Reeh-Schlieder, space-time foam, and the implications for neuroscience

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Abstract

The purpose of this paper is to explore the relationship between relativistic quantum field theory, the concept of space-time foam, cosmological models which support the existence of Boltzmann brains, and the neuroscientific understanding of brain-states as critical-point phenomena.

1 Introduction

In the past 50 years, theoretical physicists have proposed an extravagant variety of metaphysical worldviews: many believe that the world is continually branching into an uncountable infinity of parallel universes; others are convinced that there are six 'hidden' spatial dimensions; whilst another sect contends that the universe is a hologram, three-dimensional space recast as merely an illusion.

In stark contrast with this metaphysical radicalism, physicists have adopted a conservative approach to the nature of the mind, and the relevance of physics thereto: "A commonly held view is that consciousness is irrelevant to physics and should therefore not be discussed in physics papers. One oft-stated reason is a perceived lack of rigor in past attempts to link consciousness to physics," (Tegmark 2015).

The purpose of this paper is to take physicists at their word, and to use their metaphysical radicalism to explore the relationship between relativistic quantum field theory, space-time foam, cosmological models which support the existence of Boltzmann brains, and the neuroscientific understanding of brain-states as critical-point phenomena.

To this end, the second section of the paper provides a concise introduction to algebraic quantum field theory, combined with an account of the Reeh-Schlieder theorem and its interpretation. The third section begins with a discussion of the Boltzmann brain hypothesis, and the cosmological conditions for the occurrence of replica brains. This leads to an analysis of space-time foam and the holographic conjecture, and the implication of these concepts for the spatial relationship between such replica brains. The fourth section considers critical-point phenomena and their relevance to neuroscience. The fifth section details the relationship between criticality and the brain's different 'vigilance' states, and expounds the resonant properties of neurons and neural networks.

2 Reeh-Schlieder

The Reeh-Schlieder theorem is part of Algebraic Quantum Field Theory (AQFT). This is an axiomatic approach to the representation of relativistic quantum field systems, which is more general than conventional Fock-space quantum field theory.[1] Moreover, the Reeh-Schlieder theorem implies the existence of a type of entanglement which is far more generic and robust than that considered in non-relativistic quantum mechanics or quantum information theory.

In quantum field theory, the classical fields, such as the electric field or magnetic field, are replaced with field operators. Field systems possess an infinite number of degrees of freedom, and for such systems the Canonical Commutation Relations, (which must be satisfied by the field operators), have an infinite number of inequivalent Hilbert space representations. Conventional Fock space, used to calculate the scattering amplitudes and cross-sections of particle physics, is just one of these representations.

Whilst the classical fields, (which might be scalar fields, vector fields, tensor fields or spinor fields), assign an object to each point of space-time, the quantum field operators $\Phi(f)$ are assigned to so-called 'test-functions' f, with support in open subsets of space or space-time. In the case of a canonical quantization, one starts with the symplectic space S of solutions to the classical field equations (or equivalently, the space of Cauchy data to those equations), and the test-functions are drawn from a complexified version of this space. Specifically, the real symplectic vector space S is augmented into a complex Hilbert space $\mathfrak{H}$, which becomes the 1-particle space of the quantum field theory.

Algebraic quantum field theory takes the unbounded field operators $\Phi(f)$ associated with each open subset $\mathscr{O}$ of space or space-time, and reduces these by assigning instead a C*-algebra $\mathscr{A}(\mathscr{O})$ of bounded operators to each $\mathscr{O}$.[2] Taking the closure of the union of the local algebras $\mathscr{A}(\mathscr{O})$ obtains a *quasi-local* algebra $\mathscr{A}$.

Now, an abstract C*-algebra can be homomorphically mapped into a concrete set of bounded operators $\mathscr{B}(\mathscr{H})$ acting on a Hilbert-space $\mathscr{H}$. Such a mapping $\pi : \mathscr{A} \to \mathscr{B}(\mathscr{H})$ is dubbed a 'representation'.

Given a representation π of the quasi-local algebra $\mathscr{A}$, the unbounded operators representing total energy-momentum, particle number, angular momentum and electric charge, are *affiliated* to the von Neumann algebra $\mathscr{W} = \pi(\mathscr{A})''$,

[1] David Wallace pointed out in 2010 that "Despite forty years of work... the only known physically realistic algebraic quantum field theories in four dimensions are free-field theories." Given that another decade has passed without progress in this respect, perhaps AQFT should be thought of as physics' answer to Gaudi's *Sagraga Familia*.

[2] A C*-algebra is a normed algebra, equipped with an involution operation, which is closed in the norm topology, and a complete metric space with respect to the metric induced by the norm.

obtained from $\pi(\mathscr{A})$ by taking the bicommutant. i.e., the commutant $\pi(\mathscr{A})'$ is the set of all operators in $\mathscr{B}(\mathscr{H})$ which commute with all the elements of $\pi(\mathscr{A})$, and the bicommutant is the commutant of this.

A von Neumann algebra $\mathscr{W}$ is a useful structure to work with because, unlike a mere C*-algebra, its projection operators are guaranteed to form an orthocomplete lattice. In other words, for every countable family $\{P_i\}$ of mutually orthogonal projection operators, the sum $\sum_i P_i$ exists in $\mathscr{W}$.

The unbounded operators representing global physical quantities such as energy-momentum, are affiliated to $\mathscr{W} = \pi(\mathscr{A})''$ in the sense that their spectral projection operators belong to $\mathscr{W}$. $\mathscr{W}$ is dubbed a *global* algebra.

A representation π of the quasi-local algebra $\mathscr{A}$ duly induces sub-representations $\pi_{|\mathscr{O}}$ of all the sub-algebras $\mathscr{A}(\mathscr{O})$. Hence, von Neumann sub-algebras $\pi_{|\mathscr{O}}(\mathscr{A}(\mathscr{O}))''$ can be associated with each bounded open subset.

In short, AQFT associates algebras with every bounded region, and aggregates them into an algebra associated with the entire space-time; representations of this algebra induce representations of all the sub-algebras. Moreover, there is a special representation of the global algebra induced by the vacuum state Ω for inertial observers in Minkowski space-time. This pure state on the global algebra, when restricted to each sub-algebra, reduces to a mixed-state. Specifically, it induces a thermal equilibrium ('KMS') state, with respect to a one-parameter family of automorphisms of the local algebra.

Of particular interest is the fact that spacelike-separated subalgebras commute. Thus, for any pair of open subsets in space (or space-time), $\mathscr{O}_1$ and $\mathscr{O}_2$, let $\mathscr{A}_1$ and $\mathscr{A}_2$ denote the local algebras. If $\mathscr{O}_1$ and $\mathscr{O}_2$ are spacelike separated. (i.e., so that no causal signal can connect them), then the elements of $\mathscr{A}_1$ all commute with the elements of $\mathscr{A}_2$.

Now for the Reeh-Schlieder theorem. There are effectively two versions of the theorem: an 'Active' version, and a 'Passive' version. Let's begin with the active version.

Assume we're operating with a fixed representation of the space-time algebra on a Hilbert space $\mathscr{H}$, determined by the privileged vacuum state Ω, so that $\mathscr{A}(\mathscr{O})$ now denotes a concrete algebra of bounded operators. For any vector-state $\Psi \in \mathscr{H}$ of bounded energy[3] on the global algebra, (including the vacuum state), no matter how small the region of space (or space-time) $\mathscr{O}$, the set of states $\mathscr{A}(\mathscr{O})\Psi$ generated by the action of the algebra is dense in the entire Hilbert state-space $\mathscr{H}$. Ψ is said to be a 'cyclic' vector.

Some of the elements of $\mathscr{A}(\mathscr{O})$ are so-called *Kraus* operators, representing empirical operations which can be performed in the region $\mathscr{O}$. Moreover, for any other region $\mathscr{O}'$ spacelike separated from $\mathscr{O}$, there are Hilbert-space states in $\mathscr{H}$ which are *localized* in $\mathscr{O}'$. (This means that the expectation values they generate only differ from those generated by the vacuum state in the region $\mathscr{O}'$). Hence, the cyclicity of Ψ seems to imply that actions performed in $\mathscr{O}$ can change

[3]This is a more stringent requirement than stipulating that the expectation value of the energy is finite. Rather, this requirement forbids a probability distribution over energy values with an infinitely long tail; there must instead be a finite upper-limit on the range of energy values assigned a non-zero probability.

the state in other regions of space which haven't interacted with $\mathscr{O}$, and which have no overlapping causal past with $\mathscr{O}$. This, then, is the 'Active' version of the Reeh-Schlieder theorem.

The 'Passive' version is as follows: For any pair of spacelike separated bounded regions, $\mathscr{O}_1$ and $\mathscr{O}_2$, and for any global state Ψ of bounded energy, (including the vacuum state), there is at least one $A_1 \in \mathscr{A}_1$ and at least one $A_2 \in \mathscr{A}_2$, such that:

$$\langle\Psi|A_1 A_2 \Psi\rangle \neq \langle\Psi|A_1\Psi\rangle \langle\Psi|A_2\Psi\rangle \ .$$

This failure of factorisation in the expectation value $\langle\Psi|A_1A_2\Psi\rangle$ of the product A_1A_2 entails that there are non-classical correlations between separated regions of space, even if those regions haven't interacted, and even if they don't share overlapping causal pasts.

The failure of factorisation is exemplified by the 'Weyl operators', $W(f) = e^{(i\Phi(f))}$, the exponentiated versions of the field operators. Suppose $\mathscr{O}_1$ and $\mathscr{O}_2$ are space-like separated subsets in space, suppose that $f \in \mathfrak{H}$ has support in $\mathscr{O}_1$, and suppose that $g \in \mathfrak{H}$ has support in $\mathscr{O}_2$. The inner product on the 1-particle space of a massive scalar field contains an 'anti-local' operator, associated with the complex structure J used to turn the real symplectic space S into the complex 1-particle Hilbert space $\mathfrak{H}$, (Halvorson 2001). As a consequence, f and g are not orthogonal, i.e. $(f, g) \neq 0$, and the vacuum state Ω is not a product state across $\mathscr{A}_1$ and $\mathscr{A}_2$:

$$\langle\Omega|W(f)W(g)\Omega\rangle = \langle\Omega|W(f)\Omega\rangle \langle\Omega|W(g)\Omega\rangle e^{(-\mathrm{Re}(f,g)/2)} \ .$$

There is a more general version of this, often dubbed 'cluster decomposition'. It begins with the translation covariance of the local algebras: for space-time translations $\mathbf{a}$, there are unitary operators $U(\mathbf{a})$ such that:

$$U(\mathbf{a})\mathscr{A}(\mathscr{O})U(\mathbf{a})^{-1} = \mathscr{A}(\mathscr{O} + \mathbf{a}) \ .$$

This result enables us to map the elements $W(f)$ of one bounded local algebra into the elements $U(\mathbf{a})W(f)U(\mathbf{a})^{-1}$ of other algebras.

Specialising to the case where $\mathbf{a}$ is a spacelike vector, cluster decomposition entails that the strength of the Reeh-Schlieder correlation diminishes with increasing space-like separation $|\mathbf{a}|$:

$$\lim_{|\mathbf{a}|\to\infty} \langle\Omega|W(f)U(\mathbf{a})W(f)U(\mathbf{a})^{-1}\Omega\rangle - \langle\Omega|W(f)\Omega\rangle \langle\Omega|U(\mathbf{a})W(f)U(\mathbf{a})^{-1}\Omega\rangle = 0 \ .$$

Cluster decomposition concisely explains why localised subsystems can be defined and identified in physical science, even though the Reeh-Schlieder theorem entails that everything is correlated.

Another way of quantifying the degree of entanglement between regions of space or spacetime, is by using the Bell correlations. Bell operators B can be defined in the tensor product algebra $\mathscr{A}_1 \otimes \mathscr{A}_2$:

$$B = \frac{1}{2}[X_1 \otimes (Y_1 + Y_2) + X_2 \otimes (Y_1 - Y_2)] \ ,$$

where $X_i \in \mathscr{A}_1$, $Y_i \in \mathscr{A}_2$, and the norm of X_i and Y_i are less than or equal to 1. The Bell correlation between the space-like separated subsets $\mathscr{O}_1$ and $\mathscr{O}_2$ in the state ω is defined to be:

$$\beta(\omega, \mathscr{O}_1, \mathscr{O}_2) = \sup_B |\omega(B)| \ .$$

The Bell inequalities require that $\beta(\omega, \mathscr{O}_1, \mathscr{O}_2) \leq 1$, so the Bell inequalities are violated when $\beta(\omega, \mathscr{O}_1, \mathscr{O}_2) > 1$. The maximal violation occurs for the case where $\beta(\omega, \mathscr{O}_1, \mathscr{O}_2) = \sqrt{2}$. Maximal violation is realised in the case of two spacelike-separated 'double cones', (i.e., spacetime regions formed by the intersection of a future-directed light-cone with a past-directed light-cone), which are mutually tangent. If there is a non-zero spacelike separation between $\mathscr{O}_1$ and $\mathscr{O}_2$, then the violation of the Bell inequalities is less than maximal (Summers 2011).

Expressing the distance $d(\mathscr{O}_1, \mathscr{O}_2)$ between the two regions in terms of the maximum timelike distance $\mathscr{O}_1$ can be translated before it is no longer spacelike separated from $\mathscr{O}_2$, the Bell correlations of a mass m field decline as follows:

$$\beta(\Omega, \mathscr{O}_1, \mathscr{O}_2) \leq 1 + 2e^{-m \cdot d(\mathscr{O}_1, \mathscr{O}_2)} \ ,$$

for the vacuum state Ω.[4] Summers concludes from this that:

"If $d(\mathscr{O}_1, \mathscr{O}_2)$ is much larger than a few Compton wavelengths of the lightest particle in the theory, then any violation of Bell's inequality in the vacuum would be too small to be observed," (ibid.).

In terms of applying these results in a cosmological setting, Wald acknowledges that "the Reeh-Schlieder theorem has been proven only in the context of flat spacetime quantum field theory, although some generalizations to curved spacetime have been given,...so, at the very least, in linear field theory correlations over spacelike separations similar to those occurring in flat spacetime case always must be present. Indeed, the strength and generality of the Reeh-Schlieder theorem in flat spacetime is such that it seems inconceivable that similar correlations could fail to be present for essentially all states and over essentially all regions in any curved spacetime, including cosmological spacetimes with horizons," (Wald 1992, p220).[5]

To conclude this brief exposition of the Reeh-Schlieder theorem, it might be helpful to recall some of the distinctions between entanglement in non-relativistic quantum mechanics/quantum-information theory, and the more radical type of entanglement defined here.[6]

The simplest type of system used to introduce entanglement in non-relativistic quantum mechanics and quantum-information theory consists of a pair of systems, each of which has a two-dimensional complex Hilbert state space $\mathbb{C}^2$. In quantum information theory, this is referred to as a pair of 'qubits'. In

[4]In the case of a massless field, the decay is less rapid, being proportional to $d(\mathscr{O}_1, \mathscr{O}_2)^{-2}$.

[5]There have been some recent attempts to prove the existence of Reeh-Schlieder states on curved spacetimes. See, for example, Sanders (2009).

[6]See Clifton and Halvorson (2001) for further details.

typical textbook presentations, the 2-dimensional state spaces are the spin-states of an electron, or the polarization states of a photon.

In this simple system, the joint Hilbert-space is $\mathscr{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$, the global algebra is the set of all bounded operators on the Hilbert space, $\mathscr{R} = \mathscr{B}(\mathbb{C}^2 \otimes \mathbb{C}^2)$, and the sub-algebras associated with the sub-systems are $\mathscr{R}_1 = \mathscr{B}(\mathbb{C}^2) \otimes I \cong \mathscr{B}(\mathbb{C}^2)$ and $\mathscr{R}_2 = I \otimes \mathscr{B}(\mathbb{C}^2) \cong \mathscr{B}(\mathbb{C}^2)$. These sub-algebras are mutually commuting, (just like the algebras associated with spacelike separated bounded regions of space in algebraic quantum field theory).

Some of the states of this simple bi-partite system are entangled, while others are not. In the latter category, there are numerous pure states ω of $\mathscr{B}(\mathbb{C}^2 \otimes \mathbb{C}^2)$ which are *product* states; i.e., pure states which factorise: $\omega(AB) = \omega(A)\omega(B)$, for $A \in \mathscr{R}_1$, $B \in \mathscr{R}_2$. These product states are *not* entangled. But there are also pure states which don't factorise; these induce mixed states on the sub-algebras $\mathscr{R}_1$ and $\mathscr{R}_2$. For example, the well-known 'singlet' state, which yields maximal violation of the Bell inequalities, is a pure state which doesn't factorise. Such states *are* considered to be entangled states.[7]

In contrast with non-relativistic quantum theory, where some pure states are entangled and some are not, in relativistic quantum field theory *all* pure states of bounded energy on the global algebra fail to be product states on pairs, $\mathscr{A}_1$ and $\mathscr{A}_2$, of mutually commuting space-like separated sub-algebras, and the Bell inequalities are necessarily violated. Hence, there is generic rather than selective entanglement in relativistic quantum field theory.[8]

Relativistic quantum field theory is the most fundamental, empirically-verified theory currently known to us, and the only such theory which unifies quantum theory and relativity. Hence, our most fundamental theory is telling us that all parts of space, no matter how distant, are correlated to some degree.

These correlations support a hypothesis entertained in the final section of the paper, but in the next section we turn to consider some patterns in the macroscopic world which can emerge from such a system of relativistic quantum fields.

3 Boltzmann brains, space-time foam, and the holographic conjecture

Boltzmann brains are disembodied brains which form spontaneously, by chance, in random spatial locations, rather than forming as a consequence of biological development within a species which has evolved by natural selection on the

[7]Or, at least, to be entangled relative to the choice of subsystem decomposition. See Earman (2014) for the ambiguity of entanglement in non-relativistic quantum mechanics.

[8]From a mathematical perspective, there is a fundamental difference between the algebras of non-relativistic quantum mechanics, and the local algebras of algebraic quantum field theory: the algebras of the former are Type I factors, (which means that they possess 1-dimensional projectors), whilst the local algebras $\mathscr{A}(\mathscr{O})$ of relativistic quantum field theory are Type III factors, (which means that all their projectors are onto infinite-dimensional subspaces).

surface of a planet. They are a consequence of statistical mechanics. They form with the anatomy and neurophysiology of a familiar biological brain, and possess initial memory states resembling our own, but their stream of experience is randomly generated in the sensory regions of the cortex, rather than being stimulated by an external environment.

In the modern cosmology literature, Boltzmann brains are typically postulated to exist in a universe which exists forever. In a past-finite, but ever-expanding universe, such as that which we currently believe our universe to be, Boltzmann brains are postulated to form in the cold, dark, empty expanses of the remote future. A fairly typical justification for believing in the existence of Boltzmann brains is provided by Christian Loew (2017):

"A plausible cosmological assumption is that our universe is temporally infinite. Statistical Mechanics predicts that the entropy of our universe is extremely likely to keep increasing until it reaches a state of thermodynamic equilibrium, which has maximal entropy. But once it has reached equilibrium, there still is a non-zero chance that fluctuations from thermodynamic equilibrium into states of lower entropy will happen. In fact Poincare's recurrence theorem says that if our universe is appropriately bounded, then it is extremely likely to come arbitrarily close to every possible macrostate. So it is extremely likely that over an infinite amount of time every possible fluctuation into states of lower entropy will happen, including fluctuations that lead to pianos, solitary brains ('Boltzmann brains'), fully-formed persons, and entire galaxies. It might take billions of years until any particular such fluctuation will occur, but it will almost certainly occur eventually."

However, it is seldom acknowledged that Boltzmann brains should also exist in a spatially infinite universe. One often finds extremely large numbers quoted for the length of time required for a Boltzmann brain to appear, but this is not the minimum time, it is the *expected* time; i.e., the mean time. There will be a probability distribution over time, with tails on either side of the mean. In a spatially infinite universe, random processes will sample from this distribution an infinite number of times throughout space, and Boltzmann brains will form after arbitrarily short lapses of time. In a spatially infinite universe, an infinite number of Boltzmann brains exist at all times. In particular, an infinite number of Boltzmann brains exist right now, in regions spacelike separated from us.

Moreover, in a spatially infinite universe, there will also be an infinite number of replicas of you, existing as stable biological systems, belonging to species which have evolved by natural selection on the surface of a planet:

"If space is infinite and the distribution of matter is sufficiently uniform on large scales, then... there are infinitely many other inhabited planets, including not just one but infinitely many with people with the same appearance, name, and memories as you," (Tegmark 2004, p461). As Barrow puts it, "in a universe of infinite size and material extent, anything that has a non-zero probability of occurring somewhere must occur infinitely often," (2011, p245).

Tegmark points out that, "for every copy of you that has evolved and lived a real life, there are infinitely many delusional disembodied Boltzmann brains who think that they've lived that same real life... for every set of false memories that could pass as having been real, very similar sets of memories with a few random crazy bits tossed in... are vastly more likely... because there are vastly more ways of getting things almost right than getting them exactly right," (2014, p307). In fact, these ways of getting things almost right, with crazy bits tossed in, sound not dissimilar to most dream-experiences.

"A crude estimate suggests that the closest identical copy of you is about $\sim 10^{10^{29}}$m away. About $\sim 10^{10^{91}}$m away, there should be a sphere of radius 100 light years identical to the one centred here, so all perceptions that we have during the next century will be identical to those of our counterparts over there. About $\sim 10^{10^{115}}$m away, there should be an entire Hubble volume identical to ours," (ibid., p464).

Tegmark obtains these estimates by counting the number of protons which are permitted, by the Pauli exclusion principle to occupy a volume of space, at a temperature less than 10^8K. In the case of the last figure, for example, he estimates that 10^{115} protons can be packed into a Hubble volume. Each of these 'slots' can be occupied or unoccupied (there are only 10^{80} protons in our own Hubble volume), so he infers from this that there are $N = 2^{10^{115}} \sim 10^{10^{115}}$ possible quantum states of a Hubble volume. He then assumes that the expected distance to the nearest identical Hubble volume will be given by the cube-root, $N^{1/3} \sim 10^{10^{115}}$.

However, the last step in the reasoning here tacitly assumes that the physical universe possesses the simply-connected topology of three-dimensional Euclidean space $\mathbb{R}^3$. If, on the contrary, space is multiply-connected, this assumption cannot be made. The distance between a pair of points in a Riemannian manifold is defined to be the infimum of the length of all smooth paths between that pair of points. Hence, if space is permeated by a dense network of wormholes, macroscopic or microscopic, then regions which otherwise would be extremely distant, will actually be extremely close.

There is, in fact, a long-standing conjecture within quantum gravity that, when inspected on a sufficiently short length-scale, space-time is a fluctuating multiply connected network of wormholes. This is the notion of a 'space-time foam',[9] first devised by J.A. Wheeler:

"Because it is the essence of quantum mechanics that *all* field histories contribute to the probability amplitude, the sum... not only may contain doubly or multiply connected metrics; it must do so... General relativity, quantized, leaves no escape from topological complexities," (1955, p535).

Wheeler's concept seems to have been inspired by an analogy with the Feynman diagram approach to calculating scattering amplitudes in quantum field theory. In this approach, one sums over Feynman diagrams with many different one-dimensional topologies, each interpolating between an initial and final state.

Visser claims that "Wheeler wormholes, deriving their existence from the assumed vacuum fluctuations taking place in the spacetime foam, are definitely

[9]Sabine Hossenfelder (2015) complains that "The most abused word in science writing is 'space-time foam'." However, apart from being a phrase rather than a word, it seems premature to award victory to 'space-time foam', if only because the entry-list to this particular competition is more than a little over-subscribed.

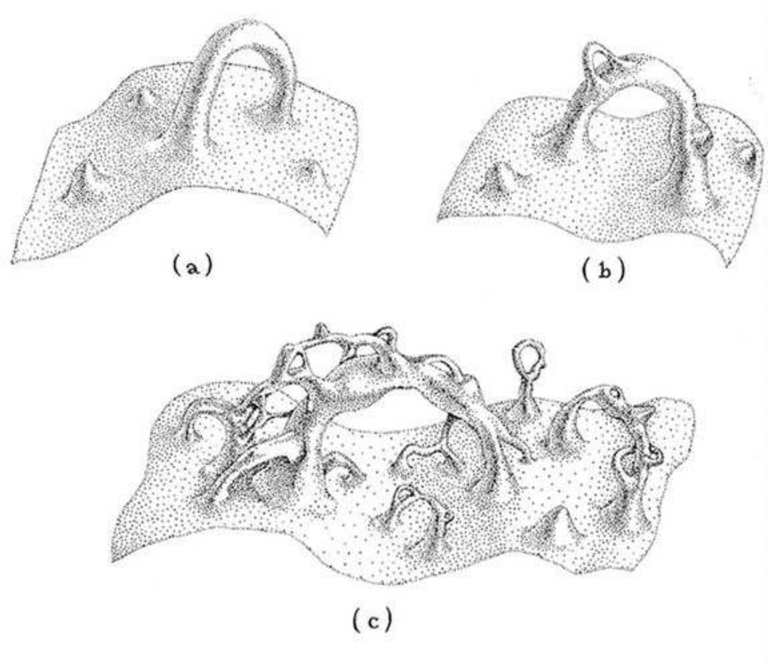

Figure 1: Space-time foam. Three topologies are illustrated here, each assigned a different probability (amplitude). From Thorne (1994).

microscopic in nature. They are typically transient, though by sheer luck might arise with the topology suitable to be considered quasipermanent,"[10] (1996, p92).

Whilst the throat-radius of these microscopic wormholes might be very small, they are not constrained to join points which are microscopically close to each other in the classical space-time. In fact, there is no limit to the macroscopic distance which might be bridged by a quasi-permanent wormhole of much shorter length.

Kip Thorne considers space-time foam to represent 'gravitational vacuum fluctuations', by analogy with the fluctuations of the electromagnetic vacuum:

"Quantum foam... is everywhere: inside black holes, in interstellar space, in the room where you sit, in your brain. But to see the quantum foam, one would have to zoom in with a (hypothetical) super-microscope, looking at space and its contents on smaller and smaller scales...

"Since the quantum foam is everywhere, it is tempting to imagine an infinitely advanced civilization reaching down into the quantum foam, finding in it a wormhole,... and trying to grab that wormhole and enlarge it to classical

[10]Visser defines a 'quasipermanent' wormhole to be a compact region Ω with a non-trivial topology which can be foliated as $\mathbb{R} \times \Sigma$. i.e., the wormhole topology remains stable for some period of time. Compact wormholes which fail to satisfy this condition are dubbed 'transient'.

size." (1994, p494-496). In this respect, the multiply connected nature of the quantum foam was assessed as a speculative method for both time-travel and distant space-travel.

As well as being omnipresent on the smallest scales as vacuum fluctuations, space-time foam has been postulated to play a cosmological role as the initial geometrical state of the universe: "in the past, the Universe went through the quantum stage when the temperature exceeded the Planckian value and the fluctuations were strong enough to form a non-trivial topological structure of space. In other words, on the very early, quantum stage Universe had to have a foam-like structure. During the cosmological expansion, the Universe cools, quantum gravity processes stop, and the topological structure of space freezes. There is no obvious reason why the resulting topology has to be exactly that of $\mathbb{R}^3$ - relics of the quantum stage foam might very well survive, thus creating a certain distribution of wormholes in space," (Kirillov and Turaev, 2007, p1).

Kirillov has vigorously argued that the properties of dark matter (DM) can be explained in terms of the relics of this space-time foam. He claims that there should be a gas-like distribution of wormholes in a Friedmann-Robertson-Walker background:

"There are no convincing theoretical arguments of why such a foamed structure should decay upon the quantum stage - relics of the quantum stage foam might very well survive the cosmological expansion, thus creating a certain distribution of wormholes in the Friedman space. Moreover, the inflationary stage in the past should enormously stretch characteristic scales of the relic foam... parameters of the foam may arbitrary vary in space to produce the observed variety of DM halos in galaxies (e.g., the universal rotation curve for spirals constructed for the foamed Universe perfectly fits observations). Moreover, the topological origin of DM phenomena means that the DM halos surrounding point-like sources appear due to the scattering on topological defects and if a source radiates, such a halo turns out to be luminous too," (Kirillov and Savelova, 2011, p1710-1711).

By virtue of surviving until the period of inflationary expansion, these primordial wormholes would be stretched to enormous distances. In particular, they could join replica Hubble volumes containing replica brains.

Kirillov acknowledges that "there remains a strong scepticism in accepting the existence of actual wormholes. It bases, in the first place, on the fact that spherical wormholes are highly unstable (they collapse during the characteristic time $\sim R/c$, where R is the size of the throat section and c is the speed of light). Therefore, to be more or less stable (and traversable) they require the presence of an exotic violating the averaged null energy condition matter. It is possible to find a source of such matter at Planck scales (e.g., due to the Casimir effect) but such a matter does not exist at laboratory and astrophysical scales.

"It turns out that the problem of the stability of cosmological wormholes can be easily solved when we consider less symmetric wormhole configurations. In this case the presence of an exotic matter is not the necessary condition for a wormhole to be traversable in both directions and be stable, while the less symmetry gives rise to the fact that cosmological wormholes have the neck sections in the form of tori or even more complex surfaces," (Kirillov and Savelova, 2016).

Initial attempts were made to rigorously formulate the concept of space-time foam in path-integral quantum gravity. As with any sum-over-histories approach to the dynamics of a quantum theory, the transition amplitude between an initial state and a final state is calculated by integrating or summing over all the possible interpolating histories. In path-integral quantum gravity, each of the space-times integrated over were smooth manifolds. The space-time topology could be fixed, or a sum over different topologies could be included. The transition amplitude between an initial spatial configuration (Σ_1, γ_1) and a final spatial configuration (Σ_2, γ_2) was 'formally'[11] written as:

$$\sum_{\mathcal{M}} \nu(\mathcal{M}) \int e^{iA[\mathcal{M},g]/\hbar} \, d\mu[g] \, .$$

The sum here is understood to be over 4-manifolds $\mathcal{M}$ bounded by a disjoint pair of 3-dimensional geometries (Σ_1, γ_1) and (Σ_2, γ_2). A is the action associated to each interpolating space-time history, and $\nu(\mathcal{M})$ is a weight assigned to each interpolating 4-manifold. As John L. Friedman remarks, "the path-integral approach to quantum gravity suggests that the microscopic topology of space-time, as well as its geometry, will fluctuate because smooth Lorentzian four-geometries interpolate between space-like three-geometries with different spatial topologies," (1991, p540).[12]

More recently, the concept of a space-time foam has been recast in the Loop Quantum Gravity programme as a 'spin foam'. In this approach, one sums over discrete 2-complexes, consisting of faces, edges and vertices, with a representation of the Lorentz group $SL(2, \mathbb{C})$ assigned to each face. For each 2-complex, one sums over all possible combinations of representations assigned to the faces. Each 2-complex is topologically dual to a 4-dimensional simplicial complex, hence each 2-complex corresponds to a type of discrete space-time history. One can fix the space-time topology, or include different topologies in the spin-foam sum.

The notion of a space-time foam entails that the metric-space structure of space is radically different from that which our macroscopic experience conditions us into believing it to be. There are wormhole networks connecting all regions of space-time, irrespective of the apparent macroscopic spatial distance between them. Moreover, the actual distance between a pair of points is determined by the distance through the wormhole network, (possibly calculated in terms of the spin foams dual to the triangulated wormhole network). The length and connectedness of the wormhole network is envisaged to be constantly fluctuating or interfering. There is no lower limit on the length of the wormholes joining an arbitrary pair of points, hence there is no lower limit on the

[11] There are problems obtaining a tractable version of this expression, not the least of which is integration over an infinite-dimensional space of 4-dimensional histories.

[12] A topology-changing space-time must, however, be either time non-orientable or contain closed timelike curves.

actual distance between an arbitrary pair of points. The distance might even momentarily fluctuate to zero, hence the structure of space may be that of a degenerate metric space.[13]

Quite distinct from the notion of a 4-dimensional space-time foam, there is a conjecture within superstring theory which also holds that space-time is multiply connected by means of wormholes. This is the so-called 'ER=EPR' conjecture[14], which holds that entangled particles are connected by a non-traversable wormhole: "ER=EPR tells us that the immensely complicated network of entangled subsystems that comprises the universe is also an immensely complicated (and technically complex) network of Einstein-Rosen bridges," (Susskind, 2016). If this conjecture is extended to Reeh-Schlieder correlations, then the ubiquitous and robust nature of Reeh-Schlieder correlations entails that space-time must be multiply connected by a dense network of wormholes.

However, it's important to understand the larger theoretical context within which the ER=EPR conjecture sits. Recall first that superstring theory represents the physical universe to be a 10-dimensional space-time. Within superstring theory, there is a sub-genre of work devoted to the so-called AdS/CFT correspondence.

The correspondence revolves around one particular 10-dimensional space-time, $\mathrm{AdS}_5 \times S^5$. This is obtained by first taking 5-dimensional anti-de Sitter space-time, an empty, static space-time of constant negative curvature, equipped with a negative cosmological constant. As Penrose points out, the conventional definition of anti-de Sitter space-time yields a manifold of topology $S^1 \times \mathbb{R}^4$, which possesses closed timelike curves, (2016, p113). The universal cover of this has topology $\mathbb{R}^5$. One can take the conformal compactification of each of these.

The conformal boundary of the anti-de Sitter space-time with topology $S^1 \times \mathbb{R}^4$ is compactified Minkowski space-time (see Penrose 2016, p114). The conformal boundary of the universal cover anti-de Sitter space-time is conformally equivalent to the Einstein static universe, with topology $\mathbb{R} \times S^3$. Indeed, the 'cylindrical' topology of the Einstein static universe is the one typically used in diagrammatic representations of the AdS/CFT correspondence.

This conformal boundary $\mathscr{M}$ is variously dubbed the boundary of AdS_5, or the boundary of $\mathrm{AdS}_5 \times S^5$. Penrose emphasises that as a 4-dimensional space-time, $\mathscr{M}$ cannot be the boundary of a 10-dimensional space-time, (ibid., p108-109). Moreover, $\mathscr{M}$ is the boundary of the conformal compactification of AdS_5, not the boundary of AdS_5. Furthermore, the product $\mathscr{M} \times S^5$ is not the conformal boundary of $\mathrm{AdS}_5 \times S^5$.

Maldacena (1998) conjectured that string theory on the 10-dimensional space-time $\mathrm{AdS}_5 \times S^5$, is equivalent to a supersymmetric conformal gauge field

[13]The multiply connected nature of space-time foam is crucial to a hypothesis proposed later in this paper. Given that Reeh-Schlieder correlations diminish in an exponential fashion with distance, to support the hypothesis it will be necessary to assume that space is multiply connected by means of a dense network of wormholes, with the consequence that identical copies and Boltzmann brain replicas of each person are separated by only very short distances.

[14]i.e., Einstein-Rosen wormhole = Einstein-Podolsky-Rosen entanglement.

theory[15] on the 4-dimensional boundary, in the sense that there is a one-to-one mapping between the states of the two theories.[16] This is often referred to as a 'holographic' duality, on the basis that the five-dimensional interior 'bulk' AdS_5 can be represented by the state of its 4-dimensional boundary.

However, as Penrose argues (ibid., p107), the analogy with real-world holograms is not valid. The optical holograms we're familiar with don't represent a 3-dimensional volume with the information on a 2-dimensional surface; rather, they represent the stereoscopic surface appearance of a 3-dimensional object with the information on a flat 2-dimensional surface. The 3-dimensional interior of an object is not represented by a hologram.

Nevertheless, within the AdS/CFT programme, Susskind and Maldacena subsequently conjectured that entangled black holes in the 4-dimensional boundary $\mathscr{M}$, correspond to non-traversable wormholes in the 5-dimensional bulk AdS_5. This was then generalized to the conjecture that all entangled systems in the 4-dimensional boundary are connected by non-traversable wormholes in the bulk.

Allied to this is the Ryu-Takayanagi formula and associated concepts (see Jaksland, 2020). Given a spacelike slice Σ of the 4-dimensional boundary $\mathscr{M}$, the Ryu-Takayanagi formula holds that the entanglement entropy between two disjoint spatial regions $B_1 \cup B_2 = \Sigma$ is proportional to the minimal area surface in the bulk whose boundary separates B_1 from B_2. In conjunction with this, the entanglement between a pair of systems in the 4-dimensional boundary is inversely proportional to the distance between them in the 5-dimensional bulk AdS_5. (The distance in the bulk is defined in the conventional manner of differential geometry, i.e., in terms of the infimum of the length of all the geodesics connecting them). For the algebras $\mathscr{A}_1$ and $\mathscr{A}_2$ associated with the complementary spatial regions B_1 from B_2, the two-point correlations for $A_1 \in \mathscr{A}_1$ and $A_2 \in \mathscr{A}_2$ are such that:

$$\langle \Psi | A_1 A_2 \Psi \rangle - \langle \Psi | A_1 \Psi \rangle \langle \Psi | A_2 \Psi \rangle \sim e^{-mL} ,$$

where Ψ is a state of the quantum CFT, L is the bulk distance and m is a constant.

Which is certainly interesting. However, those not socially conditioned by being a member of the string-theory community might want a *bona fide* theoretic 'duality' to be something which pertains to all the models of a theory, not just one particular space-time. Moreover, the static, empty space-time in question, equipped with a negative cosmological constant, seems rather alien to our own Friedmann-Roberston-Walker space-time, replete with mass-energy and a positive cosmological constant.

Nevertheless, both the concept of space-time foam, and the ER=EPR conjecture, open up the possibility that space-time is multiply connected, from which

[15] CFT = Conformal Field Theory.

[16] Maldacena's paper has been cited 20,000 times. Nevertheless, at the time of writing, Gilbert Toyne's invention of the rotary clothes line in the 1920s still provides a greater contribution to society.

it follows that spatial regions separated by apparently enormous distances might not be so distant after all. In a spatially infinite multiply connected universe, one might not be so far away from one's own replicas or near-replicas.

But *is* the universe spatially infinite? There is, first of all, a degree of ambiguity to this question: space-times can often be foliated in many different ways by a 1-parameter family of spacelike hypersurfaces, and these hypersurfaces can be of infinite volume in one foliation, but of finite volume in another. Aguirre provides a nice example of this in the case of an inflationary model in which a bubble of true vacuum (a Coleman-De Luccia, or 'CDL' bubble) nucleates by quantum tunelling from a false vacuum state: "The structure of a CDL bubble... looks entirely different depending upon how spacetime is 'foliated' into space and time. In the foliation outside that gives [the de Sitter metric appropriate to the false vacuum], the bubble is finite, non-uniform, and growing; in the foliation that [represents the interior of the bubble as a Friedmann-Robertson-Walker spacetime with homogeneous spacelike hypersurfaces of constant negative curvature] it is infinite," (Aguirre 2008).

However, if there is at least one foliation of a cosmological model in which the hypersurfaces are (i) homogeneous, and (ii) spatially infinite, then we can deem the universe represented by such a model to be spatially infinite. The homogeneity condition supports the Boltzmann brain hypothesis because it entails that the spatial universe is not just infinite, but randomly infinite. The condition of exact spatial homogeneity in a cosmological model of general relativity corresponds to *statistical* homogeneity in terms of the distribution and motion of matter in the physical universe. General relativistic cosmology only applies above a certain length-scale L, so homogeneity means that the distribution and motion of matter in each spatial volume of size $\sim L^3$ is sampled from the same statistical distribution.

Spatially infinite cosmological models can be found in the class of Friedmann-Robertson-Walker (FRW) models of 'big-bang' cosmology, and within the models of inflationary cosmology. Tegmark comments that "a spatially infinite universe is a generic prediction of the cosmological theory of inflation," (2004, p462-463). In terms of the FRW models, the spatially infinite cases are typically associated with the ever-expanding models in which the spatial curvature k is either zero, $k = 0$, or negative $k < 0$. These are the FRW models currently believed to represent our own universe. This should be qualified, however, by noting that one can obtain spatially finite models with $k = 0$ or $k < 0$ by taking quotient topologies, (Ellis 1971).

All of the FRW models with positive spatial curvature, $k > 0$, are spatially finite. So if our universe belongs to this class, the Boltzmann brain hypothesis would have to be relegated to the far future, and the conjecture in the final part of this paper would have to be rejected.

4 Critical phenomena and the brain

As a bridge between the cosmological and the neurological, we now turn to the concept of criticality. In physics, criticality has typically been used to represent phase transitions in condensed matter systems. For example, the dynamics of metallic crystalline solids as they transition between paramagnetic and ferromagnetic states, or the dynamics of a fluid as it transitions between liquid and vapour states, both exhibit critical behaviour.

In many critical systems, there is a *correlation length* ξ, which measures the length-scale over which the constituent elements exist in the same phase. For example, many systems undergoing critical phase transitions are represented by the Ising model, where binary values $s(x_i)$ are assigned to a discrete lattice of sites x_i. In these models, the coupling between adjacent sites tends to correlate their values, but thermal fluctuations tend to randomize them, so that the correlation coefficient $\Gamma(r) = \langle s(x_i)s(x_i + r)\rangle$ drops off as an isotropic inverse exponential function of radial distance r (Hughes 2010, p168):

$$\Gamma(r) = \exp(-r/\xi) \ ,$$

where ξ is the correlation length. For systems approaching a critical state, however, the correlation function has a power-law form:

$$\Gamma(r) = \left(\frac{r}{\xi}\right)^{-\gamma} \ .$$

In a fluid, the correlation length might measure the size of liquid droplets within a background of vapour. Alternatively, in a magnetizable crystalline solid, ξ measures the size of domains whose units are magnetically aligned in the same direction.

Batterman defines the correlation length as a *typical* length-scale over which correlation exists: "For a fluid in equilibrium at a given temperature away from criticality, there is a well-defined average size for the droplets. One can now introduce the correlation length, ξ, which, roughly speaking, characterizes the spatial extent of the average droplets of liquid. (Put slightly differently: The correlation length is the typical distance over which the behavior of one microscopic variable or degree of freedom can be correlated with the behavior of another.)" (Batterman 2011).

Hughes, however, defines the correlation length as the *maximum* length-scale over which correlation exists: "Effectively this parameter provides a measure of the maximum size of locally ordered islands. . . it is worth emphasizing that ξ gives a measure of the *maximum* size of locally ordered regions," (Hughes 2010, p168).

In these examples, with all other variables fixed, the parameter controlling the phase transition is the temperature T. As it approaches a critical temperature T_c, the correlation length diverges:

$$\xi(T) \to \infty \ .$$

"Near the critical temperature, islands of all sizes up to the correlation length coexist, and participate in the behaviour," (Hughes 2010, ibid). Moreover, as the size of the correlated domains approaches the size of the entire system, "external perturbations applied to any part of the system may lead to a change in the state of the whole system," (Cocchi *et al*, 2017).

So criticality involves: (i) Long-range correlations; and (ii) Maximal sensitivity to small perturbations.

Criticality has also been used to represent patterns of activity in the brain: "A set of recent observations... show that the neural functioning of the brain is close to criticality, which is confirmed by the distributions of avalanches and neural activity with a scaling that fits this regime and therefore its behavior is very sensitive to small perturbations. The states of the brain that exhibit critical behavior are the most suitable for more efficient neural processing information. The critical point characterizes the maximum capacity of exchanging information without bottlenecks, and as in every second order phase transition, it exhibits long range correlations of neural function... close to the critical point the system is maximally sensitive to small fluctuations, for instance a change of one neuron can trigger an avalanche of activity," (Gambini and Pullin, 2019).

Recent empirical research in neuroscience has established that whilst the neural network of the brain exists close to criticality, in a healthy subject at least it remains subcritical at all times: "computational optimality may not have been the only evolutionary constraint, but stability might have been an additional goal. Stability is compromised in the supercritical state as the supercritical state was linked to epileptic behaviour. It may well be that the brain in all its vigilance states maintains a safety margin to the supercritical state, because supercriticality allows for runaway activity, which is pathological, energy demanding and may induce erroneous learning," (Priesemann *et al*, 2013).

Clusters of neurons which exhibit heightened levels of activity over a period of time are referred to as 'neural avalanches'. In the close-to-critical state, the size-distribution of neural avalanches possesses the form of a power-law distribution. The distribution is said to be 'scale-free': "A power-law indicates that the activity between the units is correlated, but the units don't form strongly interconnected subgroups," (Priesemann *et al*, 2013). In the range between the inter-unit separation, and the correlation length ξ, there is no privileged length-scale.

Priesemann and colleagues analysed the criticality of brain-states under conditions of both wakefulness and sleep, which brings us to the study of dreams.

5 Dreams

Modern sleep science partitions the process of sleep into a number of successive stages, each identified by distinctive brain-wave patterns recorded by Electro-Encephalography (EEG), a non-invasive method of detecting patterns of electric potential on the outer surface of the skull.[17] Some stages of sleep involve dream experiences, the scientific status of which has been succinctly summarised as follows:

"A first reason for thinking that dreams are experiences during sleep is the relationship between dreaming and REM (rapid eye movement) sleep. Researchers in the 1950s discovered that sleep is not a uniform state of rest and passivity, but there is a sleep architecture involving different stages of sleep that is relatively stable both within and across individuals. Following sleep onset, periods of non-REM (or NREM) sleep including slow wave sleep (so called because of the presence of characteristic slow-wave, high-voltage EEG activity) are followed by periods of high-frequency, low-voltage activity during REM sleep. EEG measures from REM sleep strongly resemble waking EEG. REM sleep is additionally characterized by rapid eye movements and a near-complete loss of muscle tone.

"The alignment between conscious experience on the one hand and wake-like brain activity and muscular paralysis on the other hand would seem to support the experiential status of dreams as well as explain the outward passivity that typically accompanies them. Reports of dreaming are in fact much more frequent following REM (81.9%) than NREM sleep awakenings (43%). REM reports tend to be more elaborate, vivid, and emotionally intense, whereas NREM reports tend to be more thought-like, confused, non-progressive, and repetitive," (Windt 2019, Section 2.3).

Recent empirical research in neuroscience has also revealed the different correlation structure of the brain's 'vigilance' states. Specifically, neural avalanche statistics enable us to define and differentiate between: (i) a state of full wakefulness; (ii) Slow-Wave-Sleep (SWS), during which it is believed that memories are consolidated and the waste products of neural metabolism are scavenged; and (iii) REM sleep, during which vivid dreams are experienced.

Slow-Wave-Sleep corresponds to the third stage of NREM sleep, dubbed N3 sleep. "Only few reports contained elements of dreaming after awakenings from N3 sleep early during the night, when large slow waves are most prevalent in the EEG signal…It is important to note that reports of dreams after awakenings from NREM sleep are not merely a recall of dreams that occurred during the REM sleep phase, because (1) dreaming has been reported after awakenings from the first period of NREM sleep before the occurrence of REM sleep and (2) individuals reported dreams after awaking from short naps that consisted of NREM sleep only. As such, it has been suggested that dreaming during NREM sleep relates to 'covert REM' brain activation processes, which occur outside polysomnographically scored REM sleep. In line with this view, it is important to realize that wakefulness, REM, and NREM sleep are not necessarily mutually exclusive phenomena; sleep is far from being homogenous in terms of mental experiences. Hence, dreaming might be described along a continuum, ranging from thought-like mentation that is typical of the early stages of NREM sleep to very vivid dreams that are more typical of REM sleep," (Mutz and Javadi,

[17]Brain-waves are macroscopic oscillations in which clusters of neurons engage in synchronized firing patterns.

2017).

EEG data simply detects patterns on the surface of the skull, and therefore under-determines the pattern of activity in the interior of the brain. A more informative technique is to record Local Field Potentials[18] (LFP) using intracranial electrodes which penetrate into deep brain structures, with several spatially separated contacts on each shaft. Priesemann *et al* (2013) studied LFPs in human subjects, and found that in each vigilance state, the distribution of neural avalanche sizes could be fitted by a power-law distribution with cutoff, corroborating the notion that the brain is in a state close-to-criticality in each case.

However, it transpires that whilst all three vigilance states are subcritical, it is the SWS state which exhibits the greatest degree of correlation. REM sleep has the weakest degree of correlation, in relative terms, with a lower frequency of larger neural avalanches, (as evident in Figure 2). Full wakefulness exhibits an intermediate degree of internal correlation.

Priesemann *et al* conclude: "Our analyses of avalanche dynamics from human intracranial depth recordings indicated that the human brain operates close to criticality from wakefulness to deep sleep, as indicated by a power-law like distribution of avalanche sizes for each vigilance state. However, the sizes of neuronal avalanches changed with vigilance states: SWS showed larger and longer avalanches, wakefulness showed intermediate ones, and REM showed smaller and shorter ones. The larger avalanches of SWS confirm the correlated character of SWS dynamics across brain areas, while the smaller avalanches of REM revealed a fragmented organization of brain dynamics compared to wakefulness and SWS," (ibid.). (Priesemann *et al* model the differences between vigilance states to be "mediated by tiny changes in effective synaptic strength.")

So, when we dream, whilst the brain still resides in a state close to criticality, a state where it is still sensitive to small perturbations, it has retreated into the condition where it possesses the smallest degree of internal correlation.[19]

[18]These are electric potentials in the extracellular medium, generated by neurons in the local neighbourhood of the recording electrode.

[19]One might submit the conjecture that human dream experiences during REM sleep are the resonantly amplified manifestation of Reeh-Schlieder correlations between space-like separated replica brains. Such a hypothesis holds that dreams are external perturbations of a neural network, induced when the brain is still sensitive to small perturbations, but when the strength of its internal correlations are at their weakest, and therefore most sensitive to small *external* perturbations. Dream experiences include coherent alternative personal episodes and histories, as well as experiences with no temporal or spatial coherence, and even experiences which are unconstrained by the laws of physics. The hypothesis entertained here is that some of your coherent vivid dreams may be induced by the experiences of your replicas on distant worlds, whilst other dreams, the ones which are most disconnected, or which violate the laws of physics, are induced by the random experiences of your Boltzmann brain replicas. By definition, each replica brain possesses exactly the same modes of oscillation, and exactly the same power-law distribution of neural avalanches. The hypothesis is that when the internal correlations of one brain drops below a threshold in REM sleep, the distant correlations become detectable, in the first-person experiential form of dreams.

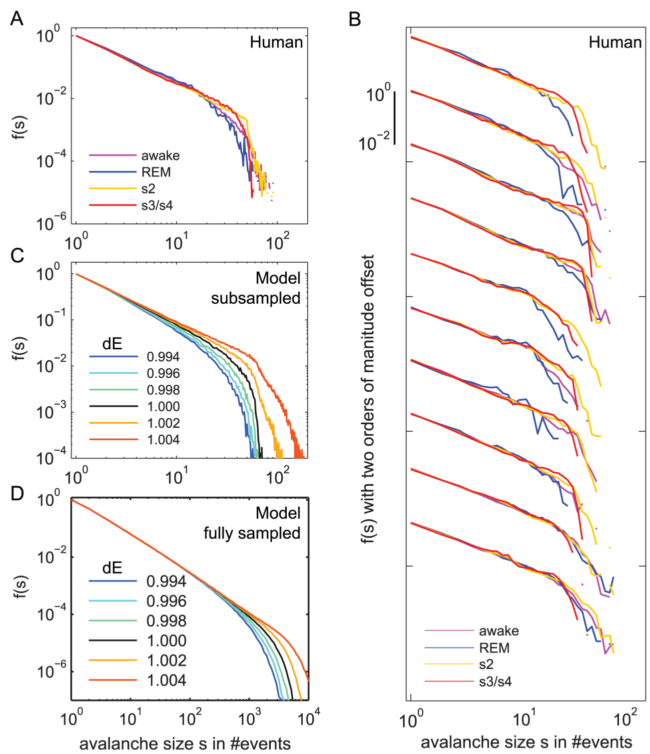

Figure 2: Charts A and B plot the distribution $f(s)$ of avalanche size s for 5 patients monitored over two nights, in four vigilance states: wakefulness, REM sleep, and two types of Slow Wave Sleep, s2 and s3/s4. Chart B plots the data for each patient-night. (From Priesemann *et al*, 2013).

5.1 Neurons and neural networks

The brain can be represented as a neural network, which on an abstract level consists of a set of nodes, and a set of connections between the nodes. The nodes possess activation levels; the connections between nodes possess weights; and the nodes have numerical rules for calculating their next activation level from a combination of the previous activation level, and the weighted inputs from other nodes. A negative weight transmits an inhibitory signal to the receiving node, while a positive weight transmits an excitatory signal.

The activation levels in a neural network are also referred to as 'firing rates', and in the case of a biological brain, generally correspond to the frequencies of the so-called 'action potentials' (or 'spikes') which a neuron transmits down its output fibre, the axon. The neurons in a biological brain are joined at synapses, and the neural network weights in this case correspond to the synaptic efficiency. The latter is dependent upon factors such as the pre-synaptic neurotransmitter release rate, the number and efficacy of post-synaptic receptors, and the availability of enzymes in the synaptic cleft, (Bickle, Mandik and Landreth 2019).

In physical terms, the propagation of each action potential is a pattern in the ionic electric field $\mathbf{E}(x)$ of the axon. The state of a nerve fibre, such as an axon, is defined by the distribution of positive and negative ions inside and outside the fibre. In the rest state of a nerve fibre, there is an excess of negative ions inside the fibre, and a net positive charge on the outside. In its rest state, the interior of a nerve fibre is at a voltage of approximately -70mV relative to the exterior.

When the nerve fires, a region of charge imbalance reversal propagates down the fibre. As the signal approaches, sodium gates open in the cell membrane, permitting the flow of positive sodium ions from the outside to the inside, reversing the charge imbalance so that a region of net positive charge is created inside the fibre, with a region of net negative charge on the outside; when the region of charge reversal has passed, potassium gates open to permit the flow of positive potassium ions from the inside to the outside, restoring the excess of negative charge on the inside.

If a signal reaches an excitatory synapse, it releases a neurotransmitter which injects a current pulse that makes a positive contribution to the electrical potential difference between the inside and outside of the next neuron; if it reaches an inhibitory synapse, it releases a neurotransmitter which contributes a negative potential difference.

In terms of the conditions under which a neuron can be triggered into firing, there are two different types: integrators and resonators. For integrators, there is a threshold voltage below which the neuron will not fire. The contributions from incoming synapses are summed ('integrated'), and if the net result exceeds the threshold, the neuron fires. In contrast, resonators exhibit subthreshold oscillations, and fire when current pulses are injected with the same frequency as that of the subthreshold oscillations. As such, resonators can fire even when they receive inhibitory pulses of current.

Cell membranes are perforated by ionic channels. These are proteins containing aqueous pores, through which ions can flow, driven either by concentration gradients or electric potential gradients. These channels can be opened or closed. When the state of a channel is dependent upon the membrane potential V, it is said to be 'voltage gated'. A channel can be opened or closed by voltage-dependent changes in the conformation of the protein; these are referred to as 'activation' gates. However, a channel can also be closed by other means, such as the presence of a channel-blocking particle; these are referred to as 'inactivation' gates, (Izhikevich, 2006, p32-33).

Inward currents correspond to the transfer of positive ions inside the cell, increasing the membrane potential (i.e., making it more positive, a change referred to as 'depolarization'). Outward currents correspond to the transfer of positive ions from inside to outside, decreasing the membrane potential (i.e., making it more negative, a change referred to as 'hyperpolarization').

As a function of membrane potential, let $m(V)$ denote the probability of an activation gate being open, and let $h(V)$ denote the probability of an inactivation gate being open. The voltage-dependence of these gates works to produce spikes as follows:

"The *amplifying* gating variable is the activation variable m for voltage-gated inward current or the inactivation variable h for voltage-gated outward-current. These variables amplify voltage changes via a positive feedback loop. Indeed, a small depolarization increases m and decreases h, which in turn increases inward and decreases outward currents and increases depolarization. Similarly, a small hyperpolarization decreases m and increases h, resulting in less inward and more outward current, and hence in more hyperpolarization.

"The *resonant* gating variable is the inactivation variable h for an inward current or the activation variable [m] for an outward current. These variables resist voltage changes via a negative feedback loop... To get spikes in a minimal model, we need a fast positive feedback and a slower negative feedback," (Izhikevich, 2006, p129-130).

The resonant characteristics of neurons were spelt out by Hutcheon and Yarom (2000): "there are three classes of frequency-dependent mechanism in central neurons: (1) solitary resonances caused by unaided resonant currents; (2) amplified resonances that arise from the interaction of resonant and amplifying mechanisms; and (3) spontaneous oscillations caused when a resonant current interacts so strongly with an amplifying current that the resting membrane potential becomes destabilized. Only in this last class is the frequency preference of the neuron overtly displayed as a pacemaker oscillation. In the first two classes the frequency preference of the neuron is latent and revealed only in the presence of inputs."

In general terms, the resonance of an electrical system can be defined as a peak in the impedance profile. If an electrical circuit is probed with an input current $I(t)$ of constant amplitude and varying frequency, the corresponding voltage $V(t)$ is deemed to be the output response. Both the amplitude and phase of the voltage response can differ from that of the input current. Taking the Fourier transform of the current $\mathscr{F}I(k)$ and the voltage $\mathscr{F}V(k)$, the impedance is defined to be the ratio $Z(k) = \mathscr{F}V(k)/\mathscr{F}I(k) = |Z(k)|e^{i\theta_Z(k)}$, with the modulus $|Z(k)|$ representing the ratio of the voltage-current amplitudes, and the argument $\theta_Z(k)$ representing their phase-difference. The impedance profile provides the ratio of the amplitudes at each frequency/wave-number $|\mathscr{F}V(k)|/|\mathscr{F}I(k)|$, (see Figure 3).

The resonant property of neurons is a consequence of two mechanisms: "one that attenuates voltage responses to inputs that occur at high frequencies and another that attenuates responses to inputs arriving at low frequencies. The resulting combination of low- and high-pass filtering behaviour effectively creates a notch filter that is capable of rejecting inputs at frequencies outside the pass-band," (ibid., p218).

The low-pass filter property is a consequence of the fact that the outer membrane acts like a capacitor and conductor in parallel. The high-pass filter property is a consequence of transmembrane ionic currents which activate slowly relative to the membrane time constant,[20] and which oppose low-frequency changes

[20]This is the product of the resistance and capacitance, RC.

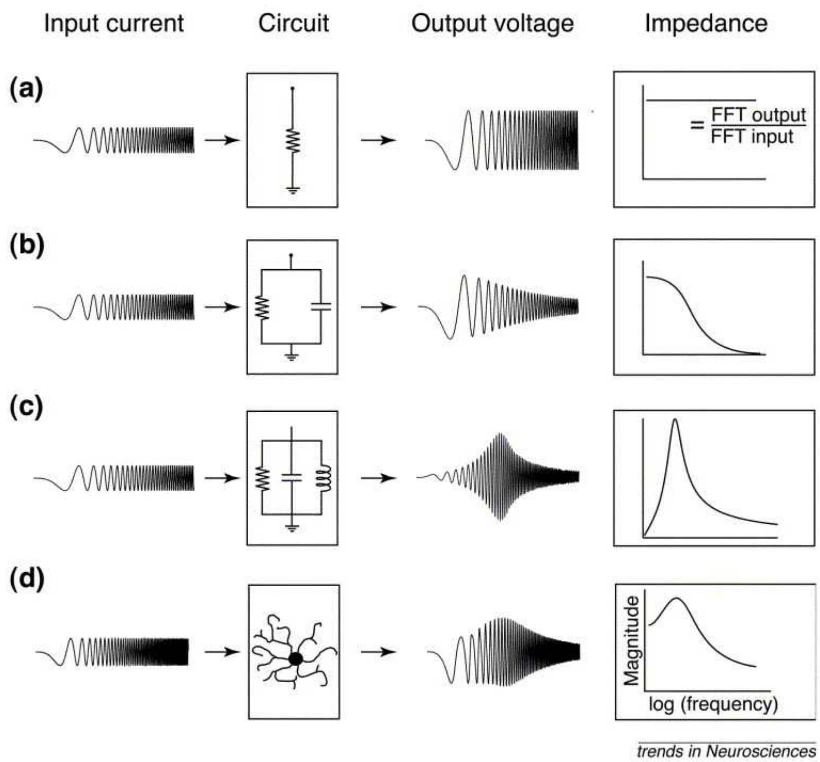

Figure 3: Impedance amplitude as a function of frequency in electrical circuits and neurons. (From Hutcheon and Yarom, 2000).

in membrane voltage.

Hutcheon and Yarom claim that amplification occurs when there is a resonant current, and another ionic current which, in contrast to the former, enhances voltage fluctuations of the membrane, and activates quickly. Amplifying currents of sufficient strength are capable of triggering self-sustained oscillations of the membrane potential.

As an example, Hutcheon and Yarom cite pyramidal neurons in the neocortex, which possess a 5-20Hz amplificatory resonance (the frequency is voltage-dependent), occuring at membrane potentials more positive than -55mV. The oscillation can be eliminated with TEA, a K^+ channel blocker, which switches off the resonant current, and it can be attenuated with TTX, a Na^+ channel blocker which switches off the amplificatory current.

REM sleep is characterised in terms of EEG traces by a so-called 'theta' oscillation at 4-12 Hz. This oscillation is believed to originate from the hippocampus: "Supporting evidence includes that the average magnitude of theta power measured by multisite recordings along the hippocampus-neocortex axis monotonically decreased with distance from the hippocampus and that the distribution of theta power on the neocortical surface reflects the physical layout of the underlying hippocampus," (Yamada and Ueda, 2020).

Yamada and Ueda reason that "isolated hippocampal neurons can exhibit oscillations at the theta frequency band in vitro when it is bathed in acetylcholine or kainate receptor agonist... Therefore it is plausible to assume that the neurons of [the] hippocampus possess an intrinsic ability to generate the theta oscillation. Interestingly, a recent study showed that a majority of hippocampal neurons are self-oscillatory, and the properties of oscillation, including frequency, are affected by environmental ions and cellular Ca^{2+}. This effect occurs without changes in synaptic connectivity or neural circuit, suggesting that the intrinsic neural properties directly affect circuit-level oscillation.

"Together, the body of evidence suggests that the brain oscillations, including hippocampal theta oscillation, originate from intrinsic cellular properties. The intrinsic oscillation resonates and is amplified in neural circuits," (ibid.).

The Reeh-Schlieder theorem entails that there are correlations in the quantized ionic electric field $\widehat{\mathbf{E}}(f)$ of spacelike separated replica brains, so that:

$$\langle \Psi | \widehat{\mathbf{E}}(f) \widehat{\mathbf{E}}(g) \Psi \rangle \neq \langle \Psi | \widehat{\mathbf{E}}(f) \Psi \rangle \langle \Psi | \widehat{\mathbf{E}}(g) \Psi \rangle \ ,$$

where f and g have compact support within spacelike-separated volumes occupied by replica brains. The brains are *structural* replicas in the sense that their network graphs and synaptic weights are identical, even if their firing patterns happen to be different.

Given that the firing patterns of the neuronal network of a brain are determined by the pattern in the ionic electric fields, and given that there are correlations between the ionic electric fields of spacelike separated replica brains, it follows that the firing patterns of one such brain are, in principle, capable of influencing the firing patterns of another.

However, in the real-world of neurobiological systems, weak electric fields have to compete with various sources of noise: "Stochastic fluctuations arise from many biochemical processes, such as aqueous diffusion and protein 'breathing motions', in addition to transmembrane voltage noise. Thus, molecular changes within biochemical pathways are thoroughly randomized, with average values controlled but containing an inherent molecular shot noise uncertainty," (Weaver *et al* 1998).

On the other hand, the sensitivity of neurons is enhanced by a surprising phenomenon called 'stochastic resonance' (SR):

"SR is a nonlinear phenomenon whereby the addition of a random interference ('noise', as it is almost universally called) can enhance the detection of weak stimuli or enhance the information content of a signal (e.g. trains of action potentials or signals generated by a neuronal assembly). An optimal amount of added noise results in the maximum enhancement, whereas further increases in the noise intensity only degrade detectability or information content. The phenomenon does not occur in strictly linear systems, where the addition of noise to either the system or the stimulus only degrades the measures of signal quality. In its simplest manifestation, referred to as 'threshold SR' or 'non-dynamical SR', stochastic resonance results from the concurrence of a threshold, a subthreshold stimulus, and noise," (Moss *et al*, 2004).

The phenomenon is illustrated in Figure 4 for the case of a system with a symmetric double-well potential. The perturbations due to noise alone are capable of causing transitions back and forth between the two local minima. There is a typical time-scale for this. When the weak, sub-threshold periodic signal has a period equal to twice the time-scale of the noise-induced transitions, stochastic resonance occurs. In effect, the noise boosts the weak signal to detectable levels.

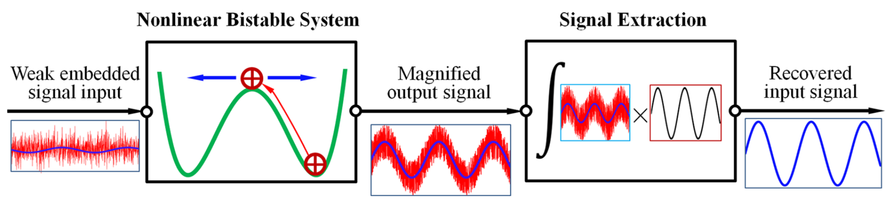

Figure 4: Stochastic resonance in the presence of noise. (From Lin *et al*, 2019).

Including the presence of stochastic resonance, Weaver *et al* obtained a theoretical estimate of the lower limit of the sensitivity of a neuron to an electric field of $\sim 100\mu$V/mm. A subsequent experimental study by Francis *et al* verified that networks of pyramidal cells from the mammalian hippocampus are sensitive to electric fields of this order:

"The mammalian hippocampus has several unique features that render it particularly sensitive to electric fields. Cellular packing is so dense that it can display epileptiform events even in the absence of functioning chemical synapses, a condition under which electric fields likely play a significant role in ensemble activity. Hippocampal pyramidal cells have somata asymmetrically placed with respect to their dendritic trees, and the sensitivity of a neuron to firing rate modulation from an imposed electric field is related to the amount of positional asymmetry of the soma with respect to the dendritic tree. In addition, the individual pyramidal cells are aligned such that adjacent cells have parallel dendrites, which favor interaction with fields aligned along the collective somatodendritic axes," (Francis *et al*, 2003).

In particular, Francis *et al* discovered that "neuronal networks respond to fields more sensitively than single neurons. Whether this is a manifestation of simply increasing the numbers of neuronal detectors or is from array-enhanced signal detection caused by coupling remains to be determined," (ibid.).[21]

[21] The presence of ionic and macromolecular noise places a severe constraint on the hypothesis that distant replica brains are capable of influencing each other. Whilst replica brains will, by definition, share *exactly* the same power spectrum of noise, REM dreams can only be caused by correlations with replica brains if the replica brains are permitted to exercise contingently different firing patterns. *Ex hypothesi*, each neuron in a replica brain can be mapped to a particular neuron in one's own brain, but different firing patterns entail that the

6 Conclusions

In summary, the reasoning in this paper runs as follows:

1. The most fundamental theory currently available to us is quantum field theory, the only such theory which includes the axioms of both quantum theory and relativity. This theory predicts that there exist correlations between apparently distant spacelike-separated regions of space, irrespective of how far apart those regions appear to be. The strength of these Reeh-Schlieder correlations, however, decays exponentially with distance, suggesting they could only be detectable with the most sensitive systems imaginable.

2. Our best available cosmological theories suggest that space is randomly infinite. Moreover, our understanding of statistical mechanics suggests that under randomly infinite conditions, all possible states of all possible bounded physical systems will be realised an infinite number of times. This includes all possible states of all possible brains. Some of these replica brains have developed as part of biological species which have evolved by natural selection; others have formed spontaneously, by chance. The latter are referred to as 'Boltzmann brains'.

3. The topological connectivity of space, and hence its structure as a metric space, remains an open question. Under the 'ER=EPR' conjecture, for example, entangled systems are connected by non-traversable wormholes. Given the ubiquity of Reeh-Schlieder entanglement, this would entail that space-time is multiply connected by a dense network of wormholes, bringing the apparently distant parts of entangled systems into arbitrarily close proximity. This mitigates the exponential decay of Reeh-Schlieder entanglement with distance.

4. The human brain is the most complex physical system known to us. It exists in a state close-to-criticality, which entails that it possesses a high level of internal correlation, and is extremely sensitive to small perturbations. The human brain cycles through different states of 'vigilance' during the course of each day, during which its proximity to a critical state varies. In the state known as REM sleep, the human brain drops to a minimal level of internal correlation, whilst still residing in a state close-to-criticality. During REM sleep, humans experience vivid dreams. These dreams includes experiences of other personal histories, as well as experiences with no temporal or spatial coherence.

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pattern of ionic and macromolecular noise in those corresponding neurons will be different.

Nevertheless, if space-time possesses a multiply-connected structure, such as that entailed by the concept of quantum space-time foam, or the 'ER=EPR' conjecture, then replica brains can exist in arbitrary proximity to each other, offsetting the difficulties posed by the exponential decay in the strength of the Reeh-Schlieder correlations. Thus, it is hypothesized that the threshold sensitivity of $\sim 100\mu\text{V/mm}$ is sufficient for a mammalian brain in a sub-critical REM sleep state to resonantly amplify the Reeh-Schlieder correlations with replica brains.

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