The Past Hypothesis and the Nature of Physical Laws

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> Therefore I think it is necessary to add to the physical laws the hypothesis that in the past the universe was more ordered, in the technical sense, than it is today—I think this is the additional statement that is needed to make sense, and to make an understanding of the irreversibility.
>
> Richard Feynman (1964 Messenger Lectures)

Abstract

If the Past Hypothesis underlies the arrows of time, what is the status of the Past Hypothesis? In this paper, I examine the role of the Past Hypothesis in the Boltzmannian account and defend the view that the Past Hypothesis is a candidate fundamental law of nature. Such a view is known to be compatible with Humeanism about laws, but as I argue it is also supported by a minimal non-Humean "governing" view. Some worries arise from the non-dynamical and time-dependent character of the Past Hypothesis as a boundary condition, the intrinsic vagueness in its specification, and the nature of the initial probability distribution. I show that these worries do not have much force, and in any case they become less relevant in a new quantum framework for analyzing time's arrows—the Wentaculus. Hence, both Humeans and minimalist non-Humeans should embrace the view that the Past Hypothesis is a candidate fundamental law of nature and welcome its ramifications for other parts of philosophy of science.

Keywords: time's arrow, counterfactuals, laws of nature, vagueness, objective probabilities, typicality, scientific explanation, Past Hypothesis, Statistical Postulate, Humeanism, non-Humeanism, minimal primitivism, the Mentaculus, the Wentaculus, quantum statistical mechanics, density matrix realism

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1 Introduction

One of the hardest problems in the foundations of physics is the problem of the arrows of time. If the dynamical laws are (essentially) time-symmetric, what explains the irreversible phenomena in our experiences, such as the melting of ice cubes, the decaying of apples, and the mixing of cream in coffee? Macroscopic systems display an entropy gradient in their temporal evolutions: their thermodynamic entropy is lower in the past and higher in the future. But why does entropy have this temporally asymmetric tendency? Following Goldstein (2001), we distinguish between two parts of the problem of irreversibility:

1. The Easy Part: if a system is not at maximum entropy, why should its entropy tend to be larger at a later time?

2. The Hard Part: why should there be an arrow of time in our universe that is governed by fundamental reversible dynamical laws?

The Easy Part was studied by Boltzmann (1964)[1896]. Key to Boltzmann's answer is this:

- Key to the Easy Part: states of larger entropy occupy much larger volume in the system's phase space than those states of lower entropy.

So far, answering the Easy Part does not require any time-asymmetric postulates.[1] Boltzmann's program is primarily focused on closed subsytems of the universe. But its success leads us to expect that a Boltzmannian account can work at the universal level. If we model the universe as a mechanical system, we expect that typically the non-equilibrium state of the universe will evolve towards higher entropy at later times.

However, why is the entropy lower in the past? That is the Hard Part. A proposed answer suggests that it has to do with the initial condition of the universe:

- Answer to the Hard Part: the universe had a special beginning.

We can introduce this as an explicitly time-asymmetric postulate in the theory, by using the Past Hypothesis:

Past Hypothesis (PH) At the initial time of the universe, the microstate of the universe is in a low-entropy macrostate.[2]

Given that some microstates are anti-entropic, it is standard to introduce a probability distribution over the microstates compatible with the low-entropy macrostate:

Statistical Postulate (SP) The probability distribution of the initial microstate of the universe is given by the uniform one (according to the natural measure) that is supported on the macrostate of the universe.

However, a detailed probability distribution may be unnecessary. In the typicality framework, we just need to be committed to a typicality measure:

Typicality Postulate (TP) The initial microstate of the universe is typical inside the macrostate of the universe.[3]

Unlike SP, TP is compatible with a variety of measures that agree on what is typical. PH, SP, and TP are physical postulates that have an empirical status. The answer to the Hard Part of the problem of irreversibility requires PH and one of SP and TP. In fact, we also need to assume (an unconditionalized) notion of probability or typicality to answer the Easy Part. I call the answers to the Easy Part and the Hard Part the *Boltzmannian account* of the arrow of time.

How to characterize the initial macrostate of PH remains an open question. We know that the matter distribution is more or less uniform in the early universe, which is opposite from the usual conception of low entropy. However, the initial gravitational

[1]Boltzmann's *Stosszahlansatz* (hypothesis of molecular chaos) is often blamed for introducing an illicit time asymmetry. But it is an innocent theoretical postulate if we understand it correctly—as a typicality or probability measure over initial conditions. See (Goldstein *et al.* 2020, section 5.5).

[2]The Past Hypothesis was originally suggested in (Boltzmann 1964)[1896] (although he seems to favor another postulate that can be called the *Fluctuation Hypothesis*) and discussed in (Feynman 2017)[1965]. For recent discussions, see (Albert 2000), (Loewer 2020), (Callender 2004, 2011), (North 2011), (Lebowitz 2008), (Goldstein 2001), and (Goldstein *et al.* 2020). (The memorable phrase 'Past Hypothesis' is coined by Albert (2000).)

[3]For more on the notion of typicality and its application in statistical mechanics, see (Goldstein 2012), (Lazarovici & Reichert 2015), and (Wilhelm 2019).

degrees of freedom are in a special state, providing a sense that the total state of the early universe has low entropy. This observation led Penrose (1979) to postulate a geometric version of PH called the *Weyl Curvature Hypothesis*: the Weyl curvature vanishes at the initial singularity. The urgent question is, of course, how to understand this in terms of quantum theory or quantum gravity. Some steps have been taken in the Loop Quantum Cosmology framework by Ashtekar & Gupt (2016). This is compatible with a Boltzmannian account, but the final details will depend on the exact theory of quantum gravity, which is currently absent.

In the philosophical literature, a number of objections have been raised against the Boltzmannian account. First, some criticize the answer to the Easy Part: the explanation is too hand-wavy and not completely rigorous (Frigg 2007). Second, some criticize the answer to the Hard Part, such as the charge that PH is not even false because the entropy of the early universe is not well-defined (Earman 2006), the charge that PH is not sufficient to explain the thermodynamic behaviors of subsystems (Winsberg 2004), and the charge that it is *ad hoc* and therefore not explanatory (Price 2004; Carroll 2010, p.346). There are some responses in the literature, and more work on these issues are welcome. However, my interest here is different. I take the Boltzmannian account as my starting point. My aim is to explore the conceptual and scientific ramifications of accepting the PH and its explanation of time's arrow.

In this paper, I focus on the connection between PH and our concept of fundamental laws of nature. What is the status of PH, if the Boltzmannian program turns out to be successful? Can PH be accepted as a candidate fundamental law even though it is a boundary condition of the universe? What differences does it make to our concepts of laws, chances, and possibilities? Can PH be completely expressed in mathematical language? What is the relevance of quantum theory to these issues? I argue for the following theses:

Nomic Status The Past Hypothesis is a candidate fundamental law of nature.[4]

Axiomatic Status The Past Hypothesis is a candidate axiom of the fundamental physical theory.[5]

Relevance Whether the Past Hypothesis has the nomic status (and the axiomatic status) is relevant to the success of explaining time's arrows, the metaphysical account of laws, the nature of objective probability, and the mathematical expressibility of fundamental physical theory.

Some of these ideas have been defended along Humean lines, but I think we should accept them regardless of whether we think fundamental laws supervene on the matter distribution or are part of the fundamental facts in the world. We can embrace all

[4]A *candidate* fundamental law of nature has all it takes to be a fundamental law of nature, but it may not turn out to be the true law of the actual world if it makes false predictions. For example, Newton's dynamical law $F = ma$ is a candidate fundamental law of nature but it is not the actual fundamental law.

[5]The status of a candidate fundamental law of nature and the status of a candidate axiom of the fundamental physical theory may be equivalent. I distinguish the two theses because some people may be happy to accept the axiomatic status of PH but deny the nomic status of PH. They may be reluctant to call PH a fundamental *law*, perhaps due to it being a boundary condition.

of the above even accepting certain non-Humean frameworks.[6] My methodology below is naturalistic and functionalist. *Whatever plays the role of a fundamental law can be a fundamental law.* Whatever that cannot be derived from more fundamental laws *and* plays the right roles in guiding our inferences about the past and the future, underlying various scientific explanations, the high-level regularities, our manifest image of influence and control, and so on is a candidate fundamental law.[7] To borrow a phrase from Loewer (2020), PH "looks, walks and talks" like a fundamental law. So, we should interpret it as such.

Hence, I disagree with people who think that even if the PH is true and plays all the roles we suggest, it still cannot be a fundamental law—it may just be a special but *contingent* initial condition. I also disagree with people who think that whether or not the PH is a fundamental law makes no substantive difference.[8]

Here is the roadmap. In §2, we provide more details of the Boltzmannian account and discuss the subtle changes in PH when we move from classical mechanics to quantum mechanics. The variations result in three types of physical theories: the Classical Mentaculus, the Quantum Mentaculus, and the Wentaculus theories. In §3, we provide positive arguments for the nomic and axiomatic statuses of the PH. Some of these have been mentioned in the literature, but it is worth emphasizing and clarifying the exact argumentative structure. I also put forward a novel argument based on considerations of the nature of the quantum state. In §4, I discuss some apparent obstacles from recognizing the PH as a fundamental law. This has to do with its nature as a boundary condition, the status of the Statistical Postulate and the Typicality Postulate, and the intrinsic vagueness in their specifications. I argue that these worries do not have much force even on some non-Humean views, and they become even less worrisome in the Wentaculus theories.

2 Variations on a Theme from Boltzmann

Before we get into the philosophical and conceptual issues, let us be more explicit about what the Boltzmannian account is and how to state the Past Hypothesis in that account. Although the Boltzmannian account is more or less the same in classical and in quantum theories, the exact form of the Past Hypothesis is subtly different. We will exploit this feature in §4 to dissolve some of the worries about the classical version of the Past Hypothesis. Readers familiar with the standard Boltzmannian statistical mechanics can jump to §2.3, where a new framework called the *Wentaculus* is introduced.

[6] This view is, I think, in the same spirit as the suggestion made by Demarest (2019).

[7] I do not claim that we should reduce laws to these roles; that would be the strategy of metaphysical functionalism about laws. Rather, I am merely appealing to the methodology in naturalistic metaphysics of science that I think many people accept independently of the issue of the arrows of time.

[8] Maudlin (2007) §4 seems to regard PH as an important boundary condition but does not think of it as a fundamental law. The disagreement is based on a different view about how laws govern that I call Dynamical Law Primitivism. I discuss this in §3.4. Carroll (2010) (p.345) suggests that there is no substantive difference between the statements "the early universe had a low entropy" and "it is a law of physics that the early universe had a low entropy." Carroll seems to be worried about the distinction between boundary conditions and laws; I discuss this in §4.1.

2.1 The Classical Mentaculus

Let us start with the Boltzmannian account in classical statistical mechanics. Here we summarize the basic elements of classical statistical mechanics from the "individualistic viewpoint."[9] Let us consider a classical-mechanical system with N particles in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian $H = H(X) = H(\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n})$ that specifies the standard interactions in accord with Newtonian gravitation, Coulomb's law, and other forces obeyed by the classical system.

1. Microstate: at any time t, the microstate of the system is given by a point in a $6N$-dimensional phase space,

$$X = (\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n}) \in \Gamma_{total} \subseteq \mathbb{R}^{6N}, \tag{1}$$

   where Γ_{total} is the total phase space of the system.

2. Dynamics: the time dependence of $X_t = (\boldsymbol{q_1}(t), ..., \boldsymbol{q_N}(t); \boldsymbol{p_1}(t), ..., \boldsymbol{p_n}(t))$ is given by the Hamiltonian equations of motion:

$$\frac{d\boldsymbol{q_i}(t)}{dt} = \frac{\partial H}{\partial \boldsymbol{p_i}}, \frac{d\boldsymbol{p_i}(t)}{dt} = -\frac{\partial H}{\partial \boldsymbol{q_i}}. \tag{2}$$

3. Energy shell: the physically relevant part of the total phase space is the energy shell $\Gamma \subseteq \Gamma_{total}$ defined as:

$$\Gamma = \{X \in \Gamma_{total} : E \leq H(x) \leq E + \delta E\}. \tag{3}$$

   We only consider microstates in Γ.

4. Measure: the measure μ_V is the standard Lebesgue measure on phase space, which is the volume measure on $\mathbb{R}^{6N}$. The Lebesgue measure on a finite volume can be normalized to yield a probability distribution.

5. Macrostate: with a choice of macro-variables, the energy shell Γ can be partitioned into macrostates Γ_ν:

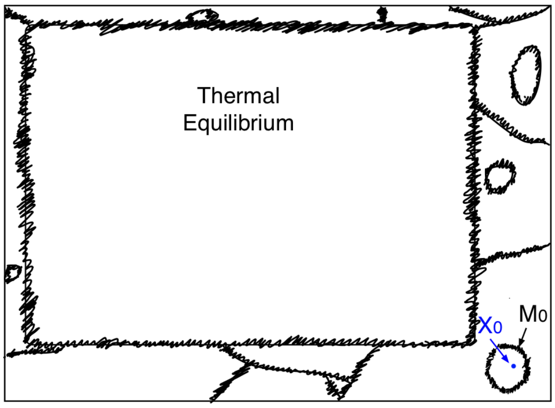
$$\Gamma = \bigcup_\nu \Gamma_\nu. \tag{4}$$

   A macrostate is composed of microstates that share similar macroscopic features (similar values of the macro-variables), such as volume, density, and pressure.

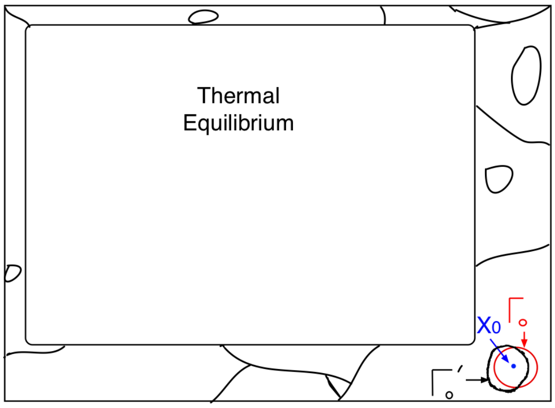
   Caveat: the partition of microstates into macrostate is exact only after we stipulate some choices of the parameters for coarse-graining (the size of the cells) and correspondence (between functions on phase space and thermodynamic quantities). We call these *C-parameters*. Without the exact choices of the C-parameters, the partition is inexact and the boundaries between macrostates are vague.[10] See

[9] I follow the discussion in (Goldstein & Tumulka 2011). These do not intend to be rigorous axiomatizations of classical statistical mechanics. What is presented here differs in emphasis from (Chen 2018), as here we are explicit about the sources of vagueness, which will be discussed in §4.

[10] For more discussions, see (Chen 2020b).



(a) Vague boundaries



(b) Exact boundaries

Figure 1: The partition of microstates into macrostates on phase space with exact choices (as in b) or without exact choices (as in a) of the C-parameters. X_0 represents the actual microstate of the universe at t_0. M_0 represents the vague boundaries of the Past Hypothesis. Γ_0 represents an admissible precisification of M_0, where Γ_0' represents another admissible precisification. The diagrams are not drawn to scale.

Figure 1. It is also expected that, given the nature of the actual forces, some partitions will be superior to others in supporting generalizations in the special sciences.

6. Unique correspondence: given exact choices of the C-parameters, the macrostates partition the energy shell, and as a consequence every phase point X belongs to one and only one Γ_ν. (This point is implied by #5. But we make it explicit to better contrast it with the situation in quantum statistical mechanics.)

7. Thermal equilibrium: typically, there is a dominant macrostate Γ_{eq} that has almost the entire volume with respect to μ_V:

$$\frac{\mu_V(\Gamma_{eq})}{\mu_V(\Gamma)} \approx 1. \tag{5}$$

   A system is in thermal equilibrium if its phase point $X \in \Gamma_{eq}$.

8. Boltzmann Entropy: the Boltzmann entropy of a classical-mechanical system in microstate X is given by:

$$S_B(X) = k_B \log(\mu_V(\Gamma(X))), \tag{6}$$

   where $\Gamma(X)$ denotes the macrostate containing X. The thermal equilibrium state thus has the maximum entropy.

   Caveat: Without exact values of the C-parameters, there will be many admissible choices of the $\Gamma(X)$'s. Moreover, what is admissible is also vague. Furthermore, since k_B is a scaling constant that plays no direct dynamical role, its value is also vague. Hence, the Boltzmann entropy is of a microstate should be understood as a vague quantity. If we stipulate some C-parameters and the value of k_B, we can arrive at an exact boundary for the macrostate that contains X and an exact value of Boltzmann entropy for the system.

9. Low-Entropy Initial Condition: on the assumption that we can model the universe as a classical-mechanical system of N point particles, we postulate a special low-entropy boundary condition, which David Albert calls *the Past Hypothesis*:

$$X_{t_0} \in \Gamma_{PH} \, , \, \mu_V(\Gamma_{PH}) \ll \mu_V(\Gamma_{eq}) \approx \mu_V(\Gamma), \tag{7}$$

where Γ_{PH} is the Past Hypothesis macrostate with volume much smaller than that of the equilibrium macrostate. Hence, $S_B(X_{t_0})$, the Boltzmann entropy of the microstate at the boundary, is very small compared to that of thermal equilibrium. Here, Γ_{PH} is underspecified; we can add further details to specify the macroscopic profile (temperature, pressure, volume, density) of Γ_{PH}.

The answer to the Easy Part of the problem of irreversibility lies in the first eight bullet points, which make plausible the typical tendency for a system to evolve towards higher entropy in the future. Even though microstates are "created equal," macrostates are not. Their volumes are disproportionate and uneven. Macrostates with higher entropy have much larger volume in the energy shell. Moreover, by far the largest macrostate is that of thermal equilibrium. It is plausible that, unless the dynamics is extremely contrived, a typical microstate starting from a medium-entropy macrostate will find its way through larger and larger macrostates and eventually arrive at thermal equilibrium. That is a process where a system gradually increases in entropy until it reaches the entropy maximum.[11] Of course, for the actual universe, there can be exceptions to the entropy increase, such as short-lived decrease of entropy.

However, this solves only half the problem. If typical microstates compatible with a medium-entropy macrostate will, at most times, increase in entropy towards the future, then typical micorstates compatible with the same macrostate will also at most times increase in entropy towards the past. Hence, given the resources so far we have shown that the medium-entropy macrostate is at an entropy minimum, and it is produced by a thermodynamic fluctuation from equilibrium. We are led to the Hard Part of the problem: why is the entropy so much lower in the past direction of time? Enter the Past Hypothesis. Given PH, the actual microstate starts in an atypical region of the energy shell, in a low-entropy macrostate M_0. Suppose we choose a precisification Γ_0. Given the Easy Part, typical initial microstates compatible with Γ_0 will evolve towards macrostates of higher entropy in the future direction. But there is nothing earlier than t_0, as it is stipulated to be the initial time, say, the time of the Big Bang.[12]

[11] Boltzmann's original H-theorem (Boltzmann 1964)[1898] is an attempt to show this. Lanford (1975) produces an exact result for a simple system of hard spheres where the Boltzmann equation is shown to be satisfied for a short duration of time and hence Boltzmann entropy is shown to be increasing towards the future. However, it is plausible that the equation continues to be valid and Boltzmann entropy continues to rise afterwards.

[12] It can also be stipulated that t_0 is some time close to the Big Bang, in which case some anti-thermodynamic behavior can be displayed in the short duration before t_0.

Therefore, if we find the universe to be in a medium-entropy macrostate Γ_t, say the state we are in right now, then the actual microstate is not like a typical microstate inside Γ_t, but a special one that is compatible with Γ_0. The reason that the entropy was lower in the past is because the universe started in a special macrostate, a state of very low Boltzmann entropy. Assuming PH, it is reasonable to expect that it is *with overwhelming probability* that entropy is higher in the future and lower in the past, and the sense of probability is specified by bullet point #4. There are two ways to understand the measure:

- A measure of probability: the natural measure picks out the correct probability measure of the initial condition. This interpretation yields the Statistical Postulate.
- A measure of typicality: the natural measure is a simple representer of a vague "collection" of measures that are equivalent as the measure of typicality of the initial condition. This interpretation yields the Typicality Postulate.

The Past Hypothesis together with the Statistical Postulate supports the following classical-mechanical version of the Second Law of Thermodynamics (this is adapted from the Mathematical Second Law described in (Goldstein *et al.* 2020) §5.2):

The Second Law for X At t_0, the actual phase point of the universe X_0 starts in a low-entropy macrostate and, with overwhelming probability, it evolves towards macrostates of increasingly higher entropy until it reaches thermal equilibrium, except possibly for entropy decreases that are infrequent, shallow, and short-lived; once X_t reaches Γ_{eq}, it stays there for an extraordinarily long time, except possible for infrequent, shallow, and short-lived entropy decreases.

The Second Law can be stated also in the language of typicality. For simplicity, I will conduct the discussion below mostly in the language of probability.

The Second Law above is stated for the behavior of the universe, but it also makes plausible what Goldstein *et al.* (2020) call a 'development conjecture' about isolated subsystems in the universe:

Development Conjecture Given the Past Hypothesis, an isolated system that, at a time t before thermal equilibrium of the universe, has macrostate ν appears macroscopically in the future, but not the past, of t like a system that at time t is in a typical microstate compatible with ν.

Classical mechanics with just the fundamental dynamical laws (expressed in equations (2)) are time-symmetric. Introducing the probability measure takes care of the Easy Part of the problem of irreversibility, but to solve the Hard Part of the problem—the retrodiction to the past, we need to explicitly introduce something that breaks the time symmetry. PH is a simple postulate that does the job. The bullet points about energy shell, macrostate partition, unique correspondence, and the dominance of thermal equilibrium are supposed to follow from the basic postulates about fundamental dynamics (including the structure of the Hamiltonian function) and the probability measure. Adapting the terminology of Albert (2015) and Loewer (2020), we call the collection of basic postulates the *Classical Mentaculus*:

The Classical Mentaculus

1. **Fundamental Dynamical Laws (FDL):** the classical microstate of the universe is represented by a point in phase space that obeys the Hamiltonian equations of motion described in equations (2).

2. **The Past Hypothesis (PH)**: at a temporal boundary of the universe, the microstate of the universe lies inside M_0, a low-entropy macrostate that, given a choice of C-parameters, corresponds to Γ_0, a small-volume set of points on phase space that are macroscopically similar.

3. **The Statistical Postulate (SP)**: given the macrostate M_0, we postulate a uniform probability distribution (with respect to the standard Lebesgue measure) over the microstates compatible with M_0.

If the uniform measure is given a status of objective probability, it delivers more than just the Second Law. It provides an exact probability for any proposition formulable in the language of phase space. This is the reason that Albert and Loewer regard the Mentaculus as providing a "probability map of the world." Hence, the Mentaculus has an ambitious scope: it is possible to recover all the non-fundamental regularities, including the special science laws (such as laws of economics), and other arrows of time such as the epistemic arrow, the records arrow, the influence arrow, and the counterfactual arrow.

Whether the Albert-Loewer project can succeed in their ambitious goal of recovering all the non-fundamental regularities and arrows of time is an interesting question. Nonetheless, the Classical Mentaculus as formulated provides an underpinning for the thermodynamic arrow of time. Given the universality and importance of the Second Law, the Mentaculus should be taken as a serious contender as a promising framework for the structure of fundamental physical theory. In the next subsection, we examine how to adapt the Classical Mentaculus to the quantum domain. In §3, we discuss the suggestion that the Past Hypothesis should be taken as a candidate fundamental law as well as some ramifications of the more ambitious project.

2.2 The Quantum Mentaculus

Let us turn to the Boltzmannian account of quantum statistical mechanics from the "individualist viewpoint." We consider a quantum-mechanical system with N fermions (with $N > 10^{20}$) in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian $\hat{H}$. (Here I follow the discussions in (Goldstein *et al.* 2010a) and (Goldstein & Tumulka 2011).)

1. Microstate: at any time t, the microstate of the system is given by a normalized (and anti-symmetrized) wave function:

$$\psi(\boldsymbol{q_1}, \dots, \boldsymbol{q_N}) \in \mathscr{H}_{total} = L^2(\Lambda^N, \mathbb{C}^k) \ , \ \| \psi \|_{L^2} = 1, \tag{8}$$

where $\mathscr{H}_{total} = L^2(\Lambda^N, \mathbb{C}^k)$ is the total Hilbert space of the system.

2. Dynamics: the time dependence of $\psi(q_1, ..., q_N; t)$ is given by the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi. \tag{9}$$

3. Energy shell: the physically relevant part of the total Hilbert space is the subspace ("the energy shell"):

$$\mathscr{H} \subseteq \mathscr{H}_{total} \; , \; \mathscr{H} = \text{span}\{\phi_\alpha : E_\alpha \in [E, E + \delta E]\}, \tag{10}$$

This is the subspace (of the total Hilbert space) spanned by energy eigenstates ϕ_α whose eigenvalues E_α belong to the $[E, E + \delta E]$ range. Let $D = \dim\mathscr{H}$, the number of energy levels between E and $E + \delta E$.

We only consider wave functions ψ in $\mathscr{H}$.

4. Measure: given a subspace $\mathscr{H}$, the measure μ_S is the surface area measure on the unit sphere in that subspace $\mathscr{S}(\mathscr{H})$.[13]

5. Macrostate: with a choice of macro-variables,[14] the energy shell $\mathscr{H}$ can be orthogonally decomposed into macro-spaces (subspaces):

$$\mathscr{H} = \oplus_\nu \mathscr{H}_\nu \; , \; \sum_\nu \dim\mathscr{H}_\nu = D \tag{11}$$

Each $\mathscr{H}_\nu$ corresponds to small ranges of values of macro-variables that we have chosen in advance.

Caveat: similarly to the classical case, the decomposition of Hilbert space into macrostate requires some stipulation of the exact values of the C-parameters. But in the quantum case, these parameters includes coarse-graining sizes, correspondence of functions, and also the cut-off values of how much support a quantum state needs to be inside a subspace to be counted towards belonging to the macrostate (see the next bullet point). Without the exact choices of the C-parameters, the decomposition is inexact and it is vague which microstate belongs to which macrostate. Again, it is also expected that, given the nature of the actual forces, some decompositions will be superior to others for supporting generalizations in the special sciences.

6. Non-unique correspondence: typically, a wave function is in a superposition of macrostates and is not entirely in any one of the macrostates (even if we represent macrostates with exact subspaces). However, we can make sense of situations where ψ is (in the Hilbert space norm) very close to a macrostate $\mathscr{H}_\nu$:

$$\langle \psi | P_\nu | \psi \rangle \approx 1, \tag{12}$$

[13] For simplicity, here we assume that the subspaces we deal with are finite-dimensional. In cases where the Hilbert space is infinite-dimensional, it is an open and challenging technical question. For example, we could use Gaussian measures in infinite-dimensional spaces, but we no longer have uniform probability distributions.

[14] For technical reasons, Von Neumann (1955) suggests that we round up these macro-variables (represented by quantum observables) so as to make the observables commute. See (Goldstein *et al.* 2010b, section 2.2) for a discussion of von Neumann's ideas.

where P_ν is the projection operator onto $\mathscr{H}_\nu$. This means that $|\psi\rangle$ lies almost entirely in $\mathscr{H}_\nu$. In this case, we say that $|\psi\rangle$ is in macrostate ν.

7. Thermal equilibrium: typically, there is a dominant macrostate $\mathscr{H}_{eq}$ that has a dimension that is almost equal to D:

$$\frac{\dim \mathscr{H}_{eq}}{\dim \mathscr{H}} \approx 1. \tag{13}$$

   A system with wave function ψ is in equilibrium if the wave function ψ is very close to $\mathscr{H}_{eq}$ in the sense of (12): $\langle\psi| P_{eq} |\psi\rangle \approx 1$.

8. Boltzmann Entropy: the Boltzmann entropy of a quantum-mechanical system with wave function ψ that is in macrostate ν is given by:

$$S_B(\psi) = k_B \log(\dim \mathscr{H}_\nu), \tag{14}$$

   where $\mathscr{H}_\nu$ denotes the subspace containing almost all of ψ in the sense of (12). The thermal equilibrium state thus has the maximum entropy:

$$S_B(eq) = k_B \log(\dim \mathscr{H}_{eq}) \approx k_B \log(D), \tag{15}$$

   where eq denotes the equilibrium macrostate.

9. Low-Entropy Initial Condition: on the assumption that we can model the universe as a quantum-mechanical system, we postulate a special low-entropy boundary condition on the universal wave function—the quantum-mechanical version of the Past Hypothesis:

$$\Psi(t_0) \in \mathscr{H}_{PH} \text{ , } \dim \mathscr{H}_{PH} \ll \dim \mathscr{H}_{eq} \approx \dim \mathscr{H} \tag{16}$$

   where $\mathscr{H}_{PH}$ is the Past Hypothesis macro-space with dimension much smaller than that of the equilibrium macro-space.[15] Hence, the initial state has very low entropy in the sense of (25). More details can be added to narrow down the range of choices of $\mathscr{H}_{PH}$.

The quantum Boltzmannian account is similar to the classical one. The higher-entropy macrstates have much higher dimensions than lower-entropy ones, and the equilibrium macrostate has by far the largest dimension. It is plausible that, unless the dynamics is very contrived, a medium-entropy wave function will find its way through larger and larger subspaces and eventually arrive at the equilibrium subspace. Again, if typical wave functions in a non-equilibrium macrostates will evolve towards higher entropy in the future, then typical ones also come from higher entropy states in the past. The quantum mechanical Past Hypothesis blocks that inference. The reason that there is a thermodynamic arrow in a quantum universe is because the universal wave function started in a special state, a subspace with very low entropy (the one described

[15] Again, we assume that $\mathscr{H}_{PH}$ is finite-dimensional, in which case we can use the surface area measure on the unit sphere as the typicality measure for # 10. It remains an open question in QSM about how to formulate the low-entropy initial condition when the initial macro-space is infinite-dimensional.

by the quantum PH). Assuming the quantum PH, it is reasonable to expect that *with overwhelming probability* the entropy is higher in the future and lower in the past, with the probability measure specified in bullet point #4. Again, we can understand it as a measure of probability or a measure of typicality.

Together, the probability distribution and the Past Hypothesis support the quantum-mechanical version of the Second Law:

The Second Law for Ψ At t_0, the actual wave function of the universe Ψ_0 starts in a low-entropy macrostate and, with overwhelming probability, it evolves towards macrostates of increasingly higher entropy until it reaches thermal equilibrium, except possibly for entropy decreases that are infrequent, shallow, and short-lived; once Ψ_t reaches $\mathscr{H}_{eq}$, it stays there for an extraordinarily long time, except possibly for infrequent, shallow, and short-lived entropy decreases.

(This makes plausible a similar Development Conjecture for typical isolated subsystems.)

Note again we make three basic postulates in the quantum version of the Boltzmannian account: the fundamental dynamical laws, the Past Hypothesis, and the Statistical Postulate. We call this the Quantum Mentaculus:

The Quantum Mentaculus

1. **Fundamental Dynamical Laws (FDL):** the quantum microstate of the universe is represented by a wave function Ψ that obeys the Schrödinger equation (9).

2. **The Past Hypothesis (PH)**: at a temporal boundary of the universe, the wave function Ψ_0 of the universe lies inside a low-entropy macrostate that, given a choice of C-parameters, corresponds to $\mathscr{H}_{PH}$, a low-dimensional subspace of the total Hilbert space.

3. **The Statistical Postulate (SP)**: given the subspace $\mathscr{H}_{PH}$, we postulate a uniform probability distribution (with respect to the surface area measure on the unit sphere of $\mathscr{H}_{PH}$) over the wave functions compatible with $\mathscr{H}_{PH}$.

The Quantum Mentaculus, as a candidate fundamental theory of physics, faces the quantum measurement problem. To solve the measurement problem, there are three promising options: Bohmian mechanics, GRW spontaneous collapse theories, and Everettian quantum mechanics. We have three distinct kinds of the Quantum Mentaculus.

First, the Everettian version is completely the same as the original Quantum Mentaculus in terms of the basic postulates. However, it diverges greatly from common sense: we have to give up the expectation that experimental outcomes are unique and determinate. Instead, our experiences are to be understood as experiences of agents in an emergent multiverse (see (Wallace 2012)).

Second, in the Bohmian version, in addition to the wave function that evolves unitarily according to the Schrödinger equation, particles have precise locations, and their configuration $Q = (Q_1, Q_2, ..., Q_N)$ follows the guidance equation, which is an additional law in the theory:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_i\psi(q)}{\psi(q)}(q = Q) \tag{17}$$

Moreover, the initial particle distribution is given by the quantum equilibrium distribution:

$$\rho_{t_0}(q) = |\psi(q, t_0)|^2 \tag{18}$$

Adding the above two postulates to the Quantum Mentaculus completes the Bohmian Mentaculus.

Third, the GRW version requires revisions to the linear evolution represented by the Schrödinger equation. The wave function typically obeys the Schrödinger equation, but the linear evolution is interrupted randomly (with rate $N\lambda$, where N is the number of particles and λ is a new constant of nature of order 10^{-15} s^{-1}) by collapses:

$$\Psi_{T^+} = \frac{\Lambda_k(X)^{1/2}\Psi_{T^-}}{||\Lambda_k(X)^{1/2}\Psi_{T^-}||} \tag{19}$$

where Ψ_{T^-} is the pre-collapse wave function, Ψ_{T^+} is the post-collapse wave function, the collapse center X is chosen randomly with probability distribution $\rho(x) = ||\Lambda_k(x)^{1/2}\Psi_{T^-}||^2 dx$, $k \in \{1, 2, ...N\}$ is chosen randomly with uniform distribution on that set of particle labels, and the collapse rate operator is defined as:

$$\Lambda_k(x) = \frac{1}{(2\pi\sigma^2)^{3/2}}e^{-\frac{(Q_k-x)^2}{2\sigma^2}} \tag{20}$$

where Q_k is the position operator of "particle" k, and σ is another new constant of nature of order 10^{-7} m postulated in current GRW theories. The GRW Mentaculus replaces the deterministic Schrödinger evolution of the wave function by this stochastic process. It still requires PH. However, as Albert (2000) §7 points out, it is plausible (though not proven) that SP is no longer needed, and the GRW collapses suffice to make anti-entropic trajectories unlikely (in the sense of the quantum probabilities stipulated by the GRW stochastic process). (See Ismael's contribution in this volume.)

2.3 The Wentaculus

In this subsection, we consider the Boltzmannian account of quantum statistical mechanics with a very special "fundamental density matrix." The account is inspired by (Dürr *et al.* 2005), proposed in (Chen 2018), and discussed at length in (Chen 2019a, 2020d). It is another variation on the same theme from Boltzmann, but it suggests some astonishing possibilities, one of which is the Initial Projection Hypothesis that will be introduced shortly.

The density matrix can play the same *dynamical role* as the wave function does in the previous theories. In a quantum system represented by a density matrix W, W is the complete characterization of the quantum state; it does not refer to a statistical state representing our ignorance of the underlying wave function. In general, W can be a pure state or a mixed state. A density matrix $\hat{W}$ is pure if $\hat{W} = |\psi\rangle\langle\psi|$ for some $|\psi\rangle$. Otherwise it is mixed. For a spin-less N-particle quantum system, a density matrix of the system is a positive, bounded, self-adjoint operator $\hat{W} : \mathscr{H} \to \mathscr{H}$ with $\mathrm{tr}\hat{W} = 1$, where $\mathscr{H}$ is the Hilbert space of the system. In terms of the configuration space $\mathbb{R}^{3N}$, the density matrix can be viewed as a function $W : \mathbb{R}^{3N} \times \mathbb{R}^{3N} \to \mathbb{C}$. In the unitary case, $\hat{W}$ always evolves deterministically according to the von Neumann equation:

$$i\hbar \frac{d\hat{W}(t)}{dt} = [\hat{H}, \hat{W}]. \tag{21}$$

Equivalently:

$$i\hbar \frac{\partial W(q, q', t)}{\partial t} = \hat{H}_q W(q, q', t) - \hat{H}_{q'} W(q, q', t), \tag{22}$$

where $\hat{H}_q$ means that the Hamiltonian $\hat{H}$ acts on the variable q. The von Neumann equation generalizes the Schrödinger equation (9).

In theories with a fundamental density matrix W, our discussions of Boltzmannian quantum statistical mechanics in terms of ψ will need to be adapted. Here are the key changes:

- Being in a macrostate: typically, a density matrix is a superposition of macrostates and is not entirely in any one of the macrospaces. However, we can make sense of situations where W is very close to a macrostate $\mathscr{H}_\nu$:

  $$\mathrm{tr}(W I_\nu) \approx 1, \tag{23}$$

  where I_ν is the projection operator onto $\mathscr{H}_\nu$. This means that almost all of W is in $\mathscr{H}_\nu$. In this situation, we say that W is in macrostate $\mathscr{H}_\nu$.

- Thermal equilibrium: typically, there is a dominant macro-space $\mathscr{H}_{eq}$ that has a dimension that is almost equal to D:

  $$\frac{\mathrm{dim}\mathscr{H}_{eq}}{\mathrm{dim}\mathscr{H}} \approx 1. \tag{24}$$

  A system with density matrix W is in equilibrium if W is very close to $\mathscr{H}_{eq}$ in the sense of (23): $\mathrm{tr}(W I_{eq}) \approx 1$.

- Boltzmann entropy: the Boltzmann entropy of a quantum-mechanical system with density matrix W that is very close to a macrostate ν is given by:

  $$S_B(W) = k_B \mathrm{log}(\mathrm{dim}\mathscr{H}_\nu), \tag{25}$$

  for which W is in macrostate $\mathscr{H}_\nu$ in the sense of (23).

Next, let us consider how to adapt the Past Hypothesis for density matrices. The wave-function version says that every initial wave function is entirely contained in the Past Hypothesis subspace $\mathscr{H}_{PH}$. Similarly, for density-matrix theories, we can propose that every initial density matrix is entirely contained in the Past Hypothesis subspace:

$$\mathrm{tr}(W(t_0) I_{PH}) = 1 \text{ , } \mathrm{dim}\mathscr{H}_{PH} \ll \mathrm{dim}\mathscr{H}_{eq} \approx \mathrm{dim}\mathscr{H} \tag{26}$$

where I_{PH} is the projection operator onto the Past Hypothesis subspace. Assuming $\mathscr{H}_{PH}$ is finite-dimensional, there is also a natural probability distribution over all density matrices inside this subspace. See (Chen & Tumulka 2020) for a mathematical characterization. The probability distribution and the density-matrix Past Hypothesis support a Second Law for W, which is similar to the Second Law for Ψ.

However, there is a much more natural way to implement the idea of the Past Hypothesis in the density-matrix framework, which I favor. The Past Hypothesis picks out a particular subspace $\mathscr{H}_{PH}$. It is canonically associated with its projection I_{PH}. In matrix form, it can be represented as a block-diagonal matrix that has a $k \times k$ identity block, with $k = \dim\mathscr{H}_{PH}$, and zero everywhere else. There is a natural density matrix associated with I_{PH}, namely the normalized projection $\frac{I_{PH}}{\dim\mathscr{H}_{PH}}$. Hence, we have picked out the natural density matrix associated with the Past Hypothesis subspace. We propose that the initial density matrix is the normalized projection onto $\mathscr{H}_{PH}$:

$$\hat{W}_{IPH}(t_0) = \frac{I_{PH}}{\dim\mathscr{H}_{PH}}. \tag{27}$$

I call this postulate the *Initial Projection Hypothesis* (IPH) in (Chen 2018). Crucially, it is different from (16) and (26); while IPH picks out a unique quantum state given the Past Hypothesis, the other two permit infinitely many possible quantum states inside the Past Hypothesis subspace. Remarkably, we no longer need a fundamental postulate about probability or typicality for the quantum state. We know that we can decompose a density matrix *non-uniquely* into a probability-weighted average of pure states, and in the canonical way we can decompose $\hat{W}_{IPH}(t_0)$ as an integral of pure states on the unit sphere of $\mathscr{H}_{PH}$ with respect to the uniform probability distribution:

$$\hat{W}_{IPH}(t_0) = \int_{\mathscr{S}(\mathscr{H}_{PH})} \mu(d\psi) \left|\psi\right\rangle \left\langle\psi\right|. \tag{28}$$

The decomposition here is not an intrinsic expression of what $\hat{W}_{IPH}(t_0)$ is. But the expression is something that can nonetheless be used fruitfully in statistical analysis. (See (Chen 2020d, section 3.2.3).)

By doing away the need for an extra postulate about initial quantum states, we only need two basic postulates, and we call the theory *the Wentaculus*:

The Wentaculus

1. **Fundamental Dynamical Laws (FDL):** the quantum state of the universe is represented by a density matrix $\hat{W}(t)$ that obeys the von Neumann equation (21).

2. **The Initial Projection Hypothesis (IPH)**: at a temporal boundary of the universe, the density matrix is the normalized projection onto $\mathscr{H}_{PH}$, a low-dimensional subspace of the total Hilbert space. (That is, the initial quantum state of the universe is $\hat{W}_{IPH}(t_0)$ as described in equation (27).)

Similar to the Quantum Mentaculus, the Wentaculus as it is also suffers from the quantum measurement problem. There are three promising solutions, each of which gives rise to a distinct version of the Wentaculus.

First, there is the Everettian Wentaculus that looks exactly like the original Wentaculus. For this theory, we need to embrace the idea that there is a (vague) multiplicity of emergent worlds that is similar on the original Everettian theory. What is interesting about the Everettian Wentaculus is that it suggests an astonishing possibility. If the Initial Projection Hypothesis is interpreted as a fundamental law, then the theory is *strongly deterministic*, in the sense of (Penrose 1989) that the laws pick out a unique micro-history of the fundamental ontology (represented by $W(t)$). This theory does not postulate any objective probability.

Second, the Bohmian Wentaculus postulates that, in addition to the universal density matrix W that evolves unitarily according to the von Neumann equation, there are actual particles that have precise locations in physical space, represented by $\mathbb{R}^3$. The particle configuration $Q = (Q_1, Q_2, ..., Q_N) \in \mathbb{R}^{3N}$ follows the guidance equation (written for the i-th particle):[16]

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} W(q, q', t)}{W(q, q', t)} (q = q' = Q), \tag{29}$$

Moreover, the initial particle distribution is given by the density-matrix version of the quantum equilibrium distribution:

$$P(Q(t_0) \in dq) = W(q, q, t_0) dq. \tag{30}$$

Third, the GRW Wentaculus postulates that the universal density matrix typically obeys the von Neumann equation, but the linear evolution is interrupted randomly (with rate $N\lambda$, where N is the number of particles and λ is a new constant of nature of order 10^{-15} s^{-1}) by collapses:[17]

$$W_{T^+} = \frac{\Lambda_k(X)^{1/2} W_{T^-} \Lambda_k(X)^{1/2}}{\mathrm{tr}(W_{T^-} \Lambda_k(X))} \tag{31}$$

where W_{T^-} is the pre-collapse density matrix, W_{T^+} is the post-collapse density matrix, with k uniformly distributed in the N-element set of particle labels and X distributed by $\rho(x) = \mathrm{tr}(W_{T^-} \Lambda_k(x))$, where the collapse rate operator is defined as before in (20).

In this section, we presented several versions of PH. They can all be traced back to the original Boltzmannian idea that the initial state of the universe is special and has low entropy. Differences arise when we move to the Wentaculus framework where IPH selects a unique and simple initial *microstate* of the universe. The different initial-condition postulates—(7), (16), and (26)—form a family, and I shall continue using the generic label the "Past Hypothesis" to refer to them and only use their specific labels when their differences are relevant.

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[16]This version of the guidance equation is first proposed by Bell (1980), then discussed for a fundamental density matrix in Dürr *et al.* (2005).

[17]To my knowledge, the W-GRW equations first appear in (Allori *et al.* 2013).

3 Why the Past Hypothesis is Law-Like

It is clear that PH has a special status in the Boltzmannian account. It has been suggested that PH is like a law of nature. For example, this is emphasized by Feynman (2017)[1965] as quoted in the epigraph. Making a similar point about classical statistical mechanics, Goldstein *et al.* (2020) suggest that PH is an interesting kind of law:

> The past hypothesis is the one crucial assumption we make in addition to the dynamical laws of classical mechanics. The past hypothesis may well have the status of a law of physics—not a dynamical law but a law selecting a set of admissible histories among the solutions of the dynamical laws.

In this section, I offer four types of positive arguments to support the view that PH is a candidate fundamental law of nature. These arguments also support the weaker thesis that PH is a candidate axiom in the fundamental theory. My methodology is naturalistic and functionalist. We argue for Nomic Status of PH by locating the roles of the fundamental laws in our physical theories and by showing that PH plays such roles. These roles include backing scientific explanations, constraining nomological possibilities, and supporting objective probabilities.

Some of these arguments (§3.1–3.3) are related to ideas that have appeared in the literature. I try to make the premises explicit, in the hope that the arguments are clear enough for others who disagree to examine and criticize. Usually the arguments are made in the Humean framework, but as I argue they can also be made on behalf of non-Humeans who have a minimalist conception of what it is for laws to really *govern*. That is the account I favor. Of course, the minimalist account is at odds with the idea about "dynamical governing":

Dynamical Governing Only dynamical laws can be fundamental laws of nature.

In §3.4, I offer a new argument for Nomic Status of PH based on considerations of the nature of quantum entanglement.

3.1 Arguments from the Second Law

The (fundamental) nomic status of PH is supported by the nomic status of the Second Law of Thermodynamics. Clearly, the Second Law is a law of nature. Whatever underlies a law is a law. A law that cannot be derived from other laws is a fundamental law. Therefore, PH is a fundamental law. Let us spell out the argument in more detail:

P1 The Second Law of Thermodynamics is a law of nature.

P2 A law of nature can be scientifically explained only by appealing to more fundamental laws of nature and laws of mathematics.

P3 The Second Law of Thermodynamics is scientifically explained (in part) by the Past Hypothesis, and the Past Hypothesis is not a law of mathematics.

C1 So, the Past Hypothesis is a law of nature and is more fundamental than the Second Law.

P4 The Past Hypothesis is not scientifically explained by fundamental laws.

P5 A law of nature that is not scientifically explained by fundamental laws is a fundamental law.

C2 So, the Past Hypothesis is a fundamental law of nature.

Comments on P1. First, the Second Law of Thermodynamics summarizes an important regularity: the tendency for things to become more chaotic and more decayed as time passes. It is part of our concept of lawhood that this irreversible tendency is law-like. We learn about this law much more directly in our experiences than the microscopic equations of motion. Second, the irreversible tendency is encoded in our concept of physical necessity. For example, we learn that it is physically impossible that a metal rod left by itself will spontaneously heat up on one side and the cool down on the other; it is physically impossible to create a perpetual motion machine of the second kind, and this is impossible whoever tries to do it whenever and wherever.[18] Hence, the Second Law is not an accidental feature of the world. Of course, the usual formulation of the Second Law in terms of the absolute monotonic increase of entropy is too strong. It should be modified in two ways: it holds for the overwhelming most nomologically possible initial conditions, and for each entropic trajectory there can be short-lived, shallow, and infrequent decreases of entropy (see Second Laws for X and for Ψ).

Recognizing the importance of the Second Law, Eddington (1928) suggests:

> If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations—then so much the worse for Maxwell's equations. If it is found to be contradicted by observation—well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

This may be too strong. Nevertheless, we should accept that the Second Law is nomologically necessary. It is not merely an accidental feature of the world, such as the contingent fact that all gold spheres are less than one mile in diameter. Third, counterfactuals are backed by laws of nature; laws are the things we hold fixed when evaluating counterfactuals. The Second Law backs counterfactuals about macroscopic processes that display a temporal asymmetry: if there were a half-mixed ink drop in water right now, it would have been more separated in the past and more evenly mixed in the future. (See §3.2 for more on the counterfactual arrow.)

Comments on P2. The notion of scientific explanation here is not a fully analyzed notion. What is relevant to this argument is that the Second Law is supposed to be *derived as a theorem* from the basic postulates of the Mentaculus or the Wentaculus.

[18] This becomes more complicated if a "Albertian demon" turns out to be physically possible. See (Albert, 2000, §5) and Maudlin's contribution in this volume.

It is expected that, assuming the laws of mathematics, the fundamental dynamical laws, PH, and SP, with overwhelming probability an initial microstate starting from the initial macrostate will travel to macrostates of increasingly higher entropy until it reaches thermal equilibrium (except possibly for short-lived and infrequent decreases of entropy). So it is mathematical derivation from basic postulates of the physical theory that is the relevant notion here.[19]

A standard response to P2 is that a non-fundamental law (such as those in the special sciences) can be explained (in part) by some contingent boundary conditions. Examples may include the laws of genetics and laws of economics. However, it is not clear what does the explanatory work in those cases. Let us suppose that those special science laws S arise from boundary conditions B. Suppose if B obtains then there is a high objective probability that S obtains. What is the origin of these objective probabilities? What is the physical explanation? If everything is ultimately physical, and the physics is informationally complete in so far as the motion of objects go, then it seems that the objective probabilities are ultimately backed by some postulates in physics. The probabilities in physics may supply conditional probabilities based on which $Pr(S|B)$ is high. What does the explanation, then, is the probability supplied by physics, and the real law should be the high probability of S obtaining given B, which is consistent with the physicalist picture we started with.

Winsberg (this volume) offers another potential counterexample to P2. He suggests that due to the near certainty of the existence of Boltzmann brains and large fluctuations in future epochs of the universe, it is important to postulate also a Near Past Hypothesis (see also (Chen 2020c)):

Near Past Hypothesis (NPH) We are inside the first epoch of the universe between the initial time and the first relaxation to thermal equilibrium.

Winsberg argues that NPH should have the same status as the PH and SP because it is also a necessary postulate to derive the Second Law (among other special science laws). However, NPH is an indexical statement about our location in time; as such, it cannot be a candidate fundamental law of nature. So a non-law can be part of the explanation for the Second Law, then there is no reason to think that only laws can back laws. This is an interesting insight, but I do not think Winsberg's argument undermines P2. If indexical statements cannot be laws, then the Second Law should not be stated in an indexical way. We should use the non-indexical version of the Second Law (and special science laws), such as the Second Law for X, the Second Law for Ψ, and the Second Law for W. In those versions, fluctuations are already taken into account (in a non-indexical way). Hence, we do not need to invoke NPH to derive those versions of the Second Law from PH, SP, and dynamical laws.

Comments on P3. This premise is true if we grant the explanatory success of the Boltzmannian account, which is assumed in this paper. It is clear that PH is not a law of mathematics.

[19] This is different from the notion of metaphysical explanation. See (Loewer 2012), (Hicks & van Elswyk 2015). On some views, a fundamental law of nature is metaphysically explained (but not scientifically explained) by the matter distribution.

Comments on P4. P4 is an open scientific question. Perhaps some future theory (e.g. along the lines of (Carroll & Chen 2004)) can explain PH using some simple and satisfactory dynamical laws. Still, it is also a scientific possibility that PH remains a fundamental law in the final theory and it is not explained further. Given the openness of P4, we should accept C2 only to the degree as acknowledging that PH is a *candidate* fundamental law.

Comments on P5. This follows from our concept of a fundamental law of nature.

The argument above supports the (fundamental) nomic status of PH. If Nomic Status implies Axiomatic Status, then the argument also supports the idea that PH is an axiom in the fundamental physical theory. But there is another more straightforward argument for the Axiomatic Status. The predictive consequences of a physical theory should come entirely from its axioms and their deductive consequences. A good physical theory aims at capturing as many regularities as possible using simple axioms. The Second Law describes an important regularity. Therefore, we postulate the PH and SP in addition to the fundamental dynamical laws. These postulates have an axiomatic status in the Mentaculus.

The Mentaculus is a good theory; it is better than Mentaculus−, the theory without PH and SP. The Mentaculus predicts not only the motion of planets, but also that it is with overwhelming probability that my table will not spontaneously rearrange itself into the shape of a statue. The Mentaculus− can tell us everything about the motion of planets but is silent about many macroscopic regularities we see around us. Even so, the Mentaculus is a pretty simple theory. Someone might suggest that to achieve maximal predictive power, we can add the statement about the exact microstate of the universe at t_0 as an additional axiom to Mentaculus−. But that is a detailed fact that will likely complicate the Mentaculus – such that its axioms will no longer be simple enough. (In the Wentaculus, IPH pins down a quantum microstate, but its informational content and simplicity level are the same as those of PH in the Mentaculus.)

3.2 Arguments from Other Asymmetries

The thermodynamic arrow of time described by the Second Law is best explained by PH, which provides the first type of arguments for the Nomic Status and the Axiomatic Status. What about other arrows of time? In this subsection we present arguments based on the counterfactual arrow, the records arrow, the epistemic arrow, and the intervention arrow. The upshot is that they can also be traced back to the nomic status of the Past Hypothesis, without which they would be left completely mysterious. Many of these ideas can be found in (Albert 2000, 2012) and (Loewer 2007), and they are also discussed in (Frisch 2005, 2007), (Demarest 2019), (Fernandes, this volume), (Callender 2004), (Horwich 1987), and (Reichenbach 1956).

The records arrow. We have photographs and videos of WWII but no photographs of the next major world war. We have detailed accounts of the life of President Washington but no detailed accounts of the life of the 65th president of the United States. There are craters on the moon indicating past meteorite impacts but no craters indicating future meteorite impacts. Similarly, there are fossils, rocks, ice sheets, all of which tell us the state of our planet in the past, but we do not have similarly abundant records that tell us the state of our planet in the future.

What is it about our world such that there are abundant records about the past but few records about the future? One could appeal to some A-theory of time, according to which the future does not yet exist and the past has happened. So there cannot be records about the future because there are no facts about the future. But it does not seem to provide a satisfactory scientific explanation. In any case, on a block universe picture compatible with the B-theory of time, the past, the present, and the future are all equally real; all events exist tenselessly. There are strong probabilistic correlations between physical records (e.g. fossils) that exist at a particular time and physical systems (e.g. dinosaurs) that exist at an earlier time, but no strong correlations between physical records and events at a later time.

The Past Hypothesis offers an explanation. Albert (2000) suggests that a record is a relation between two temporal ends of a physical process. A record enables us to infer what happens inside the temporal interval. For example, in a lab, the record of an electron passing through a small slit is the relation of the "ready state" of the measuring instrument at t_1 and the "click" state of the measuring instrument at t_2. If the instrument moves from "ready" to "click," then we can infer that an electron has passed through the slit between t_1 and t_2. But if the instrument was not at "ready," we cannot infer that. However, to know that the instrument was indeed "ready," we also need to rely on earlier records. This seems to go back in time *ad infinitum*, to records about the lab, and to records about the larger environment, and eventually to records earlier states of the universe. To know the CMB data is reliable, we also need to postulate that there is some "ready" state at the beginning of the universe. The Past Hypothesis, stipulated at (or around) t_0, is the "mother of all ready states." It provides the underpinning that our inferences based on records can be carried out.[20]

However, it is not enough that PH be true. We also need to justify the important fact that physical records are reliable. For this, we need the PH to have the status of a law. (If PH is not derived from other laws, it will have the status of a fundamental law.) If a theory predicts that it is unlikely physical systems that look like records reliably indicate past events, then the theory would undermine the rational justification for believing in it. Such a theory would be *epistemically self-undermining*, because we believe in physical theories based on records about past experiments and observations.[21] Mentaculus without PH is such a theory. It would predict that most "records" come about from random fluctuations. If we dig out a shoe of Napoleon, most likely it came about from random fluctuations and not from a low-entropy past state. Postulating PH as a law avoids that. The objective probabilities will be the uniform probability distribution conditionalized on PH, which will predict that most physical systems that look like "records" will be reliable records about the past (here we set aside the problems of future large fluctuations).

The epistemic arrow. Given the information about the present, there is some sense of which our knowledge about the past is more vast, detailed, and easily gained than

[20]More details are needed to fully explain the records asymmetry; Rovelli (2020) provides an interesting analysis that adds additional constraints on the initial condition.

[21]This is somewhat parallel to the situation of empirical adequacy and records in EQM. See (Barrett 1996) on the latter.

our knowledge about the future. We know that the sun will (likely) rise tomorrow, but we do not know who will win the US presidential election of 2028, and when exactly the stock market will crash over the next 20 years. But we know exactly who won the election of 1860, when exactly the stock market crashed over the last 20 years, and so on. Similar to the records arrow, the epistemic arrow is especially puzzling in the block universe picture compatible with a B-theory of time. All facts about past, present, and future are out there. Why is our knowledge so skewed towards one temporal direction?

Albert (2000) explains the epistemic arrow in terms of the records arrow, which in turn is explained by the nomic status of PH. The basic idea is this. We distinguish between inferences based on records from inferences based on predictions or retrodictions. The latter uses only the current macrostate together with the dynamical laws plus SP (construed as an unconditionalized uniform probability distribution) to the past (retrodictions) and to the future (predictions). Inferences based on predictions and retrodictions will tell us that with overwhelming probability the sun will rise tomorrow and the ice cubes in my coffee will melt in the next hour, but they will get most things wrong about the past. For example, retrodictions will tell us that the ice cubes in my coffee were actually smaller in the past (they spontaneously got larger in my coffee); all the photographs and books about someone named Lincoln winning the 1860 election came about from random collision of particles. However, inferences based on records are much more powerful and demand much less detailed information about the macrostate of the world right now. We can infer to past states reliably by assuming that the records are reliable. Such an inference is backed by the assumption that the recording device was "ready" at a time before the event, and there was another recording device measuring the "ready" state of that one, and so on. As discussed before, the records arrow is explained by PH. Moreover, if the epistemic arrow has physical necessity (or high objective probability), as is required to avoid epistemic self-undermining, whatever underlies it also has a physical necessity. In a way similar to the records arrow, the epistemic arrow has to be non-accidental, otherwise our beliefs about the past experiments would have a low probability of being accurate.

The counterfactual arrow. I am at home right now. If I had been in my office, the future would be somewhat different but the past would have been pretty much the same. Trump did not press the nuclear button on Independence Day this year. If he had, events on Labor Day would be dramatically different but events on Memorial Day would have been pretty much the same. Why is there a temporal asymmetry of counterfactual dependence? The semantics for counterfactuals is a controversial issue. It is not clear if there is a unified theory that explains every instance of ordinary language counterfactuals. But if we focus on the counterfactuals that are important for control, decision, and action, it is often accepted that such counterfactuals are backed by laws. This is made explicit in Lewis (1979)'s metric for comparing similarity relations among worlds, but it should also be compatible with a strict-conditional approach. If the laws are temporally asymmetric, and if laws entail that changes in the present macrostate would lead to vast differences in the future but not much in the past, then the counterfactual arrow has an explanation.

However, given equations (2) in classical mechanics or equations (9, 17) in unitary quantum mechanics, changes to the current state (such as the location of Trump's index finger and the location of the nuclear button) will lead to macroscopic differences in both directions of time. It is only by assuming PH as a fundamental law can we explain the following: most physically possible trajectories compatible with the current macrostate will be such that if Trump had pressed the button, most future trajectories would be macroscopically different from the actual ones but the past trajectories would be pretty much the same. This is also due to the records arrow. Assuming PH, there will be abundant records about the past. In so far as PH makes it very likely that those records are reliable, PH constrains the past histories of the trajectories even if certain macroscopic features get changed in the present state. However, few records exist about the future, so there is no such constraint about future macrostates.

The correct counterfactual semantics will no doubt involve context sensitivity and other parameters. Still, it is hard to deny that laws of nature play an important role in determining the truth values of counterfactuals.

The intervention arrow. We can exert influences towards future events but we can no longer act to bring about changes in the past. The intervention arrow is intimately connected to the counterfactual arrow, and it is not clear which is conceptually prior. Some contemporary analyses of influence and intervention are couched in the causal modeling framework. In that framework, we postulate directed acyclic graphs with variables representing events and arrows representing the direction of effect. But if the fundamental dynamical laws are time-symmetric, what is the scientific explanation for these arrows? Often the arrows are taken to be primitives in the causal model, left unexplained. However, if the Past Hypothesis explains the counterfactual arrow, then it can also explain the arrows of intervention. We can flesh out the language of intervention in terms of intervention counterfactuals, and the arrows of intervention counterfactuals can be explained in a similar way by the records arrow and PH. Loewer (2007) provides such an account.

The arguments from the entropic arrow (the Second Law) and the other arrows can be taken together as an inference to the best explanation. PH (and SP) ground these asymmetries of time. Moreover, to explain them satisfactorily, we need to postulate PH as a fundamental law and we need SP to provide objective probabilities. An opponent may take all of these arrows to be fundamental features of the world, and they can postulate them as primitives in the theory. But that would strike many as a disunified and unsatisfactory view. Postulating PH and SP in addition to the dynamical laws is a much simpler and more unified way to think about the various arrows of time: from a set of simple axioms we can derive the temporally asymmetry regularities—we get a big bang for the buck.

3.3 Arguments from Metaphysical Accounts of Laws

Humeanism provides a natural home for PH to be a fundamental law and for SP to specify objective probabilities. According to Lewis, the fundamental laws and postulates of objective probabilities are the *axioms* of the best system that are true about the mosaic and optimally balances various theoretical virtues such as simplicity, informativeness, and fit. On this account, the dynamical equations such as equations (2, 9, and 17) could be axioms of the respective best systems and the GRW chances could be the objective probabilities in a GRW world. Can the classical Mentaculus, quantum Mentaculus, and the Wentaculus count as axiomatizations of the best system? This depends on whether PH can count as a fundamental Lewisian law and whether SP can count as objective probabilities on the best-system account. Anticipating the need to add a boundary condition into the best system, Lewis (1983) writes,

> A law is any regularity that earns inclusion in the ideal system. (Or, in case of ties, in every ideal system.) The ideal system need not consist entirely of regularities; particular facts may gain entry if they contribute enough to collective simplicity and strength. (For instance, certain particular facts about the Big Bang might be strong candidates.) But only the regularities of the system are to count as laws. (p.367)

But a statement such as PH is axiomatic in the best system, why not count it towards laws? In the same paper (p. 368), Lewis distinguishes between fundamental laws and derived laws. He suggests that fundamental laws are those statements that the ideal system takes as *axiomatic* and invokes only perfectly natural properties. But PH certainly is axiomatic in the Mentaculus and the Wentaculus. Moreover, PH can be stated in the fundamental language of the respective theory. (There will be some residual vagueness, which we discuss in §4.3.) So it seems that Lewis should be open to the idea that PH is a fundamental law according to the best-system account.[22]

Hence, if we are committed to the Humean conception that laws supervene on the mosaic in the way specified by the best-system account, we are led to accept the fundamental nomic status of PH. (Similarly, as Loewer (2001) argues, Lewis's version of Humeanism also has room to recognize the probabilities specified by SP as objective.)

On the Humean theory, given facts about the mosaic, the theoretical virtues *metaphysically determine* what the laws are. They are constitutive of laws. Laws are just certain ideal summaries of facts in the world. Laws are nothing over and above the mosaic.

On non-Humean theories, laws do not supervene on the mosaic. Laws may be as fundamental as the mosaic itself. Following Hildebrand (2013), we can distinguish between two types of non-Humean theories:

1. Primitivism: fundamental laws are primitive facts in the world.
2. Reductionism: fundamental laws are analyzed in terms of something outside the mosaic.

Carroll (1994) and Maudlin (2007) maintain primitivist versions of non-Humeanism. Hildebrand (2013) provides a survey of the reductionist versions according to which laws are further explained by relations among universals, dispositions, essences, or some other more fundamental entities. It is not clear what the further analysis buys us. It is not clear to me how to reformulate various modern physical laws and objective probabilities in terms of those entities, and it is less clear to me what advantages there

[22] For a related point, see (Callender 2004).

are to reduce laws to something further.[23] Here I agree with Maudlin that the concept of laws seems more familiar to us than the concepts employed in the further analysis (such as in terms of dispositions, universals, and the like). Maudlin's version of primitivism is influential in contemporary discussions of the metaphysics of laws in philosophy of physics. Maudlin (2007) seems to focus on a more restrictive version of primitivism that I call *Dynamical Law Primitivism*:

Dynamical Law Primitivism Fundamental laws are primitive facts in the world, and only dynamical laws can be fundamental laws.

This view is connected to Maudlin's view about the intrinsic and primitive arrow of time. The spirit of the present project is to analyze time's arrow in terms of something else. However, the extra commitment about the primitive arrow of time can be disentangled from the basic idea about how laws govern.

The basic non-Humean idea is simply that laws really *govern*. They metaphysically explain why the nomological possibilities are the way they are and why things are as constrained as they are. The metaphysical explanation can take the form of constraints: given $S(t_0)$, some complete specification of the state of the universe at some time, there is a constraint on what the history of the world is like. If the theory is deterministic, then there is only one microscopic history compatible with the $S(t_0)$. A fundamental dynamical law is a kind of conditional constraint. Constraints can take other forms, such as by selecting a space of possible histories. This is the form of certain equations in general relativity and Maxwellian electrodynamics. PH also selects "a set of admissible histories among the solutions of the dynamical laws." There is conceptual space for a minimalist conception of primitivism that places no restriction on the form of fundamental laws and in particular not all of them have to be dynamical laws. The basic view is this:

Minimal Primitivism Fundamental laws are primitive facts in the world; there is no restriction on the form of fundamental laws. In particular, boundary conditions can be fundamental laws.

Even though fundamental laws can take on any form, we expect them to be relatively simple and informative. These theoretical virtues are no longer *constitutive* of what laws are, but they can serve as our best guides to find the primitive laws[24]:

Epistemic Guides Even though theoretical virtues such as simplicity and informativeness are not constitutive of fundamental laws, they are good epistemic guides for discovering the fundamental laws.

Chen & Goldstein (2020) develop this idea in more detail. It seems to me that Minimal Primitivism is a good version of non-Humeanism, and it may well be one that best fit our scientific practice and the actual conception of laws. The minimal primitivist view does

[23] Hildebrand (2013) suggests that reductionist theories have an advantage over primitivist theories for answering the problem of induction. I will discuss this in future work.

[24] Why is this expectation rational? I do not know of any non-circular justification. We can appeal to the success of physics and the discovery of simple and informative laws in the past. We may also appeal to a deeper "meta-law" that metaphysically constrain the physical laws.

not commit to an intrinsic and irreducible arrow of time, making it compatible with the current project of analyzing time's arrow in terms of the entropy gradient. PH, as we have discussed already, is virtuous in the right ways. It provides a simple explanation for the restrictions of physical possibilities and the overwhelming probability of irreversibility. According to the Epistemic Guides on the Minimal Primitivist conception, we have identified clear *symptoms* that PH is a candidate fundamental law.

Hence, both Humeanism and non-Humeanism (in the minimal form) support the idea that PH is a candidate fundamental law.

3.4 New Light on Quantum Entanglement

I suggest that there is a new reason to take PH as a fundamental law: it can help us solve a long-standing puzzle in the foundations of quantum mechanics. One of the chief innovations of quantum theory that has no classical analog is quantum entanglement. It is also the origin of the quantum measurement problem. If we solve the measurement problem using one of the three strategies discussed in §2: along the lines of Bohm, GRW, and Everett, we are still left with the quantum state that plays an important dynamical role in the respective theories. Hence, the puzzle of quantum entanglement can be traced to the nature of the quantum state. What does the quantum state represent physically? What is it in the world? Given the role of Ψ in formulating well-posed initial-value problems in BM and EQM, and its role in dynamical collapses in GRW, it is reasonable to think that Ψ represents something objective. Here are some options for a realist interpretation (see (Chen 2019b) for more detail):

1. High-dimensional field: this view is proposed by Albert (1996). Even the defenders acknowledge that this view has highly counter-intuitive consequences. It is also an open question whether it really succeeds in recovering the manifest image of low-dimensional objects.

2. Low-dimensional multi-field: this view has been explored by Forrest (1988), Belot (2012), Chen (2017) and Hubert & Romano (2018). However, this view has awkward consequences. For the Bohmian framework, the multi-field will be guiding particles but there is no influence (back-reaction) of the particles on the multi-field, even though both particles and multi-field are fundamental entities in the multi-field interpretation of Bohmian mechanics. In the multi-field interpretation of Everett, since entanglement relations are still in the 4-dimensional mosaic, its Lorentz-invariance comes at a surprising cost—the failure of what Albert (2015) calls *narratability*. This also arises in Wallace & Timpson (2010)'s spacetime state realist interpretation of Everett.

3. Nomological interpretation: given the problems of the above two approaches, the nomological interpretation remains a promising solution. See (Dürr *et al.* 1996), (Goldstein & Teufel 2001) and (Goldstein & Zanghì 2013). However, it faces problems of a different kind. First, Ψ_t is time-dependent. Can nomological entities change in time? I do not see why laws can't change in time, but as defenders of this view have long recognized, if the universal quantum state obeys

the Wheeler-DeWitt equation $H\Psi = 0$ then Ψ will be time-independent and not changing. Second, the universal wave function is a very detailed function and may be too complicated to be a law. Call this the problem of complexity.

Taking PH to be a fundamental law provides a solution to the problem of complexity in the nomological interpretation. This works in the Wentaculus framework, where we can take IPH to be a fundamental law. For IPH, the normalized projection onto $\mathscr{H}_{PH}$ contains no more and no less information than $\mathscr{H}_{PH}$, specified by PH in the Mentaculus. If $\mathscr{H}_{PH}$ is simple and informative enough to be nomological, then so is its normalized projection, which is $W_{IPH}(q, t_0)$. That is, we can afford the same status of a fundamental law to $W_{IPH}(q, t_0)$. In the Bohmian Wentaculus, we can interpret $W_{IPH}(q, t_0)$ as similar to the Hamiltonian function $H(p, q)$: as providing a velocity field of particle trajectories. In the Everettian Wentaculus, we can interpret $W_{IPH}(q, t_0)$ as a law that determines the "local beables," such as a matter-density field in physical space. In the GRW Wentaculus, we can interpret $W_{IPH}(q, t_0)$ as providing conditional probabilities for the configurations of"local beales" such as a matter-density field or flashes in space-time.

However, to solve the complexity problem, it is not sufficient for IPH to be a contingent initial condition; it is crucial that IPH has the status of a fundamental law. If it is nomologically possible that W_0 differ from the state specified by IPH, then the initial quantum state could (i.e. physically possible) well be too complicated to be regarded as nomological. Hence, only by assuming the nomic status of IPH do we obtain a solution to the problem of complexity, thereby arriving at a satisfactory way to understand the nature of quantum state. The nomological interpretation of the quantum state locates the origin of entanglement in the laws, and by doing so in the Wentaculus framework we see a unified solution to two problems in foundations of physics: the problem of irreversibility and the nature of the quantum state. Again, this is compatible with both Humeanism and (the minimal form of) non-Humeanism about laws. (For more detail, see (Chen 2018, 2020a).)

4 Apparent Conflicts with Our Concept of Laws of Nature

In the previous section we have provided positive arguments for the fundamental nomic status of PH. In this section, we discuss three apparent conflicts between that and our concept of laws of nature. These may explain some people's hesitation for accepting PH but denying its fundamental nomic status. However, as I argue, these conflicts are merely apparent if we adopt the Mentaculus framework, and at any rate they become even less worrisome in the Wentaculus framework.

4.1 Boundary Condition Laws?

It is often said that the Past Hypothesis is merely a boundary condition. The contrast is with the paradigm cases of fundamental laws—the dynamical equations such as equations (2, 9, 17, 21, and 29). A boundary condition selects a subclass of the dynamically possible trajectories. As such, it does not *directly* play a dynamical role.

It is not clear how to make this idea precise. First, it is unclear why every law has to be dynamical. Second, by restricting the possible initial conditions, a boundary-condition law can be an important ingredient in the theory, as in the case of the Mentaculus. In fact, in the Wentaculus framework, the boundary condition IPH plays a *direct* dynamical role, akin to the dynamical role of the Hamiltonian function in classical mechanics. For example, in the Bohmian version, the IPH (27), the von Neuman equation (21), and the guidance equation (29) can be combined into one equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i} W_{IPH}(q,q',t)}{W_{IPH}(q,q',t)}(Q) = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i} \langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}{\langle q| e^{-i\hat{H}t/\hbar}\hat{W}_{IPH}(t_0)e^{i\hat{H}t/\hbar}|q'\rangle}(q = q' = Q) \quad (32)$$

Hence, in the Bohmian version, IPH does not select a subclass of velocity fields; it pins down a unique velocity field. In the Everettian version, IPH does not select a subclass of possible multiverses; it pins down a unique one. In the GRW version, IPH is directly involved in setting the chances of collapses.

There is a related worry about admitting boundary condition laws. Typically we distinguish between dynamical laws and initial conditions. If initial conditions can be laws, then how can we distinguish between laws and contingent initial condition data? It seems that the distinction would collapse. However, that is not the case. Some boundary conditions such as PH are special, but it does not follow that all boundary conditions are special. This is because not all boundary conditions exemplify the required theoretical virtues (either as constitutive of what laws are or as epistemic guides to primitive laws) of contributing to the optimal balance of simplicity, informativeness, and fit.

One may worry that it is odd to have a fundamental law that refers to a particular time (t_0). Our most familiar fundamental laws are all general statements that do not refer to particular time or place. But why is that a requirement that laws cannot refer to particular events? Suppose some place or time are in fact physically distinguished, then it seems appropriate for laws to refer to them. Think about the Aristotelian tendency for things to move towards the center of the (geocentric) universe. If that is indeed the case, then we should have a fundamental law describing that motion, and a simple candidate would just state that the center of the world C_0 is where things evolve towards. Similarly, if the initial time t_0 is indeed special (as the initial state accounts for a great many regularities), then it is appropriate to postulate a law that refers to t_0.

4.2 Non-Dynamical Chances?

Another worry concerns the nature of objective probabilities. The Mentaculus postulates both PH and SP. PH and SP share the explanatory burden, so they should have the same status. It is important that the probabilities of SP be objective. However, if the dynamics are deterministic (as in the Bohmian and the Everettian versions), objective probabilities seem to be either 0 or 1. How can non-trivial probabilities be objective and represent something beyond subjective credences? (In so far as typicality plays a similar explanatory role as probability, TP may face the same *prima facie* problem as SP.)

Humeanism has resources to solve this problem. Loewer (2001) suggests that deterministic "chances" can gain entry in the Lewsian best system by the informativeness they bring and the simplicity of the postulate, such as the uniform measure specified by SP. On non-Humeanism, how to understand deterministic "chances" is an open problem. On Minimal Primitivism, perhaps the notion of primitive constraining can come in degrees that can be represented by probabilities. Another way is to use the notion of typicality specified by TP, which can be interpreted as only allowing the initial conditions that are *typical*. On this understanding, abnormal anti-entropic initial conditions are not nomologically possible. (A similar problem arises in Bohmian mechanics where a quantum equilibrium distribution will be required, in addition to the deterministic dynamical laws, to deduce the usual Born rule for subsystems.)

In the Wentaculus framework, it is clear that there is an objective anchor for SP (and TP). Since IPH selects a unique initial quantum state of the universe, we no longer need a probability distribution over initial quantum states. However, as a purely mathematical fact, the $W_{IPH}(t_0)$ induces a "uniform" probability distribution over pure states. This is not a fundamental postulate of probability in the theory, unlike SP or TP in the Mentaculus. It arises as a mathematical consequence of the objective quantum microstate of the universe. This emergent probability distribution, though not fundamental, can play the same role in statistical analysis about typical behaviors. For example, the usual conjecture about typical pure states will approach equilibrium can be translated into: most parts of the density matrix will be very close to the equilibrium subspace, which is equivalent to the claim that the density matrix will approach equilibrium. Hence, the Wentaculus framework provides an objective anchor for SP and TP.[25]

4.3 Nomic Vagueness?

The final worry about the fundamental nomic status of PH has to do with the fact that PH is vague, and an exact version of PH would be arbitrary in an unprecedented way. In Figure 1 of §2.1, we made clear that the boundaries of macrostates are fuzzy, and the macrostates only form an exact partition if we stipulate some arbitrary choices of the parameters for coarse-graining. These are the C-parameters: the size of coarse-graining cells, the exact correspondence between macroscopic quantities and functions on phase space, and (in the quantum case) the cut-off value for macrostate inclusion. There are better or worse ways to choose the C-parameters. But it is implausible that there are some exact values of C-parameters as known to Nature. The vagueness comes up in the Classical Mentaculus and the Quantum Mentaculus: where the PH selects an initial macrstate that constrains the initial microstate. In the quantum case, PH only selects an exact subspace in Hilbert space when we choose some arbitrary C-parameters.

In the Quantum Mentaculus, can we stipulate that there is an exact subspace $\mathscr{H}_{PH}$ as known to Nature? This leads to what I call *untraceable arbitrariness*. There is an infinity of *admissible* changes[26] to the boundary of $\mathscr{H}_{PH}$ that do not change the nomological

[25]However, it does not answer the related question about the nature of the quantum equilibrium distribution in Bohmian theories.

[26]Admissibility here is vague and rightly so. It can be interpreted as some measure of simplicity of theories. We want PH to be simple enough, and different ways of carving out the boundary will lead to different exact versions of PH. But we only want to consider those versions that are sufficiently simple (and not too gerrymandered).

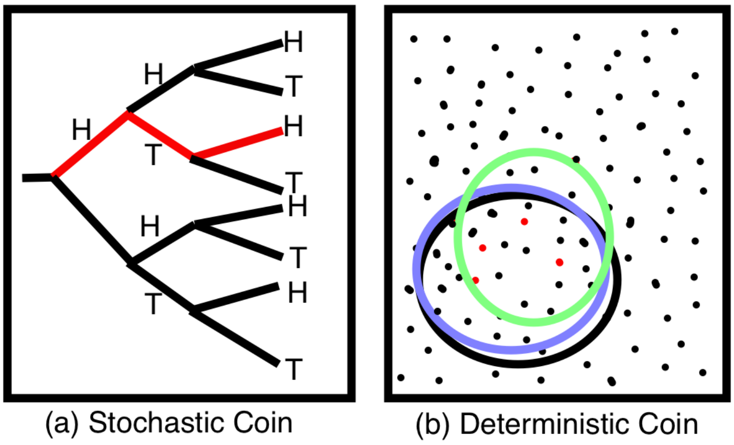

Figure 2: Illustration of nomic vagueness using the contrast between a stochastic coin and a deterministic coin. What is actually observed is this sequence: Heads, Tails, Heads.

status of most microstates compatible with $\mathscr{H}_{PH}$. This is unlike the kind of arbitrariness of natural constants or other fundamental laws. For example, any change to the value of the gravitational constant in Newtonian theory will make most worlds (compatible with the original Newtonian theory) impossible. What about the case of stochastic theories? Are the dynamical chances traceable? Yes they are, but not in the way of changing status from physically possible to physically impossible. The traceable changes are reflected in the probabilistic likelihood of most worlds.

I discuss this in more details in (Chen 2020b). Here we provide a simple illustration by considering mechanisms for flipping a coin (see Figure 2(a)). Suppose we have a stochastic coin and it is flipped three times. It landed Heads, Tails, and Heads. In the diagram, the possible sequences are marked in black and the actual sequence is marked in red. The simplest chance hypotheses are going to be:

- H_α: the chance of landing Heads at each flip is the same, and it is α.

So this is a one-parameter family of chance hypotheses. $H_{0.5}$ is the hypothesis that the coin is fair, H_1 is the hypothesis that the coin lands at Heads surely, and so on. Among these hypotheses, a sequence of coin flips will select exactly one of the chance hypotheses from this class based on which receives the highest likelihood value. In this case, it is $H_{2/3}$ with the highest likelihood value being 4/27. Conversely, $H_{2/3}$ will assign a determinate chance to every sequence of coin flips. If Nature stochastically acts according to $H_{2/3}$, then changing the chance of Heads even slightly, say to 0.65, will change the chance of every sequence. So it is in this sense that stochastic theories are traceable. Of course, we can imagine a more gerrymandered chance hypothesis according to which half the time the coin operates with a 5/6 chance of landing Heads and the other half the time the coin operates with a 3/6 chance of landing Heads. This will predict the same chance as $H_{2/3}$ to every sequence, but it is far less simple. The gerrymandered chance hypothesis is not a serious competitor to $H_{2/3}$. The actual hypothesis is *by far* simpler and more fit than any competitor.

Traceability is lost in the case of the deterministic coin. In this case, there are no dynamical transition chances. The objective probabilities come from probabilities over initial conditions. Suppose a deterministic coin is flipped and it landed Heads, Tails, and Heads. In Figure 2(b), the red dots represent initial conditions of the coin (which also include details about the flipping mechanism) that deterministically lead to the sequence HTH, and the black dots represent initial conditions that lead to other outcomes (such as HHH, TTT, and so on). To simplify things, suppose the sample space is finite, so there are only finitely many initial conditions to consider. Then we can draw different probabilistic hypotheses as different "circles" over initial conditions. Suppose the Black Circle encloses 4 red dots and 23 black dots. Then it represents a probabilistic hypothesis that all and only the dots within the Black Circle are possible initial conditions and each dot has equal probability. Black Circle has the highest likelihood given the data of HTH. If the Green Circle encloses 4 red dots and 25 black dots, then Green Circle has less likelihood than Black Circle given HTH. However, it is easy to have a nearby circle, say Blue Circle, that (like Black Circle) has 4 red dots and 23 black dots. This is possible if the red dots are sufficiently localized in state space such that it is easy to keep their proportion to black dots while changing the boundary of the circle.

Moreover, specifying Blue Circle need not be more complicated than specifying Black Circle. They are the same kind of probabilistic hypotheses, and there is no reason to think that one is more gerrymandered than the other, unlike the situation with the stochastic coin where to recover the same likelihood one has to resort to time-dependent chances. This reasoning can generalize if the state space gets richer and the sequence of coin flips gets longer. There may be infinitely many ways to slightly change the boundary of the Black Circle and keep the relative proportion *constant*. This means that there will be a large class of probabilistic hypotheses that have the same likelihood given a particular history of coin flips. No particular hypothesis is far simpler and more fit than all competitors. Hence, super-empirical virtues such as simplicity will become more relevant. Furthermore, since the variation of the boundary is incremental, comparing simplicity can generate a sorites series: is there a determinate class of hypotheses that are simple enough? It is implausible for there to be a sharp line. Hence, we have a vague "collection" of hypotheses that pass the simplicity bar.

The contrast between the stochastic coin and the deterministic coin is analogous to the comparison between GRW theories and a vague law such as PH. GRW chances are traceable, but the boundary of PH macrostate and the exact probability distribution are untraceable. Hence, there are reasons to think that if PH is a fundamental law in the Mentaculus, then it is a vague law. I call this phenomenon *nomic vagueness*.

However, it is not clear why vagueness disqualifies a statement from being a fundamental law. After all, we should be led by empirical evidence and scientific practice to consider what the laws are, and our metaphysical commitments to precision and exactness should not be given an absolute priority. It is a surprising consequence that a fundamental law can be vague. This gives us reason to think that perhaps the final theory of the world will not be completely mathematical expressible, in so far as vagueness and higher order vagueness defy classical logic and set-theoretic mathematics. This is a radical consequence about nomic vagueness that deserves more attention.

Nevertheless, the situation is different in the Wentaculus. It gets rid of nomic vagueness without introducing untraceable arbitrariness. On IPH, the initial macrostate and the initial microstate is represented by the quantum state of the universe—$W_{IPH}(t_0)$. It enters directly into the fundamental micro-dynamics. Hence, $W_{IPH}(t_0)$ will be traceable, from the perspective of two realist interpretations of the quantum state (Chen 2019b):

1. $W_{IPH}(t_0)$ is ontological: if the initial density matrix represents something in the fundamental material ontology, IPH is obviously traceable. Any changes to the physical values $W_{IPH}(t_0)$ will leave a trace in every world compatible with IPH.

2. $W_{IPH}(t_0)$ is nomological: if the initial density matrix is on a par with the fundamental laws, then $W_{IPH}(t_0)$ plays the same role as the classical Hamiltonian function or fundamental dynamical constant of nature. It is traceable in the Everettian version with a matter-density ontology as the initial matter-density is obtained from $W_{IPH}(t_0)$. It is similarly traceable in the GRW version with a matter-density ontology. For the GRW version with a flash ontology, different choices of $W_{IPH}(t_0)$ will in general lead to different probabilities of the macro-histories. In the Bohmian version, different choices of $W_{IPH}(t_0)$ will lead to different velocity fields such that for typical initial particle configurations (and hence typical worlds compatible with the theory) they will take on different trajectories.

The traceability of $W_{IPH}(t_0)$ is due to the fact that we have connected the low-entropy macrostate (now represented by $W_{IPH}(t_0)$) to the micro-dynamics (where $W_{IPH}(t_0)$ appears). Hence, $W_{IPH}(t_0)$ is playing a dual role at t_0 (and only at that time): it is both the microstate and the macrostate. In contrast, the untraceability of Γ_0 in the classical mechanics is due to the fact that classical equations of motion directly involve only the microstate X_0, not Γ_0. Similarly, the $\mathscr{H}_{PH}$ in the standard wave-function version is untraceable because the Schrödinger equation directly involves only the wave function, not $\mathscr{H}_{PH}$. There are many changes to Γ_0 and to $\mathscr{H}_{PH}$ that make no changes whatsoever in typical worlds compatible with those postulates. The Mentaculus but not the Wentaculus faces a dilemma between nomic vagueness and untraceable arbitrariness.

5 Conclusion

I have argued that, in the Boltzmannian framework, the Past Hypothesis is a candidate fundamental law of nature. This is supported by various theoretical roles it plays in the theory. In arguing for the Nomic Status and the Axiomatic Status of PH, we see that whether it is a law makes a difference to many issues. Moreover, its nomic status calls for some re-thinking about the nature of physical laws. I suggest that, according to a minimal version of non-Humeanism, boundary conditions can be fundamental laws, SP and TP can be objective, and fundamental laws and chances can be vague. The conflicts with our concept of laws of nature are merely apparent, and in any case they become much less worrisome if we adopt the Wentaculus framework.

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