# On the Relationship Between Modelling Practices and Interpretive Stances in Quantum Mechanics

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**Abstract**: The purpose of this article is to establish a connection between modelling practices and interpretive approaches in quantum mechanics, taking as a starting point the literature on scientific representation. Different types of modalities (epistemic, practical, conceptual and natural) play different roles in scientific representation. I postulate that the way theoretical structures are interpreted in this respect affects the way models are constructed. In quantum mechanics, this would be the case in particular of initial conditions and observables. I examine two formulations of quantum mechanics, the standard wave-function formulation and the consistent histories formulation, and show that they correspond to opposite stances, which confirms my approach. Finally, I examine possible strategies for deciding between these stances.

#### **1** Interpreting Quantum Mechanics

Can the literature on scientific representation, which is primarily concerned with modelling practices, inform us on the interpretation of quantum mechanics?

Debates in metaphysics of science about the interpretation of scientific theories (in particular the theories of physics) are usually approached from a perspective that does not really take into account pragmatic aspects. Rather the question being asked is "What is the world like if the theory is true?", assuming the theory is "interpreted literally", or "taken at face value". Generally, the whole universe is considered as the proper object of inquiry, even though in practice, theories are more often used to represent bounded systems. Scientific realism is implicitly assumed, at least as a working hypothesis: scientific models are taken to accurately represent a mind-independent reality. However, in practice, model construction is sensitive to the purposes of epistemic agents, and the models that are used to represent physical systems are idealised, and not taken to represent their objects with perfect accuracy in all respects. In this

sense, the metaphysics of science takes an idealistic stance that abstracts away from the messiness of concrete representational activities. A justification for this approach could be that this messiness is not an essential feature of representation, but a contingent aspect inherent to our practical limitations, and that it could be overcome in principle. The debates on scientific representation would not be directly relevant to metaphysics for this reason.

This stance could seem unproblematic if understood as a working hypothesis allowing us to inquire into what exists independently of our activities, a "view from nowhere". In the case of quantum mechanics, however, the fruitfulness of this working hypothesis is questionable. It is not clear what a "literal interpretation" of the theory means. On the one hand, the standard textbook formulation of quantum mechanics has instrumentalist flavours: it does not really present us with a realist interpretation of the theory. The anthropocentric notion of measurement is explicitly mentioned and left unanalysed. On the other hand, so-called realist interpretations do not always take this formulation "at face value". They often complete the formalism with additional structure, for example particle positions in Bohmian mechanics, and they disagree with each other. Non-realist interpretive programs, such as QBism, are developed alongside realist ones, and they give us interesting insights into the theory. There is not one "literal interpretation" of the theory. What was supposed to be the ground for metaphysical analysis, stemming from a harmless working hypothesis, ends up being open to controversies.

For these reasons, it might be interesting to take a step back and have a closer look at representational practices, for a better understanding of what "interpreting" really means: how exactly are physical systems represented in quantum mechanics? Asking what interpreting means is just asking for the semantics of a scientific theory, and here, I wish to advocate an approach that is analogous to attempts, in philosophy of language, to elucidate meaning in terms of use (Wittgenstein 1953; Grice 1968). The idea is the following: in order to understand what a scientific theory says, one should first have a look at how the theory is used to represent the world. Pragmatist aspects are no substitute for a metaphysical picture, and perhaps some of these aspects are indeed contingent rather than essential. But this doesn't mean that pragmatics is not informative at all. In a situation where interpretive approaches proliferate without any resolution in sight, there is no good reason to abstract pragmatics away.

Examining the imports of the pragmatics of representation for interpretive issues in metaphysics is an ambitious project that would not fit in a single paper. My more modest aim in this article is to outline a strategy that could be employed for this purpose. It consists in establishing relationships between interpretive stances and modelling practices. I will argue that certain ways of constructing models in order to represent the world are more compatible with some interpretive stances than others. I will illustrate these connections between interpretive stances and modelling practices by examining two formulations of quantum mechanics. Establishing such connections opens the way for new argumentative strategies for deciding between one or the other interpretive stance, which I will sketch at the end of this article.

This article makes heavy use of the philosophical notion of modality, which refers to discourse about the possible and the necessary. There are various types of modality: logical, conceptual, epistemic, practical, metaphysical or nomological. The reason why I am employing this philosophical tool is that I am convinced that interpreting the modal structures of a theory in terms of these various types of modality is key to understanding "what the theory says". This is particularly true in quantum mechanics, where, for example, probabilities are interpreted either

as objective chance (nomological possibilities) or subjective degrees of credence (epistemic possibilities), depending on the interpretation (Frigg and Hoefer 2007; Wallace 2007). My methodology consists in examining the role that different types of modality play in representation. Then, by examining how theoretical structures are used in model construction, one can give a certain interpretation to these structures in modal terms. This is how I propose to establish connections between modelling practices and interpretive stances.

In section 2, I present recent developments in the philosophical literature on scientific representation. In section 3, I provide a short analysis of the role of various types of modalities (conceptual, epistemic, practical and natural) in representation. In section 4, I attempt to identify which modal structure of non-relativistic quantum theory plays which representational role, so as to match them with types of modalities. This attempt fails, but in an interesting way: there is a latency over how to build a quantum mechanical model, but each option can be associated with a particular interpretive stance. I call the two main stances that one can adopt the objectivist and the perspectivist stance. In section 5, I show that the objectivist stance corresponds to the standard formulation of quantum mechanics, where models describe wavefunctions evolving in time, and that the perspectivist stance corresponds to the consistent histories formulation, where models describe correlations between possible "contextual events". I examine how the two formulations differ in terms of natural ontological commitments, and argue that these differences confirm the validity of the methodology adopted. Finally, in section 6, I argue that this methodology offers new potential strategies for evaluating interpretive options.

# 2 Scientific Representation

Scientific theories were once conceived of as general statements about the world expressed in a theoretical language. The question of interpretation boiled down to the question of providing a semantics for theoretical terms, by connecting them to observational terms, and this was a question that fell under the scope of philosophy of language. The difficulties of this positivist project are well known, and nowadays, scientific theories are generally conceived of as families of models, where models are abstract entities, for example, set-theoretical structures, that represent target systems. In empirical confrontation, theoretical models are directly compared to other structures: data models (Suppe 1972; van Fraassen 1980).

This model-based approach does not really solve nor dissolve the problem of what theories "mean" in a general sense: it merely pushes back this problem, which now falls under the scope of the topic of scientific representation. The question concerns the relation between models and what they represent. Initially, it was thought that relations such as isomorphism or similarity could be enough to account for representation, but these accounts have been criticized for not being directional (the relation involved is symmetric and reflexive, while the representation relation is not), and for conflating representation and accuracy (it is possible to represent something inaccurately (Suárez 2003)).

These shortcomings have motivated a move towards user-centred accounts of representation. Representing is at least a three-place relationship between a user, a vehicle and a target. Some would add other components, for example, purposes (Bailer-Jones 2013; Giere 2010a). The user takes the model to denote its target, which ensures directionality and does not

guarantee accuracy. Models are generally idealised: some aspects of the target take more importance than others, some are wilfully neglected, and this depends on the aims of the modeller.

According to a minimalist understanding of scientific representation, representing a system is, for an epistemic agent, using a source (for example, an abstract mathematical structure described by equations on paper) to make inferences on a target. Whether one can say something more substantial is a subject of controversy. Suárez (2004) claims that representational activities are too disparate to give necessary and sufficient conditions for the representation relation to take place, while others attempt to provide more substantive accounts. Contessa (2007) for example, claims that it is sufficient that a mapping be given between components of a model (objects, properties, relations) and components of the target system for the model to represent the target. Any mapping will do. Even more liberally, Callender and Cohen (2006) have defended that stipulation by the agent is enough for the representation relation to take place. But others have objected that such liberalism is unfit for accounting for epistemic representation in general, and scientific representation in particular. Epistemic vehicles have a distinctive role, which is to "allow their users to have access [...] to aspects of their targets" (Liu 2015). Stipulation is not enough to fulfil this role. These liberal conceptions of representation also fail to account for communal aspects: what a given scientific model can represent depends on the history of the model, its construction and reception by scientific community. A model must be licensed by the community as a representation of its target (Boesch 2017). This licensing aspect is responsive to empirical and theoretical aims: not any mapping between the vehicle and target will do, and something more than stipulation is involved.

There is still no consensus on the exact nature of scientific representation. However, it is possible, without taking sides, to give a list of potential constraints on the establishment of a representation relation between a model and a target system. These constraints can be understood as limiting the sets of interpreted models that are apt to represent a target of a particular type in a particular context (where the context includes the purposes of epistemic agents). They can be classified as follows:

- 1) **Conceptual constraints**: Is the model a legitimate vehicle of representation according to the epistemic community? Is it a proper use of scientific concepts?
- 2) **Practical constraints:** Is the model appropriate for the particular purpose of the user in context? Are objects and properties of interest for this purpose represented? Are standards of precision sufficient? Is the model simple enough to be used?
- 3) **Epistemic constraints**: Does the model correspond, in relevant respects, to the particular target, given the empirical inputs available to the user?

Constraints of the first type are a-contextual. Whether or not scientific concepts are properly used does not depend on the particular target system being represented, nor on the purposes of the user. Constraints of the second and third type are contextual. They differ in the direction of fit: for the second type of constraint, the model must fit our aims, while for the third one, it must fit the world.

Does considering these constraints shed light on the way a theory should be interpreted in terms of ontological commitments? I believe that it does. Contextual (practical and epistemic) constraints can be interpreted as being a matter of correctly identifying the target of representation, and not as a matter of content: the user wants the model to represent one particular target system for one particular purpose: for example, a concrete pendulum of a certain length, with a focus on its position along a particular axis. In this sense, these constraints are merely selective. They do not determine "what the model says", but rather "what the model is about". Conceptual constraints correspond to the content of a theory: its laws and principles (for example, the laws of Newtonian mechanics). They determine the cognitive content of the model more directly. However, they can only be applied once the target of representation has been identified in context (these conceptual constraints can be analysed as a function from context to content). So, it is of crucial importance, for delineating the cognitive content of models, to examine the interplay of these three types of constraints.

#### **3** Modalities

Let us connect the aspects just mentioned to modalities, so as to outline the methodology of this paper. Interestingly, the collection of models of a theory, for example, the set of Newtonian systems, is sometimes viewed as a modal structure, each model representing a possible world according to the theory<sup>1</sup>. This way of putting things has metaphysical overtones, in particular if the modality involved is interpreted as nomological (what must be true according to the laws of nature). Furthermore, scientific models do not, in general, represent the universe as a whole. However, we can make the more neutral claim that each model of the theory is a possible way a target system could, in principle, be represented within the theory. The set of models of the theory thus provides a set of *conceptual* possibilities for representing a concrete system. Conceptual possibilities correspond, in general, to what is not precluded by a proper grasp of the concepts involved in a description. This is indeed the case for the models of a theory, insofar as they respect the conceptual content of the theory: its general laws and principles.

This idea of identifying the models of a theory with conceptual possibilities provides a direct way of analysing the constraints listed in the previous section in modal terms. A set of models is conceptually relevant in virtue of respecting the general laws and principles of the theory, which are conceptual constraints. It corresponds to the set of all models of the theory (for example, the set of Newtonian systems). Then a subset of this first set is practically relevant given our purposes, and another subset is epistemically relevant given our prior knowledge of the particular target<sup>2</sup>, assuming a certain interpretation of these models in terms of the target (for example, models of pendulum with specific parameter values, interpreted in terms of a concrete pendulum). The intersection of these two subsets gives us the set of interpreted models that are apt to represent the target in a particular context. In sum, each type of constraint gives us a set of possibilities for representing the target, and these possibilities are conceptual, practical and epistemic respectively. Formally speaking, these constraints act as relations of necessity that limit the set of available possibilities, since they limit the set of acceptable models<sup>3</sup>.

<sup>1</sup> This is at least the case for the theories of physics, which are the main focus of this paper.

<sup>2</sup> This is consistent with the idea that epistemic possibilities are a subset of conceptual possibilities, which are a subset of logical possibilities, which, in this case, would correspond to the set of all mathematical structures.

<sup>3</sup> Note that I am not trying to capture the process of model construction, but the final product. Presumably, scientists start from epistemic and practical constraints and then use conceptual inferences to construct an appropriate model. In retrospect, the constraints from which they started

Applying these constraints does not guarantee that the model is an accurate representation of the target: only that the representation relation takes place. However, we can presume that they determine *conditions* of accuracy. If, by analogy with truth-conditional semantics in philosophy of language, we identify conditions of accuracy and cognitive content, then these constraints determine the cognitive content of the model. Presumably, whether these conditions of accuracy are satisfied by the target system depends on a form of correspondence between relevant parts of the model and relevant aspects of the target, or perhaps relevant parts of the relation between the target, the user and the environment.

One further aspect should not be neglected: scientific models generally have an internal modal structure. They often describe various possibilities for the target system. This is notably the case for probabilistic models, where various possible states for the target system are represented and weighted. This internal structure is determined by the constraints above, in the sense that applying these constraints allows the user to select a particular model with a particular internal structure. However, contrarily to the possibilities analysed so far, the associated possibilities do not correspond to the choice of the right model for representing a given target: they are represented *within* models. This means that conditions of accuracy are not extensional, but intensional: the content of a model is, in general, modal.

If these "internal" possibilities were epistemic, that is, if they corresponded to our ignorance of the real state of the target system, I think it would make sense to understand them as having to do with model choice, and to interpret the corresponding probabilities as weighing up different possible sub-models (at least for philosophical analyses). The probabilities associated with these possibilities would not represent aspects of the target, but rather our knowledge or expectation of the target. They would typically be evaluated by taking into account measurement errors, for example. However, when it comes to representing the target alone, and not our belief state, the proper units of representation are the weighted structures, that is, the sub-models, and not their combination. So, epistemic modalities can in principle be "externalised", and interpreted in terms of our ignorance about which model actually represents the target. But once this is done, the resulting units of representation might still have an internal modal structure that should be interpreted, and I think that in this case, the proper interpretation is in terms of natural or nomological modalities: what the model represents is a set of possibilities "out there, in the world", a set of ways the target could be in virtue of natural constraints. Such a model is able to support counterfactual reasoning of the type "if this were to happen, this would result". If we didn't accept this, it would be hard to account for the explanatory role of scientific models and for causal inferences, given that explanations and talk of causes and effects often involve counterfactual reasoning (Saatsi and Pexton 2013).

Note that I am not trying to impose a metaphysical interpretation of scientific models here. I am merely proposing an association between their content and this interpretation, that is, a *semantics*. A modal sceptic who thinks that the world is a mosaic of actual facts should consider that the right units of representation are models with no internal modal structure, because this is

can be seen as restrictions on the space of available conceptual possibilities. Relations of necessity can be approached inferentially, or semantically, and the two approaches are complementary. Also note that this is a simplification, because models do not always strictly satisfy the laws of a theory, for example when approximations are used, and they often incorporate domain-specific postulates. However, I will neglect these complications here.

what the world is like. In so far as models do have this internal structure, either they should be decomposed into more basic models, or they should be interpreted in terms of natural or nomological possibilities.

I should also stress that I don't consider this association between models and interpretations to be as strict as, say, logical entailment. Of course, it is always possible for someone to integrate a modal structure within a model and then claim: "actually, I take this structure to represent my state of knowledge, not real possibilities". The point is that this claim does not go without saying: this person is simply acting as if she was assuming that this structure represents natural possibilities, whereas if she was pondering several models, she would be acting as if in a state of uncertainty about which model represents the target. So, some modelling choices naturally induce a certain metaphysical picture of the represented object, and conversely. This can be turned into a normative claim: modelling choices are not necessarily innocuous, and when it comes to philosophical analysis, one should use the model structures that naturally match their intended metaphysical interpretation, so as to avoid being misled by modelling choices. Having said that, I will argue that this connection is robust enough to correspond to the way that scientific models are typically interpreted in the scientific and philosophical literature.

Accepting this analysis, there seems to be an important divide between two kinds of modalities:

- **Mind-independent** (natural/nomological) modalities are *represented*; they correspond to the internal structure of models. They describe what could happen for a given target.
- Mind-dependent (epistemic, practical) modalities are about *choosing the right model*; they correspond to possible targets or possible ways of representing a target.

Conceptual modalities connect these two kinds of modalities: given epistemic and practical constraints, one can deploy conceptual resources so as to construct or select the right model for a particular target. If the selected model has an internal modal structure, then it represents natural constraints on the phenomena. In this sense, the conceptual necessity associated with the content of a theory is a function from context to content: it tells us which natural constraints one should posit given epistemic and practical constraints.

The relevant question in order to decide if a given modal structure, for example a range of possible values for a parameter, corresponds to the first or the second type of modality appears to be: does fixing this value amount to identifying the target of representation, eliminating our ignorance about the target, for example, or is it merely about considering one possible way the target, already identified, could be in virtue of its nature, irrelevant of how it actually is? Given the remarks made in this section, in the former case, said value should be *fixed within models*, such that changing the value would correspond to changing the target and model, and in the latter case, it should remain *variable* in the model, representing natural possibilities for the target. This value might be fixed later, for example, when using the model for inferences (including counterfactual inferences), but it is not fixed during the process of model construction.

#### 4 Application to Quantum Mechanics

We now have a tool for interpreting the modal structures of a theory. The structures that are fixed within models should be interpreted as corresponding to mind-dependent (epistemic or

practical) possibilities, having to do with the correct identification of the target of representation, since the corresponding possibility is fixed by the process of selecting the "right" model. The structures that are variable, such that various possibilities are represented within the model, should be interpreted as mind-independent (natural) possibilities for identified targets. Let us apply this tool to the case of non-relativistic quantum mechanics, so as to test its validity.

The following elements are typically involved in model construction and inferences in quantum mechanics:

- A state space (configuration space or Hilbert space) and an algebra of observables, representing the relevant degrees of freedom of the target system.
- Dynamical constraints on the state space (Schrödinger equation, a Hamiltonian, boundary conditions), associated with the specific physical configuration of the target system.
- Specific observables or combinations thereof indexed in time, generally representing "what is measured" on the system.
- Initial conditions (vector or density matrix), representing the initial state in which the system is prepared or first observed<sup>4</sup>.

One can make probabilistic predictions by fixing all these elements and then applying the Born rule.

These four elements have a modal character, in the sense that they can be instantiated in various possible ways: possible state spaces, possible dynamics, possible values for dynamical parameters, possible observables, etc. The question is: how shall we interpret this modal character? Which of these elements are part of the identity conditions for target systems, and which are naturally contingent aspects of identified targets? Which should be interpreted in terms of epistemic or practical possibilities, and which should be interpreted in terms of natural possibilities? Applying our new tool, these questions can be reframed as: which elements should be fixed within a model, and which should remain variable?

The answer to this question might depend on the context of use, and in particular, on the level of abstraction involved. Perhaps, for example, a scientist working on foundational issues in quantum mechanics would like to keep all elements variable, so as to consider any possible type of target, while a scientist making predictions will fix all components. A scientist presenting an explanation for some phenomena would probably work with a model in between these two extremes, fixing enough elements to identify the phenomena to be explained, but keeping enough components variable for counterfactual reasoning. In some contexts, only certain aspects of the algebra of the observables could be fixed (for example, their tensor-product form). There seems to be a hierarchy of models, as observed by Giere 1999 (ch. 6), where each level in the hierarchy makes the level above more precise by fixing the value of a component that was kept variable at this upper level. So, there seems to be no absolute fact of the matter as to whether a given theoretical structure should be considered a representation of natural possibilities, with all possibilities appearing in the model, or as an epistemic or practical possibility that should be fixed by the relevant inputs.

<sup>4</sup> They can be replaced by final conditions in the case of retrodictions, or by any relevant input concerning the properties of the system at a particular time. In the following, I will only consider initial conditions, but my analyses can easily be transposed.

Note that this is not inconsistent with the analysis of the previous section. It only means that a structure that corresponds to natural possibilities in an abstract context can become an epistemic or practical possibility in a more concrete context. Once we start wondering which out of a set of natural possibilities is realized, for instance, we trade a natural modality for an epistemic one.

Having said that, Giere also claims that there is a more salient level of abstraction, what he calls a basic level. It corresponds to a level that is sufficiently abstract to be useful in various contexts, and sufficiently concrete for the various potential targets of the model to share a high degree of similarity. Giere bases this claim on research in cognitive psychology on concepts and categories. We could also suspect that there is a privileged way of selecting a path down the hierarchy of models. For example, it does not make sense to fix initial conditions in a model without fixing the state space, because initial conditions are defined as regions of the state space. Observables and dynamical constraints are also expressed in terms of the state space. So, a model should at least have a fixed state space to be of any use. I would say that dynamical constraints should also be fixed prior to initial conditions and observables, at least for bound systems, because without dynamical constraints, initial conditions and observables are idle (except perhaps for static systems, but then an implicit dynamics is provided)<sup>5</sup>.

Let us assume that there is a salient level of abstraction for representation in quantum mechanics, that it is the proper level for philosophical analysis, and that the models at this level incorporate at least a fixed state-space and fixed dynamical constraints. This means that these two components are associated with a correct identification of the target system, given the purposes and prior knowledge of the user of the model. This idea makes sense: in general, a user will be interested in particular quantities, which determine a particular state space. Typically, quantum mechanical models can represent an electron by focusing only on its spin, without integrating position and velocity in Hilbert space. The user will also be interested in a particular physical configuration, which implies the choice of a dynamics: as noted above, in general, a model would not be very informative without it.

Things are less clear for the two remaining elements. Should a model at the basic level of abstraction incorporate fixed observables? Fixed initial conditions? Or should we keep these elements variable? Are they a matter of identification of the target of representation, or do they support counterfactual reasoning?

Let us first consider observables. On the one hand, it is quite intuitive to associate them with the particular interests of users: users are interested in this or that quantity of a physical system that they plan to measure. In such a context, the set of possible observables would correspond to practical possibilities, that is, to a choice of model, and they will be fixed within every selected model. One could also associate observables with epistemic possibilities,

<sup>5</sup> This observation cannot be generalised to unbounded systems. A counterexample (that I owe to an anonymous reviewer) is scattering theory, where the Hamiltonian is variable while initial conditions and observables are fixed. This could entail that the conclusions of this article are limited to the representation of some systems, notably bound systems. Further analyses beyond the scope of this paper would be required to address this potential limitation. However, the representation of bound systems remains canonical in physics, in particular when it comes to interpretive issues, and I hope that the analyses provided in what follows are still valuable as a demonstration of the fruitfulness of the methodological approach presented in this article..

remarking that what is measured can be determined by considering a larger situation that comprises the initial target and the environment. Observables can be fixed by invoking the theory of decoherence, for example. The two approaches are not incompatible: the direction of fit for observables (purposes or empirical inputs) could depend on the context.

On the other hand, it is also quite intuitive to think that what is measured on a system is external to the system, and that this measurement is not part of its nature or identity. It could therefore seem odd to associate observables with a correct identification of the target of representation (Bell (2004)'s disdain of the anthropocentric notion of measurement can be invoked to this effect). We can imagine that scientists are generally interested in a particular target without presuming that something specific will be measured on this target. In this case, we would naturally consider that observables are variable rather than fixed. From the point of view of the target system itself, measurements are natural contingencies: various systems of the same type can interface with their environment in various ways. So, observables should not be fixed at the basic level of abstraction<sup>6</sup>.

What has just been said about observables could also be said about initial conditions. On the one hand, initial conditions could correspond to either practical possibilities in a context where a system is prepared in a certain state for specific purposes, or to epistemic possibilities if the purpose is to learn about a system in a certain state. On the other hand, it is in general possible, in quantum mechanics, to consider that the initial state of a system is nothing but the outcome of a first measurement (preparing a system in a particular state then implies filtering targets on the basis of this first measurement), and we could consider this outcome a natural contingency. So, there does not seem to be a clear answer to our question as to whether observables and initial conditions should be fixed or kept variable.

I hope that the idea that different interpretive stances can be associated with different modelling practices starts to make sense at this point. Various ways of identifying the relevant targets of representation yield different modelling choices, depending in particular on whether or not measurement setups and preparation procedures should be considered relevant inputs for this identification. What is at stake is a choice concerning the hierarchy between observables and initial conditions, or the path that should be followed down the hierarchy of models. If observables take priority over initial conditions, then they can be fixed by epistemic or practical constraints, while possible initial conditions would be natural contingencies represented within models. This choice corresponds to a certain way of identifying targets of representation: as particular physical configurations viewed "from a perspective", that is, relative to a measuring environment. In this case, representation is perspectival in the sense that an anthropocentric element, associated with measurements, is involved in the identification of the target. We could also talk about relational identity, since the target is identified in relation to its measuring

<sup>6</sup> I should note that another option consists in fixing a privileged observable at the conceptual level, and not at the practical or epistemic level. This option corresponds to Bohmian mechanics, which specifies that the positions of particles have definite values, whatever the model. In this case, the right observable could be considered a matter of natural (or even metaphysical) necessity, because it has nothing to do with the identification of the target. Note, however, that it is debatable whether fixing a "privileged basis" such as position at the conceptual level amounts to fixing observables: a distinct notion of observable that is independent from this privileged basis can be seen as supervening on the relation between a system and its environment.

environment. If, on the other hand, initial conditions take priority over observables, then they could be fixed by epistemic or practical constraints, while observables would correspond to natural contingencies. The target of representation is now identified as an autonomous object or type of object in a determinate state that can be measured in different ways, and the way it is measured is a natural contingency. Its identity is intrinsic<sup>7</sup>.

Two main stances emerge from this analysis<sup>8</sup>. Let us call them the perspectivist and the objectivist stance:

- **Perspectivist stance:** At the basic level of abstraction, observables are fixed but not initial conditions, because objects are represented from a perspective, associated with a focus on particular quantities being measured. The initial conditions in which physical systems are found are natural contingencies.<sup>9</sup>
- **Objectivist stance:** At the basic level of abstraction, initial conditions are fixed, but not observables, because targets of representation are objects in a determinate state existing autonomously. What is measured on them is not part of their identity: it is a natural contingency.

We can see how the analysis of the role of modalities in representation from section 3 bears fruit. I claimed that the content of a theory (the conceptual constraints on representation) could be analysed as a function from context (epistemic and practical constraints) to cognitive content (natural possibilities). The two stances presented above differ in what they take to be part of the context, and what they take to be part of the content, and so, they imply different interpretations of possible initial conditions and observables, as either epistemic/practical or natural possibilities. This, in turn, implies different ways of considering the target of representation (see figures 1 and 2). We will see in the next section that it also implies different ontological interpretations of the theory.

<sup>7</sup> The fact that the Hilbert space and algebra of observables are fixed, and potentially reflect a focus from the user of the model on particular properties of interest, could let us think that representation is perspectival in all cases. However, it is perspectival only in a weak sense, because of the selective interests of users, but properties of interest could still be objective. One can reasonably assume that the Hilbert space could be completed with new properties without affecting the initial content of the model. Fixing an observable makes the model perspectival in a stronger sense, insofar as observables are associated with the anthropocentric notion of measurement, and affect the content of the model.

<sup>8</sup> Two more stances could have been considered: fixing everything, or fixing nothing at the basic level of abstraction (and perhaps still others considering unbounded systems: see footnote 5). However, they are less interesting because they do not specify a priority between observable and initial conditions, so I will leave them out.

<sup>9</sup> This perspectivist stance could be compared to perspectivist positions in epistemology, such as Giere (2010b) and Massimi (2018)'s perspectival realism. The notion of perspective involved here is arguably more local, since it concerns a focus on some properties for a concrete target of representation, while perspectival realism puts emphasis on relativity to a conceptual scheme, a theoretical lexicon or epistemic norms of justification at a more general level. Nevertheless, there could be interested connections between the two, notably because one of the motivations of perspectival realism is to account for the successful use of incompatible models to represent a single target system. There is no place to explore these connections here.





Figure 2: Perspectivist model

We could have hoped that our association between modalities and roles in representation would provide us with a straightforward interpretation of scientific theories in general, and of quantum mechanics in particular, but our approach appears to be insufficient: rather than helping us determine the right way of interpreting modal structures, it leaves several options open. However, there is still an improvement, since we are now able to associate specific kinds of models with specific interpretive stances, and we can have a look at the kinds of models that are actually used by scientists to see what they imply.

### **5** Stances and Formulations

Let us examine two types of quantum mechanical models. The first type of model is the one found in textbooks: the wave-function evolving with time. This kind of model is probably the most widely used by physicists and philosophers of physics. It is the one generally used for presenting the theory and examining foundational issues. The second type of model is the one proposed by the consistent histories formulation of quantum mechanics (Griffiths 2003). It has been applied to quantum cosmology (Craig 2016), quantum gravity (Carlip 2001) and quantum computation (e.g. Arrasmith et al. 2019; Brun and Hartle 1999). Although less often used than the wave-function formulation, it played a prominent role in the development of the notion of decoherence, which is now central in contemporary physics. It is very similar to Feynman's path integral formulation is used for more concrete applications of the theory. What I will show is that these two types of models, the wave-function and consistent histories, make opposite choices with regards to the status of observables and initial conditions, and these modelling choices correspond precisely to the objectivist and the perspectivist stances described in the previous section.

A standard wave-function model can be constructed roughly as follows: take (1) a Hilbert space and an algebra of observables, (2) a vector or density matrix defined on this Hilbert space representing the initial state of the system and (3) a Hamiltonian, which is an operator on Hilbert space. Then apply a law of evolution to the initial state using the Schrödinger equation with this Hamiltonian. This gives a state (vector or density matrix) for the system at any instant, the wave-function. Now, in order to connect the model to empirical data, the Born rule should be applied, taking an observable as input. The Born rule gives a probability distribution for all the eigenvalues of the observable (interpreted as all possible outcomes for a measurement corresponding to this observable). But the observable is not part of the model: the same model

can be used for any choice of observable. In this respect, the observable remains variable in the model: it is only fixed when particular inferences, including counterfactual inferences, are made using the model. The structure of possible observables is indirectly represented in the model: it is given by the algebra of observables from which the wave-function is defined.

We have a perfect illustration of the objectivist stance: the idea is that what is measured is external to the target system itself. If our account of the role of modalities in representation of section 3 is correct, this means that the user of such a model should consider what is measured on the target to be a natural contingency: all measurements are naturally possible. Initial states should correspond to epistemic or practical modalities that are fixed by empirical or practical inputs. They are part of the identity of the target. The target is taken to be an object that exists independently of its environment, and whose intrinsic properties determine the outcome of possible measurements. Since measurements could be made at any time, the target must continuously have a state in time, the evolution of which is given by a law. Interpreting the model "literally" naturally gives us an ontology of physical states and laws of evolution.

This approach is plagued with interpretational difficulties. In particular, the ontological status of the Born rule (whether or not it corresponds to a physical process) is unclear, which gives rise to the infamous measurement problem (Maudlin 1995). The wave-function is infinitely extended in time, which seems idealistic. Since the correspondence between the content of the model and measurement outcomes is mediated by the Born rule, the ontological interpretation of the wave-function is also unclear: the wave-function does not attribute properties to entities that are well-located in space, and several ontological options are available (Belot 2012). However, any attempt to complete the theory to solve these issues results in nonlocality, which is in contradiction with relativity theory, and the way of completing it is underdetermined. Finally, extracting causal relations from this representation is problematic (Elby 1992), and since causality is a central concept in higher-level disciplines, the relation between the fundamental theory and higher-level representations, such as those found in chemistry, biology, or classical physics, remains unclear. Yet this is the approach usually followed for interpretive purposes. Also note that if the initial state of the system can be given by a first measurement, then assuming that the system exists independently of all measurements introduces an epistemic uncertainty with regards to this initial state.

Let us now examine the consistent histories formulation. In this formulation, one also starts from a Hilbert space and algebra of observables, but a model is constructed first by defining a *framework*, which is a set of possible *histories* for the represented system. A history is a finite sequence of projectors in Hilbert space indexed at particular instants. Each projector represents a possible property for the system, for example, being located in a particular spatial region, or having spin up. A history therefore represents a time sequence of properties in which the system could be found at particular instants (typically, a sequence of possible measurement outcomes). A framework is a set of mutually exclusive histories, one of which actually occurs (in technical terms, these histories are projectors on a tensor product of Hilbert spaces indexed in time, and these projectors are orthogonal and sum to unity). This defines a sample space for probabilistic reasoning, which can be used to generate an *event algebra* by considering histories that are the disjunction of other histories (coarse-graining). Finally, the histories of a framework must satisfy a consistency condition, which depends on the Hamiltonian of the system. This consistency condition (or "decoherence condition") will guarantee that the inferences made

within the framework respect standard probability calculus. With respect to the standard formalism of quantum mechanics, providing a framework is roughly equivalent to providing a set of observables indexed at particular times, where the observables can be coarse-grained and must respect the consistency condition. Properties then correspond to the eigenvalues of these observables, and the histories of the framework correspond to all possible sequences of properties.

Once the framework is provided, the Hamiltonian and a generalisation of the Born rule can be used to determine the conditional probabilities of any two projectors in the framework. These conditional probabilities constitute the model. What we have is, roughly speaking, a set of correlations (or transition probabilities) between the possible outcomes (eigenvalues) of every observable specified by the framework. These correlations respect standard probability calculus. As said earlier, this is guaranteed by the consistency condition that the framework must respect, which depends on the Hamiltonian. Note, importantly, that assigning conditional probabilities to pairs of projectors does not amount to assigning probabilities to the histories of the framework directly. This would require having more information, such as initial or final conditions or a probability distribution on initial or final conditions, but such information is not given by the dynamics of the system itself, and it is not part of the model (Griffiths 2003, ch. 9).

As we can see, the modelling choices adopted here are exactly the opposite of the previous ones, and they correspond to the perspectivist stance. In both cases, the Hilbert space and Hamiltonian are fixed in the model, but whereas in the previous case, the initial condition was fixed and observables served as inputs for making inferences from the model, in the present case, the observables are fixed in the model, and the initial condition (which here is just the outcome of the first measurement) is variable and can serve as input for inferences, including for counterfactual inferences. According to our analysis in section 3, observables (the framework) correspond to practical or epistemic possibilities in the consistent histories formulation, while initial conditions are natural contingencies (which is indeed how Griffiths (2003, ch. 9) considers them).

We have seen that a wave-function model is naturally interpreted in terms of an autonomous state evolving according to a law. The ontology that one could read off a consistent histories model is quite different. First, the target is not an autonomous object, but it is defined relative to a framework. According to Griffiths, the latter is entirely associated with a pragmatic choice from the user of the model (its correspondence with what is measured in an experimental situation is actually a matter of convenience). The identity conditions of the system thus represented are relational. Changing what is measured, or what choice of framework is made by the user, yields a different target represented by a different model. This is made explicit by the one framework rule: the formalism forbids reasoning outside of a framework, for example, by switching between two frameworks, so as to avoid logical contradictions. Loosely speaking, we are not allowed to ask "What would have happened with this system if we had measured this instead of this?", because it would not be the same target of representation. The resulting structure is quite different too. Properties are not defined at all times, but only at discrete times. This suggests an ontology of events (or what Griffiths calls "contextual events", since they are relative to the framework). The dynamics is not given by a law, but by transition probabilities between possible events. Given that these probabilities respect standard probability calculus, the model can be straightforwardly interpreted as a causal network, assuming a counterfactual

theory of causation. In sum, the ontology that comes out naturally from interpreting this kind of model "literally" is an ontology of causal relations between contextual events.

This approach does not encounter the difficulties of the previous approach. The Born rule is baked into the model, so its ontological status is clear, and as long as we restrict considerations to a single framework, the dynamics can be considered local. Distant correlations are always explained by a common cause (Griffiths 2003, ch. 23). Systems are bounded in time. Contextual events are well-defined entities that can be directly connected to measurement outcomes, so there is no need to complete the theory for interpretational purposes. Finally, the connection with higher level theories is also straightforward if the model is interpreted as a causal network. One can introduce "quasiclassical frameworks" to account for this connection in particular cases, and the predictions of classical physics are recovered (Griffiths 2003, ch. 26-27). But all this comes at a price, which is a relativity of the representation to a framework.

The problem is that the framework is a holistic, non-local, teleological entity. Future measurements are defined in advance, so to speak, but they determine transition probabilities for events in their past. In light of our discussion on scientific representation, and in light of Griffiths's contention that the choice of framework is pragmatic, this framework could be associated with the intentions and aims of modellers. The associated modality would be practical. Then the fact that future measurements are defined "in advance" would not be so much a problem. This approach is compatible with pragmatist stances towards quantum mechanics (Healey 2012). However, it requires giving up a naturalistic picture of the world, since "what the theory says" is now relative to intentional entities. Dowker and Kent (1996, section 5.2) argued that consistent histories leaves us "puzzled as to why the world appears quasiclassical and unable to predict that this quasiclassicality will persist", since it is always possible, in principle, to model a system using a non-classical framework (see also Okon and Sudarsky 2014). Arguably, this criticism is ultimately a demand for theoretical predictions that are not conditioned on intentional aspects, but correspond to a mind-independent reality that would be "quasiclassical". So, consistent histories, thus understood, contradicts the implicit realist assumptions of metaphysicians mentioned in the introduction. I guess that this is the main reason why the formulation is not often adopted when discussing foundational issues, even though a similar formulation, the path integral formulation, is often used by scientists for practical purposes.

	Perspectivist Stance	Objectivist Stance
Observables	Fixed (practical/epistemic possibilities)	Variable (natural possibilities)
Initial conditions	Variable (natural possibilities)	Fixed (practical/epistemic possibilities)
Identity conditions	Relational	Intrinsic
Ontology	Contextual events + causal network	Objective states + law of evolution

The following table summarises the differences between the two stances:

We started in sections 2 and 3 from an account of the role of modalities in representation, which implies that the possibilities that are fixed within models should be interpreted as

epistemic or practical possibilities, while those that are not should be interpreted as natural possibilities. This account entails an association between modelling choices and interpretive stances that was examined in the case of quantum mechanics in section 4. We can see that these modelling choices correspond to the wave-function formulation and to consistent histories, and that they imply different ontologies: evolving states or causal networks of events. Does the present analysis confirm the validity of our methodology? Griffiths is quite explicit that choosing a framework in the consistent histories approach corresponds to adopting a certain perspective on the represented system (Griffiths 2003, pp. 240–241) and that the represented system is intrinsically stochastic (ch. 9.3). The wave-function formulation is the formulation used for metaphysical analysis by most realists, who assume that represented objects exist mind-independently. A no-go theorem even precludes interpreting the wave-function as an epistemic entity (Pusey, Barrett, and Rudolph 2012). So, the association implied by our methodology seems to correspond to the positions defended by proponents and users of the formulations of quantum mechanics that we have examined.

The fact that the standard textbook presentation of quantum mechanics is couched in the wave-function formulation, and that it has instrumentalist flavours, seems to constitute a counterexample<sup>10</sup>. However, as noted in section 3, I take the association between models and interpretive stances to be rather loose: it is always possible, but less natural, to interpret models in a way that does not match this association. It should be noted that formulations of quantum mechanics are flexible enough to be used in various ways. One can use a mathematical variable for representing the initial state of a system in the wave-function formulation without specifying its value, for instance, which allows for counterfactual reasoning on initial states, and make inferences using only one fixed observable, as if it were fixed in the model, and conversely for the consistent histories. The idea defended here is merely that some formulations naturally match or induce some interpretive stances, without requiring qualifications of the kind "I take the model to represent an objective state / my perspective on the object", because these qualifications come out naturally. I would argue, in this respect, that the standard presentation of quantum mechanics is still impregnated with an objectivist stance. The wave-function is generally characterised as representing an evolving "state". The only problem with this presentation is that the notion of measurement is required for predictions, but left unanalysed, which makes the picture incomplete and creates an inherent tension. The Copenhagen interpretation (Bohr style) can be seen as an attempt to alleviate this tension by interpreting the wave-function instrumentally, taking observables to correspond to epistemic or practical possibilities (so, according to our analysis, they should be fixed within models, but they are not). However, this interpretation requires an explicit qualification to the effect that some specific structures used in representation are not actually representational. So, it is not necessarily the most natural interpretation of the bare formalism. It might have been considered unsatisfactory for this reason, and the Copenhagen interpretation is rarely defended today. Griffiths (2019) notably claims that consistent histories, which indeed fixes observables in the model, is nothing but "Copenhagen done right".

Our analysis is limited to two formulations for lack of space. It would be interesting to examine other formulations, such as Bohmian mechanics, GRW and QBism, and perhaps to make finer distinctions between various metaphysical interpretations of quantum mechanics.

<sup>10</sup> I am grateful to a reviewer for raising this objection.

However, the results of our analysis are enough to demonstrate the fruitfulness of the association between modalities and representational roles established in this article.

#### 6 Conclusion: Assessing the Two Stances

In this article, I have proposed to analyse representational activities in terms of various types of modalities, and I have attempted to demonstrate the fruitfulness of this approach for interpreting scientific theories, by applying it to non-relativist quantum mechanics. The main idea is that by looking at the role played by theoretical structures in modelling practices, and in particular, whether they are fixed or variable within models, one could provide a modal interpretation of these structures, in terms of epistemic/practical and natural possibilities respectively. This approach appears to be underdetermined because of the plurality of modelling practices. However, it can help associate these modelling practices with distinct interpretative stances. In this respect, two main stances towards quantum mechanics can be distinguished: what I have called the objectivist and the perspectivist stance. The first one considers targets of representation to be autonomous objects with intrinsic properties, while the second one takes them to be physical configurations identified relationally. These two stances can be associated with the wave-function and consistent histories formulations respectively, and we can see how they imply different ontologies: objective states evolving according to laws in the first case, and causal networks of contextual events in the second. Although these formulations are flexible, their typical uses in foundational inquiry seem to confirm our association between interpretive stances and modelling choices. This demonstrates the fruitfulness of this approach, which could be applied to other formulations, interpretations and theories.

By means of conclusion, and in order to move forward, I wish to examine to what extent the considerations of this paper could advance metaphysical inquiry. Taking a pragmatist approach towards scientific theories does not solve metaphysical problems, but it is informative. I believe that one crucial merit of the present methodology is that it offers a way of evaluating different metaphysical options by examining the virtues of associated formulations and models.

One way of assessing our options could consist in just observing scientific practice and seeing what choices are made by scientists. If, as is likely the case, scientists generally use the wave-function formulation for their purposes, and if they are successful in their purposes, then perhaps the wave-function formulation is the right one, and the right metaphysics is objectivist.

Scientific practice might be indicative indeed. However, one could suspect that modelling choices also reflect a disciplinary tradition rooted in historical contingencies, such as continuity with past theories, or persistent metaphysical prejudices. It is far from obvious that using the consistent histories instead of wave-functions would have impaired the empirical successes of quantum theory, since they are two formulations of the *same theory*. Given the flexibility of formulations, a more relevant question could be: do scientists often perform counterfactual inferences on observables and/or initial conditions for their purposes? If observables are generally fixed for practical purposes, and if counterfactual inferences on initial conditions are often made, then this would play in favour of a perspectivist stance. The two formulations could also be compared on their heuristic strength, that is, their capacity to lead to new theoretical developments, but this strength might be hard to assess given that one formulation is

predominant (and as noted at the beginning of section 5, the consistent histories has some successes on its side, in particular with regard to the theory of decoherence).

Alternatively, one could take a more systematic approach and evaluate the capacity of various formulations to fulfil particular aims attributed to science. One such aim is unity. I have given arguments in the previous section to the effect that the consistent histories formulation is more compatible with other scientific theories (including relativity theory) than the wave-function formulation, because its models display a local causal network. Another aim often invoked in philosophy of science is explanatory power. Even an author such as van Fraassen (1980), who would deny that science aims at producing true theories, accepts that the role of explanations in science is important, to the extent that he felt compelled to develop a pragmatic theory of explanation that does not require theories to be true. I think that another virtue of perspectival models in quantum mechanics is that they are straightforwardly explanatory.

It is generally recognised that explanations rest on counterfactual reasoning: in order to explain how a spark produced a fire, we can consider what would have happened if there hadn't been a spark. This suggests that initial conditions should be variable for a model to be explanatory. Explanations are also focused on a particular property of interest, so an observable should be given. Take, for example, the explanation of absorption and emission spectrums in quantum mechanics. They are explained by the fact that electrons emit or absorb photons when moving from one energy state to another, with frequencies corresponding to the energy gaps between initial and final energy states. If we want to explain spectrums in general, we have to take into account all possible initial and final energy states for the electron, so fixing specific initial conditions in the model would be inappropriate. However, an observable is implicit in this explanation: it corresponds to the possible energy states of the electron, because they are involved in the explanation, whereas positions, for example, are not. This means that the models of a perspectivist stance, and in particular the models of consistent histories, are naturally fit for explanatory purposes. Note, in this respect, that the explanation of spectrums implicitly involves discrete events and transitions, which is consistent with the natural ontology of consistent histories. More generally, the fact that the resulting dynamics corresponds to a causal network is also consistent with the tight relationship between causality and explanations (Salmon 1984).

Wave-function models rather than consistent histories are generally used in classrooms to explain emission spectrums. However, one could suspect that there are implicit steps in the standard explanation (presumably, an application of the Born rule) that are not provided by the model alone, but require specifying an observable. So, the wave-function formulation, although generally used, could actually be less fit for explanatory purposes, and this might explain part of the unease people feel when confronted with quantum theory in its standard formulation.

This is a rather crude analysis for the purpose of illustration. Although I am personally convinced of the virtues of a perspectivist stance, the point that I wish to defend here is only that by connecting interpretive stances, modelling choices, ontological commitments and pragmatic virtues, we are potentially able to evaluate various interpretive options in a way that is not available to traditional metaphysical approaches. In other words, this methodology opens the way for new argumentative strategies that could inform metaphysics.

The pragmatist methodology presented in this paper is not the only one available. Traditional metaphysics of science is generally focused on extracting ontological commitments from theories without paying attention to pragmatic aspects or to model construction, but rather, for example, to the general structure of these theories. However, as observed in the introduction, these approaches lead to unresolvable issues. Furthermore, they seem to neglect the fact that models are more directly representational than theories, and that different modelling choices within the same theory induce different interpretive stances. They might implicitly narrow the range of interpretive options for this reason, in favour of an objectivist stance. The lesson from this article is therefore that metaphysicians should have a closer look at pragmatic aspects, and in particular, modelling practices. Not only does this broaden the space of interpretative possibilities, but it also offers a tangible way of assessing them.

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