordingly slower the greater is the mass of the ponderable matter in its neighbourhood" [Einstein 1922, 352]. The lack of a full development of these ideas in the early days of his metrical field theories might also be part of the reason for Einstein not to notice that the Riemannian space-time, which is locally Minkowskian and in which coordinates have no direct physical meaning, imposes, so to speak, a concrete geodesy. Whatever might be the causes, it seems that from the start Einstein's efforts on the development of a metrical field theory were permeated by a perverted geodesy as we can notice in the following excerpt from the first paper on the Entwurf theory:

From the foregoing, one can already infer that there cannot exist relationships between the space-time coordinates x_1, x_2, x_3, x_4 and the results of measurements obtainable by means of measuring rods and clocks that would be as simple as those in the old relativity theory. [...] We note in this connection that ds is to be conceived as the invariant measure of the distance between two infinitely close space-time points. For that reason, ds must also possess a physical meaning that is independent of the chosen reference system. We will assume that ds is the "naturally measured" distance between the two space-time points [...] The immediate vicinity of the point (x_1, x_2, x_3, x_4) with respect to the coordinate system is determined by the infinitesimal variables dx_1, dx_2, dx_3, dx_4. We assume that, in their place, new variables $d\xi_1$, $d\xi_2$, $d\xi_3$, $d\xi_4$ are introduced by means of a linear transformation in such a way that $ds^2 = d\xi_1{}^2 + d\xi_2{}^2 + d\xi_3{}^2 - d\xi_4{}^2$ […] the ordinary theory of relativity holds in this elementary $d\xi$ system. […] ds is the square of the four-dimensional distance between two infinitely close space-time points [… measured] by means of unit measuring rods and clocks […] From this one sees that, for given dx_1, dx_2, dx_3, dx_4, the natural distance that corresponds to these differentials can be determined only if one knows the quantities $g_{\mu\nu}$ that determine the gravitational field. This can also be expressed in the following way: the gravitational field influences the measuring bodies and clocks in a determinate manner [Einstein & Grossmann 1913, 156-7].

As it is, that space-time is locally Minkowskian, that space-time coordinates do not have a direct physical meaning, and that ds is an invariant that can be determined by the rods and clocks of a local inertial reference frame, do not have to be seen as implying Einstein's adoption of a concrete

18 The brief treatment of Nordström's theory and the reference to the post-Newtonian approximation in footnote 16 was motivated by one of the Reviewers, who mentioned, in relation to perverted geodesy, the "perturbative methods which Einstein used and which are still used today" and "the influence of Nordstrom theory on Einstein's point of view".

19 While in his review paper from 1916 Einstein acknowledges that "we must choose the acceleration of the infinitely small ("local") system of coordinates so that no gravitational field occurs" [Einstein 1916a, 154], he still considers a clock "arranged to be at rest in a static gravitational field" [Einstein 1916a, 197].

geodesy.[20]

5. Coda

In this work, we have made the case that Darrigol's view regarding Einstein's reliance on a perverted geodesy is quite plausible, by showing in section 3 that Einstein's views on geometry are permeated by references to perverted geodesy. In section 4 we have considered some further evidence for this in Einstein's early work. This is not to say that there might not be other explanations that do not rely on a perverted geodesy reading of Einstein's work. At first it might seem that we are dealing with a dichotomy here. One position could be: (1) Einstein believed in a perverted geodesy. Another position could be: (2) Einstein believed in the concrete geodesy and used the perverted geodesy at most as a heuristic tool (by considering the "effect" of the gravitational field on rods and clocks). Previous to address position (2) we must notice that in this work we do not adopt position (1). From our point of view there is no reason to consider that Einstein had a clear "believe" in any clear position. The presence of inconsistent elements in his work and views seem to us as an indication of that. Our position is that the elements mentioned in [Darrigol 2015] complemented with the elements presented in this work make it very plausible that Einstein's views on geometry where permeated by a perverted geodesy, but this does not mean that Einstein was adhering consistently to a perverted geodesy instead of a concrete geodesy.

Regarding position (2) we must notice first of all that while it would avoid the cumbersome situation of accepting that part of Einstein's work was, inconsistently, permeated by a perverted geodesy, we would need to face the equally cumbersome situation of having to explain how Einstein made such inconsistent claims (and derivations) is so many of his works, even if we consider them as resulting from adopting a "heuristic tool".[21] In arguing for position (2), we might try to consider Einstein's references and calculations related to a perverted geodesy as "occasional deviations" from the standard interpretation in terms of a concrete geodesy. But this standard interpretation is nowhere to be found in Einstein's presentation of general relativity. We have a non-mathematical presentation of practical geometry, in terms of what we now identify as a concrete geodesy, in the text "geometry and experience" from 1921, not before. And the only example in this period of a scientific text where concrete geodesy is implicitly adopted is in a little work where Einstein develops a theory along the lines of Weyl's theory of gravitation and electromagnetism [Einstein 1921b]. The other case we have found that might relate to a concrete geodesy is a brief reference, in his Princeton lectures (published in 1922), in relation to the invariance of ds, along the lines of what Einstein wrote in his correspondence with Weyl in 1918. However, as we have seen, in this work we find several elements that make reference to a perverted geodesy. In fact, as we have seen in sections 3 and 4, in a large number of Einstein's texts on gravitation – independently of classifying these texts as scientific, semi-popular, didactic, etc.–, we actually find (direct or indirect) references to perverted geodesy not concrete geodesy.

There is another view, more nuanced than (2), in relation to which the textual evidence might seem to make it difficult to distinguish it from the one defended in this work. As mentioned, in his paper, Darrigol writes about Einstein's reliance on a perverted geodesy being confirmed [Darrigol 2015, 172]. However, there is a commentary by Darrigol regarding §22 of Einstein's 1916 review paper that might seems to open the door for an alternative view.

In this section, Einstein considers the "behavior" of rods and clocks in a static gravitational field. Einstein considers a unit measuring rod, for which $ds^2 = -1$, laid in the radial direction, in which case $-1 = g_{11} dx_1{}^2$. According to Einstein, "the unit measuring-rod appears a little shortened in relation

20 This last paragraph address worries set forward by another Reviewer that defends a view different from ours. We will consider it in the next section where we address views proposed by this Reviewer.

21 In this discussion of possibility (2) we will consider that the only element giving rise to a perverted geodesy is the effect of the gravitational fields on rods and clocks and disregard that the Newtonian approximation might also be related to a perverted geodesy as mentioned in [Darrigol 2015]. In this respect see also footnote 16.

to the system of coordinates by the presence of the gravitational field" [Einstein 1916a, 197]. In this way:

Euclidean geometry does not hold even to a first approximation in the gravitational field, if we wish to take one and the same rod, independently of its place and orientation, as a realization of the same interval. [Einstein 1916a, 197][22]

For the case of a unit clock "arranged to be at rest in a static gravitational field" [Einstein 1916a, 197], the time coordinate dx_4 is equal to $1-(g_{44}-1)/2$. Accordingly:

The clock goes more slowly if set up in the neighbourhood of ponderable masses. From this it follows that the spectral lines of light reaching us from the surface of large stars must appear displaced towards the red end of the spectrum. [Einstein 1916a, 198]

As we can see the time coordinate is given a direct physical meaning as the time gone by the clock under the influence of the gravitational field: this is incompatible with a concrete geodesy.

Regarding this determination by Einstein of "the influence exerted by the field of the mass M upon the metrical properties of space" [Einstein 1916a, 196], in particular with respect to the calculations regarding rods, Darrigol considers that:

This way of discussing the non-Euclidean properties of space is somewhat strange: On the one hand, it is based on the sound idea that true metric relations are defined by the metric element ds^2; on the other hand, it seems to retain a meaning for the concept of length with respect to a system of coordinates, as if the coordinates still had an independent physical existence. [Darrigol 2015, 172]

In terms of a concrete-perverted geodesy "debate" regarding Einstein's position (in which we engage in this last section), we can read Darrigol's remark in two ways:

A) Evidently, parts of Einstein's calculations are based on sound mathematical properties of the Riemannian space-time, even if it went unnoticed by Einstein that this "imposes" a concrete geodesy (in relation to this recall the last part of the previous section).

B) With the adoption of a metric field theory of gravity based on Riemann geometry and tensor calculus (i.e. already in the context of the Entwurf theory), Einstein adopts concrete geodesy. In this case, we might consider that Einstein's standard position is *based on* concrete geodesy. However, Einstein *retains* perverted geodesy elements to present or even to obtain some important results.

The evidence presented in this work lead us to consider that Darrigol's remarks should be read as in A). As we have seen in the final part of the last section, the adoption of some basic features of a Riemannian space-time (like for a unit measuring rod having $ds^2 = -1$, for which, when laid in the radial direction, $-1 = g_{11}{dx_1}^2$) does not have to be seen as implying that Einstein had any clear position based on concrete geodesy. On the contrary, as we have just seen, in §22 of his review paper we find references to a perverted geodesy. This is not to say that while we find position B) unconvincing, an even more nuanced view might be impossible to distinguish from ours in relation to the textual evidence taken into account here. For that purpose, one might need further studies, possibly made from perspectives different from the one adopted in this work.

22 This is in our view a clear reference to a perverted geodesy. Compare with Einstein's remarks related to the marble slab example or the rotating disk: a) "The propositions of Euclidean geometry cannot hold exactly on the rotating disk, nor in general in a gravitational field, at least if we attribute the length 1 to the rod in all positions and in every orientation" [Einstein 1917, 334]; b) "With reference to our little rods – defined as unit lengths – the marble slab is no longer a Euclidean continuum" [Einstein 1917, 338]. On this point see the discussion on section 3. Notice also that Einstein explicitly considers that the rod is shortened due to the presence of the gravitational field.

Abbreviations

CPAE Albert Einstein (1987-). *The Collected Papers of Albert Einstein*. Ed. by John Stachel et al. 14 Vols. Princeton: Princeton University Press.

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