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PROPENSITIES IN A NON-DETERMINISTIC PHYSICS*

ABSTRACT. Propensities are presented as a generalization of classical determinism. They describe a physical reality intermediary between Laplacian determinism and pure randomness, such as in quantum mechanics. They are characterized by the fact that their values are determined by the collection of all actual properties. It is argued that they do not satisfy Kolmogorov axioms; other axioms are proposed.

1. PURPOSE

The word 'probability' covers several concepts, but not all concepts have something to do with the probability of a non-necessary physical event. The Kolmogorov axioms accept several interpretations, but not all probability concepts satisfy the Kolmogorov axioms. In this article I am interested in the meaning of the word 'probability' as used by physicists when they describe an event that may happen in the future. More specifically, I am interested in the peculiar meaning that this word has in quantum mechanics and the axiomatization of this concept. My thesis can be summarized as follows: quantum probability is closely related to classical determinism; its axiomatization should thus contain classical determinism as a particular case.

The search for a conceptual framework allowing us to speak and to think about a metaphysical non-determinism is the main motivation for this article. The importance of such a conceptual framework is crucial, since many physicists (and also many philosophers), think that a real non-determinism (one that applies also to Laplace's demon) cannot be thought of and would be the end of physics. This prejudice is caused by some confusion about causality, physical probabilities and Kolmogorov axioms. The identification of the latter two, for instance, leads to the search of the probability space (hidden variable) and to the desperate attempt to restore a deterministic causality on that level. It also leads to the claim that quantum mechanics describes only ensembles. One thus tries to describe non-determinism with a formalism adapted for epistemic probabilities. My point of view is that the Kolmogorov axioms, although very illuminating in statistical mechanics and epis-

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temic probabilities, are not adapted for the description of the probability of truly non-deterministic events.

In the next section I shall introduce my concept with an example. The concept will be formalized in Section 3. In the conclusion I stress that the necessary mathematical structure already exists in today's classical and quantum physics. Moreover, if one starts from this formalization of what I shall call 'propensity', the structure of classical physics (phase space) and of quantum physics (Hilbert space) emerge naturally.

I shall use the word 'propensity' for the probabilistic generalization of classical determinism that I have in mind, although it may be confusing, since Sir Karl Popper (1959), who advocated a propensity interpretation of probabilities, claimed that classical mechanics is not deterministic. This is true from an operational point of view. However, the way classical and quantum mechanics are non-deterministic are very different: for the Laplace demon, classical physics is deterministic, whereas quantum physics remains non-deterministic. I propose to use the word 'propensity' for the probability of a non-necessary (non-predetermined) physical event. This choice is supported by several of Popper's citations as, for instance: "I propose a new physical hypothesis. The two slits experiment convinced me that probabilities . . . are physical propensities, comparable to Newtonian forces . . . to realize singular events" (Popper 1959). I accept each word of this citation, but I would like to add: "there is no reason why this new physical hypothesis should satisfy the old Kolmogorov axioms". At this point I should also mention D. H. Mellor: "A genuine, methaphysical, indeterminism must enter into a chance set-up. . . . If propensities are ever displayed, determinism is false" (Mellor 1971); D. A. Gillies: "if we can correctly assert that at least one probability system exists in reality, then it follows that at least some objective randomness exists in the world, and so complete determinism is shown to be false" (Gillies 1973); and R. N. Giere: "Propensities are . . . causal connections with varying strengths" (Giere 1973).

2. INTRODUCTION

Let me start with classical determinism versus quantum non-determinism. Probabilities enter in both theories due to uncertainties about the exact initial state of the physical system or the exact state of the environment. But I like to concentrate on the irreducible non-determinism remaining once everything in the initial state and in the environment is assumed to be exactly known. Maybe in a future theory nothing would remain, but this is clearly not the case with today's physics, and it is reasonable to think that today's situation is likely to last and that it calls for a better understanding. Consequently, the question is whether it is legitimate to associate one pure state with a physical system.

In classical mechanics the evolution of a pure state is deterministic, whereas the evolution of a non-pure state may be chaotic in the sense that a state well localized in the state space may undergo very different evolutions, apparently erratic, due to the sensitivity to the initial condition. There is clearly no way to prepare a system in a pure state (i.e., to prepare it in such a way as to know in which pure state it is), nor to measure it with infinite precision. Hence, the concept of a system in a pure state is not operational. But it is nevertheless a very useful and meaningful concept for a realist: the system is in a pure state even if the physicist has no way to know in which one exactly. This epistemic probability is so closely related to the mathematical theory of measure that I hardly see any problem with it.

In quantum mechanics the situation is very different: even a pure state can undergo non-deterministic evolutions. There remains, however, the problem of whether it is legitimate to describe a system by a pure state. I consider that it is relevant to describe at least some systems by pure states. Experiments about EPR-like correlations (Einstein et al. 1935; Clauser and Shimony 1978; Aspect et al. 1982) taught us that one cannot arbitrarily cut the world into pieces: even spatially-separated systems can be genuinely correlated such that only the whole can be described by a pure state. But, precisely, the whole can be described by a pure state, and this is used by a physicist when computing the correlations. It seems to me that a realist can accept that there is a level between an electron (let's say) and him, where one can cut the world in two such that the electron together with its surroundings is in a pure state, without genuine (quantum) correlations between the electron and him; when a realist physicist becomes conscious of an experimental result, he really learns something about the electron, in contrast to the hypothetical physicist for whom the experiment would merely reveal some pre-existing correlations between the electron and himself.

I take seriously the idea that the physical world is non-deterministic:

an event may happen in a closed physical system without necessity. The present state of quantum mechanics versus hidden variables strongly supports that the investigation of this idea is interesting.

The fact that an event is not necessary does not mean that it has no cause. A counter that clicks, for instance, may be a non-necessary event, but it is clearly caused by a particle detected by the counter. Simply, the same counter, in the same circumstances, could also have not detected the same particle. The particle is in a certain state, (unknown, at best known to a finite precision, but this ignorance will clearly not affect the counter), and the counter tests the presence of the particle in a certain region of space. Together the state of the particle and the counter determine the propensity for the event 'click' to take place. Actually, the counter could have a poor efficiency, failing to count particles that are really in the space region that the counter is supposed to cover. We are not interested in the study of these defects, but in the limiting case of ideal counters. We are thus led to the conclusion that the propensity of the event is completely determined by the state φ of the particle and the property 'a' of being in a certain space region. But this raises the question: What is the state φ of the particle? If the answer was, 'The state is the collection of all the propensities of all possible events', then the previous sentence would be tautological. (Note that this concept of state is very common among physicists. In this way they are led to represent the state of a quantum system by a density operator – a mathematical object containing all probabilities without distinction between epistemic ones and the 'real' ones, that is, propensities.) For me the state of a system is something that is really engraved into the system; the system really possesses its state in act. Hence, following C. Piron (Piron 1976 and 1983; see also Aerts 1981 and 1982), I define the state as the collection of all the actual properties of the system. What else could it be? At least the state has to contain all the actual properties. Our definition is thus a minimal one.

Let me argue further that the above definition of state is not only a possible one, but is the natural one. The distinction between the state, i.e., the collection of all actual properties, and the kinematics, i.e., the structure of the state space that determines the propensities of possible reactions to different external circumstances, can be illustrated as follows. Imagine a (classical) ball attached to a rail. Its state is given by its position and velocity along the rail (the state space is two-dimensional). The kinematics determines the possible reactions of the ball to different external forces. To introduce the propensities in the definition of state would be similar to the inclusion of the boundary conditions in the state of the ball. The inclusion of all the propensities to react to any possible external circumstances would overload the concept of state. It would reduce its relevance by confusing the structure of the state space with the elements of the state space. At least one should have good arguments to put so much into that concept. It is not necessary; the collection of actual properties is enough, moreover, it is close to the concept of state in classical mechanics.

Let me summarize. First, the state contains only actual properties, or, in Einstein's terms, elements of reality. Next, the state, i.e., these actual properties, determine uniquely and completely the propensities of all the potential properties.[1] Hence, the non-deterministic aspect is completely characterized by the deterministic aspect. This is true in classical physics (we shall see that only the propensity 0 and 1 exist) and in quantum physics (the propensity can take any value between 0 and 1), and this is the characterization that I propose as the definition of a propensity (probability of non-necessary physical events). In the next section I make this characterization more concrete.

Let me emphasize that propensities have the same reality as pure states and properties of classical or quantum systems. But, in real experiments propensities will always be mixed with epistemic probabilities, due to ignorance of the exact state of the system and its environment.

3. FORMALIZATION

In order to present my concept of propensity as sharply as possible, I would like to formalize it into a mathematically-rigorous framework. I chose the lattice theory as framework, with a rather general interpretation. Other frameworks are clearly possible. I shall be happy if the reader admits that my choice is a possible one and that the concept can be made precise. Only the outline of the proofs are presented below, since the theorems are already published in Gisin (1984), but the presentation of the theorems is new.

Let me start with the assumption that the set of properties of a physical system has the structure of a complete lattice $\mathscr{L}$. Piron (1976 and 1983) and Aerts (1981 and 1982) gave very strong arguments in favor of this assumption. We shall also assume that the lattice $\mathscr{L}$ is ortho-modular; this is a non-obvious assumption[2] (Aerts 1981 and 1982), but it is so widely used that I feel free to assume this structure without elaborating on it. Notice that this assumption holds in classical and quantum mechanics (with or without superselection rules). Recall that a lattice is a set with an order relation '$\leqslant$' such that the upper '$\vee$' and lower '$\wedge$' bounds exist. The interpretation of '$a \leqslant b$' is: whenever the property a is actual, the property b is also actual. Ortho-modular means that the lattice $\mathscr{L}$ is equipped with a map $\mathscr{L} \rightarrow \mathscr{L}$ such that (1) $a'' = a$, (2) $a \leqslant a' \Rightarrow a = 0$, (3) $a \leqslant b \Rightarrow a' \geqslant b'$, (4) $a \leqslant b \Rightarrow \exists c \leqslant a'$, $c \vee a = b$. Denote $a \perp b$ (read a orthogonal to b) whenever $a \leqslant b'$. As usual, the interpretation of $a \perp b$ is that the properties a and b can be tested simultaneously, but they are never simultaneously actual. In classical mechanics $\mathscr{L} = \mathscr{P}(\Gamma)$, the set of all subsets of the phase space Γ, the order relation is given by set inclusion, the upper and lower bounds by set union and intersection, $a' \equiv \Gamma \backslash a$, and $a \perp b$ if and only if their intersection is empty. In quantum mechanics $\mathscr{L} = P(\mathscr{H})$, the set of closed linear subspaces of the Hilbert space $\mathscr{H}$, the order relation is given by set inclusion, the upper and lower bounds by union and closure and by set intersection, $a' \equiv \{p \in \mathscr{H} \mid p \perp a\}$, and $a \perp b$ if and only if the vectors of a and b are mutually orthogonal.

When working in this framework one should always have the following important mathematical result (Piron 1976) in mind:

THEOREM: If $\mathscr{L}$ is a complete atomic ortho-modular lattice with at least four orthogonal atoms and if $\mathscr{L}$ satisfies the covering law (i.e., $p \leqslant q \vee s \Rightarrow q \leqslant p \vee s$ for all triplet of atoms p, q, s), then $\mathscr{L}$ is isomorphic to the direct union over a set Γ of Hilbertian space lattices:

$$\mathscr{L} \approx \vee \{P(\mathscr{H}_a) \mid a \in \Gamma\}.$$

This theorem identifies the lattices that correspond to classical (all $\mathscr{H}_a$ of dimension 1) and quantum ($\Gamma = \{a\}$, a singleton) physics; the intermediate cases correspond to quantum mechanics with superselection rules.

We are now ready for the definition of a propensity function within this context and some related theorems.

DEFINITION: A *measure* is a function f: $\mathscr{L} \rightarrow [0, 1]$ such that:

(1) $\forall a_i \in \mathscr{L},\ a_i \perp a_j,\ i \neq j \Rightarrow f(\vee a_i) = \Sigma f(a_i)$,
(2) $\forall a_i \in \mathscr{L},\ f(a_i) = 1 \Rightarrow f(\wedge a_i) = 1$.
(3) $f(1) = 1$.

Note 1: The interpretation is: $f(a) = 1 \Leftrightarrow$ the property a is actual.
Note 2: Condition 2 expresses that $a \wedge b$ is actual $\Leftrightarrow$ the properties a and b are both actual. This condition has been criticized. Bell (1966), for instance, presented a deterministic spin 1/2 model. In this model the properties a and b that the spin points in direction $\vec{a}$, respectively $\vec{b}$, can be simultaneously actual (i.e., the measurements of the spin in directions $\vec{a}$ and $\vec{b}$ have both predetermined positive outcomes). In this model, the property $a \wedge b$ can thus be actual, contrary to quantum mechanics where $a \wedge b = 0$. This only proves that the property lattices $\mathscr{L}$ of quantum mechanics and of hidden variable models are different. Notice, however, that condition 2 holds in both lattices. Note also that some properties of the hidden variable model cannot, even in principle, be tested.
Note 3: Since $a \leqslant b \Rightarrow b = a \vee c$ for some $c \perp a$, one has $a \leqslant b \Rightarrow f(a) \leqslant f(b)$.
Note 4: Since $0 \perp a \ \forall\, a \in \mathscr{L}$, one has $f(0) = 0$.

Notations: For all measure f, I denote by a_f the smallest element of $\mathscr{L}$ with value one:

$$a_f \equiv \wedge \{a \in \mathscr{L} \mid f(a) = 1\}.$$

Note:

$$f(a_f) = 1.$$
$$f(b) = 0 \Leftrightarrow b \perp a_f.$$

DEFINITION: A *propensity* function is a measure f such that:

$$\forall \text{ measure } g,\ a_g = a_f \Rightarrow g = f.$$

Note 1: The idea is that a propensity function is uniquely determined by the set of elements on which it assumes the value 1.
Note 2: In the algebraic approach to classical and quantum physics (Primas 1983), a propensity function can be characterized by its collection of dispersion-free observables: a propensity function is a state such that there are no other states with the same collection of dispersion-

free observables. This characterization leads to the description of propensities by the values of pure states (Anderson 1979).

LEMMA: Let f and g be two measures.

$$\forall \lambda \in]0, 1[,\ h = \lambda.f + (1-\lambda).g \text{ is a measure and } a_h = a_f \vee a_g.$$

Proof: $h(a) = 1 \Leftrightarrow f(a) = g(a) = 1$. Hence, h is a measure and $a_h \geqslant a_f \vee a_g$. Finally, $h(a_f \vee a_g) = 1$; hence, $a_h \leqslant a_f \vee a_g$.

THEOREM: Let f and g be two measures. If $a_f \leqslant a_g$, then g is not a propensity function.

Proof: Let $h = \lambda.f + (1-\lambda).g$, $a_h = a_g$ and $h \neq g$.

AXIOM 1: $\forall a \in \mathscr{L}$, $a \neq 0$, $\exists$ a measure f such that $f(a) = 1$.

THEOREM: If g is a propensity function, then a_g is an atom of $\mathscr{L}$.

Proof: If $0 \neq p \leqslant a_g$, then $\exists f$ s.t. $a_f \leqslant p \leqslant a_g$.

AXIOM 2: $\forall a \in \mathscr{L}$, $a \neq 0$, $\exists$ a propensity function g such that $g(a) = 1$.

Note: The propensity function associated to an atom in Axiom 2 is necessarily unique. This axiom assumes thus that a 'Gleason theorem' (Gleason 1957) holds in $\mathscr{L}$.[3]

THEOREM: Let g be a measure.

If a_g is an atom of $\mathscr{L}$, then g is a propensity function.

Proof: Let f be the propensity function associated to a_g by Axiom 2. $a_f \leqslant a_g$. But $f(a_f) = 1 \Rightarrow a_f \neq 0$. Hence, $a_f = a_g$. And $f = g$ since f is a propensity function.

THEOREM: $\mathscr{L}$ is atomic.

Proof: Let $0 \neq a \in \mathscr{L}$. $\exists$ a propensity function f s.t. $f(a) = 1$, i.e., $a_f \leqslant a$. And a_f is an atom.

Note: Since $\mathscr{L}$ is weakly modular, it is also atomistic, i.e., $\forall a \in \mathscr{L}$, $a = \vee \{p \in \mathscr{L} \mid p \text{ is an atom and } p \leqslant a\}$.

Notations: For all atoms p I denote g_p the unique propensity function such that $g_p(p) = 1$.

THEOREM: $\forall a \neq b \in \mathscr{L}$, $\exists$ propensity function such that $g(a) \neq g(b)$.

Proof: $a \neq b \Rightarrow \exists$ atom p s.t. ($p \leqslant a$ and not $p \leqslant b$) or ($p \leqslant b$ and not $p \leqslant a$). Hence, $g_p(a) = 1 \neq g_p(b)$ or $g_p(a) \neq 1 = g_p(b)$.

THEOREM: $\mathscr{L}$ is distributive $\Leftrightarrow$ $\forall a \in \mathscr{L}$ and $\forall$ propensity functions g, one has $g(a) = 0$ or $g(a) = 1$.

Proof: If $\mathscr{L}$ is distributive, the following defines a propensity function for all atoms p: $g_p(a) = 1$ if $p \leqslant a$, 0 if not. For the "$\Leftarrow$" part, it is enough to prove that $\forall a, b \in \mathscr{L}$, $(a \vee b) \wedge b' \leqslant a \wedge b'$. Let p be an atom s.t. $p \leqslant (a \vee b) \wedge b'$. Since $g_p(a) \in \{0, 1\}$, $p \perp a$ or $p \leqslant a$. But $p \perp a \Rightarrow p \perp a \vee b$, a contradiction. Hence, $p \leqslant a$.

Note: $\mathscr{L}$ is distributive $\Leftrightarrow$ $\mathscr{L} = \mathscr{P}(\Gamma)$ where Γ is the set of atoms of $\mathscr{L}$. The above theorem states thus that classical mechanics is characterized by the fact that all the propensity functions assume only the values 0 and 1. In this sense propensity is a generalization of classical determinism.

4. CONCLUSION

I introduced and formalized the metaphysical assumption that there is an intermediate level between determinism and randomness, namely, a kind of non-determinism which is completely characterized by the actual properties of the system. I gave some arguments in favor of this assumption, based on quantum physics, and I proposed a definition of propensity functions in a rather general mathematical framework. Adding a very natural axiom to this framework, one gets a mathematical structure containing both classical and quantum mechanics. The question of whether the framework contains other structures is open.[4] The discovery of a lattice satisfying the axioms of Section 3, but not the covering law and without a physical content, would be a point against my propensity concept.

NOTES

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[1] Notice that the propensity that will actually show up in a specific case depends on the

property which is tested. In the case of a counter, for instance, the state of the particle determines the propensities of all properties, and the counter determines that it is the propensity of "the event click" that expresses itself.

[2] Actually, the assumption of ortho-complementarity would be enough, since ortho-modularity follows from the existence of measures (Gisin 1984).

[3] Axiom 2 is also true in non-separable Hilbert spaces (S. P. Gudder: 1975, *Int. J. Theor. Phys.* **13**, p. 419) and C*-algebras (Anderson 1979).

[4] I tried without success to find a lattice satisfying the axioms of Section 3, but not the covering law. In particular the lattices based on octonionic spaces (M. Günaydin, C. Piron and H. Ruegg: 1978, *Comm. Math. Phys.* **61**, p. 69) or "verifiable" spin-1 properties (B. O. Hultgren and A. Shimony: 1977, *J. Math. Phys.* **18**, p. 381) reinforce the assumption that no such lattice exists. Indeed the first one satisfies Axiom 2 and the covering law, while the second one does not satisfy Axiom 2 nor the covering law. Navara (1987, *Czechoslovak Math. J.* **37**, pp. 188–96) has presented an example of a finite ortho-modular lattice with exactly one state per atom evaluating it to one, but these states do not satisfy condition 2 of our measures.

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