The explanatory nature of constraints:
Law-based, mathematical, and causal

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This paper provides an analysis of explanatory constraints and their role in scientific explanation. This analysis clarifies main characteristics of explanatory constraints, ways in which they differ from "standard" explanatory factors, and the unique roles they play in scientific explanation. While current philosophical work appreciates two main types of explanatory constraints, this paper suggests a new taxonomy: law-based constraints, mathematical constraints, and causal constraints. This classification helps capture unique features of distinct constraint types, the different roles they play in explanation, and it includes causal constraints, which are often overlooked in this literature.

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1 Introduction Why are there limitations on the size of animals and the height of skyscrapers? In cities with multiple bridges why is it sometimes impossible to find a single path that crosses each bridge exactly and only once? When breast cancer spreads through the human body, why does it always present in the axilla first, as opposed to the leg, stomach, brain, or anywhere else? Phenomena that we seek to understand in the world are often limited in their presentation–they show up in some variations and not others. Some outcomes seem off-limits, not possible, or very uncommon, while others much more common, possible, and likely. What explains this?

Scientific explanations are often viewed as answers to explanatory-why questions, but the why-questions above look a bit different. Usually we are interested in explaining why a system takes on one possible value as opposed to others. For example, we might be interested in explaining why a patient has a fever versus a normal temperature, why a guinea pig has brown fur as opposed to white or black, and why a block slides down an incline at 1 meter per second instead of faster or slower. In these cases, the outcome we want to explain is contrasted with other outcomes that are also possible. Explaining such a target is often said to involve citing the factors it depends on, whether these are causal, mathematical, or of some other variety (Woodward 2019). However, some explanations appear different from this standard framework, as suggested by the questions above. In these cases we are not explaining the occurrence of one possible outcome over another, but instead why some values of the explanandum are not likely, or flat out impossible, while others are possible. These cases involves explaining the limits of an explanatory target.

According to standard views, these explanations are provided by constraints in what are sometimes alled "constraint-based explanations" or "explanation by constraint" (Green and Jones 2016; Lange 2017). These constraints are (i) claimed to have a mathematical, formal, or physical, character and they are (ii) said to provide non-causal explanations. Two types of constraints that are often discussed in this work are design constraints and topological constraints (Green and Jones 2016). Design constraints limit the process of designing, creating, or developing a final product,

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such as the evolutionary development of species in biology and the manufacture of products in engineering. These constraints clarify what systems it is possible (and impossible) to construct in a given context. On the other hand, topological constraints are mathematical features of a pre-existing system that limit the behavior the system is able to produce. These are said to capture various "constraints on mechanisms" in the biological sciences and the explanatory nature of the well-known Königsberg bridge example.[1] Both of these constraints are taken to be non-causal and they are said to figure in non-causal explanations, where they derive their explanatory power from formal or mathematical relations.[2]

Mainstream characterizations of explanatory constraints encounter various problems. First, these discussions characterize constraints and constraint-based explanation as non-causal, despite the fact that promising causal candidates exist. This is seen in various sociological explanations in which structural factors structural–often called "structural causes"–explain the limitations of some outcome of interest (Dretske 1988; Haslanger 2016) A similar pattern is found in examples of "physical" constraints such as: a bowl's surface constraining a rolling marble, river banks constraining a flowing river, and blood vessels constraining the movement of blood (Hooker 2012). Second, current design and topological classifications fail to clarify how explanatory constraints differ from "standard" explanatory factors and they fail to group constraints on the basis of similar features. For example, design constraints appear to come in both mathematical and empirical-law varieties, which have different features, sources of explanatory power, and roles in explanation. Furthermore, mathematical constraints appear to cross-cut this binary classification, as they are found in both the design and topological categories. Third, as scientists routinely state, the term "constraint" is often unclearly defined and inconsistently used. For example, while the notion of a "developmental constraint" has received significant attention in evolutionary biology, this "popularity has also bred confusion" as the term is often used in "distinctly different ways" (Gould 1989, 516).[3] Providing a clear and compelling definition of constraint–perhaps one with distinct subtypes or usages in different fields–is necessary for clear communication, effective theorizing, and progress in science and philosophy. How should we best understand explanatory constraints? What distinguishes explanatory constraints from standard explanatory factors? Are there different types of explanatory constraints and, if so, how should we understand their differences?

This paper addresses these questions by providing an analysis of explanatory constraints and their role in scientific explanation.[4] First, this analysis clarifies main characteristics of explanatory constraints, ways in which they differ from standard explanatory factors, and the unique roles they play in scientific explanation. Second, this work suggests a new taxonomy for distinguishing

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[1]For discussion of constraints on mechanisms see: (Chemero and Silberstein 2008; Silberstein and Chemero 2013; Huneman 2018b, 2010, 2018a) and for discussions of the Königsberg case see: (Euler 1956; Woodward 2019; Ross 2020).

[2]For example, Green and Jones (2016) discuss constraint-based explanation as distinct from causal mechanistic explanation. In these constraint-based explanations "general laws and principles–rather than causal, mechanistic details–carry the explanatory burden" (Green and Jones 2016). For further claims that design and topological constraints are non-causal see: (Wouters 2007; Huneman 2018a).

[3]However, simply accommodating all possible definitions–such that all of biology is a result of constraints–is also problematic because then "the meaning of the word would vanish" (Stearns 1986, 35). For more on the "chaos of constraint terminology" see (Antonovics and van Tienderen 1991).

[4]In this paper I am exclusively concerned with explanatory constraints, although other types of constraints receive attention. For example, my analysis does not apply to constraints that are used to narrow down a search space for some actual causal process in a system (Green and Jones 2016).

constraint types. This new taxonomy includes: law-based constraints, mathematical constraints, and causal constraints. This classification helps capture unique features of distinct constraint types, the different roles they play in explanation, and it introduces discussion of causal constraints, which are often overlooked in this literature.

2 Beginning an analysis. Consider potential examples of constraint-based explanation. A first example involves design-constraints in evolutionary biology that limit the possible body size of an animal. In this case the square-cube law, law of gravity, and features of musculoskeletal strength limit how large animal can be, while the square-cube law and principles of heat transfer limit how small the animal can be. In a second case, the topological structure of a city's bridges constrain potential walking paths, such that walking a path that traverses each bridge only once (an Eulerian path) is either possible or not. Third, various physical structures constrain the flow and movement of materials: river banks constrain the movement of water, blood vessels guide the flow of blood through the body, and various anatomical barriers dictate the spread of cancer through the body. In all of these cases some constraint is identified and said to limit, guide, or channel outcomes of the system of interest. These constraints are said to explain *why* the outcome can take on some states and not others and, in some cases, why particular states are impossible.

This analysis relies on a framework in which explanations are understood as involving: an explanandum (i.e. explanatory target), explanans (i.e. factors that do the explanatory work), and some dependency relation that connects the two. This framework has been supported by many philosophers and it is said to accommodate various types of scientific explanation, including causal and non-causal varieties (Jansson and Saatsi 2017; Woodward 2019). In this framework, the explanandum can be framed as an explanatory why-question, while the explanans addresses this question and explains the target of interest. Differences in the explanans explain differences in the explanatory target. For example, a gene variant explains the presence of a disease because differences in the variant (mutated or not) explain differences in the disease state (present or absent). In other non-causal cases, the stable nature of an ecosystem (in the face of random species extinction) is explained by its topological structure–namely, the fact that it has the mathematical property of being scale-free (Huneman 2018a). In both cases, the explanans "makes-a-difference" to the explanandum and it does so in a way specified by the dependency relation connecting them (Jansson and Saatsi 2017; Woodward 2019).

I am going to suggest that constraint-based explanations have at least four characteristic features that distinguish them from "standard" scientific explanations. In constraint-based explanations, constraints are factors that: (1) limit the values of the explanatory target of interest, (2) are often conceived of as separate from or external to the process they limit, (3) are considered relatively fixed compared to other explanatory factors, and (4) structure or guide the explanandum outcome, as opposed to triggering it. I briefly describe these four features before analyzing examples of these constraints in more detail.

First, explanatorily relevant constraints (1) limit the set of values an explanatory target can take on, given some initial set of values. This is often understood in terms of a constraint reducing the degrees of freedom of an outcome (Hooker 2012)–they clarify what values the explanatory target is limited to and why it is so limited. This limiting influence reveals an important feature–constraint-based explanations contrast possible versus impossible outcomes of the explanatory target, while standard explanations consider only possible outcomes. For this reason, constraints can provide

impossibility explanations, which explain why some outcome (or set of outcomes) are not possible.[5] Although constraints are often described as negatively limiting an outcome, they can also have a "positive meaning" in which they are viewed as positively "channeling" or guiding a system to some outcomes over others (Gould 1989, 518-9). Constraints can limit to different degrees making a larger or smaller set of outcomes available given some initial set of values. If the constraint limits to a single value or a very small number of values, the outcome is said to be "fully determined" by the constraint (Huneman 2018a, 131).

Second, (2) instead of directly limiting the explanatory outcome of interest, the constraint is often said to limit some distinct *process or system* that produces this outcome. The constraint is characterized as conceptually distinct from and even somewhat external to the process it influences–it is through this influence that the constraint channels, guides, and limits the behavior of the system. In this sense, the constraint plays an overseer, regulator role and has a "directing force" (Gould 1989, 518) over the system and outcome it produces. By explaining which explanatory outcomes are available and which are off-limits, the constraint has more of a coarse-grain control, relative to other explanatory factors that determine which specific outcome, among all those available, the system will exhibit.

Third (3), explanatory constraints are often characterized as relatively fixed or difficult to change relative to other explanatory factors. This is because they are either harder to change or they are held in an unchanged state for a long amount of time, relative to other factors. The fixed nature of constraints is reflected by the fact that they are sometimes referred to as "structural" factors or as "structuring" various outcomes (Dretske 1988; Haslanger 2016). This compares them to a building's frame and an organism's skeletal structure, which are fixed, physical scaffolds that limit various outcomes of these systems. This seemingly "fixed" nature of constraints can lead them to be overlooked as explanatory factors and can lead to the assumption that they cannot be manipulated to change the effect of interest.

A fourth (4) feature of constraints is that instead of triggering the occurrence of some effect, they more often shape its presentation(Dretske 1988). This is related to the fact that constraints are often fixed factors, which limit some separate, external process. As constraints are typically unchanging, they are set in place and ready to guide outcome whenever its triggering factors are set off. As an ordinary life example of this, consider a toy boat that is dropped into the upstream portion of a river. The riverbanks operate as constraints on the trajectory taken by the boat, while its launch triggers the start of this causal process.

While explanatorily relevant constraints have these features in common, they also appear to have particular differences. It will be helpful to get a sense of what different types of constraints figure in scientific explanation, how exactly they meet the four criteria specified, and how they differ from each other in terms of their characteristics and explanatory power.

2.1   Law-based constraints. In order to see which constraint types are found in scientific explanations, let us first consider examples of design explanations in evolutionary biology. In the biological sciences, design explanations involve the evolutionary development of an organism and its various characteristics. These cases are often analogized to engineering examples, in which the manufacture of some final product is constrained by factors such as time, materials, and overall

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[5]While how-possibly explanations often focus on whether a particular explanatory outcome is possible or not, the constraint-based explanations considered here focus more on a general division between impossible and possible states.

cost. In a similar way, organism development and evolution can be constrained by various factors, often called "developmental constraints," which limit the organism's final set of morphological and other features (Maynard Smith et al. 1985).

Consider an example involving developmental constraints, which concerns blood vessel size. Particular constraints limit the upper and lower bounds of blood vessel size given the goals of these structures, which include the delivery of nutrients and removal of wastes from different sites of the body. The supply and removal of these materials takes place via diffusion, in which materials travel across walls of these vessels. Given this goal, two main constraints dictate blood vessel size: Poiseuelle's law and the laws of diffusion. Poiseuelle's law states that as blood vessel radius decreases, resistance to flow increases by a power of four. In this manner, small changes in radius drastically increase blood flow resistance, as "a blood vessel that is half as wide as another has a resistance to flow 16 times greater" (Gilbert 2010, 457). This drastic increase in resistance will limit how small the vessels can be, as high enough levels of resistance will become impossible for the pumping heart to overcome and stagnant blood can clot. This law might seem to indicate that relatively large vessels will be ideal for transporting blood and, thus, selected for in organisms. However, the size of vessels are also limited in the upward direction, by the laws of diffusion. If vessel size is too large, blood flow will be too fast to allow for the diffusion of nutrients and wastes into and out of the circulating blood.[6] A small enough vessel ensures that the vasculature can serve this purpose. Thus, Poiseuelle's law places a lower limit on blood vessel size, while the laws of diffusion place a higher limit. In other words, "the constraints of diffusion mandate that vessels be small, while the laws of hydraulics mandate that vessels be large" (Gilbert 2010, 457).

To say that these laws are explanatory means that changes to these laws, would result in changes to these limits. If blood resistance increased with the square of vessel radius, as opposed to the fourth, smaller vessels would have less resistance, which would decrease the lower limit on vessel size. Relatedly, increasing the radius-resistance-relationship (such that, for example, resistance increased to the fifth power of the radius) would increase this lower limit on vessel size. These empirical laws serve as the explanans and they relate to the explanandum (limits on vessel size) via a change-relating dependency relation. This dependency relation explains the boundary between available and unavailable vessel sizes and it shows how this boundary would change if the empirical laws were different. Of course, we do not typically think that we can change the laws of nature or that they change on their own–this unchangeable characteristic of the explanans and the "boundary" feature of the explanandum helps distinguish these cases from standard scientific explanations.

In clarifying this distinction, it is helpful to first explicate how these laws are *constraints*, as opposed to standard explanatory factors. Notice that the scientific laws in this example meet the explanatory constraint criteria listed in section 2. These empirical laws (i) constrain or limit the values of the explanatory outcome and they do so by (ii) influencing a distinct process–evolutionary development–that produces this outcome. In this sense these constraints are represented as external to the system of interest and as providing a guiding, regulatory force on it. Furthermore, these constraints are (iii) fixed in the sense that they are not viewed as manipulable–while we might consider the consequences of Poiseuelle's law scaling to the fifth power as opposed to the fourth, this is not a change that we can implement (or even conceive of implementing). Finally, Poiseuelle's law and the law of diffusion are viewed as constant, "structuring" factors, which guide the evolu-

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[6]Within these upper and lower bounds on size, the circulatory system has a hierarchy of vessels of different sizes that support distinct goals. Larger vessels are specialized for transport and smaller ones for diffusion (Gilbert 2010).

tionary process once it starts up, as opposed to triggering its start. These laws are an ever-present background condition that structures the outcome as opposed to initiating it.

While the laws in this case are explanatory constraints, they differ from other constraints in terms of their features and explanatory power. The laws in this example are best understood as empirical law constraints as opposed to mathematical or causal constraints. While these empirical laws can be represented with math, they are not directly or exclusively derived from mathematics. These empirical laws capture relationships that must be discovered in the natural world, while mathematical properties do not have this feature (as they can be known *a priori* or identified without empirical study). So the explanans is an empirical law (as opposed to a mathematical property) and the dependency relation involves empirical information, which prevents it from qualifying as a mathematical explanation (Woodward 2019). This captures an important difference between two different types of constraints that are referred to as "formal" and that are conflated in the philosophical literature–namely, empirical and mathematical constraints. While both constraint types can be represented with mathematics, empirical law constraints are discovered through empirical study of the natural world and they fail to meet our standard conception of a "manipulable" explanans. As will be discussed further in the next subsection, mathematical constraints can be identified without empirical study and are often viewed as manipulable.

Some empirical relationships capture causal regularities–why not view the laws in this example as causal constraints? These laws are not well-understood as causal because they fail to meet basic criteria for causal relevance. An interventionist conception of causal explanation involves an explanans that can be manipulated or changed through intervention, but this strains our conception of the explanatory role of these laws. We can consider manipulation of a gene variant and how this would change an explanatory outcome, but it is difficult to conceptualize what it would mean to manipulate a law of nature (the laws of diffusion, for example). To be clear, laws of nature can capture various change-relating relationships–this is not the manipulation we are considering. In this case we are considering changing the law entirely, in terms of the regularities it contains and how they relate various properties. The explanatory role of these empirical law constraints fits a first type of non-causal explanation discussed by Woodward, in which the explanans is non-manipulable and the dependency relation involves empirical information (Woodward 2019). Thus, these empirical laws are a sort of non-causal, non-mathematical explanatory constraint.[7]

Perhaps one of the most useful aspects of law-based constraints is that they help explain why we find particular patterns of phenomena in the world–why some outcomes are more common than others (Wimmsatt 1986; Kauffman 1993). This is often discussed in the context of evolutionary biology, where there is an interest in explaining why–given the potential for such a large number of traits–only a narrow set of traits are repeatedly found. An interest in explaining these "limitations on phenotypic variability" has been expressed by many scientists in this community (Maynard Smith et al. 1985, 269). These patterns have been viewed as motivating and informing a study of the concept of constraint as "the strong bounds that nature places on diversity provide our best starting point for a study of limits" (Gould 1980, 39). Notice, again, how constraints address a different

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[7]Biologists sometimes distinguish "universal" (or "formal") constraints from "local" (or "historical") constraints (Maynard Smith et al. 1985; Gould 1989). Universal constraints are taken to apply to "all physical systems..., to all things build out of the materials in question..., and to all physical systems of the requisite complexity," while local (or historical) constraints that "are confined to particular taxa" (Maynard Smith et al. 1985, 267). For further work on global and local constraints in evolutionary biology, see: (Forber 2010)

type of explanatory target than is standardly examined in philosophical literature on explanation. Instead of explaining a particular outcome in the world–the height of a flagpole's shadow, red eye color in fruit flies, or the speed of a block sliding down an incline–constraints explain why a set of outcomes are available versus unavailable (or impossible). This focus on explaining limits of the explanatory target is likely to be more common in some scientific contexts than others, likely cases in which there is some limited presentation of an outcome that is surprising or in need of explanation. While many law-based constraints are found in design explanations they arise in other domains as well. Examples of this include the role of gravity (and other constraints) in limiting an organism's jumping height and various motor patterns or the laws of motion, diffusion, and conservation of energy in limiting various biological processes (Haywood and Getchell 2009, 31).[8]

2.2 Mathematical constraints A second type of constraint that figures in scientific explanation are mathematical constraints. Paradigmatic examples of this constraint type are found in cases of topological explanation. As an example of this, consider the Königsberg bridges, in which seven water-spanning bridges connect up two central islands to nearby landmasses. In the eighteenth century, there was interest in identifying whether it was possible to walk a single path that traversed each of these bridges exactly and only once. After various individuals failed to find such a path, Euler provided a mathematical proof demonstrating that such a single-pass route (what we now call and Eulerian path) was impossible, given the topological structure of the bridge system. When this bridge system is represented graphically, particular topological features of the system are evident, which constrain available walking paths and explain why some paths are not possible (and off-limits). In particular, when bridges are represented as edges and landmasses as nodes, it can be deduced that an Eulerian path is possible if all nodes are connected to each other and there are "either zero or two nodes of odd degree" (Euler 1956; Woodward 2019).

In this bridge example, the topological properties are explanatory in the sense that they "make-a-difference" to the explanandum of interest–changing these properties would change the behavior of the system. If the Königsberg bridge system were modified such that there were either zero or two landmasses with an odd number of bridges, walking an Eulerian path would then be possible. Not only are these mathematical properties explanatory, they are explanatory constraints in the sense that they limit the behavior of the system–they constrain which routes are available and unavailable to walkers. Again, notice that these topological properties meet the criteria for explanatory constraints listed in section 2. By constraining which routes are available to walkers, these topological properties (1) limit the values of the explanatory target (available walking paths) and (2) they are represented as external to the process that they limit (individuals walking along bridges). While the bridge structure in a given city can be changed, we typically view it at (3) relatively fixed, especially compared to other changing factors (such as moving travelers). Finally, (4) the bridge structure "shapes" the explanatory target, without triggering a particular outcome. The bridge structure is distinct from factors that initiate walking or the causal process that is ultimately constrained–this structure controls which outcomes are available and unavailable to the system, not which particular available outcome is chosen, in a given setting, or when exactly this outcome takes place.

How should we understand the mathematical constraint in this example? In this example, topology is a mathematical constraint as opposed to a law-based or causal constraint. This is

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[8]For example, gravity is a constraint that "encourages certain motor patterns while eliminating others" (Haywood and Getchell 2009, 31).

apparent in the sense that the explanans (topological structure) is a mathematical property and that the dependency relation is provided by mathematics, as opposed to empirical information. Once the topology of the system is specified, the explanation is provided by mathematics alone– the explanandum is derived from mathematical understanding as opposed to empirical study or observation in the world. This is contrasted to causal explanation, in which the dependency relation consists of empirical information and cannot be derived exclusively from mathematics. Whether a gene mutation causes a disease or not, cannot be determined through mathematical analysis alone, as some empirical input is needed.[9] These mathematical constraints fit a second type of non-causal explanation discussed by Woodward (2019). These cases fail to qualify as causal explanations, but this is not because the explanans is unmanipulable, like the law-based constraints–we can make perfect sense of altering bridge structure and how this changes the explanatory outcome. These cases fail to provide causal explanations, because their dependency relation is mathematical, as opposed to empirical.

Mathematical explanation is popular in current philosophy of science, as the literature corrects for a decades-long preoccupation with causal explanation. Mathematical explanations are standardly viewed as noncausal (Batterman 2002, 2010; Lange 2013) and they are often associated with constraints (Lange 2017).[10] However, not all mathematical explanations are constraint-based explanations– this is because not all explanatorily relevant mathematical properties are constraints. One class of mathematical explanations that make this clear are optimality explanations. Optimality explanations are commonly discussed in the context of evolutionary biology, in which mathematical techniques from engineering and economics (namely, optimization theory) are used to explain phenotypic outcomes in organisms (Maynard Smith 1978). These cases involve explaining why some phenomenon manifests in the world on the basis of the fact that it captures an optimal solution, given a specified set of contextual factors, constraints, and pressure to optimize (from natural selection, for example) (Rice 2013). This framework is used to explain phenomena such as (i) the hexagonal shape of bee's honeycombs on the basis that this shape is more efficient in using a smaller amount of wax than other shapes, (ii) the prime number life-cycle of cicadas on the basis of the fact that this number minimizes intersections with predators and competing species, and (iii) the fact that sunflower heads have their exposed seeds packed in a Golden ratio distribution as this maximizes seed density (Lyon and Colyvan 2008; Baker 2005; Lyon 2012). Each of these explanations involves a conception of evolutionary fitness and a mathematical characterization of how this fitness is best attained.

While these optimality cases are mathematical explanations, they are not constraint-based explanations because they do not explain a hard limitation of the explanatory target.[11] In these optimality cases, the less optimal phenotypes are not strictly off-limits, as organisms can manifest suboptimal traits as they evolve toward more optimal ones (and even as they don't). This is discussed by Lyon who describes the potential evolutionary history of a bee community's honeycomb structure. In this example, the bees might first build their honeycombs with triangles, then move on to squares, and ultimately hexagons, as each of these subsequent shapes is more efficient and fit than the former. The efficiency and fitness of the final phenotype explains why "we only see hexagon bees" (Lyon 2012).[12] But even if some organisms are inching toward these optimal states,

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[9]This is supported by claims that causal relationships cannot be know *a priori* (Hume 1748, 19).

[10]Lange (2017) provides a rich analysis of constraints and their role in mathematical and other explanations.

[11]This is related to the fact that the mathematics they employ do not function as constraints.

[12]Similar evolutionary stories could be told for cicadas converging on prime number life-cycles and sunflowers

they still spend signifiant time in less than optimal ones, showing that they are not strictly off-limits. Furthermore, other organisms seem not to optimize at all, which is evident by the fact that "maladaptive traits" are present and even appear to be fixed in the population (Maynard Smith 1978, 37). Again, this shows that these less-than-optimal traits are not strictly off-limits and that they are more permanent than is sometimes suggested (Seger and Stubblefield 1996). Now scientists do cite "constraints" in these optimality cases, but they are used to specify available phenotypes to optimize on, so that optimal states can ultimately be explained and distinguished from sub-optimal ones.[13] If these cases focused on explaining possible versus impossible phenotypes, they would fit a constraint-based explanation framework, but this is not their focus. Instead of explaining this impossible-possible distinction, they use specification of possible phenotypes to divide them into optimal and suboptimal states, which is the primary explanatory target. As these optimality explanations do not specify explananda outcomes that are strictly off-limits–but merely more or less optimal given various criteria–they fail to qualify as constraint-based explanations.

In order for mathematical explanations to qualify as constraint-based they must strictly limit some explanatory target. This often takes the form of providing an impossibility explanation, in which the explanandum includes a possible-impossible contrast. Consider the commonly discussed mathematical explanation of a mother dividing strawberries. In this example, a mother has twenty-three strawberries and she aims to divide them evenly among her three children, but she is unable to do this. In fact, it is impossible for her to evenly divide in this case. Why is it impossible for her to do this? The fact that this is impossible is explained by mathematical facts–basic division specifies that twenty-three cannot be evenly divided by three.[14] Notice that these mathematical facts do not just explain what is optimal, likely, or favored to happen. They specify what is and is not possible–these are strong "necessities" that indicate which outcomes are available and which others are simply off-limits. As Lange states, "[m]other's strawberries were not distributed evenly among her children because they *cannot* be" and "[t]he Königsberg bridges as so arranged were never crossed because they *cannot* be crossed" (Lange 2013, 491). In these cases a strict constraint explains why some outcomes are impossible or off-limits.

Mathematical constraints are important, in part, because they capture a uniquely, non-empirical way of identifying influences on outcomes in the world. In some cases, these mathematical constraints address explanatory why-questions when empirical methods cannot. The Königsberg case is a helpful example of an explanatory why-question that cannot be sufficiently addressed with empirical study. The inability to find a single pass route through this bridge structure–after many "empirical investigations" and attempts–was no guarantee that this was impossible. These empirical studies could not decidedly rule out such a future finding. Mathematical analysis, however, could definitively exclude this outcome and provide an impossibility explanation. This captures an important difference among factors that are considered "formal constraints" in much of the literature. While law-based and mathematical constraints can be represented with math, they differ with respect to their reliance on empirical information.

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evolving toward a Golden ratio seed distribution in their flower heads.

[13]In particular, they capture the "strategy set" which is "the set of possible phenotypes on which selection can operate" (Maynard Smith 1978, 32).

[14]My analysis of constraints as explaining impossibilities is related to Lange's analysis of mathematical explanation as showing how something is strongly "necessary" and why particular outcomes are impossible (Lange 2013).

2.3 Mixed constraint explanation. While it helps to categorize constraints into different types (e.g., law-based, mathematical, and causal), this does not mean that individual explanations only rely on one constraint at a time or multiple constraints of the same "type." For some explanations, multiple constraints, and sometimes different types of them, are necessary to explain the outcome of interest. In fact, some of the most well-known examples of constraint-based design explanation fall into this category. In order to see this, consider well-known design explanations for limitations on animal body size. Although animal species on our planet exhibit various types of phenotypic diversity, there is significant similarity when it comes to body size. In particular, for terrestrial animals there seem to be hard limits on maximum and minimum body size and these limits are especially constraining when it comes to animals with particular "shapes." For example, our planet does not have any terrestrial organisms that are the size of skyscrapers or freight airplanes. In terms of body shapes, our planet lacks "spindly-legged creatures" that are large–there are no ants the size of elephants (or even the size of humans). While fictional characters such as Godzilla, huge spiders, and giant humans do not exist, we often hear that it is, in fact, impossible for them to exist. Why is this? What explains this upper limit on animal body size?

The upper limitation on terrestrial animal body size is explained by two main constraint types: mathematical constraints, such as the square-cube law, and law-based constraints, such as gravity and mechanical properties of the body (including materials and shape). The square-cube law is a mathematical constraint, which describes differences in how surface area and volume scale as a shape changes in size. The square-cube law was first discussed by Galileo, who applies it to load-bearing considerations in animals and engineered structures (Galilei 1632).[15] According to the square-cube law, when a square with length "l" is enlarged, its surface area will scale with the *square* of its length, while its volume will scale with the *cube* of its length. This has some interesting consequences for terrestrial animals–if their size were to increase while keeping their shape and proportions the same (isometric scaling), their strength would quickly diminish and, due to the force of gravity, their bones would be crushed under their own weight. This is because their mass would quickly overwhelm their musculature and bone strength, as their mass would increase by a power of three, while the cross-section of their muscle (and bone) would only increase by a power of two. Or as Galileo states:

> "you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art of in nature...if one wishes to maintain in a great giant the same proportion of limb as that found in an ordinary man he must either find a harder and stronger material for making the bones, or he must admit a diminution of strength in comparison with men of medium stature; for it his height be increased inordinately he will fall and be crushed under his own weight" (Galilei 1632, 130-1).

This reasoning is related to claims that the fictional Godzilla character and elephant-sized ants are both impossible–both would be crushed under their own body weight. In these cases, it is not just the square-cube law alone, which explains the limits on the size of animals and man-made structures. This limitation is explained by the conjunction of this mathematical principle[16] with various empirical laws, such as gravity and its relation to musculoskeletal properties, bone-strain,

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[15]This Galilean example is discussed by Green (2015) who considers a type of constraint-based generality in biology.

[16]Notice that while the square-cube law is referred to as a "law" it is a mathematical principle that can be derived without empirical studies.

and mechanical properties.[17] In order to see the explanatory relevance of both constraints, consider situations in which gravity is perceived to change, while the square cube law is held fixed. As an example of this consider aquatic environments, in which the force of gravity is lessed due to the offsetting forces of buoyancy. In this environment, marine animals can reach a larger body size as they experience decreased bone-strain (and their limbs need not support their entire body weight) (Goldbogen 2018). As buoyancy offsets the perceived influence of gravity, this reduces bone-strain for aquatic animals versus terrestrial ones, which increases the upper-limits on marine animal body size.

3 Causal constraints. In the literature on constraint-based explanation, causal constraints have received little to no attention.[18] Instead, this work focuses on "formal constraints," including mathematical properties and general principles, which are considered non-causal in nature. The assumption that constraints are non-causal is evident by claims that constraint-based explanation and causal-mechanical explanation are "diverging scientific practices" (Green and Jones 2016, 343).[19] Similar views are found in the literature on mathematical explanation, in which constraints provide a unique form of necessity that is said to be absent from cause-effect relationships (Lange 2013). However, given the definition of constraint provided in this paper's analysis, there is nothing that prevents causal factors from serving as explanatory constraints. In fact, numerous examples of these constraints exist in scientific and ordinary life contexts. In relation to this, many cases that are taken to be representative of constraints are well understood as causal in nature–this is seen in cases such as marbles rolling along a bowl and rivers flowing along riverbanks (Hooker 2012).

This analysis suggests that explanatory constraints have at least four main features, outlined in section 2. In the literature on causal explanation, some causal factors have been flagged as having one or more of these features and, due to this, they have been viewed as playing a unique explanatory role. This is seen in the work of Garfinkel (1981), Dretske (1988), and Haslanger (2016), in which "structural causes" are viewed as structuring, guiding or shaping a final explanatory outcome of interest. These structural causes are often distinguished from "triggering causes," which do not dictate the particular form or shape of the outcome, but determine when in time it takes place (Dretske 1988). In medicine, "structural cause" often refers to some larger-scale, physical entity– such as a tumor, vascular routes/pathways, tissue malformation, etc.–which produces a medical outcome through larger-scale, physical forces (e.g. compression, occlusion, herniation, etc.) as opposed to effects at the cellular level (Singhal 2019). In sociological and political theory, structural

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[17]A similar set of mixed constraints are used to explain lower limits on animal body size. As specified by the square-cube law, smaller animals will have a larger surface area to volume than larger animals. This larger ratio combined with laws of heat transfer, makes it more difficult for smaller animals to retain heat–heat is lost through surface area, animal needs to keep internal temperature within particular range. As this ratio increases (with decreases in body size) the smaller body size in untenable, as the animals cannot produce enough heat to offset losses, and maintain a high enough (and stable) internal temperature. Square-cube law is a mathematical constraint, heat transfer laws are empirical–need both together to explain this lower limitation on animal body size.

[18]While Bechtel (2018), Winning and Bechtel (2018), and Winning (2018), discuss the role of constraints in mechanisms, they do not interpret constraints themselves as causal. Instead, constraints are viewed as factors that "ground causal powers" (Winning and Bechtel 2018, 293)(Winning 2018, 1401).

[19]This is also suggested by Huneman's claim that his topological explanations–which are taken to be an example of constraint-based explanation–are non-causal, as they are "seemingly distinct from mechanistic explanations" (Huneman 2018b, 44).

causes are "large-scale social forces" that constrain and limit the behavior of individuals, such as poverty, racism, and gender inequality (Farmer et al. 2006, 1686).

Consider two types of causal constraints, which can fall under the heading of structural causes. A first example are causal constraints that guide changes in the spatial location of some entity over time. Examples include the marble and river cases, in which the curved edge of the bowl and winding boarders of the riverbank guide movement of the rolling marble and flowing river, respectively. These constraints reduce the potential movement of some entity from some unconstrained three-dimensional space to a more narrow set of available routes. This is similar to the manner in which roadways constrain traffic and maze-structures constrain the movement of an entering traveler. Other examples of this constraint type are found all throughout the biological sciences–these include blood vessels constraining flow of blood, nerve tracts constraining the flow of nerve signaling, and lymph vessels constraining the spread of cancer. These constraints are physical and non-specific in the sense that they provide a sort of blunt physical barrier or "anatomic avenue" that encourages movement in particular directions and prevents movement in others (Meyers et al. 1987). In the context of breast cancer, initial spread to the axilla is explained by the fact that lymphatic vessels channel fluid here first, before other areas. In the biological sciences, these constraints are typically illustrated with pathway maps, circuit diagrams, and connectomes, which outline the different possible routes that some entity is confined to and can travel along. These are similar to roadmaps, which outline the possible routes that cars and other vehicles can travel along.

A second type of causal constraint are factors that guide changes in the makeup or constitution of some entity over time. Examples include constraints that guide metabolic pathways, stem cell pathways, and developmental pathways. In each of these cases, some entity moves through a sequence of constitutional changes until it turns into a final product. This is seen in a factory assembly line, in which some initial substrate is sequentially converted into a downstream product. These sequential changes are guided by constraints, which limit which types of downstream products the upstream substrate is converted into. In this second set of cases, instead of constraining changes in the spatial location of some entity, these factors constrain changes in the makeup or constitution of the type of downstream entity that is produced. These constraints determine which types of downstream forms the product can and cannot be converted into. These constraints are also often represented with pathways maps, but in these cases the steps along the pathway represent changes in an entity's makeup over time, as opposed to changes in its spatial location.

Causal constraints can limit more than just spatial location and constitution. In social and political contexts, causal constraints can limit various behaviors or decisions of individuals, which do not fall into one of these two explananda types. However, what is common across all of these causal constraints types is that they "structure" and limit the explanatory outcome–they determine which set of outcomes are available to the system. In fact, the "structure" reference in "structural cause" likely draws on a number of related meanings. First, some of these constraints are "structural" in the sense that they are a "physical" cause, which limits outcomes by physically blocking or reducing options. We see this with marble, river, blood vessel cases, in which physical barriers guide the flow or moment of a causal process. Notice how different this is from law-based and mathematical constraints, which both limit the outcome, but are not as tangible as physical barriers. Other "structural" causes, which lack this overt, physical presence–such as particular social cause–are still called structural in order to highlight their similarity to the dominating guidance of these more barrier-like causal constraints. Second, structure can also refer to a perceived fixed nature of the cause, in the sense that it does not change much (or at all) relative to other explanatory factors.

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Just as there is a fixed and character to a building's frame and an organisms skeletal system, these causal constraints have a fixed, stable nature.

In the two cases above, the constraints reduce available values of the effect of interest, in particular, they limit the location of movement and type of product formed. Similar to law-based and mathematical constraints, these factors dictate which outcomes are off-limits and which are actual possibilities. However, unlike law-based and mathematical constraints, the constraints in these examples are causal in nature. These constraints meet both criteria for causal explanation that law-based and mathematical constraints could not fulfill–they involve a (a) manipulable explanans and a (b) dependency relation that contains empirical information (Woodward 2019). Notice that if the surface of the bowl or the curvature of the riverbanks were changed, this would change the downstream movement of these causal processes. This captures a dependency relation which specifies how changes in the constraints "make-a-difference-to" and explain, physical location of the marble and flowing river. However, for this dependency relation, (a) the explanans is manipulable (we can consider changing the bowl's surface and the riverbanks curvature) in a way that is not available to us for laws of nature. Furthermore, (b) the dependency relation in this case is derived from empirical investigation, as opposed to mathematical derivation alone. In order to know how the bowl's surface interacts with the marble, we need to perform studies in the world as opposed to relying on mathematics alone.

This suggests that these factors are causal, but are they really causal *constraints*? Why not just view them as standard causal factors? While these constraints are causal, they are importantly different from standard causal factors that figure in scientific explanation. This difference is well-captured by the fact that these causal constraints meet the four constraint criteria specified in section 2, while standard causal factors need not meet these criteria. A first obvious difference here is the role that causal constraints play in limiting the explanatory target or reducing the potential values it can take on. Standard causal factors dictate (or control) which of a set of possible outcomes some system *will* take on, while constraints determine which set of outcomes is possible or available, versus impossible. If a light switch is wired to a light bulb and not a fan or a toaster, the result of flipping the switch is limited to turning the bulb on/off, as opposed to producing outcomes in the other systems. The circuit captures a causal constraint in that it limits the flow of electricity to a particular downstream system and explains what downstream outcomes are possible or not. If this switch is flipped, turning on the fan or toaster is not a possibility, while turning on the light bulb is. This example also well captures the "structuring" feature of causal constraints, which differs from the "triggering" feature that other causes can have (Dretske 1988). In this case, the electrical circuitry is a structuring cause as it structures the final outcome and dictates what form the final outcome will take, but is does not have the capacity to "trigger" this outcome, or control when it takes place, which is a feature of the switch. The circuit is represented as more "fixed" and unchanging relative to the switch, which is more easily manipulated.

Causal constraints are particularly significant for explanations of biological systems, in a way that law-based and mathematical constraints are not. In order to sustain life, biological systems need to produce highly specific outcomes, with a high degree of accuracy, and in a way that can be repeated with high fidelity. Development of cells, tissues, and organisms often follows a very precise sequence of steps that are highly-regulated. Any misstep in this process can produce an unexpected product or outcome that can easily lead to organism death or pathology. Causal constraints are responsible for ensuring that these systems have fine-grained control over the specific type of product or outcome produced, when it is produced, and that this can be repeated faithfully

during the organism's lifetime. Essentially, causal constraints can be considered a prerequisite for the order required by biological life. Despite their important role, causal constraints are often overlooked in scientific and ordinary life explanations. When explaining signal propagation in neurons we more often cite the triggering stimulus than the cellular membrane that guides flow. When explaining why light bulb in room turned on, we are more likely to cite the switch on the wall than the circuitry that connects them. Indeed, structural causes are often a topic of interest because they are so often overlooked in explanations of individual and societal outcomes (Krieger 2011). Reasons for this inattention to causal constraints are likely numerous, complicated, and deserving of serious attention and analysis. At the very least, the conception of constraints as relatively fixed, unchanging, or difficult to manipulate are likely to play a significant role. Excellent work on structural and causal factors exists (Dretske 1988; Haslanger 2016; Krieger 2011) and will likely motivate further work on questions in this area.

4   Conclusion   This paper has considered the role of constraints in scientific explanation. A new taxonomy of constraints has been proposed, which includes law-based, mathematical, and causal constraints. While there are likely other constraint types or constraints which fail to comfortably fit these categories, the aim of this analysis is to provide useful framework for many explanatory constraints, even if it is not exhaustive. Advancing the discussion on what constraints are, how they figure in explanation, and how they differ, if at all, from standard explanatory factors will likely involve a careful, piecemeal process of making distinctions among constraint types. Such analyses should be open to the use of constraints in different scientific contexts, ways in which they are differently defined, and the roles they play outside of scientific explanation.

In engaging in the topic of constraint-based explanation, this paper makes three main contributions to this literature. First, it outlines four criteria that explanatory constraints will often meet. These criteria help clarify the role constraints play in scientific explanation and how they differ from other "standard" explanatory factors. Second, this analysis distinguishes among three different types of explanatory constraints, namely, law-based, mathematical, and causal constraints. These distinctions have to do with where these constraints derive their explanatory power and how they relate to the explanatory target of interest. Finally, this paper brings attention to a type of explanatory constraint that has been largely ignored in this literature, namely, causal constraints. Careful attention to the role these factors play in explanation helps reveal their different characteristics, how they differ from "standard" explanatory factors, and how they explain the limited presentation of phenomena in the world.

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