How Much Change is Too Much Change? Rethinking the Reasons Behind the Lack of Reception to Brouwer's Intuitionism

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Abstract

The paper analyzes Brouwer's intuitionistic attempt to reform mathematics through the prism of Leo Corry's philosophical model of "body" and "image" of knowledge. Such an analysis sheds new light on the question of whether Brouwer's intuitionism could at all be attractive to broader groups of mathematicians. It focuses on three characteristics that are unique to Bouwer's reformation attempt and suggests that when considered together, they combine to provide a more complex understanding of the reasons behind the lack of reception to Brouwer's intuitionism than any of the three can offer alone.

1. Introduction

Brouwer's intuitionistic program was an intriguing attempt to reform the foundations of mathematics and was probably the most controversial one within the contours of the foundational debate during the 1920s (van Stigt 1990; Hesseling 2003). Historians and philosophers of mathematics have tried to account for the reasons why Brouwer's intuitionism did not prevail. Some associate the demise of Brouwer's intuitionism with his dismissal from the editorial board of the *Mathematische Annalen* in 1928 (van Atten 2004). Others suggest that the lack of reception derived from technical difficulties within Brouwer's mathematical arguments (Epple 2000) or due to his awkward and too-technical style of writing (van Dalen 2013).

In the following pages, I wish to discuss a specific aspect of the question of whether Brouwer's far-reaching intuitionistic program could at all be attractive to broader groups of mathematicians. In order to do that, I would like to consider the story of Brouwer's intuitionism in light of Leo Corry's model of image and body of knowledge, and alongside Corry's compelling analysis of Van der Waerden's *Moderne Algebra*, which created new knowledge from mathematical notions that already existed. I intend to focus on three significant differences between the stories of Brouwer and Van der Waerden: on their different motivations for change, on the scope of the change, and on the implications of using familiar mathematical concepts (as opposed to introducing completely new notions). The variations between the two stories, I shall argue, offer a new perspective on the lack of reception to

Brouwer's intuitionism, that is owed not only to technical difficulties within the theory but to a combination of a deep philosophical motivation (with which mathematicians were less sympathetic), a too comprehensive reformation, and contradicting new mathematical concepts.

The terminology of 'image of knowledge' and 'body of knowledge' is borrowed from Yehuda Elkana's work. According to Elkana, the 'body of knowledge' is where the research is being done; thus, it consists of different theories, concepts, and mechanisms (Elkana 1978, 315). The 'images of knowledge' governs particular aspects of scientific activity that the 'body of knowledge' does not address, like: sources of knowledge, the legitimization of knowledge, the audience of knowledge, and relatedness to prevailing norms and ideologies. Building on Elkana's theory, the process of scientific progress can be described as engaging with two different types of questions: the first addresses the methods used in the process of making a discovery or forming a new theory, and the second addresses the guiding principles and normative boundaries of the discipline itself.

Unlike other disciplines, mathematics is uniquely endowed with a special interconnection between its body and image of knowledge. The reflexive aspect of mathematics enables it to examine the nature of the discipline itself by applying the same framework that is used in everyday methodological practice[1] (Corry 1989). Some mathematical theories can be easily

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[1] Consider, for example, proof theory. No other discipline has a dedicated practical doctrine about how its methods should be properly done.

classified into one of the two realms, while other arguments may encompass an aspect of both. Upon considering past attempts to transform a constituting mathematical framework, a series of questions arise regarding the place of such revolutionizing theories: do they evolve from the body of knowledge, the image of knowledge, or both? Is there a specific path or order for changes to occur? Does a shift in one layer must proceed the other?

The historian Leo Corry suggests that there is not only one direction in which mathematical transitions can occur (Corry 2001). As a case study, Corry examines the structural image of a specific mathematical discipline, namely, algebra, by analyzing van der Waerden's *Moderne Algebra*, which presents the body of algebraic knowledge as deriving from a single unified perspective, and all the relevant results in the field are achieved using similar concepts and methods (Corry 2001, 172). The systematic study of different varieties of algebra through a common approach is what Corry calls a structural image of algebra, and whereas the transition to a new structural image in the case of van der Waerden's *Moderne Algebra* was enabled due to changes in the body of knowledge, it does not imply that this is mandatory. Thus, transitions between images of knowledge are unique and distinct processes from transitions in the body of knowledge. Corry perceives the body and image of knowledge as organically interconnected domains in the history of a discipline, but he does not regard their relation as a cause and effect.

In the case of van der Waerden's *Moderne Algebra*, the newly proposed image had firm roots in the then-current body of knowledge. Though the textbook presents an original perspective regarding the algebraic structure, it uses as cornerstones several mathematical notions such as groups, fields, and ideals that have already been introduced to the mathematical community, and it builds upon already developed theories of renown algebraists (such as Emmy Noether and Ernst Steinitz). Van der Waerden took mathematical concepts (such as Isomorphism) that were previously defined separately for different mathematical notions (such as groups, rings, or fields) and showed that they could be a-priori defined for each algebraic system (Schlote 2005; Corry 2001). The mathematical entities van der Waerden discussed were familiar and acceptable within the mathematical discourse; the novelty he introduced lied in the relations between them.

The notions van der Waerden applied in Moderne Algebra did not appear there for the first time: the concept of 'group' was already found in algebra textbooks from 1866, and the notions of ideals and fields were introduced by Dedekind in 1871. Brouwer, on the other hand, introduced new, original concepts and theories that were meant to replace the old, classical, non-constructive ones.

To suggest an alternative to the set-theoretical notions of a class of numbers, Brouwer employed two intuitionistic analogs: ‘species’ and ‘spread.’ A species is a property that mathematical objects can have, and objects with this property are called the elements of the species. A spread is a collection of sequences called the nodes of the spread and is defined by a ‘spread function’ which performs a decidable procedure on finite sequences (Troelstra 1969; Dummett 1977). Another new concept that Brouwer introduced was ‘choice sequences,’ also called ‘infinitely proceeding sequences’ (Troelstra 1977; van der Hoeven & Moerdijk 1984). Such sequences need be neither law-like (that is, governed by computable recipes for generating terms) nor even fully determinate in advance. Nothing about the future course of the sequence may be known, other than the fact that its terms are freely and independently chosen[2].

The problematic aspect of spread, species, and choice sequences (that are only taken here as representatives among several other new intuitionistic concepts Brouwer had introduced[3]), does not lie solely in their novelty. It is entwined with Brouwer’s motive to develop these new intuitionistic concepts, namely, his philosophical views that put philosophy before mathematics (and not the other way around). Brouwer was willing to forego significant parts of mathematics in order to refrain from the paradoxes of set theory, but for mathematicians, the scope of the change was far too comprehensive. Practicing mathematicians wish to solve problems at the core of the discipline, not to contemplate philosophical conundrums. Here lies another significant difference between Brouwer’s and van der Waerden’s stories: due to Brouwer’s philosophical views, the whole foundational basis of mathematics had to change.

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[2] Brouwer permitted restrictions imposed by a spread law, but nothing beyond that.

[3] From these notions, together with the new definition of the natural numbers as mental constructs, Brouwer goes on to formulate additional intuitionistic concepts and theories such as bar theorem, fan, and fan theorem (Dummett 1977, van der Hoeven & Moerdijk 1984).

Brouwer's intention was primarily to reform the foundations of mathematics, but van der Waerden's agenda was utterly different. Even tough *Moderne Algebra* turned out to be an influential book that had a significant impact on algebra as a discipline, 'reformation' was not what van der Waerden had in mind. To take seriously the question of whether intuitionism could at all appeal to a broader mathematical audience, we must consider the combination of several differences between the two stories. It is not only the use of familiar or non-familiar mathematical notions, but also the philosophical motives for change (or lack thereof), and the scope of the change that shape the way mathematicians read and respond to new ideas. In order to gain a better understanding of the differences between Brouwer's and van der Waerden's motives and scope of change, let us explore the contours of Brouwer's intuitionism.

2. The scope of change as suggested by Brouwer's intuitionistic program

Brouwer's intuitionism holds that the existence of an object is equivalent to the possibility of its construction in one's mind. There is an important philosophical distinction between objects like finite numbers and constructively given denumerable sets, which are objects that we finite beings can intuitively grasp, and the Cantorian collection of all real numbers, which is an infinite entity that exceeds our limited grasp. Brouwer regarded the former entities as 'finished' or 'finish-able' while the latter are 'unfinished.' A 'finished' set is produced by a recognizable process (that is, a process that one can construct), yielding some legitimate grasp of the object with all its parts (that is, that the parts are 'determined' by the initial grasp). An 'unfinished' collection is one that we cannot grasp in a way that suffices to determine all its parts (Brouwer 1952; Posy 2008).

Throughout his dissertation, Brouwer uses this differentiation to confront Cantor's perception of infinity. Brouwer accepts ω-sequences as legitimate mathematical objects since it is a sequence of discrete elements that are generated by a countably ordered process (Brouwer 1912, 85-86), but it is the only infinite object he accepted (Brouwer 1907, 142–143).

Brouwer addressed the set of real numbers as 'denumerably unfinished' from a negative perspective, pointing out that given a denumerable subset, we can straightaway find an element of the continuum that is not in the given subset, but there is no positive existence claim to support it. Hence, he proclaimed Cantor's second number class and any ranked order of increasing cardinalities as illegitimate mathematical objects, a mere "expression for a known intention" (Brouwer 1907, 148).

As for the intuition of the continuum itself, Brouwer firmly believed that we have an intuitive grasp of the continuum as a whole (Brouwer 1907, 8-9, 62). Thus, a continuum that is constructed out of a set of independently given points (like the Cantorian continuum) cannot be considered a legitimate mathematical entity. No set of points can exhaust the continuum since, in Brouwer's view, it is a unity in its own right (Posy 2005). Building on from this concept of the continuum and his notion of infinity, Brouwer postulated a separate form of intuition that delivers the continuum as a whole and generates the 'mathematics of the continuum,' thereby creating a new body of mathematical knowledge.

By virtue of Brouwer's new concept of a potential infinity, core notions like the principle of excluded middle and the concept of negation are deemed unacceptable in Brouwer's intuitionism. The principle of excluded middle can only be used as a reliable tool in finite systems where each object of the set can be examined (in principle) by means of a finite process. Within a finite system, one can eventually determine whether there is a member of the set with the property A or that every member of the set lacks the property A. However, in infinite systems it is no longer possible to examine every object of the set (not even in principle); thus, even if one never finds a member of the set with the property A, it does not prove that every member of the set lacks the property A (Brouwer 1908, 1918).

Together with the restricting concept of infinity and his demand that mathematical objects must be constructed, Brouwer introduced a new image of mathematical knowledge, which he considered as the only proper way to do mathematics. As a result of such changes, the idea of mathematical truth and its relation to the provability and refutability of a mathematical statement was redefined: in the newly proposed intuitionistic theory knowing that a statement P is true means having proof of it. Otherwise stated, to assert that a statement P is true is to claim that P can be proved; to negate P is to claim that P is refutable (i.e., that a counterexample exists), but it does not imply that "not P" is provable (Brouwer 1912; Heyting 1966; Sundholm and van Atten 2008). One of the many implications from such an utter reformation was that proofs of mathematical existence by contradiction ceased to be a legitimate technique within the discipline, inducing a change both to the image and to the body of knowledge.

Moreover, Brouwer's newly proposed image excluded several central mathematical theories, and extensively altered other widespread mathematical concepts. Among some of the classical theories Brouwer was willing to eschew, was Zermelo's axiom of choice, that was referred to by Hilbert as constituting "a general logical principle which, even for the first elements of mathematical inference, is indispensable" (Moore 1982, 253), emphasizing the considerable differences between the new and the existing bodies of knowledge.

3. Differences in the process of creating new knowledge

According to Corry, the innovative aspect of van der Waerden's book was that it created new and significant mathematical knowledge without introducing any new mathematical entities, theories, or concepts. Van der Waerden took mathematical concepts and elements that were developed within specific, different mathematical contexts and realized that within the framework of algebra, a variation of the same elements could be axiomatically defined, studied, and brought together into a new conceptual organization of the discipline. Such mathematical concepts were "different varieties of a same species ("varieties" and "species" understood here in a "biological," not mathematical term), namely, different kinds of algebraic structures" (Corry 2001, 176).

The type of knowledge created in van der Waerden's *Moderne Algebra* can be regarded, to some extent, as a continuation of the already existing mathematical body of knowledge. Brouwer's story was rather different; the extensive scope of reformation Brouwer imposed on the prevailing body of mathematical knowledge, including the restricting concept of infinity and the intuitionistic notion of the continuum as a whole, made intuitionism altogether incomparable with classical mathematics. The intuitionistic approach is not merely a restriction of classical reasoning; it contradicts classical mathematics in a fundamental way (Iemhoff 2019).

More than it was a new mathematical approach to the foundational problem of mathematics, Brouwer's Intuitionism was, first and foremost, a philosophy of mathematics. Tracing back to his 1907 Ph.D. dissertation, Brouwer intended to work out his ideas in the philosophy of mathematics, rather than to describe various views on the foundations of mathematics (van Dalen 1981). In a letter from Brouwer to his supervisor, Diederick Korteweg, Brouwer wrote that he is glad he is finally able to use mathematics in order to support his criticism of the value and usefulness of language and logic[4].

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[4] As documented in a letter from Brouwer to Korteweg from September 1906 (taken from Van Dalen 1981, 5).

Brouwer's dissertation consisted of three chapters: 'The construction of mathematics,' 'Mathematics and experience,' and 'Mathematics and logic.' In the first chapter, Brouwer constructs mathematics from the natural numbers to the negative, the rational, and the irrational numbers and introduces the continuum as an ever unfinished (Brouwer 1907, 44-52; Van Dalen 1999). The goal of Brouwer's second chapter is to improve Kant's view of the a priori, as in Brouwer's opinion the only a priori element in science is the intuition of time since the creation of the image of space is a free act of the intellect and as such cannot be part of the a priori. In the third and last chapter Brouwer touches the two themes that will become the most central issues in the foundational debate: the principle of excluded middle and mathematical existence, and directs his criticism towards Hilbert's idea of securing the foundations of mathematics by consistency proof (Brouwer 1907, 176).

Korteweg's main criticism of Brouwer's dissertation was directed towards the second chapter, as he firmly objected to the idea of philosophical mathematics as a scientific topic for a dissertation. Korteweg read parts of Brouwer's book Life, Art, and Mysticism, but he expected Brouwer to separate between his philosophical and mathematical work[5]. However, in

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[5] It should be noted that despite his criticism of Brouwer's philosophical views, Korteweg was one of Brouwer's most prominent advocates. He was a firm believer in Brouwer's mathematical abilities and did everything in his power to secure him an academic position. Throughout his attempt to get Brouwer elected to the Academy of Sciences in 1910, Korteweg

Brouwer's case, philosophy was "the basic ingredient that made the mathematics work" (van Dalen 2013, 86), and he had no intention in restricting his philosophical activity to leisure hours. Korteweg was concerned with the reception of Brouwer's work in the faculty, specifically with the philosophical and moral views Brouwer presented as part of his second chapter. He expressed his misgivings in a letter to Brouwer, where he stated:

> After receiving your letter I have again considered whether I could accept it as it is now. But really Brouwer, this won't do. A kind of pessimistic and mystic philosophy of life has been woven into it that is no longer mathematics, and has also nothing to do with the foundations of mathematics. It may here and there have coalesced in your mind with mathematics, but that is wholly subjective. One can in that respect totally differ with you, and yet completely share your views on the foundations of mathematics. I am convinced that every supervisor, young or old, sharing or not sharing your philosophy of life, would object to its incorporation in a mathematical dissertation. In my opinion your dissertation can only gain by removing it. It now gives it a character of bizarreness which can only harm it. (van Dalen 2013, 92-93)

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even approached leading international mathematicians such as Hilbert and Poincaré in order to get their recommendations for a membership to a "gifted and exceptional scholar" such as Brouwer (van Dalen 2013).

Korteweg's remarks on Brouwer's dissertation stress out the extent to which Brouwer's work deviated from the acceptable norm of a standard mathematical dissertation. Korteweg suspected that Brouwer's philosophical incentive to reform mathematics might not appeal to a wide mathematical audience, mainly as it contradicted familiar notions and stood against the theories developed by momentous mathematicians like David Hilbert. Some changes are too far-reaching for mathematicians to endure, and even the glamorous promise of consistency and non-paradoxical foundations is not enough to make them give up the methods and theories they use doing everyday mathematics.

Eventually, Brouwer revised the second chapter, leaving some of the problematic philosophical parts out of it. However, Brouwer maintained his philosophical views and continued to develop intuitionism primarily as a philosophy of mathematics that entailed a massive reformation to the foundations of the discipline rather than an extension or a continuation of classical mathematics. The role of intuition in Brouwer's mathematics is a means of introducing new mathematical structures, not 'different kind of the same species' structures as in van der Waerden's work. As David Hesseling puts it, Brouwer "started from his own ideas and looked for mathematics that fitted in, instead of working the other way around" (Hesseling 2003, 35).

4. Concluding Remarks

The current paper suggests that three factors played a significant role in the lack of reception to Brouwer's intuitionism: the first is Brouwer's philosophical agenda to reform mathematics, which was foreign and unrelatable in the eyes of most mathematicians, including Brouwer's Ph.D. supervisor, Korteweg. The second is the extensive scope of change Brouwer had imposed on all aspects of mathematics, thereby excluding major theories and acceptable proofs from the legitimate body of mathematical knowledge. The third element is the introduction of new concepts and theories that were meant to replace the non-constructive ones, a move that only further characterized intuitionism as an isolated theory, deprived of any foothold in current mathematical practices. Each factor is entwined with and explains the other two factors: Brouwer's philosophical motivation is the reason behind the massive scope of reformation he suggested, and every new concept he introduced is rooted in his philosophical views of what mathematics actually is and how it should be practiced. Taken alone, each factor is necessary but not sufficient in the attempt to explain why the intuitionistic program was not able to attract broader groups of mathematicians. However, all three factors together offer a more comprehensive picture of the lack of reception to Brouwer's intuitionism.

Among the mathematicians who did embrace Brouwer's intuitionism for a short period was Hermann Weyl, who was deeply influenced by philosophers such as Fichte and Husserl (Scholz 2000, Feferman 1998) and tried to develop his philosophical view of mathematics in his monograph *Das Kontinuum* (Weyl 1918). However, Weyl was quite a unique scholar among mathematicians and physicists that not only contributed to his own fields of research but also engaged with philosophical questions about the foundations of mathematics and the nature of mathematical entities (Weyl 1949). Most mathematicians restricted their areas of expertise to the discipline of mathematics and did not find a necessary connection between practicing mathematics and doing philosophy. Even Brouwer's prominent student, Arend Heyting, was ambivalent in regards to the inseparable connection between mathematics and philosophy that Brouwer had imposed.

Heyting's intuitionism and Brouwer's intuitionism were quite different: while Brouwer insisted on detaching intuitionism from any axiomatic method, Heyting took Brouwer's intuitionistic ideas and expressed them using a formalistic approach. Heyting's formalization comprised intuitionistic propositional and predicate logic, arithmetic, and analysis, all together in one big system (Heyting 1930; 1980). While his formalization of analysis did not derive from its classical counterpart (thus it was somewhat overlooked within the foundational debate), the parts concerned with logic and arithmetic were subsystems of their classical counterparts (expect from the principle of excluded middle, which was excluded from Heyting's theory), and were extensively discussed (van Atten 2017).

Heyting's intuitionism reached a wide mathematical audience and continued to develop over the following decades (see: Gentzen 1935; Heyting 1966; Kleene 1952; Myhill 1966; Vesley 1980). Was the reaction to Heyting's intuitionism a result of the differences between Heyting's intuitionism and Brouwer's intuitionism? Was it only the formalization of intuitionism that made Heyting's ideas more approachable to working mathematicians, or was it also the sense of detachment from Brouwer's philosophical stances that is evident in Heyting's approach to intuitionistic mathematics? There is a conflicting view regarding the way Heyting addressed Brouwer's philosophical considerations. Albeit Heyting quoted Brouwer's remarks on the relation between mathematics and logic, Heyting also claimed that philosophy is not necessary in order to understand intuitionistic mathematics, and some of Brouwer's most significant concepts (such as consciousness and mind) play no role in Heyting's approach (Heyting 1974; Placek 1999). Unlike Brouwer, Heyting did not attempt to justify the intuitionistic revision philosophically, nor to suggest that philosophical assumptions are inherent in intuitionistic mathematics. As opposed to Brouwer's viewpoint, Heyting argued that intuitionism is much simpler than any philosophy and that it would be better for the sake of intuitionism to eliminate any philosophical (metaphysical as well as epistemological) premises. As Heyting put it:

> The only philosophical thesis of mathematical intuitionism is that no philosophy is needed to understand mathematics. (Heyting 1974, 79).

The superiority of intuitionistic mathematics over classical mathematics, according to Heyting, derives from the former being free from any metaphysical or philosophical assumption. Therefore, it appears that even Brouwer's most devoted student deviated from Brouwer's philosophical positions and regarded them as alien and even irrelevant to Brouwer's intuitionistic program. What caused Heyting to abandon Brouwer's philosophical approach but to continue his intuitionistic pursue? Is it possible that Brouwer’s philosophical views combined with the massive reformation he suggested and the introduction of new concepts demanded too much mathematical compromises, even from a faithful disciple like Heyting? Can the examination of Heyting’s reception of Brouwer’s intuitionistic mathematics while discarding Brouwer’s philosophy shed yet another light on the question of why other mathematicians, less devoted to Brouwer, were unwilling to accept Brouwer’s intuitionistic program? The current paper sets the stage for exploring these questions and provides a prolific ground to start from, in its attempt to present a more inclusive perspective on how certain developments in mathematics prevailed whereas others did not.

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