The hypothesis of "hidden variables": a unifying principle, after all?

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Abstract. In the debate whether local 'hidden variables' could exist underneath quantum probabilities, the 'no hidden-variables' position is at present favored. In this article I question this consensus, by exhibiting the explanatory power of the hidden-variables hypothesis. I argue that this hypothesis can answer three foundational questions, whereas the opposing thesis ('no hidden variables') remains entirely silent for them. These questions are: 1) How to interpret probabilistic correlation ? (first analyzed by Reichenbach); 2) How to interpret the Central Limit Theorem ?; and 3) Are there degrees of freedom that could unify quantum field theories and general relativity, and if so, can we (at least qualitatively) specify them ? Question 3) arises in the context of Bell's theorem. It appears that only the hidden-variables hypothesis can provide coherent answers to these questions; answers which can be mathematically proven in the deterministic case. Finally, it is noted that the hidden-variables assumption appears to be implicit in the work of string theorists. We believe that the *onus probandi* is now on those who reject the hidden-variables hypothesis, since there are no questions or problems to which the 'no hidden-variables' hypothesis could answer, and not its opponent.

1. Introduction.

The question whether the universe is ultimately deterministic or indeterministic (probabilistic) concerns one of the oldest debates in the philosophy of physics. The atomists Democritus and Leucippus were determinists, while Aristotle was arguably one of the first indeterminists, believing in irreducible chance or hazard. Modern quantum mechanics, a probabilistic theory, has convinced many that indeterminism wins; but a more careful analysis, based notably on the interpretation of the theorems of Gleason, Conway-Kochen and Bell, shows that the debate is actually undecided (cf. e.g. Wuethrich 2011, Esfeld 2015). This may be a shared belief in the philosophy of physics community; but it surely is unpopular outside that community. A broader question can be condensed in following slogan: Can hidden variables exist underneath quantum probabilities? Can quantum probabilities be reduced to, or 'explained by', deeper-lying variables? (A precise mathematical formulation of this question will be given in Section 2.) Assuming the existence of such hidden variables is a more general hypothesis than determinism: determinism corresponds to the extremal case where the variables are deterministic, a mathematically well-defined special case of more general probabilistic variables (cf. Section 2). Now, the generally received wisdom is again that such hidden variables can*not* exist, at least if one only considers *local* hidden variables – which is what I will do throughout this article. Indeed, nonlocal variables involve, by definition, superluminal interactions, so interactions that are not Lorentz-invariant and in overt contradiction with relativity theory: this is the explicit conclusion of Bell's seminal work of 1964. Thus, I will only inquire here about the possibility of local degrees of freedom (variables). Note that the hidden variables which could, under the assumption that the hidden-variables hypothesis (HV-hypothesis) is correct, explain quantum events, could be interpreted as the *causes* of these events[1]. Other philosophers, following notably Hume and Russell, believe one should jettison the notion of cause / causality. In order to avoid controversy I will keep the discussion essentially in terms of 'variables' rather than 'causes'; even if I use causal language, it can always be translated in terms of variables, representing physical properties.

The question of the possibility of hidden variables (HV) underlying quantum probabilities is likely most clearly investigated through Bell's theorem (Bell 1964). Bell's article starts as follows: "The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality" (1964, p. 195). Now, according to Bell's theorem local HV theories contradict quantum mechanics in certain experiments. Since these experiments have vindicated the quantum prediction countless times (just two of the latest examples are Handsteiner et al. (2017) and Rauch et al. (2018)), most scholars believe now that the prospects for local hidden-variable theories, and thus for the HV-hypothesis, are dim. In other words, according to the standard view quantum probabilities are irreducible in general; they cannot be understood as emerging from a deeper-lying HV level.

My goal here is to argue that not only is the HV-hypothesis not refuted by Bell's theorem and the Bell experiments (in line, notably, with Wuethrich 2011), but this hypothesis, compared to

[1] In the literature, notably on Bell's theorem, the hidden variables λ are indeed often called 'causes', deterministic causes if x depends functionally on λ (as in (3b) below), probabilistic causes if the probability of x conditionally depends on λ (as in (1) and (2) below). Defining causes in this way has been done by several philosophers of science investigating causation (cf. Suppes 1970, Hausman and Woodward 2004, Hitchcock 2018).

its competitor ('no HV' or irreducibility), has the greater explicatory power. Specifically, I will show in Section 3 that the HV-hypothesis can provide coherent answers to three questions of the philosophy of physics, whereas irreducibility remains entirely silent for them. These questions are: 1) How to interpret probabilistic correlation ? (a problem raised by Kolmogorov and first analyzed by Reichenbach); 2) How to interpret the Central Limit Theorem?; and 3) Are there variables that could unify quantum field theories and general relativity, and if so, can we say at least something about them ? I will specify questions 1) and 2) so that they apply to both the classical and the quantum domain; question 3) arises in the context of Bell's theorem. Interestingly, it appears that in particular *deterministic* HV allow to *mathematically prove* answers to these three questions. Whether this fact privileges deterministic[2] HVs above stochastic ones, will be left here as an open question.

The article is organized as follows. In Section 2 I will extract from the literature a straightforward definition of 'deterministic' and 'probabilistic / stochastic' variables, needed in the remainder of the article. This will allow to define 'reducibility' of (quantum) variables via (yet unknown) additional variables – the so-called hidden variables. I will need these concepts throughout the article. Next, it appears that some confusion exists regarding the concept of 'objective (and subjective) probability': this concept can have two quite different meanings. To illustrate my argument, it will be helpful to look in detail at a realistic case, namely the automated tossing of a large die: an experiment that can equally well be described as a deterministic or a probabilistic system, depending on the epistemic status of the experimenter (in line with e.g. Suppes 1993). Hence it will appear useful to introduce the notion of "relativity or subjectivity of (in)deterministic ascription". This simple thought experiment does not claim to provide new results, but will allow to illustrate and clarify the notions of reducibility and determinism, and thus to throw light on the three foundational questions mentioned above. Answering the three mentioned questions is the subject of Section 3, containing the main results of this work. Section 4 will conclude, after briefly elaborating on the status of 'unifying principles' in the philosophy of physics. The philosophy of metanomological principles appears to be a quite virgin domain, but we can relate to (Bunge 2009, Ch. 6, §6.1, 6.8).

[2] For a detailed treatment of physical determinism, see notably Earman (1986), (2008), (2009); and Hoefer (2016).

2. Definitions: HV-hypothesis, reducibility, determinism, objectivity / subjectivity of probability.

Let us start by defining concepts. The mathematical representation of deterministic and probabilistic variables we will use here (definitions (1) - (3)) stems from the quantum foundations literature, specifically on Bell's theorem (see e.g. Bell (1964), Hall (2010), (2011), Friedman et al. (2019)). I will use the compact notations of these works; for a more rigorous notation, cf. Appendix. Before defining the HV-thesis or reducibility of (quantum) probability as a general principle, let us first define them *relative to some quantum probability*[3] $P(x|a)$, where 'x' is a quantum variable (say spin) and 'a' a measurement or analyzer variable (say the angle of a polarizer). In the following I will consider the case of discrete hidden variables; the continuous case is a straightforward generalisation replacing sums by integrals as usual.

Definition 1 (HV-hypothesis for x; reducibility of x)

$P(x|a)$ (or x) is reducible IFF $P(x|a)$ can be expanded (for all values of x) as a sum of conditional probabilities, where the sum runs over the values of some (hidden) variable λ, that is, IFF

$$\exists\ \lambda : \quad P(x|a) = \sum_\lambda P(x|a,\lambda).P(\lambda|a). \qquad (1)$$

In words, a variable λ exists that allows to expand $P(x|a)$ as given. In (1) all variables (x, λ and a) can be multi-component (n-tuples); these variables all represent physical properties here. Note that the expansion in (1) is *mathematically* allowed by standard rules of probability theory (the rule of total probability) if λ runs over all its possible (discrete) values; (1) is *physically* meaningful if a physical property λ exists that is correlated with x. $P(x|a)$ is then *irreducible* IFF $P(x|a)$ cannot be expanded as a sum of conditional probabilities:

Definition 2 (irreducibility of x, "no HV for x")

A probability $P(x|a)$ (or x) is irreducible IFF $P(x|a)$ is not reducible, that is, IFF

$$\nexists\ \lambda : \quad P(x|a) = \sum_\lambda P(x|a,\lambda).P(\lambda|a). \qquad (2)$$

[3] A more precise notation for $P(x|a)$ would be, e.g.: $P(X = x|A = a)$. But since no confusion is possible we will stick to our simpler notation and continue to use the symbol 'x' also for the *variable*, as is practical in a physics context. Instead of considering the conditional probability $P(x|a)$, we could equally well consider the unconditional $P(x)$, but introducing the variables 'a' now allows us to connect fluidly with *Problem 3* below. Also, as usual all variables in the following range over an interval defined by the system considered.

So in (1) λ represents hypothetical variables on which the quantum property x would depend; but the experimental confirmation of Bell's theorem has convinced most researchers that such λ and probabilities as $P(x|a,\lambda)$ do not exist.

Determinism corresponds now simply to the extremal case of reducibility (1), when the probabilities $P(x|a,\lambda)$ have values 0 or 1 for all values of x and λ; i.e. when the values of x are completely determined by the values of a and λ, so that a suitable function *f* exists for which x = *f*(a,λ). This leads to following definition (δ is the Kronecker-delta):

Definition 3 (deterministic variable x)

A variable x is deterministic IFF

$$\exists\ (\lambda, \textit{function } f):\ P(x|a) = \textstyle\sum_\lambda P(x|a,\lambda).P(\lambda|a),\ \textit{where } P(x|a,\lambda) = \delta_{x,f(a,\lambda)}, \tag{3a}$$

or, equivalently, IFF

$$\exists\ (\lambda, \textit{function } f):\ x = f(a,\lambda). \tag{3b}$$

Equating $P(x|a,\lambda) = \delta_{x,f(a,\lambda)}$ in (3a) captures well our intuitions about determinism: if we know or specify enough variables ('information' if one prefers), we know whether x occurs or not with certainty. Representation (3a,b) is widely used and uncontroversial for defining deterministic variables (compare e.g. Eq. (3b) to Eq. (1) in Bell's original paper (1964); and compare (3a,b) to Hall (2010) Eq. (6), Hall (2011) Eq. (3) and text above Eq. (22), or Friedman et al. (2019) Eq. (3)). I will further illustrate the utility of these definitions by examples.

We have thus defined reducibility (the HV-hypothesis) relative to some stochastic property / variable x. As a universal principle it can be defined as stating that *all* physical properties / variables / events are reducible in the sense (1). Determinism as a principle is then again a special case of the HV / reducibility-principle. In general, reducibility can loosely be read as stating that (quantum) variables can be "reduced to" or "explained by" additional variables, in a stochastic or deterministic way, depending on whether the $P(x|a,\lambda)$ are trivial or not. As said, in the Bell literature these additional variables λ are often called 'causes'; and it indeed seems that reducibility can be the key ingredient of, or even equated with, the hypothesis of causality (in line with the definitions of cause given by e.g. Suppes 1970, Hausman and Woodward 2004, Hitchcock 2018; see also footnote 1). But I will not explore this assumption further here. Note also that determinism, according to which all (quantum) variables would be deterministic under the surface, is still compatible with a non-trivial probabilistic distribution $P(\lambda|a)$ over the hidden variables λ, as is best seen in (3a)[4]. So in a deterministic universe, in which (quantum) variables x are deterministic functions of hidden variables, there still could be a non-trivial distribution over the (initial values of the) hidden variables. This agrees with the discussion in (Wuethrich 2011, p. 365), where the author distinguishes two roles probabilities can play in dynamical theories: 1) that of describing the dynamical evolution given a certain initial state of the physical system at stake; and 2) that of codifying a distribution over initial or boundary conditions ($P(\lambda|a)$ is an example of case 2)). Under the hardest form of determinism, this distribution $P(\lambda|a)$, taken over the 'initial values of the universe' (λ_0), would also be given by a δ-symbol (Kronecker or Dirac): $P(\lambda|a) = \delta(\lambda - \lambda_0(a))$. Then all variables in the n-tuple λ have one fixed value: Laplacianism in its purest form.

But as stated above, Bell's theorem, combined with the results of the Bell experiments, constitute a convincing physics-based argument for *ir*reducibility in the sense (2), so against the HV-assumption. Irreducibility could also be equated with the slogan "no hidden variables", neither deterministic nor probabilistic. (Of course, Bell's no-go argument only goes through if the universe is local. But we excluded the option of nonlocal HV from the start: they are in contradiction with relativity theory. Still, for completeness, it is useful to remember that the main argument for irreducibility, Bell's theorem, only holds if there really are no nonlocal interactions in this universe.)

The next conceptual clarification we need concerns the notions of 'objectivity' and 'subjectivity' of probability, which are used in the present context in two different ways. Some readers may be sympathetic to the idea that, in a sense, determinism is the intuitively most cogent form of reducibility. Notably, they make invoke an argument from homogeneity or simplicity: a universe with only one category (only deterministic events and systems) corresponds to the simplest, most homogeneous, most unified worldview – moreover the one that seems to take "the most information" or "the most adequate information" into account, at least if we can generalize from the example we will study below. This intuition lies at the basis of the historic "determinism versus indeterminism" debate. In the remainder of this Section I will comment qualitatively on this intuition (mathematical arguments are left to Section 3), while defining important concepts.

[4] From (3a) follows: $P(x|a) = \sum_{\lambda_x} P(\lambda_x|a)$, where λ_x is defined by $f(a,\lambda_x) = x$.

It is well known that probability has a remarkably complex epistemic nature; its interpretation is an intensely debated topic in philosophy of science (for reference works, see e.g. von Mises 1928/1981, 1964, Fine 1973, von Plato 1994, Gillies 2000; for recent work on the frequency interpretation, cf. Vervoort 2010, 2012). Among the many interpretations of probability on the market, such as the classical interpretation of Laplace and the propensity and frequency interpretations, there is also the *subjective* or Bayesian interpretation, which enjoys a relative popularity in quantum mechanics (Caves et al. 2002, 2007 and Bub 2007). On the subjective interpretation (of at least some schools), probability is a measure of the strength of belief, which an observer subjectively attributes to a random event (or hypothesis, depending on the specific interpretation). According to this view different observers may attribute different probabilities to the same event. *Objective* interpretations stipulate that the probability *of a well-defined event* is an 'absolute' measure, *the same for all observers* (von Mises 1928/1981, 1964, Fine 1973, von Plato 1994, Gillies 2000, Vervoort 2010, 2012) – an interpretation that resonates well with the objective character of physics. Note that in this broad philosophy of science context the question whether such objective interpretations of probability allow for a deterministic reduction is eluded – it is rarely if ever considered. So this is the first sense in which the objectivity of probability is discussed. In the philosophy of physics literature there is however a second, widely used sense of this notion: it associates subjective probability with the hypothesis of determinism, and objective probability with indeterminism (see e.g. Wuethrich 2011, p. 365). The underlying thought is that 'objective' (quantum) probability, as *irreducible (and non-trivial) probability in the sense defined above*, is the only possibility we are left with in a world that is at its most fundamental (say quantum) level indeterministic; while conceiving probability 'subjectively', as an expression of our ignorance of underlying deterministic processes, is appropriate in a deterministic world. But it is important to realize that even in the latter 'subjective' or rather 'reducible' interpretation of probability, probabilities can still be considered as objective measures in the sense used in the philosophy of probability – for instance as relative frequencies. As we will illustrate in a moment, even in a deterministic universe one can still define probabilities as frequencies that are measured and attributed the same value by agents with a radically different epistemic status. And even on an 'objective' or rather 'irreducible' reading of quantum probabilities, one might consider probability values as subjective degrees of belief, as in (Caves et al. 2002, 2007 and Bub 2007). As will become clear in the following, in the context of our present investigation it is more precise to consider not the probability *value* itself as subjective, but rather the "ascription of a deterministic or indeterministic character" *to the system or event under scrutiny*. It is this attribution that is subjective, in the sense that it depends on the observer's knowledge (cf. below). Hence also the added value of the notion of (ir)reducibility.

But sure enough, since Laplace, determinists typically understand probabilities as mathematical tools, capable of describing certain (ensembles of) objects in nature – tools that we only need when we do not have knowledge of the full set of variables and the full dynamics determining the behaviour of these objects, so when we cannot predict the full behaviour of these objects (as individual objects). But Laplace's demon knows everything so does not need probabilities: he predicts everything deterministically (he could still define and measure probabilities, though, see further). Hence on this view, the attribution of a deterministic or probabilistic character to an event / system depends on the knowledge of the subject, the observer. Let us call this the *relativity of (in)deterministic ascription*. If this Laplacian view is correct, determinism could be said to be more fundamental in that it corresponds to the enviable "truth before God's eye". In any case, according to Laplacian determinism probabilities would be reducible; or to put it still differently they would 'emerge' from a deterministic substratum – they would pop up and be useful in our human theories and experiments.

Now, there are countless examples where the relativity of (in)deterministic ascription is blatant. Consider following thought experiment, a mechanized version of a die toss, consisting of say N runs (tosses). The test involves a large (say 20x20x20 cm^3) cube made of soft plastic, positioned at the beginning of each toss with high precision in the middle of a drumhead. Underneath the drumhead a metallic pin moves quickly upward, imparting to the cube a vertical momentum so that it tosses around maybe once or twice; for each toss the mechanism can be started by a button. The outcome or event 'e' is the number $\in \{1,2,\ldots,6\}$ shown on the upper face of the cube, 'measured' after it lands on a table. Suppose that Alice wishes to ascertain whether this experiment is probabilistic and, if so, what are the values of the probabilities P(e); also suppose that Alice has no means to determine the position and force exerted by the pin (e.g., the pin-mechanism is screened off). The best she can do is to make a table and note, for a long series of tosses, the outcome e. In a trivial case she would find always the same result, say e = 6 (she is a valiant researcher and goes to N = 1000). In this case she would term the experiment or the system deterministic: she feels she can predict what would be the result of the (N+1)-th toss; so she assumes P(6) = 1 and P(1) = P(2) =… = 0. But consider now the case where each outcome looks perfectly random, unpredictable to her. Then she has to determine, based on her table, the frequencies $P_N(e) = \#_N(e) / N$, where $\#_N(e)$ is the number of times the outcome is e in N trials. Suppose she finds that for N = 1000, $P_N(e)$ comes close to 1/6 for all e, and moreover that, when comparing the $P_N(e)$ for N = 100, 200, 300,…1000, the $P_N(e)$ come (globally) closer and closer to 1/6. Realizing that this "frequency stabilization" is, besides the unpredictability of individual outcomes, the hallmark of a probabilistic system (von Mises 1928/1981, 1964; Vervoort 2010, 2012), she rather confidently concludes that the system is probabilistic and that the six probabilities P(e) are = 1/6 (to good approximation). She has done her job as a physicist.

Now, this experiment could of course be deterministic in disguise: an engineer, Bob, could have constructed the die and the system that commands the pin in such a manner that there is a perfect functional relation between the position coordinates of the pin impact and the outcome 'e'. E.g., the pin hits M = 10,000 positions on the lower surface of the die, say distributed over a square grid; once the first M grid positions have been probed the next runs will repeat the same sequence, so M is the periodicity of the impact points. If the system is well-calibrated and the die movement not too chaotic (the die is of soft material and rotates not more than once or twice), Bob can establish such a functional relation – a table – between impact position and outcome, while at the same time ensuring that the outcomes 1, 2, …, 6 arise in a random-looking sequence leading[5] to P(e) = 1/6 for all e, to excellent approximation. This example illustrates definitions (1) and especially (3): here λ is the pin position and x (= e) is the outcome of the die toss. P(x) (= P(e)) is well-defined and reduces to 0 or 1 if we specify λ ($P(x|\lambda)$ = 0 or 1; x = $f(\lambda)$ for some function *f*). (Note further that, in typical parlance, the pin position assuming a certain coordinate λ *causes* the die to show the result x. And if Bob uses enough engineering ingenuity he could even devise a system where the λ are *probabilistic* causes.)

So this is a system that the engineer Bob and any 'informed' experimenter will identify as a deterministic system – *in principle* each individual toss can be predicted in advance, more precisely there exists (before Bob's and God's eye) a functional relationship between pin positions and outcome. But Alice and any uninformed experimenter will deem the system probabilistic (recall that in this well-defined experiment the pin-mechanism is screened-off, Alice cannot peek

[5] Of course, this assumes, among other things, that there are about M/6 grid positions that lead to each of the 6 outcomes e.

behind the screens and do further experiments). So, this is an example of relativity or subjectivity of (in)deterministic ascription *regarding physical systems* – relativity with respect to the knowledge state of the agent inquiring about the *system*. Importantly, notice that this does not mean that probability *values* are subjective: both the informed Bob and the uninformed Alice will measure, when asked, the same probabilities P(e) *if they do the same experiment*[6]. Bob can put his table aside and measure and compute the same ratios $P_N(e) = \#_N(e) / N$ that Alice obtains; he should find the same results. Quite generally, in physics probabilities are measured as frequencies, even if the system is known or assumed to be deterministic deep down (see other examples below).

Note also that there is, *in principle but not always in practice*, a way for Alice to discover the deterministic nature of the system, by noting outcomes and by making N runs with N >> M, say N = n.M (n ≥ 2), with M the periodicity of the impact points. If she does so, she will notice that the outcomes repeat themselves with a periodicity M. She can then predict the individual outcomes with a likelihood that is proportional to n. In principle she can reach *quasi*-certainty about future outcomes, and that is all a physicist can ask for. But this procedure does not work if M, the number of hidden variable values, becomes too large to be practically accessed. In that case Alice, left to her devices, has no practical experimental means to discover the deterministic nature of the system. Interestingly, this variant of the experiment (M very large), is an example of experimental situations where the abundance of hidden information, the "too large number of (values of) hidden variables" prevents discovery of the deterministic nature of a system. This is essentially the same rationale used by proponents of determinism to address *Problem 3* below.

Physical examples of probabilities emerging from an underlying deterministic dynamics are ubiquitous in nature. Examples can be found e.g. in fluid mechanical systems. Fluid mechanics is governed by deterministic equations, notably the Navier-Stokes equation; yet there exist an enormous variety of probabilistic features in fluids, gases, eddies, diffusive systems etc. Such properties seem to be all examples of variables of which the random nature is induced by the distribution of the initial values $P(\lambda|a)$, as discussed in Wuethrich (2011) (recall the above-mentioned second role probabilities can play). Another example of well-defined and non-trivial

[6] Hence the importance of realizing that (physical) probability values refer to *well-defined experiments*, as many philosophers have emphasised, notably von Mises, Popper and more recently van Fraassen (1980). Popper famously interprets probability as the propensity that a certain experimental set-up has to generate certain frequencies. Van Fraassen presents in his (1980, pp. 190 – 194) a logical analysis of how to link in a rigorously precise way experiments to probabilities. This idea is discussed in more detail in (Vervoort 2010, 2012).

probabilities in deterministic systems is the following. A typical numerical method in statistical mechanics is the Monte-Carlo technique, quite faithfully reproducing probabilistic features of a wide variety of real-world stochastic systems. Now, this numerical simulation method always uses, in practice, a *pseudorandom* number generator, which allows to simulate physical randomness of some properties of the system considered. Such a generator is in reality deterministic, in that the generated numbers look randomly distributed but are actually generated by a complex function – deterministic by definition. Hence a dynamics that is fully deterministic can reproduce an enormous variety of systems of stochastic mechanics. Thus all these probabilistic systems can be understood to be deterministic under the surface[7].

3. Three Questions.

The above examples indicate that the "better informed observer" perceives the deterministic nature of phenomena that his less informed counterpart deems probabilistic. In view of such examples, it is only natural to inquire whether this deterministic reducibility as in (3) could hold for *all* probabilities; or, more generally, whether generalized reducibility as in (1) could hold for all probabilities. This is also the question that sparked Bell's inquiry (Bell was initially interested in deterministic reduction, but he and others soon realized that (1) is the more general case). I will now show that the HV-hypothesis or reducibility (1) allows to explain three fundamental problems for which irreducibility remains powerless. Mathematical proofs can be given for deterministic reduction.

Problem 1. *How to interpret statistical correlation (in the classical and quantum domain)? Or: How to interpret statistical correlation in a unified way in the classical and quantum domain?*

Probability theory can be interpreted as a purely mathematical theory, but also as a physical theory – a theory describing random physical events (cf. von Mises 1928/1981, 1964, who develops such a physical interpretation in greatest detail). In a sense, it seems that probability theory could even be considered the most general of all physical theories, since it governs countless different

[7] In this context, it would be interesting to look in detail at the question, launched notably by Suppes (1993), whether deterministic and indeterministic descriptions are observationally equivalent for micro-physical systems in general (it is clear that in the large-die experiment, they are not). However this would lead us too far: we refer to the discussion in (Wuethrich 2011, p. 367-368); we agree with its conclusion.

types of physical systems, from almost all branches of physics. Quantum mechanics is a statistical discipline and complies with probability theory; but also general relativity is, as a deterministic theory, a special case of a probabilistic theory (with probabilities having values 0 or 1). One therefore suspects that the problems of the foundations of probability theory are, or can be, also problems of the foundations of physics. Besides von Mises, several members of the prolific Russian school of probability (Kolmogorov, Chebyshev, Lyapunov, Khinchin, Markov, Gnedenko, Bernstein etc.) were interested in foundational aspects of probability theory (in this respect it is delightful to read e.g. Kolomogorov 1933/1956, Gnedenko 1967). For my present concerns the first relevant problem of the interpretation of probability is summarized in following remarkable quote by Kolmogorov in his reference work of 1933, *Foundations of the Theory of Probability* (1933/1956, p. 9):

> "In consequence, one of the most important problems in the philosophy of the natural sciences is – in addition to the well-known one regarding the essence of the concept of probability itself – to make precise the premises which would make it possible to regard any given events as independent. This question, however, is beyond the scope of this book."

Kolmogorov asks here whether there exist general conditions allowing one to know in advance (without doing a statistical test) whether two events or variables x and y are *independent* (de-correlated) or not, in symbols whether their joint probability satisfies following equation:

$$P(x, y) = P(x)P(y|x) \stackrel{?}{=} P(x)P(y). \tag{4}$$

An equality sign corresponds, by definition, to probabilistic independence. When dealing with a probabilistic question related, for instance, to the tossing of two dice, or in general to the joint occurrence of two events, one has to assume or guess whether these events / variables / systems are independent or not. Generally speaking, in real-world situations there is no rule; the only way to be sure is to experimentally verify Eq. (4). But even if no rule exists, one often 'intuits' correctly whether events and variables are dependent or not. (The present author thinks this has a touch of magic to it; or more prosaically, that causal intuitions are remarkably strongly developed in humans.) E.g., two die throws can under normal circumstances safely be treated as independent. The interesting question is: *on what basis do we intuit this* ? Surely by our intuitive understanding, our 'feeling', that these events are not *causally connected*. The point is this: while probability theory remains silent regarding the origin of stochastic (in)dependence – this is Kolmogorov's complaint –, determinism and more generally the HV-hypothesis do offer an explanation, usually attributed to Reichenbach (1956). According to it, probabilistic dependence has to do with an underlying causal stratum, with causes connecting the dependent variables. Explicitly, two events are independent if one event is not causally determining the other one, and if there is no common cause determining the two events. This analysis is sometimes termed the 'qualitative part of Reichenbach's principle' (Cavalcanti and Lal 2014, Allen et al. 2017). The quantitative part of the principle, or Reichenbach's principle in the narrow sense, states: if the correlation between x and y is exclusively due to a common cause (there is no direct causation), and if z is a complete common cause for x and y, i.e., z is the set of all variables acting as common causes, then x and y must be conditionally independent given z (Allen et al. 2017, Hitchcock and Rédei 2020). Mathematically, under the condition stated:

$$\text{If} \quad P(x,y) \neq P(x)P(y) \quad \text{then} \quad \exists\ z: \quad P(x,y|z) = P(x|z)\,P(y|z), \tag{5a}$$

where z is a complete common cause. (Recall that x and y are physical properties for us, so variables ranging over value domains.)

If (5a) would hold for all physically correlated properties, then correlation (the left-hand side of (5a)) would definitely be explained as (causal) reducibility, in other words by the HV-hypothesis: the right-hand side expresses that x and y are reducible – cf. definition (1) (z takes the role of λ; if a variable z exists so that $P(x|z)$ is defined, then (1) holds by elementary rules of probability calculus). Reichenbach's principle (5a) can also be understood as follows: correlation only exists if one does not specify enough common causes (z) – or in experiments: if one does not fix enough experimental properties z. If one specifies enough hidden reducing variables z (and conditionalizes on them), correlation disappears. On the contrary, the no-HV hypothesis does not offer an explanation for correlation.

As is well-known, Reichenbach's principle has been verified countless times in classical macroscopic systems (cf. Spirtes et al. 2001, Pearl 2009), but is generally believed not to hold in quantum systems, precisely because of the no-go theorems (Hitchcock and Rédei 2020). We will however be able to question this assumption after having treated *Problem 3*. Now, what is interesting for our argument, is that under the assumption that x and y are deterministic variables in the sense (3), one can *prove* Reichenbach's principle (5a), more precisely that the qualitative part of Reichenbach's principle implies the quantitative part. This is done in (Allen et al. 2017), so we will not repeat the proof here[8]. We prove a related result in Theorem 1 below. A corollary of its proof will turn out to be useful for *Problem 3*.

Theorem 1. If x and y are deterministic variables (if P(x) and P(y) can be deterministically reduced as in (3)), then their joint probability satisfies:

$$\exists \ z: \quad P(x,y|z) = P(x|z)\,P(y|z), \tag{5b}$$

where z is the set (n-tuple) of variables that make P(x) and P(y) deterministic.

Proof: If x and y are deterministic variables then also (x, y) is, and we have, according to (3a):

$\exists$ (λ_1, function f_1) : $P(x) = \sum_{\lambda_1} P(x|\lambda_1).P(\lambda_1)$, where $P(x|\lambda_1) = \delta_{x;f_1(\lambda_1)}$.

$\exists$ (λ_2, function f_2) : $P(y) = \sum_{\lambda_2} P(y|\lambda_2).P(\lambda_2)$, where $P(y|\lambda_2) = \delta_{y;f_2(\lambda_2)}$.

$\exists$ (λ_1,λ_2, functions f_1, f_2) : $P(x,y) = \sum_{\lambda_1\lambda_2} P(x,y|\lambda_1,\lambda_2).P(\lambda_1,\lambda_2)$, where

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))}.$$

Then we have:

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))} = \delta_{x;f_1(\lambda_1)}\delta_{y;f_2(\lambda_2)} = P(x|\lambda_1)\,P(y|\lambda_2).$$

If x is a deterministic function (f_1) of λ_1, then it is also trivially a function (f_1') of (λ_1, λ_2). Formally, we write: $f_1'(\lambda_1,\lambda_2) = f_1(\lambda_1)$, so that:

$$P(x,y|\lambda_1,\lambda_2) = \delta_{(x,y);(f_1(\lambda_1),f_2(\lambda_2))} = \delta_{(x,y);(f_1'(\lambda_1,\lambda_2),f_2'(\lambda_1,\lambda_2))} = \delta_{x;f_1'(\lambda_1,\lambda_2)}\delta_{y;f_2'(\lambda_1,\lambda_2)}$$
$$= P(x|\lambda_1,\lambda_2)\,P(y|\lambda_1,\lambda_2).$$

Thus we obtain (5b), with z = (λ_1,λ_2) the set of variables that deterministically reduces x and y.

□

This also proves as a useful corollary that, if λ_1 is a set of variables that deterministically reduces x, one has that $P(x|\lambda_1,\lambda_2) = P(x|\lambda_1)$, $\forall\ \lambda_2$: x is independent of all non-causally related variables. In short, Theorem 1 proves that in a deterministic world, any joint probability can be factorized by the full set of causes. (In the case there are only common causes, this is a weaker

[8] Allen et al. (2017) investigate whether Reichenbach's principle can be generalized for quantum systems, using standard quantum tools and remaining within quantum theory. This is an interesting matter (even if yet speculative), but a very different endeavor from ours: we investigate whether Reichenbach's principle, even for quantum correlations, can be understood as emerging from a more fundamental level than the quantum one: cf. conclusions further.

version of the theorem that proves (5a) in the deterministic case, since that theorem specifies that z are only the *common* causes.)

It is important to note that there is a body of literature that corroborates the view that determinism explains (classical) correlation, and, to start with, (classical) probability. The assumption that determinism underlies probabilities is actually a ubiquitous guidance principle in the statistical theory of causal modeling (Pearl (2009)). On this view, probabilities as $P(x|y)$ can ultimately be reduced to deterministic functions from (y, λ) to x, where λ are unobserved variables; and (y, λ) are termed the causes of x (Pearl 2009, p. 26; note the coherence with the conclusions we drew from the artificial die tossing experiment of the previous Section). Pearl states:

> "In this book, we shall express preference toward Laplace's quasi-deterministic conception of causality and will use it, often contrasted with the stochastic conception, to define and analyze most of the causal entities that we study. This preference is based on three considerations. First, the Laplacian conception is more general. Every stochastic model can be emulated by many functional relationships (with stochastic inputs), but not the other way around; functional relationships can only be approximated, as a limiting case, using stochastic models. Second, the Laplacian conception is more in tune with human intuition. […] Finally, certain concepts that are ubiquitous in human discourse can be defined only in the Laplacian framework. We shall see, for example, that such simple concepts as 'the probability that event B occurred because of event A' and 'the probability that event B would have been different if it were not for event A' cannot be defined in terms of purely stochastic models (Pearl 2009, p. 26)."

Recall that we do not need to invoke determinism to explain correlation: the assumption of general hidden variables, deterministic or stochastic, is enough: see the explanation in terms of the more general stochastic case under (5a). Even without a deterministic interpretation, we can causally interpret correlation and (5a): x and y are caused by z and z 'screens off' the correlation between them.

In sum, irreducibility or the no-HV-hypothesis, offers, it seems, no answer to the question what correlation is. But only determinism allows to prove what appears to be the essence of correlation, namely the quantitative part of Reichenbach's principle (5a). Of course, it is often believed that Reichenbach's principle and reducibility do not work for quantum correlations. But there is a way out of this conundrum, as argued under *Problem 3*.

Problem 2*. How to interpret the Central Limit Theorem ? Or more precisely: Why are there so many stochastic phenomena, both in the classical and quantum realm, described by an (approximately) Gaussian distribution?*

Note first that Gaussian distributions are ubiquitous in quantum mechanics too, including for pure states (see, for example, Shumaker 1986 and Griffiths 1995 Problem 2.22, p. 50 and Problem 2.40, p. 69, and §3.4.2, p. 111). Just as *Problem 1*, *Problem 2* is most interesting in the quantum case; can we craft an explanation that works for both quantum and classical systems? The history of the Central Limit Theorem (CLT), one of the corner pieces of probability theory, is extremely rich and many variants of it exist. Let us pay tribute here to only a few of the great names of mathematics having contributed to it: de Moivre, Laplace, Chebyshev, Lyapunov, Pòlya, Gnedenko, Kolmogorov (it seems that Lyapunov was the first to have given the exact proof of the common variant we consider here, cf. Tijms 2004, p. 169). As even a quick glance in probability books shows, there is an aura of venerability surrounding the CLT: statisticians are struck by the regularity this theorem brings in the randomness of probabilistic systems. Witness the eloquent words of Francis Galton[9] (1894, p. 66); Tijms calls the CLT the "unofficial sovereign of probability theory" (Tijms 2004, p. 169). The version of the CLT we will consider here (cf. e.g. Gnedenko 1967, Feller 1971, Tijms 2004) states that under broad conditions (notably the Lindeberg-condition, cf. Feller 1971, p. 262):

> **CLT**. Any random variable that is the average, i.e. the weighted sum, of many independent random variables, has a normal distribution independently of the distribution of the contributing variables.

In pure mathematics this is treated, of course, as a purely mathematical theorem, but in textbooks for practitioners of probability theory the CLT is given a real-world *interpretation*. These textbooks typically present the theorem as an explanation of why the Gaussian distribution is so overwhelmingly present in the physical world, describing an unlimited variety of stochastic phenomena and properties (distribution of height; countless biological parameters; fabrication errors; diffusion phenomena; accident rates; meteorological data; etc.). The textbook rationale is straightforward: many phenomena are the average of many other (unobserved) independent phenomena. This is a direct interpretation of the mathematics of the CLT, and an answer to *Problem*

[9] "I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the Law of Frequency of Error. The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshalled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along" (Galton 1894, p. 66)

2. Now, if a Gaussian variable x is a sum-function of other (unobserved[10]) variables, the latter can be considered the causes of x. (So a more general answer to our question is that many physical properties are the *effect* of many (independent) *causes*.) Now this again sneaks in causal terms in the interpretation of probability. Again, key ingredients of probability theory are most naturally understood in terms of an underlying causal reality. As far as I know, irreducibility does not help to understand the CLT. Note, again, that determinism [cf. definition (3b)], assuming $x = f(\lambda_1,\ldots,\lambda_N)$ where $(\lambda_1,\ldots,\lambda_N)$ are hidden variables, and for the special case that f is a sum-function, allows to mathematically prove the answer given above – through the CLT.

At this point one wonders whether the answer "many variables are *sums* of many hidden causes" could be generalized to "many variables are effects, i.e. *functions*, of many hidden causes". So our causal (deterministic) interpretation makes one wonder whether a generalized CLT-gen can be proven:

> **CLT-gen**. Under broad conditions, a stochastic variable that is a *function* of many independent variables will be normally distributed, independently of the distribution of the contributing variables.

I do not know if this theorem can be proven (the proof would include, crucially, the specification of the mentioned 'broad conditions'), but it is tempting to suggest following path for a proof in a hand-waving way. According to Taylor's theorem one can approximate a function $f(x_1,\ldots,x_N)$ as follows:

$$f(x_1,\ldots,x_N) = f(0,\ldots,0) + [\,\partial f/\partial x_1\,]_0 \,.\, x_1 + \ldots + [\,\partial f/\partial x_N\,]_0 \,.\, x_N + O[x_i^2]\,.$$

According to the CLT the dominant terms, linear in $(x_1,\ldots,x_N)$, converge to a Gaussian[11].

Problem 3*. Are there degrees of freedom that could unify quantum field theories (QFT) and general relativity (GR), and if so, can we specify them (even a little more, qualitatively) ?*

Of course, we are looking here for a qualitative answer within philosophy of physics, not a detailed mathematical description of these variables within a physics theory. The physics problem underlying the question is considered as one of the greatest unresolved problems of the discipline.

[10] If the variables would all be observed, x would not be a stochastic variable, but a deterministic one.

[11] There are functions however that do not lead to a Gaussian distribution. For instance, one can show that a product of independent variables leads to a lognormal distribution, which is not Gaussian. As said, the main task would reside in the identification of the precise mathematical conditions under which functions lead to a Gaussian distribution.

Indeed, GR and QFT are generally believed to be mutually incompatible, at least in regions of extremely high mass and small dimensions, such as exist in black holes or at the Big-Bang. This incompatibility parallels the standard interpretation of Bell's theorem, which thus seems to constitute a severe obstacle to the unification program. Indeed, on its standard interpretation, Bell's theorem shows that quantum mechanics cannot be derived from an underlying deterministic theory (as GR), in general not from any theory that introduces (local) hidden variables for quantum properties in the sense (1). In particular, deterministic and Lorentz-invariant degrees of freedom that could describe GR cannot describe[12] quantum mechanics as an irreducibly probabilistic theory in the sense (2).

Now, it is known that the no-go verdict of Bell's theorem is only valid if one adheres to *a particular form of determinism / reducibility, not* if one adheres to full determinism or rather full reducibility – often called 'superdeterminism' in the Bell literature. Indeed, the proof of Bell's theorem is based on an assumption of 'measurement independence', a condition of probabilistic independence or de-correlation stating that:

$$P(\lambda|a, b) = P(\lambda). \tag{6}$$

Here 'a' and 'b' are the left and right analyzer directions in a Bell experiment, and λ are the hidden variables (deterministic or stochastic) that would 'explain' and determine the quantum probabilities $P(x, y|a, b)$ measured in the Bell test (as in Eq. (7) below, which is an instance of (1)). Here 'x' and 'y' are the left and right polarizations or spins and $P(\lambda)$ is the probability distribution of the λ. As is well known, assumption (6) can only be justified by metaphysically tainted arguments (based notably on the notion of free will, subject to philosophical debate since millennia); for a detailed overview of these arguments, against or in favor of (6), see e.g. Lewis (2006)[13] and Wuethrich (2011). Most people accept (6) on the basis of the idea that freely chosen parameters as a and b cannot be stochastically determined by particle properties – such a 'superdeterministic' assumption would contradict free will or amount to a cosmic conspiracy[14]. First note that reducibility or the HV-hypothesis applied *at face value* to the Bell correlation implies (cf. (1)):

[12] According to Bell's theorem only *non-local* HV variables could describe quantum mechanics, but such a theory would involve superluminal, non-Lorentz-invariant interactions (see the explicit statement in the conclusion of Bell 1964). As stated, this possibility is not considered here.
[13] Lewis proposes a new argument in favor of (6). His main conclusion is that this assumption needs far more work before it can be accepted as uncontroversial.
[14] 'Superdeterminism' is a somewhat infelicitous term in that (6) or its negation do not necessarily assume determinism; in general (6) is a probabilistic assumption. We will continue using 'superdeterminism' to indicate violation of (6).

$$P(x, y|a, b) = \sum_{\lambda} P(x, y|a, b, \lambda)\, P(\lambda|a, b). \quad (7)$$

Inserting assumption (6) in (7) leads then immediately to the Bell inequality (if also the usual factorability or 'locality' condition (8) is assumed). I do not take strong sides here, but remark that (6) inserted in (7) amounts to a *partial* reducibility. Genuine reducibility implies (7) in full, so featuring the conditional probability, as follows by direct application of probability theory (the 'law of total probability'). Expression (7) leaves open the possibility that the correlation $P(\lambda|a, b)$ could itself be reduced through (common) causes between (λ, a, b) (see further).

Some researchers, both philosophers and physicists, have contested the general validity of hypothesis (6) (cf. for instance Shimony et al. 1976, Brans 1988, Price 1996, Lewis 2006, Hall 2010, 2011, Nieuwenhuizen 2011, Vervoort 2013, 2018, 't Hooft 2014, 2016, 2017, Hossenfelder 2014); but surely superdeterminism remains a minority position. The gist of the argument, common to most of the mentioned authors, is that if one takes determinism seriously, it cannot be excluded that there exists, at least in principle, a correlation between the variables λ and (freely or randomly chosen) analyzer directions as (a, b). A straightforward way to read this rationale is that, in a fully deterministic universe, a and b are also deterministic parameters reducible to some causes (variables); these causes have their causes, etc. until the origin of the universe, the Big-Bang. It is conceivable that at and even after this singularity 'everything is correlated to everything'. (As an analogue in a world fully described by classical mechanics: consider one billiard ball hitting a dense pack of balls. After the bang the positions and velocities of all balls remain correlated, as is most easily understood in the fully deterministic picture.) We phrase the argument here using the notion of deterministic causes, but the same rationale holds for probabilistic causes (cf. the previous footnote). Back to our problem: on this non-standard view the λ are interpreted *as degrees of freedom of a theory that describes the Big Bang* (or the universe shortly after it); these degrees of freedom are common causes of x, y, a, b. So they remain correlated to 'everything', in particular (a,b), in contradiction with Eq. (6). This interpretation is schematized as Case 1 in Fig. 1, where time = 0 corresponds to the Big Bang. Or, on a closely related reading (Case 2 in Fig. 1), the λ could be 'particle' variables that share common causes with (a,b), these common causes (λ*) being the ab-initio degrees of freedom just mentioned. Note that both these interpretations are in agreement with the interpretation of the correlation $P(\lambda|a, b)$ given under *Problem 1* (the qualitative part of Reichenbach's principle), stating that correlation occurs through causes: either λ directly causes (a,b) (Case 1), in which case $P(a,b|\lambda)$ and therefore (by Bayes' rule) $P(\lambda|a,b)$ are defined; or λ and (a,b) have common causes (λ*), as in Case 2. In the latter interpretation, we have a further reduction $P(\lambda|a,b) = \sum_{\lambda*} P(\lambda|a,b,\lambda^*)\, P(\lambda^*|a,b)$. Distinguishing these two types of superdeterminism is useful, as argued in a moment. A third type is based on retrocausality, cf. Price 1996, Lewis 2006.

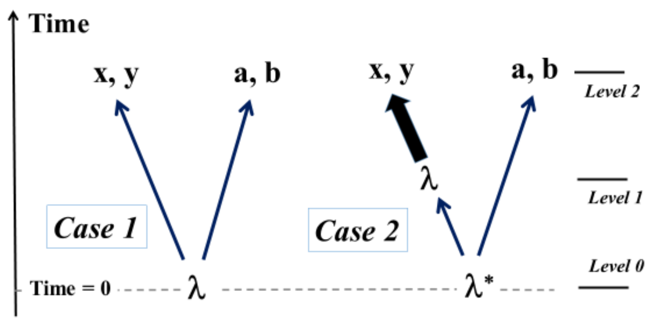

Figure 1. Two different interpretations of how the superdeterministic correlation $P(\lambda|a,b)$ could come about. Arrows indicate causal connections. Variables can be part of theories of different 'levels' (cf. text). The thick arrows symbolizes how a theory of quantum gravity could explain quantum mechanics: physics is not necessarily dead in a superdeterministic universe.

This superdeterministic view has, notably, been defended by Nobel laureate Gerard 't Hooft (2014, 2016, 2017). 't Hooft has laid the basis for developing a superdeterministic but local hidden-variable theory, even if he acknowledges, of course, that this program is not finalized. He terms his theory the "Cellular Automaton Interpretation" of quantum mechanics; it has the ambition to introduce notions that could be at the basis of a full theory of quantum gravity. In short, 't Hooft posits the existence of a basis of ultimate 'ontological states' (characterized by λ), with associated Hilbert space: a preferred basis of states of which all particles are composed and which deterministically evolve by permutations among themselves in discrete time intervals. On the nature of the hidden variables in his theory 't Hooft says: "This set consists of the states the universe can 'really' be in. At all times, the universe chooses one of these states to be in, with probability 1, while all others carry probability 0" (2016, p. 14). And more specifically on the correlations violating Eq. (6):

> "How to explain this apparent 'conspiracy'? A state considered in some experimental setup may either be a physical state, which we shall call 'ontological', or it is a superposition of ontological states. […] However, if an 'ontological basis' exists, which

we believe to be the case, then there is a conservation law: the ontological nature of a state is conserved in time. If, at some late time, a photon is observed to be in a given polarization state, just because it passed through a filter, then that is its ontological state, and the photon has been in that ontological state from the moment it was emitted by its source. It seems to be inevitable to demand that the 'ontological basis' is unobservable, that is, indistinguishable from other bases, as it is for instance in our description of string theory. 'Conspiracy' is then unobservable. […] Indeed, the same conclusion can be reached by considering the black hole microstates, which quite possibly correspond to the ultimate, classical degrees of freedom of an underlying theory, while they fundamentally arise at the Planck scale only. The problem of 'quantizing' curved space-time, quantum gravity for short, is notoriously complex and far from understood. It may well be that a complete understanding of the quantum nature of our world will have to come together with the complete resolution of the quantum gravity problem" (2014, p. 13-14).

As we understand 't Hooft, two ideas deserve special attention in our context. The first is that the ultimate ontological states may well be those that will intervene in the still unconstructed theory of quantum gravity. The second is that the 'conspiratorial' correlations $P(\lambda|a,b) \neq P(\lambda)$ may well *exist* (be based on, or derived from, ontological states), while they are the same time *unobservable*. Let us elaborate on both ideas, which appear to be related.

We suspect with 't Hooft that the second argument is essential. One does not need to ever be able to derive an *explicit* formula for the correlations $P(\lambda|a,b)$ *in practice,* neither need they be observable in practice; for contesting the standard interpretation of Bell's theorem it suffices to assume that such correlations *exist.* Bell's theorem is an *existence* theorem, not a physical theorem in the strict sense; it hinges on (6) which is not a consequence of an established physics theory[15]. Could it be that superdeterminism is not taken more seriously due to a mistaken idea, namely that it would imply that we need a theory that allows to calculate $P(\lambda|a,b)$? It may well be Bell himself who was at the origin of this assumption. A well-known statement from him that we regularly hear repeated by contemporary researchers is this: "A theory may appear in which such conspiracies inevitably occur, and these conspiracies may then seem more digestible than the non-localities of other theories. When that theory is announced I will not refuse to listen, either on methodological or other grounds. But I will not myself try to make such a theory" (Bell et al. 1985, p. 106). Physicists often seem to state "superdeterminism is not a physical theory" as a reason to reject it. But this may well be a misinterpretation of what Bell's theorem is all about.

[15] We believe it is a mixed physical – metaphysical assumption. Therefore, rejecting superdeterminism because it is metaphysical would be a category mistake: Bell's theorem *is* an existence theorem. Moreover, deeming correlations 'conspiratorial' seems an utterly anthropocentric attitude; what looks conspiratorial to our limited minds, may still be perfectly physical – and obvious to a more developed intelligence!

To counter this view, one could observe that the HV-hypothesis (schematized in Fig. 1) is a *logical precondition* for engaging in the theoretical work that thousands of physicists pursue on a daily basis: namely the attempt to unify quantum mechanics and general relativity. As ‘t Hooft recalls, this unified theory is generally believed to exist (even if it may forever elude us) and to describe, for instance, the Big Bang. So this unified theory would allow one to derive quantum mechanics (and general relativity) from more fundamental – hidden – variables. It seems that if one denies the HV-hypothesis, the unification program has no sense. The ultimate unified theory is a HV theory, and it would be a theory about the Big Bang (or instants not too far away from it), and therefore one that describes – in principle – the causes of all subsequent phenomena, including analyzer choices (cf. Case 1 in Fig. 1). Ergo, these analyzer choices are correlated with these ultimate variables. This does not mean that we will ever be able to derive this ultimate ToE that allows to predict literally all phenomena starting from time = 0, including the evolution of our choices; it seems obvious we will never be; and yet it should exist. Such an ‘ultimate’ ToE corresponds to Case 1 in Fig. 1; it could be called a theory of ‘level 0’. A more realistic ‘effective’ ToE – string theory, according to some – rather corresponds to Case 2, more specifically to the thick arrow in it: it would allow us to understand how quantum properties as (x, y) derive from string properties (λ). One could call it a theory of level 1, while quantum mechanics is of level 2. In sum, Case 2 corresponds to a workable solution – *pace* Bell. On this account the moral is clear: not all physics stops with the HV-hypothesis; rather, future physics is preconditioned on it.

Of course, theories of different levels and reduction between them is not rare in physics. Assume we live in a purely classical world in which all matter consists of hard-ball atoms (quantum effects are neglected). Then all macroscopic properties (say x,y), including statistical ones, such as described by statistical mechanics (a level-1 theory) or thermodynamics (a level-2 theory), and conceivably also all ‘freely chosen’ analyzer variables (say a, b), are – in principle – correlated with the classical variables (λ) of mechanics (a level-0 theory) describing the micro-constituents of matter. If we would specify the values of enough properties λ of enough constituents, $P(x,y|\lambda)$ and $P(a,b|\lambda)$ would be delta-functions; and more generally $P(x,y|\lambda) \neq P(x,y)$ and $P(a,b|\lambda) \neq P(a,b)$. In such a world, correlations of the type $P(a,b|\lambda)$ would exist, even if it may well be that they are unknowable, if the universe is complex enough. According to an analogous reasoning, it is legitimate to assume the existence of the correlations $P(a,b|\lambda)$ in Bell’s analysis. And notwithstanding all-pervasive and seemingly conspiratorial correlations among all variables, physics is still possible: level-1 and level-2 theories (statistical mechanics and thermodynamics in this analogy) can still be constructed.

To bring our argument to its end, we can note a connection between Bell's theorem and Reichenbach's principle (a connection which was noted from quite early on, see notably van Fraassen 1982). In order to derive Bell inequalities from the expansion (7), one has to assume, besides (6), also following factorability condition:

$$P(x, y|a, b, \lambda) = P(x|a, \lambda)\ P(y|b, \lambda)\ . \tag{8}$$

This can be seen as an application of Reichenbach's assumption (5a) together with the assumption that $P(x|a, b, \lambda)$ does not depend on b (so $P(x|a, b, \lambda) = P(x|a, \lambda)$), and $P(y|a, b, \lambda)$ not on a. The condition (8) is often termed the 'locality' condition, and adopted as a 'reasonable assumption' in agreement with relativity theory, since (x, b) as well as (y, a) are spacelike separated in advanced experiments. It is interesting to see how Bell derives (8) in his (1981) (see the discussion before and after Eq. (10) in that article): he does not mention Reichenbach but very well bases (8) on Reichenbach's idea that, if λ represents *all* the common causes of x and y, (8) is 'reasonable' (if also locality is assumed). But there is no rule from probability calculus from which (8) follows necessarily.

Now, the analysis of correlation presented under *Problem 1* allows us to *prove* (8) in the deterministic case (if also locality is assumed); in the stochastic case we can only assume (8) as a reasonable conjecture. This is a direct consequence of the fact that Theorem 1 or Reichenbach's principle can be proven in the deterministic case. Indeed, it was shown under *Problem 1* (see Theorem 1) that, in a deterministic world, and if one keeps $P(x|a, b, \lambda)$ and $P(y|a, b, \lambda)$, (8) holds necessarily true for λ = the set of all causes of (x, y); we do not invoke locality at this stage. That $P(x|a, b, \lambda) = P(x|a, \lambda)$ can also be given a causal reading: we saw in the corollary of Theorem 1 that x must be independent of b given (a, λ), since x cannot be a function of b; b cannot cause x (x and b are spacelike separated). This proves that in a deterministic scenario, (8) follows necessarily only from the locality condition $P(x|a, b, \lambda) = P(x|a, \lambda)$ (and a similar one for y), a condition which seems itself, especially in the light of Theorem 1 and its corollary, obvious. In sum, in a deterministic scenario (8) can be proven based on locality alone – it is more than 'reasonable'.

And yet, the fact that the Bell inequalities are violated in experiments, has convinced many that Reichenbach's principle does not work in the quantum realm. But this is overlooking the possibility that the culprit for the discrepancy between the experimental results and Bell's assumptions may be assumption (6) – the neglect of superdeterminism, the neglect of full reducibility, in other words of the HV-hypothesis under the full form (7). Conversely, if one assumes superdeterminism, one saves Reichenbach's principle. Regrettably, this option is rarely if ever considered (cf. e.g. Cavalcanti and Lal 2014, a recent work investigating Reichenbach's principle in the light of Bell's theorem).

Recall that genuine reducibility and the HV-hypothesis as we defined them (cf. (1) and (7)) includes the superdeterministic correlations $P(\lambda|a,b)$. It is the full-fledged form (7) that allows one to counter Bell's no-go verdict (since the proof, using (8), has often been given, we will not provide it here again). Thus, it is once more the HV-hypothesis that offers an answer to the question of *Problem 3*. Indeed, it is generally believed that the theory that can unify quantum mechanics and relativity theory should involve variables describing the Big Bang. According to the rationale recalled above, it are these variables that can at once dissolve the no-go impediment of Bell's theorem *and* potentially unify GR and QFT (as they should do).

Once more, irreducibility seems to offer no answer to *Problem 3*; rather, it leaves Bell's theorem as an obstacle to the unification of GR and QFT. Interestingly enough, deterministic reduction permits again to go farthest in the mathematical proof, since it allows for a mathematical proof of (8).

4. Conclusion.

In this article I have argued that the HV-hypothesis, or reducibility, has a greater explanatory power than its competitor, the no-HV hypothesis or irreducibility. Indeed, the HV-hypothesis can coherently answer three questions from philosophy of physics for which irreducibility remains entirely silent.

Philosophers have argued that the conclusions of Bell's theorem are independent of the question of (in)determinism (Esfeld 2015); we may never be able to know with certainty whether the universe is ultimately deterministic or probabilistic (cf. e.g. Suppes 1993, Wuthrich 2011). Our analysis is in agreement with these conclusions: the 'superdeterministic' correlations $P(\lambda|a,b)$ can be given a deterministic but also a probabilistic reading. As shown here, the discussion can synthetically be held in terms of the HV-hypothesis, or reducibility. Interestingly, it is still the case that of the two possible forms of reducibility, only deterministic reduction can provide proofs of the answers given. We leave it here as an open question whether that favors determinism over indeterminism (probabilism).

It is vital, for our argument, to realize that we do not question the standard interpretation of Bell's theorem simply by repeating existing arguments, but by pointing to the benefit that its rejection (the adoption of the HV-hypothesis) has for answering two other foundational questions. As is widely accepted in science and philosophy of science, a hypothesis gains strength if it explains a variety of problems; powerful scientific hypotheses and theories are interconnected to a variety of other hypotheses and theories; all these hypotheses are like words in a crossword puzzle, in which each correct word 'confirms' all the other words (a metaphor coming from Haack 1993). We believe that this not only holds for scientific hypotheses (and theories), but also for hypotheses of *philosophy* of science (and of philosophical theories in general).

This brings us to the question whether the HV-hypothesis can be seen as a unifying principle (or hypothesis) in the philosophy of physics, even if we restrict the principle to local variables alone. It has been noted that the philosophy of such unifying principles is an almost virgin domain (Bunge 2009, p. 419). Yet there seems little doubt that there is epistemic value in certain (possibly irrefutable yet) confirmable, grounded, and fertile metanomological principles (as there are e.g. symmetry principles such as the covariance principle, cf. Bunge 2009, Ch. 6, §6.1, 6.8). For the reasons just mentioned, we believe the HV-hypothesis belongs to this broad category: the HV-hypothesis is strengthened from *three* different, corroborating angles. At the same time, *we are not aware of problems or questions to which the no-HV hypothesis can give a coherent answer, while its competitor cannot*. Sure, this is not a definite proof, but we propose that these arguments give enough reasons to adopt, for the time being, the HV-hypothesis as a unifying principle in the philosophy (and foundations) of physics. For physics, it can serve as a heuristic principle; and it appears that it is used as such by, e.g., thousands of string theorists.

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Appendix. Definitions, more rigorously.

Let us first make a connection with basic notions of probability theory. For starters, when we say that the probability of E is p, then E is an event and p is a real number $p \in [0; 1]$. The occurrence or non-occurrence of E depends on the chain of circumstances involved: this chain is called the *experiment* or the *trial*, and the result is the *outcome* of the experiment or trial.

Definition A0. (Sample space)
The ***sample space*** *is the set of all possible outcomes of an experiment, denoted by* Ω.
□

For an experiment on a quantum system by which the quantitative value of an observable property X is measured, the sample space is thus the spectrum $\sigma(\hat{X})$ of the operator $\hat{X}$ associated to the observable property X: $\Omega = \sigma(\hat{X})$. This is the Spectrum Postulate of orthodox quantum mechanics. As explained in the main text, for defining quantum probabilities it is instrumental to explicitly introduce a parameter 'a' that quantitatively characterizes a property A of the experimental setup (or a set of such properties). $P(x|a)$ is an abbreviation for $P(X = x|A = a)$, which denotes the probability of the event that the property X is measured to have the quantitative value $x \in \sigma(\hat{X})$ under the condition that the property A is set to the value a. More rigorous formulations for definitions 1, 2 and 3 are then the following ones, where Λ is the set of all the (discrete) values λ of a physical property:

Definition A1. (Reducible probability)
A probability $P(x|a)$ is **reducible** *IFF* there is a set of constants Λ such that for all x, $P(x|a)$ is the sum over all values $\lambda \in \Lambda$ of the products $P(x|a,\lambda) \cdot P(\lambda|a)$, that is, *IFF*

$$\exists\Lambda\forall x : P(x|a) = \sum_{\lambda\in\Lambda} P(x|a,\lambda) \cdot P(\lambda|a). \tag{A1}$$
□

Definition A2. (Irreducible probability)
A probability $P(x|a)$ is **irreducible** *IFF* $P(x|a)$ is not reducible, that is, *IFF*

$$\nexists \Lambda \forall x : P(x|a) = \sum_{\lambda \in \Lambda} P(x|a,\lambda) \cdot P(\lambda|a). \tag{A2}$$

□

The idea of a determinism is then that the outcome of the experiment—the quantitative value *x* of the property *X* of the quantum system—is uniquely determined by the setting *a* and the value of a hidden variable λ. We can, then, construct a function $f : \{a\} \times \Lambda \to \Omega$ that maps the setting *a* and the value of the hidden variable λ to the thereby determined value *x* of the property *X*: $f(a, \lambda) = x$. The definition of a deterministic variable then becomes:

Definition A3. (Deterministic variable)

A variable *x* ranging over Ω is **deterministic** *IFF*

$$\exists \Lambda \exists f \in \Omega^{\{a\} \times \Lambda} : \; P(x|a) = \sum_{\lambda \in \Lambda} P(x|a,\lambda) \cdot P(\lambda|a) = \sum_{\lambda \in \Lambda} \delta_{x,f(a,\lambda)} \cdot P(\lambda|a) \tag{A3}$$

where $\delta_{u,v}$ is the Kronecker-delta.

□

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