Time-reversal invariance and ontology

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Abstract

Albert and Callender have challenged the received view that theories like classical electrodynamics and non-relativistic quantum mechanics are time-reversal invariant. According to their view of time-reversal invariance, these theories are *not* time-reversal invariant. If so, then the important metaphysical implication is that space-time must have a temporal orientation. There is a large debate on what is the best way of viewing time-reversal invariance, with many philosophers defending the standard notion contra Albert and Callender. In this paper, we will not be concerned so much with that aspect of the debate, but rather focus our attention on an aspect of the Albert and Callender view that has received little attention, namely the role of ontology. In the type of theories that are considered the ontology is actually underdetermined. We will argue that with a suitable choice of ontology, these theories *are* in fact time-reversal invariant according their view.

1 Introduction

Physics textbooks state that theories like Newtonian mechanics, classical electrodynamics and non-relativistic quantum mechanics are time-reversal invariant. Albert and Callender disagree [1, 2]. Albert claims that only the former is time-reversal invariant, while the other two are not [1, p. 14]:

> And so [classical electrodynamics] is not invariant under time-reversal. Period.
>
> And neither (it turns out) is quantum mechanics, and neither is relativistic quantum field theory, and neither is general relativity, and neither is supergravity, and neither is supersymmetric quantum string theory, and neither (for that matter) are any of the candidates for a fundamental theory that anybody has taken seriously since Newton. And everything everybody has always said to the contrary ... is wrong.

Callender discusses just non-relativistic quantum mechanics [2], but arrives to the conclusion — for the same reason as Albert — that this theory is not time-reversal invariant.

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To explain the disagreement, let us consider Albert, who gives a general discussion of what time-reversal ought to mean. Consider first Newtonian mechanics. In this case, Albert actually agrees with the standard conclusion that the theory is time-reversal invariant, but for different reasons. Newtonian mechanics is about point-particles, which have definite positions at all times. The Newtonian dynamics determines which particle trajectories are physically allowed. For Albert, the collection of positions at a time forms the instantaneous state. The temporal sequence of these instantaneous states forms a history. Albert takes the time-reversal of a history just to be the history run backwards. That is, the temporal sequence of instantaneous states is reversed. It is as if a video of the motion of the particles is run backwards. Newtonian mechanics is time-reversal invariant because the time-reversed of each dynamically allowed history is also dynamically allowed. That is, time-reversal turns solutions to Newton's equation of motion into solutions. In the example of the video, the time-reversal invariance would mean that when playing it, we would not able to tell which is the actual time order in which events have unfolded, since both are physically possible.

Also according to standard view Newtonian mechanics is time-reversal invariant, but the story is bit different. First of all, in addition to the particle positions, also the instantaneous velocities are included in the instantaneous state. In this way, the instantaneous state determines a unique solution to the Newtonian dynamics, i.e., determines a unique dynamically allowed history. Second, according to the standard view, the time-reversal amounts to reversing the temporal order of the instantaneous state *together with* flipping the sign of the velocities at each time. (So the time-reversal is more than just reversing the order of the instantaneous states.) Despite the differences, the conclusion is the same: Newtonian mechanics is time-reversal invariant.

Disagreement arises in the case of classical electrodynamics. In this case, the electric and magnetic field are included in the instantaneous state, both according to Albert and standard textbooks. So there is no disagreement concerning the electromagnetic part of the instantaneous state. However, according to the standard view, the magnetic field should flip sign under time-reversal (like the velocities in Newtonian mechanics). But for Albert, the magnetic field should not change sign under time reversal [1, p. 20]:

> Magnetic fields are not the sorts of things that any proper time-reversal transformation can possibly turn around. Magnetic fields are not—either logically or conceptually—the rates of change of anything.

As such Albert concludes that electrodynamics is *not* time-reversal invariant.

So Albert's analysis differs from the standard one on two accounts. First, there is the different notion of instantaneous state. In essence, Albert takes the instantaneous state at a time to correspond to the ontology at that time (e.g., positions, field configurations, ..., at that time). On the other hand, in the standard account, the instantaneous state is such that it determines a unique solution to the equations of motion and as such may contain more variables compared to Albert's instantaneous state (e.g., they may also include particle velocities, field velocities, ...). Second, there is the notion of time-reversal invariance, which is just the temporal reversal of instantaneous states for

Albert, whereas there might be an additional (involutive) state transformation at each instant according to the standard account. The example of electrodynamics shows that the second difference is essential for the different conclusion concerning the question of time-reversal invariance.

This issue is important because if a theory is not time-reversal invariant, then time has an objective direction according to that theory (while it has no bearings on for example the issue of the arrow of time [1]). There is a large body of interesting literature defending the standard notion [3–7]. In this paper, we will not so much be concerned with which is the better notion of time-reversal invariance.[1] Albert's notion certainly has an appeal. In particular, Albert's notion just depends on the ontology. So given a physical theory, which entails a specification of the ontology, the answer to whether the theory is time-reversal invariant or not is unambiguous. On the other hand, according to the standard notion of time-reversal, there seems to be some arbitrariness in deciding what the possible additional transformation of the instantaneous state should be. The usual attitude seems to be to consider just the additional transformation which makes the theory time-reversal invariant.

The goal of the paper is to consider the role of the ontology in Albert's account (which is an aspect that has been underexposed in the debate, exceptions are [6, 9]). At least in the case of (classical) field theories, there is a huge underdetermination of the ontology. Different ontologies yield different instantaneous states in Albert's sense. So whether a theory can be considered as time-reversal invariant depends on what is considered to be the ontology. We will show that theories like electrodynamics and non-relativistic quantum mechanics *are* time-reversal invariant in Albert's sense, provided a suitable ontology is chosen.

The outline of the paper is as follows. In the next section, we start with introducing the relevant notions. Then in section 3, we will consider the ontological implications concerning time-reversal invariance in the case of the scalar field. In section 4, we will consider ontologies for classical electrodynamics and quantum mechanics which make these theories time-reversal invariant in Albert's sense. With these choices of ontology, the time-reversal transformation happens to coincide with the standard one (just as it does in the case of Newtonian mechanics). There are also examples for which this is not the case. In section 5, we will illustrate this with scalar electrodynamics, which describes a scalar field interacting with an electromagnetic field. An ontology will be presented for which Albert's notion of time-reversal does not coincide with the standard one, but rather with the joint transformation of time-reversal and charge conjugation. We conclude in section 6.

[1] The standard model of particle physics is actually *not* time-reversal invariant even according to the standard notion. It is merely invariant under the joint transformation of time-reversal, charge conjugation and parity-reversal. The metaphysical implications have been explored in [6, 8].

2 Instantaneous state and time-reversal

Let us first formalize some notions. The instantaneous state at a certain time t is denoted by $S(t)$. As mentioned before, for Albert the instantaneous state at a time represents the ontology at that time, whereas the standard notion of instantaneous state (in this context) includes an extra specification at that time such that for a deterministic theory the instantaneous state together with the equations of motion determines a unique solution. (Albert elaborates more on the notion, but this is sufficient for our purposes.) We will add the subscripts a and s and write $S_a(t)$ and $S_s(t)$ to refer to respectively Albert's notion and the standard notion of state.

For a given history, i.e., $t \to S(t)$, the time-reversed history is denoted as $t \to T(S)(t)$. For Albert, the time-reversed history is $t \to T_a(S_a)(t)$, with $T_a(S_a)(t) = S_a(-t)$.[2] According to the standard notion, given a history $t \to S_s(t)$, the time-reversed history is $T_s(S_s)(t) = S_s^T(-t)$, where the superscript T denotes some additional involutive operation on each instantaneous state in addition to flipping the sign of the time argument.

A theory is called time-reversal invariant if for each dynamically allowed history, i.e., each possible solution to the equations of motion, its time-reversed is also dynamically allowed.

Let us give some examples. First consider Newtonian mechanics. The ontology is given by point-particles with positions $(\mathbf{X}_1, \dots, \mathbf{X}_n)$. The equations of motion read

$$m_k \frac{d^2 \mathbf{X}_k}{dt^2} = -\boldsymbol{\nabla}_k V(\mathbf{X}_1, \dots, \mathbf{X}_n). \tag{1}$$

According to the standard notion, the instantaneous state at a time t is the collection of positions and velocities at that time, i.e.,

$$S_s(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t), \mathbf{V}_1(t), \dots, \mathbf{V}_n(t)) \tag{2}$$

and the time-reversal operation is

$$T_s : S_s(t) \to S_s^T(-t) = (\mathbf{X}_1(-t), \dots, \mathbf{X}_n(-t), -\mathbf{V}_1(-t), \dots, -\mathbf{V}_n(-t)). \tag{3}$$

Newtonian mechanics is time-reversal invariant in this sense. According to Albert, the instantaneous state at time t is

$$S_a(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t)). \tag{4}$$

It does not include the instantaneous velocities. Namely, the velocities are not part of ontology, but of course they are determined by the ontology through the time derivative of the history. The time-reversal operation is

$$T_a : S_a(t) \to S_a(-t) = (\mathbf{X}_1(-t), \dots, \mathbf{X}_n(-t)). \tag{5}$$

[2] We consider only time-translation invariant theories, so that there is nothing special about $t = 0$ in the definition of a time-reversed history.

The implied transformation of the velocities is of course as in (3). So, also according to Albert, Newtonian mechanics is time-reversal invariant.

Let us now turn to classical electrodynamics. In this case, the ontology is given by point-particles together with the electric and magnetic field $\mathbf{E}(\mathbf{x},t)$ and $\mathbf{B}(\mathbf{x},t)$. The laws of motion are given by the Lorentz force law

$$m_k \frac{d^2\mathbf{X}_k}{dt^2} = e_k \left[\mathbf{E}(\mathbf{X}_k,t) + \frac{d\mathbf{X}_k}{dt} \times \mathbf{B}(\mathbf{X}_k,t)\right], \tag{6}$$

together with Maxwell's equations

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho, \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0, \tag{7}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \boldsymbol{\nabla} \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}, \tag{8}$$

where $\rho(\mathbf{x},t) = \sum_k e_k \delta(\mathbf{x}-\mathbf{X}_k(t))$ and $\mathbf{J}(\mathbf{x},t) = \sum_k e_k \frac{d\mathbf{X}_k(t)}{dt}\delta(\mathbf{x}-\mathbf{X}_k(t))$ are respectively the charge density and the charge current. The instantaneous state is

$$S_s(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t), \mathbf{V}_1(t), \dots, \mathbf{V}_n(t), \mathbf{E}(\mathbf{x},t), \mathbf{B}(\mathbf{x},t)) \tag{9}$$

and under time reversal

$$T_s : S_s(t) \to S^T_s(-t) = (\mathbf{X}_1(-t), \dots, \mathbf{X}_n(-t), -\mathbf{V}_1(-t), \dots, -\mathbf{V}_n(-t), \mathbf{E}(\mathbf{x},-t), -\mathbf{B}(\mathbf{x},-t)). \tag{10}$$

It is crucial that the magnetic field flips sign under this operation. It guarantees that the equations of motion are time-reversal invariant.

Albert takes the state to be

$$S_a(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t), \mathbf{E}(\mathbf{x},t), \mathbf{B}(\mathbf{x},t)) \tag{11}$$

and under time reversal

$$T_a : S_a(t) \to S_a(-t) = (\mathbf{X}_1(-t), \dots, \mathbf{X}_n(-t), \mathbf{E}(\mathbf{x},-t), \mathbf{B}(\mathbf{x},-t)). \tag{12}$$

There is no sign flip of the magnetic field and Albert concludes that the equations of motion are *not* time-reversal invariant; the transformation (12) is not a symmetry of the equations of motion (i.e., does not map solutions to solutions). The standard time-reversal transformation is still a symmetry of the equations of motion, but Albert just would not call that time-reversal symmetry, because the magnetic field is not the rate of change of anything.

In non-relativistic quantum mechanics, the situation is similar (and is detailed by Callender [2]). To avoid the interpretational issues that arise in this context, we will regard the Schrödinger equation as just a classical field equation. For simplicity, we will also consider just a single particle. The ontology in this case is given by the wave function $\psi(\mathbf{x},t)$ and the Schrödinger equation is

$$\mathrm{i}\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + V(\mathbf{x})\psi. \tag{13}$$

In this case,

$$S_s(t) = S_a(t) = \psi(\mathbf{x}, t). \tag{14}$$

The Schrödinger equation is invariant under the standard time-reversal operation

$$T_s : S_s(t) \to S_s^T(-t) = \psi^*(\mathbf{x}, -t), \tag{15}$$

but not under Albert's notion of the time-reversal operation

$$T_a : S_a(t) \to S_a(-t) = \psi(\mathbf{x}, -t). \tag{16}$$

3 Ontological underdetermination: the case of the scalar field

There tends to be an underdetermination in the ontology of physical theories. This is especially so in the case of field theories. In the case of, say, Newtonian mechanics, there seems to be little flexibility in the ontology. Newtonian mechanics is regarded as a theory about point-particles moving in physical space. One could consider other ontologies, but these seem to be contrived. For example, rather than considering physical space as the physical arena, one could consider, say, velocity phase space, which is the space of collections of positions and velocities, as the true physical arena. (The equation $d\mathbf{X}/dt = \mathbf{V}$ would then be regarded as a law rather than a definition of the velocity.) While such a view is arguably empirically indistinguishable from the standard view, it would be rather far removed from our everyday experience of the world. But taking the latter ontology seriously would mean that the theory is no longer time-reversal invariant in Albert's sense.

In the case of field theories there seems to be a huge underdetermination. As a simple example, consider the theory of a real scalar field. Usually the ontology is given by a scalar field $\phi(\mathbf{x}, t)$, satisfying the Klein-Gordon equation

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0. \tag{17}$$

But one could also consider a phase space representation, with an ontology given by the fields $(\phi(\mathbf{x}, t), \pi(\mathbf{x}, t))$, satisfying

$$\dot{\phi} = \pi, \qquad \dot{\pi} = \nabla^2 \phi - m^2 \phi. \tag{18}$$

While in the context of classical mechanics we consider a phase space ontology artificial because the physical arena is then phase space and not physical space, there is no such issue here, because the phase space variables are still fields in physical space. So the physical arena is still physical space. (In this case, the choice of a reference frame to define the time derivative may be regarded as unsatisfactory for a Lorentz invariant theory. This is an issue we will come back to later.)

The scalar field theory can also be written in terms of a 5-component spinor $\psi(\mathbf{x}, t)$ which satisfies the Kemmer equation [10, 11]

$$\mathrm{i}\beta^\mu \partial_\mu \psi - m\psi = 0. \tag{19}$$

This is a Dirac-like equation, which is manifestly Lorentz invariant just as (17). This theory is completely equivalent to the Klein-Gordon theory. In a particular representation of the Kemmer matrices β^μ, the spinor takes the form $\psi = (\partial_\mu \phi, m\phi)^T$. Despite the equivalence, this form of the theory is hardly used, due to its greater complexity. However, this is not a reason not to consider the Kemmer spinor as providing the actual ontology. Actually, when it comes to spin-1/2 particles, the Dirac equation which is a first-order differential equation (which is the analogue of (19)) is the one commonly used, instead of the somewhat simpler second-order Van der Waerden equation for a two-component spinor [12] (which is the analogue of (17)).

So for the scalar field theory, we have three possible candidates for the ontology and hence for the state in Albert's sense. Namely,

$$S_a^{(1)}(t) = \phi(\mathbf{x}, t), \qquad S_a^{(2)}(t) = (\phi(\mathbf{x}, t), \pi(\mathbf{x}, t)), \qquad S_a^{(3)}(t) = \psi(\mathbf{x}, t). \tag{20}$$

Only the first one yields time-reversal symmetry in Albert's sense. The time-reversal operation T_a in this case also corresponds to the standard one. According to the standard notion, the theory is time-reversal invariant for all these choices of ontology. (Similarly, in the case of a (free) spin-1/2 particle, for which the state S_a can be taken to be a Dirac spinor or a Van der Waerden spinor, only the latter will amount to time-reversal invariance in Albert's sense.)

So whether a theory is time-reversal invariant or not in Albert's sense depends on the choice of ontology. In the case of a classical field theory, different possible ontologies seem possible with no clear physical preference. (Even the requirement of manifest Lorentz covariance leaves options $S_a^{(1)}$ and $S_a^{(3)}$.) In the next section, we will show that we can exploit the ambiguity in ontology in the case of classical electrodynamics and non-relativistic quantum mechanics to choose one such that the theory is time-reversal invariant in Albert's sense.

4 Ontology of electrodynamics and quantum mechanics

Maxwell's equations imply[3]

$$\mathbf{B} = -\frac{1}{\nabla^2} \boldsymbol{\nabla} \times \left(\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right). \tag{21}$$

This expression can be used to eliminate the magnetic field from Maxwell's equation and the Lorentz force law. Maxwell's equations are then expressed as

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho, \qquad \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = -\boldsymbol{\nabla} \rho - \frac{\partial \mathbf{J}}{\partial t}. \tag{22}$$

[3]The action of $1/\nabla^2$ is defined in terms of the Green function of the Laplacian, i.e., $(1/\nabla^2) f(\mathbf{x}) = -\frac{1}{4\pi} \int d^3y f(\mathbf{y})/|\mathbf{x} - \mathbf{y}|$. The expression (21) follows if the fields fall off sufficiently fast at spatial infinity.

The resulting theory is completely equivalent to the original one, using (21). This suggest that one can take the ontology of the electromagnetic field to be given by just the electric field, so that

$$S_a(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t), \mathbf{E}(\mathbf{x}, t)), \tag{23}$$

with the field equations given by (29). This theory is time-reversal invariant in Albert's sense. There is no problem with the magnetic field because it is simply not part of the ontology (and the theory). One could define the magnetic field as (21) in terms of the particles and the electric field. From that definition it follows that under time-reversal of the state S_a, the magnetic field will flip sign. So on this view the magnetic field *does* play the role of a velocity, since it is a linear combination of the velocities of the particles (through the charge current) and the electric field.

Note that we could also have eliminated the electric field in terms of the magnetic field. But then the resulting theory would not be time-reversal invariant. So there is no technical reason why it is more natural to assume the ontology to be given by the electric field rather than the magnetic field.

Nevertheless, there is an issue with this ontology, which is clearly spelled out in [6]. It is an ontology that is suitable for Newtonian space-time, but not so much for a Minkowski space-time, which is the natural space-time in this context due to the Lorenz invariance of electrodynamics. While the theory in terms of just the electric field is still Lorentz invariant, the transformation of the electric field is rather complicated and makes the Lorentz invariance not manifest. So rather then having a 3-vector as constituting the fundamental ontology, it would be more desirable to have Lorentz-covariant objects, like the electromagnetic field tensor. However, this tensor transforms non-trivially according to the standard time-reversal transformation.

A manifestly Lorentz invariant theory that is time-reversal invariant in Albert's sense could be obtained by completely removing the fields from the ontology, so that only the particles remain. This is attempted in the Wheeler-Feynman theory [13,14]. To see how this theory is obtained, consider the covariant form of electrodynamics

$$\partial_\mu F^{\mu\nu}(x) = j^\nu(x), \qquad m_k \frac{d^2 X^\mu_k(s_k)}{ds_k^2} = e_k F^\mu{}_\nu(X_k(s_k)) \frac{dX^\nu(s_k)}{ds_k}, \tag{24}$$

where $X^\mu_k(s_k)$ is the worldline of the k-th particle, parameterized by its proper time s_k, $A^\mu(x)$ is the electromagnetic potential and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field tensor, with $E_i = F_{0i}$ and $B_i = -\epsilon_{ijk} F^{jk}/2$, and $j^\mu(x) = \sum_k e_k \int ds \frac{dX^\mu_k(s)}{ds} \delta(x - X_k(s))$ the charge current. Assuming the Lorenz gauge $\partial_\mu A^\mu = 0$, the Maxwell equations can be written as

$$\Box A^\mu(x) = j^\mu(x). \tag{25}$$

The potential can be decomposed as $A^\mu_F + A^\mu_M$ with A^μ_F a field satisfying the free Maxwell equations $\Box A^\mu_F(x) = 0$ and

$$A^\mu_M(x) = \frac{1}{\Box} j^\mu(x), \tag{26}$$

where $1/\Box$ denotes integration over a Green's function G of the d'Alembertian. There are various choices for a Green's function G; one could take the retarded G_-, the advanced G_+ or linear combinations. Different choices will correspond to differences in the free field A^μ_F. Wheeler and Feynman chose $G = (G_+ + G_-)/2$. Eq. (26) is then taken as a definition of A^μ_M, rather than as (part of) a dynamical equation. (In the Lorentz force law in (24), the self-force is subtracted to avoid infinities, but we will leave this aside here.) Furthermore, it is assumed there are no free fields, i.e., $A^\mu_F = 0$. By eliminating the fields, one is left with a theory of just particles. So,

$$S_a(t) = (\mathbf{X}_1(t), \dots, \mathbf{X}_n(t)). \tag{27}$$

This theory is time-reversal invariant under T_a, in particular since A^μ_M transforms as

$$A^0_M(\mathbf{x}, t) \to A^0_M(\mathbf{x}, -t), \qquad A^i_M(\mathbf{x}, t) \to -A^i_M(\mathbf{x}, -t) \tag{28}$$

under time-reversal of the particles, which is the standard transformation and which hence implies a sign flip of the magnetic field.[4] (For the time-reversal invariance it is also important that the particular Green function is $(G_+ + G_-)/2$ chosen. Another choice would not have made it time-reversal invariant. Concerning the free field A^μ_F, if it is introduced as part of the ontology, then this will make the theory not time-reversal invariant in Albert's sense, because of the Lorentz force law. The free Maxwell equations $\Box A^\mu_F = 0$ by themselves are time-reversal invariant. Of course, one could consider another possible ontology for the free fields, but it is unclear what could serve as a Lorentz covariant piece of ontology.)

There is debate about the empirical adequacy of the theory, but the important point for our purposes is that it is a theory that is manisfestly Lorentz invariant and that it is time-reversal invariant in Albert's sense. Possibly other choices of ontologies are possible that achieve this and perhaps also include a free field.

Allori [9] considers yet another option to have time-reversal invariance in Albert's sense, which she attributes to Horwich [15]. On this view, the electric and magnetic field are not part of the fundamental ontology. The ontology is just particles, like in the Wheeler-Feynman theory. But unlike in the latter, the electromagnetic field still appears in Horwich's account of the theory. But the field rather has a nomological character than an ontological one, i.e., the field plays a role in the dynamics of the particle. The particles are said to constitute the *primitive ontology*. The fields then transform the way they do under time-reversal just to have the primitive ontology transform the right way. The approach we consider here is different. We have not relegated some

[4] Interestingly, in his defense of the standard notion of time-reversal invariance, Earman also considers A^μ_M [3]. He proposes to take (25) as "the definition of the four-potential arising from [the current]". This could be read in the sense considered here, i.e., that there is no independent reality for the field. However, Earman seems to have had merely the intention of showing that if one accepts the usual transformation properties of the particles, then one should also accept the usual transformation properties of the vector potential. But this akin to stating that the vector potential should transform the usual way, just because that makes the theory time-reversal invariant. Because if A^μ_M (or the electromagnetic field) is taken as part of the ontology, then (25) should be taken as a law.

parts of the standard ontology to the nomological domain, but rather have eliminated them completely (i.e., also from the dynamics). For example, in our first proposal, the magnetic field was no longer part of the theory, neither on the ontological nor on the nomological level.

The non-relativistic Schrödinger equation can be dealt with similarly. We can write $\psi = \psi_r + \mathrm{i}\psi_i$, with ψ_r and ψ_i real, and eliminate the imaginary part to obtain[5]

$$\frac{\partial^2 \psi_r}{\partial t^2} = -H^2 \psi_r \qquad H = -\frac{1}{2m}\nabla^2 + V(\mathbf{x}). \tag{29}$$

The Schrödinger equation can be recovered by defining[6]

$$\psi_i = \frac{1}{H}\partial_t \psi_r. \tag{30}$$

So ψ_i roughly plays the role of the time derivative of ψ_r. Taking the ontology to be given just by ψ_r makes the theory time-reversal invariant in Albert's sense.[7]

Note that by eliminating the magnetic field in electrodynamics or the imaginary part of the wave function in quantum mechanics, we have passed from a wave equation which is a first-order in time differential equation to one that is second order. One way to obtain such equations is through the Hamiltonian formulation of the theory. In the reduced phase space formalism all redundant degrees of freedom are eliminated.

5 Time-reversal theories with different notion of time-reversal

In the previous section, we have provided examples of ontologies that make electrodynamics and the non-relativistic Schrödinger equation time-reversal invariant in Albert's

[5]This equation has been considered by a number of people, including Schrödinger himself [16]. In particular, this equation is encountered in the reduced phase space formulation of the Schrödinger theory [17]. In the reduced phase space formulation, ψ_r and ψ_i become canonically conjugate variables. An inverse Legendre transformation leads to the Lagrangian for just ψ_r, whose Euler-Lagrange equation corresponds to (29). Actually, the second order equations for electromagnetism could be obtained similarly since the electric and magnetic field are (approximately) canonically conjugate.

[6]Where $1/H$ is defined in terms of the Green function for the Hamiltonian H.

[7]Quantum mechanics actually entails much more than just the Schrödinger equation. What this is depends on the version of quantum mechanics. Let us briefly say something about the time-reversal invariance for the three main attempts that solve the measurement problem, namely the many worlds theory, spontaneous collapse theories and Bohmian mechanics. In the many worlds theory, the ontology is given by just the wave function and hence it can be considered time-reversal invariant in Albert's sense by taking the ontology to be given by just the real part of the wave function . In spontaneous collapse theories, the Schrödinger evolution of the wave function is interrupted by collapses which are stochastic. This entails further discussion of the notion of time reversal invariance which we will not consider. Usually, the theory is not considered time-reversal invariant even in the standard sense [2], but see also [18]. In Bohmian mechanics there are also actual point-particles in addition to the wave function. The Bohmian dynamics is time-reversal invariant in the standard sense [19] and also in Albert's sense if the ontology above is adopted.

sense. This was done by respectively removing the magnetic field and the imaginary part of the wave function as part of the ontology. In these cases, Albert's time-reversal transformation was in agreement with the standard time-reversal transformation. However, this need not always be the case. Yet the theory could be time-reversal invariant according to both notions. Consider for example, again a scalar field ϕ satisfying the Klein-Gordon equation (17), but now a complex scalar field one, which describes a charged spinless field. According to the standard notion of time-reversal, the field should transform as $\phi(\mathbf{x},t) \to \phi^*(\mathbf{x},-t)$, whereas according to Albert's notion, taking $S_a(t) = \phi(\mathbf{x},t)$, $T_a : \phi(\mathbf{x},t) \to \phi(\mathbf{x},-t)$. So there is disagreement about what counts as time-reversal. Yet, both transformations are symmetries of the Klein-Gordon equation (i.e., they map solutions to solutions) and hence the conclusion in both cases is that the theory is time-reversal invariant. (The same is true for the Van der Waerden equation that was mentioned in section 3. With the Van der Waerden spinor as ontology, the theory is time-reversal invariant in Albert's sense, even though his notion would be at variance with the standard notion.)

The previous example can be extended to include an electromagnetic field. In terms of the scalar field ϕ and the vector potential A^μ, this theory (called scalar electrodynamics) has the equations of motion

$$D_\mu D^\mu \phi + m^2 \phi = 0, \qquad \partial_\mu F^{\mu\nu} = j^\nu, \tag{31}$$

where $D_\mu = \partial_\mu + \mathrm{i}eA_\mu$ is the covariant derivative and

$$j^\mu = \mathrm{i}e\left[\phi^* D^\mu \phi - \phi (D^\mu \phi)^*\right] \tag{32}$$

is the charge current. The theory is invariant under the standard time-reversal operation

$$\psi(\mathbf{x},t) \to \psi^*(\mathbf{x},-t), \qquad A^0(\mathbf{x},t) \to A^0(\mathbf{x},-t), \qquad A^i(\mathbf{x},t) \to -A^i(\mathbf{x},-t). \tag{33}$$

But taking ϕ and A^μ as the ontology does not make this theory time-reversal invariant in Albert's sense.

Consider now the temporal gauge $A_0 = 0$. Then the equations of motion are

$$\ddot{\phi} - \mathbf{D}\cdot\mathbf{D}\phi + m^2\phi = 0, \quad \Box\mathbf{A} + \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{A}) = \mathbf{j}, \quad -\boldsymbol{\nabla}\cdot\dot{\mathbf{A}} = j_0, \tag{34}$$

with now $\mathbf{D} = \boldsymbol{\nabla} - \mathrm{i}e\mathbf{A}$ and the charge density and 3-current respectively given by

$$j_0 = \mathrm{i}e\left(\phi^*\dot{\phi} - \phi\dot{\phi}^*\right), \qquad \mathbf{j} = \mathrm{i}e\left[\phi\mathbf{D}\phi^* - \phi^*\mathbf{D}\phi\right]. \tag{35}$$

Taking the state to be

$$S_a(t) = \left(\phi(\mathbf{x},t), \mathbf{A}(\mathbf{x},t)\right), \tag{36}$$

then it is readily checked that the theory is invariant under Albert's time-reversal operation $T_a : S_a(t) \to S_a(-t)$. Nevertheless, this is a symmetry different from the standard time-reversal symmetry (33).

The transformation T_a considered here actually corresponds to the joint time-reversal (T) and charge (C) conjugation in the standard picture. Namely, under charge conjugation, one has $\phi \to \phi^*$ and $A^\mu \to -A^\mu$. So in this case, Albert's time-reversal transformation T_a agrees with the TC transformation of the standard picture. (This explains why under T_a, the charge current in (35) flips sign, i.e., it transforms as $j_0(\mathbf{x}, t) \to -j_0(\mathbf{x}, -t)$, $\mathbf{j}(\mathbf{x}, t) \to -\mathbf{j}(\mathbf{x}, -t)$.) That the TC transformation can actually be considered to be the time-reversal transformation is also the case in Feynman's view of time-reversal, as explained in detail in [6].

6 Conclusion

Prima facie, the views of Albert and Callender on the notion of time-reversal invariance seems to lead to the conclusion that theories like electrodynamics and quantum mechanics are not time-reversal invariant and hence imply a temporal orientation of space-time. However, their notion also depends on the ontology of the theory, which is underdetermined. We have argued that ontologies can be considered for electrodynamics and quantum mechanics so that they are time-reversal invariant. As such, whether one adopts the notion of time-reversal invariance of Albert and Callander or the standard one, the conclusion can be the same, namely that these theories do not imply a temporal orientation.

We do not want to suggest that any of these ontologies are natural. We merely wanted to point out that such ontologies do exist. In particular, with exception from the Wheeler-Feynman theory, we have paid no attention to the relativistic character of electrodynamics. The proposed ontologies for electrodynamics where often couched in a Newtonian picture of space-time rather than a Minkowskian one. So apart from the time-reversal invariance also the Lorentz invariance should be taken into account.

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References

[1] D.Z. Albert, *Time and chance*, Harvard University Press, Cambridge (2000).

[2] C. Callender, "Is Time 'Handed' in a Quantum World?", *Proceedings of the Aristotelian Society* **100**, 247-269 (2000).

[3] J. Earman, "What time reversal invariance is and why it matters", *Int. Stud. Philos. Sci.* **16**, 245-264 (2002).

[4] F. Arntzenius, "Time reversal operations, representations of the Lorentz group, and the direction of time", *Stud. Hist. Phil. Mod. Phys.* **35**, 31-43 (2004).

[5] D. Malament, "On the time reversal invariance of classical electromagnetic theory", *Stud. Hist. Phil. Mod. Phys.* **35**, 295-315 (2004).

[6] F. Arntzenius and H. Greaves, "Time Reversal in Classical Electromagnetism", *Brit. J. Phil. Sci.* **60**, 557-584 (2009).

[7] B.W. Roberts, "Three myths about time reversal in quantum theory", *Philos. Sci.* **84**, 315-334 (2017) and arXiv:1607.07388 [physics.hist-ph].

[8] H. Greaves, "Towards a Geometrical Understanding of the CPT Theorem", *Brit. J. Phil. Sci.* **61**, 27-50 (2010).

[9] V. Allori, "Maxwell's Paradox: the Metaphysics of Classical Electrodynamics and its Time-Reversal Invariance", *αnalytica* **1**, 1-19 (2015).

[10] N. Kemmer, "The particle aspect of meson theory", *Proc. R. Soc. A* **173**, 91-116 (1939).

[11] A.I. Akhiezer and V.B. Berestetskii, *Quantum Electrodynamics*, Intersience (1965).

[12] J.J. Sakurai, *Advanced quantum Mechanics*, Addison–Wesley, Reading, Massachusetts (1967).

[13] H. Spohn, *Dynamics of charged particles and their radiation field*, Cambridge University Press, Cambridge (2004).

[14] D. Lazarovici, "Against fields", *Euro. Jnl. Phil. Sci.* **8**, 145-170 (2018) and arXiv:1809.00855 [physics.hist-ph].

[15] P. Horwich, "Asymmetries in Time", Cambridge, MIT Press (1987).

[16] C. Callender, "Quantum Mechanics: Keeping It Real?", philsci-archive 17641 (2020).

[17] W. Struyve, "Pilot-wave theory and quantum fields", *Rep. Prog. Phys.* **73**, 106001 (2010) and arXiv:0707.3685v4 [quant-ph].

[18] D.J. Bedingham and O.J.E. Maroney. "Time symmetry in wave-function collapse", *Phys. Rev. A* **95**, 042103 (2017) and arXiv:1607.01940 [quant-ph].

[19] D. Dürr, S. Goldstein and N. Zanghì, "Quantum Equilibrium and the Origin of Absolute Uncertainty", *J. Stat. Phys.* **67**, 843-907 (1992) and arXiv:quant-ph/0308039.