Identical Particles in Quantum Mechanics: Against the Received View *

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31 March 2020

1 Introduction

From its beginning the history of quantum mechanics has been beset by questions concerning the nature of the objects with which the theory deals. Controversy started soon after Planck (1901) had introduced discrete energy packets $h\nu$ in his theoretical derivation of the law of black body radiation (which determines the distribution of energy over the frequencies of electromagnetic radiation, in thermal equilibrium).

Planck had set out to calculate the average energy of the electromagnetic oscillators in the wall of a box containing radiation, because these oscillators emit and absorb radiation and establish thermal equilibrium with the field. To determine this average energy Planck needed the entropy of the system of oscillators, for which he used the celebrated Boltzmann expression $S = k \log W$, with S the entropy and W the number of possible configurations of the system. In the case at hand this number of configurations is the number of ways a given amount of energy can be distributed over N oscillators—a number that is difficult to get a grip on, since energy is a continuous quantity. Planck's introduction of discrete energy packets was first of all meant to reduce this distribution problem to the more tractable one of distributing P

*Draft of a chapter in *Individuals and Non-Individuals in Quantum Mechanics*, to appear with Springer.

energy packets over N oscillators. The latter is a combinatorics problem, in which Planck treated the oscillators as distinct physical individuals, so that permuting them lead to different configurations. By contrast, Planck treated the energy elements $h\nu$ as mathematical quantities without physical identity: he assumed that the interchange of two or more energy elements $h\nu$ in the formulas did not correspond to any physical change.

The physical motivation for this different treatment of oscillators and energy packets is clear. The oscillators are entities that differ in one or more of their physical properties: at least their positions in the walls of the container are different. So if, for example, an energy distribution is considered in which oscillator 1 has energy E_α and oscillator 2 energy E_β , and the two oscillators are interchanged, we obtain a distribution with oscillator 1 possessing energy E_β, which represents a different situation.

But when we start contemplating what we have to do in order to replace a part of the energy of oscillator 1 with the same amount of energy now residing in oscillator 2, we run into conceptual difficulties. Classically, energy is a continuous quantity, and there is no clear and natural way of subdividing the total energy of each oscillator into parts; let alone parts possessing their own identity, so that it would become meaningful to say that there is a difference between the situation in which the total energy of oscillator 1 is E_α while all this energy is where it was before, and the situation where this total energy is still E_α, but now part of it was originally in oscillator 2.

But, of course, the supposition that the amount of energy can be varied continuously is typical of *classical* mechanics. In classical mechanics it does not make sense to think, *e.g.*, of the speed of a material body as consisting of an individual first and a second half, and likewise it is meaningless to ask about individual parts of the energy. However, in Planck's calculation the energy is quantized: Planck assumed that the oscillators can only possess energy values $n.h\nu$, with n an integer. So the question arises if this doesn't make an essential difference for the problem of whether it makes sense to switch energy elements.

The answer is that the quantization of energy does not automatically imply that there exist individual energy units. Think of the analogy of a collection of jugs, each filled with a quantity of liquid that is an integer multiple of deciliters, because the filling mechanism can deliver only these discrete volumes. In spite of the fact that the amount of liquid in each jug is "quantized", there is no first, second, third, etc., deciliter in any given jug, once it has been filled.

This analogy might be considered imperfect: one might argue that in a microscopic description there do exist individual constituents of the fluid (molecules or atoms) and that in principle it would be possible to follow them during the filling process. In this way one could define the first deciliter that came in, and so on, by distinguishing between molecules that entered first and those that came in later. Note, however, that this counterargument does not show that the quantized nature of the volumes entails that these volumes are composed of individual units: on the contrary, the argument *adopts as its premise* that there exist individual fluid constituents, quite independently of the volume quantization, and its conclusion is consequently independent of whether or not fluid amounts are discrete. Nevertheless an alternative analogy, first introduced by Schrödinger (1950), avoids objections of this kind and captures the situation in quantum mechanics better.

Schrödinger discusses the case of money in a bank account—scriptural (deposit) money, not physical coins and banknotes in a safe. Suppose the balance is 100 euros; does it make sense to ask for the first euro in the account? Clearly not: there are no individual euros in the account. Assume that I take a 1 euro coin to a bank branch and deposit it in the account. The account balance will change to 101 euros, but it does not make sense to pose the question which of these 101 euros corresponds to my coin: there is no hundred-and-first euro. This is different from the case of the fluid particles of the earlier analogy, where we could—in principle—follow particles over time. (In fact, the coin will still physically exist somewhere; it has not been absorbed by the bank account—this is a remaining disanalogy with quantum cases in which amounts of energy are absorbed into a bigger whole.) The balance of the bank account represents a total amount of buying power that does not consist of individually existing units.

Does this mean that the account is filled with a collection of entities of a previously unknown kind, namely entities that lack individuality and identity? Entities that are completely indistinguishable from each other, and to which the notion of identity does not apply, so that each of them cannot even be said to be identical to itself? In order to discuss such putative objects in a consistent way we should employ a logic reflecting the rules of a non-standard set theory, since in standard (ZF) set theory each member of a set automatically has its own identity (namely the property represented by the singleton set of which it is the sole member). Going this way seems an extraordinary and highly artificial measure. After all, there is nothing mysterious or unsatisfactory in the usual descriptions of situations of the bank account type: for this, both bankers and clients only need standard logic and standard set theory. Common-sense dictates that there *are no* constituent parts of the total buying power of a bank account. It is usually true that it is possible to withdraw only discrete sums of money from bank accounts, for example integer numbers of euro cents. But as we already have seen this does not imply that there were individual cents in my account.

It is true that under certain conditions amounts of money may acquire an identity. Different withdrawn amounts can be told apart if they land in different bank accounts, and similarly flows of money can sometimes be followed over time. In a scenario in which a collection of bank accounts can each contain only 1 or 0 euro, euros can be identified individually by the account they are in (see (Dieks and Versteegh 2008), also for comparison with the quantum case). This is not because we are here facing objects that are generally without identity but now suddenly obtain an identity. There are no euros independent of accounts in these cases—there are only accounts, identifiable by their account numbers, and possessing definite values.

In the following we will discuss and defend a way of dealing with "identical quantum particles" that is very similar to the story we just sketched for bank money. That is, we will consider a collection of "identical particles that are in exactly the same state" as *one* object, not consisting of individual parts; this quantum object is identified by the state and its occupation number. Under certain circumstances it will happen, though, that distinguishing characteristics are created (analogously to what may happen in bank transfers) and that the notion of an individual quantum particle becomes applicable.

By going this way we will deviate from what in the philosophy of physics has become known as the "Received View" regarding the nature of identical quantum particles (French and Krause 2006). This Received View is motivated by developments that took place after Planck's introduction of discrete oscillator energies, starting with Einstein's suggestion that black body radiation might be considered to consist of "light quanta" with energy $h\nu$, even in the absence of any energy exchanges with material oscillators (Einstein 1905). This suggestion remained controversial, but started to gain acceptance when two decades later Einstein (1924) showed that Planck's statistical treatment of energy packets could also be applied successfully to the atoms of a quantum gas. The basic statistical feature that permutations of the elements "filling a state" leave the state invariant remained in place in this new application of Planck's statistics—but now these elements were interpreted as *particles*, instead of the earlier energy packets. This permutation invariance of particles led to the core idea of the Received View: quantum particles of the same kind are physical objects of a previously unknown sort, namely objects without identity.

2 The Received View

2.1 Quantum statistics

As mentioned, Planck's derivation of the black body radiation law hinged on the calculation of the entropy of the oscillators that exchange energy with the radiation inside a container. Planck assumed that this energy exchange could only take place in discrete amounts, namely multiples of $h\nu$, with ν the radiation's frequency. Since the entropy is proportional to the logarithm of the number of possible states, it has to be determined how many ways there are for N oscillators to jointly possess a total (quantized) energy $P\epsilon$, with ϵ the "energy element" $h\nu$. Put differently, the question is in how many ways the total energy $P\epsilon$ can be distributed over N recipients that each can hold only an integer number of energy elements. Planck (1901) had been very brief about this question: he had simply written down the answer

$$C_P^N = \frac{(N-1+P)!}{(N-1)!P!}, \tag{1}$$

referring to a text on combinatorics in which a rather complicated line of reasoning for a similar situation was presented. This did not satisfy Ehrenfest and Kamerling-Onnes (1914), who set out to provide a deduction of formula (1) in which the physical premises would be perspicuous. In order to do so, they represented the possible states of the oscillator system by "symbols"; for the case in which oscillator 1 possesses the energy 4ϵ, oscillator 2 the energy 2ϵ, oscillator 3 0ϵ (no energy) and oscillator 4 the energy ϵ, this representative symbol takes the form (Ehrenfest and Kamerling-Onnes 1914, 870-871):

$$\Big\| \quad \epsilon \quad \epsilon \quad \epsilon \quad \epsilon \quad \bigcirc \quad \epsilon \quad \epsilon \quad \bigcirc \quad \bigcirc \quad \epsilon \quad \Big\|$$

The small circles indicate boundaries between the individual oscillators; the oscillators themselves are ordered from left to right and could be given individual labels or names. The background of formula (1) now becomes clear: the number of possible energy distributions, given fixed values of P and N, equals the number of different symbols of this kind that can be written down with those values of P and N. In each such symbol there are P instances of $\boldsymbol{\epsilon}$ and $(N-1)$ instances of the sign $\bigcirc$. As Ehrenfest and Kamerling-Onnes (1914) write:

> first considering the $(N-1+P)$ elements $\boldsymbol{\epsilon}...\boldsymbol{\epsilon}$, $\bigcirc...\bigcirc$ as so many distinguishable entities, they may be arranged in
>
> $$(N-1+P)! \tag{2}$$
>
> different manners between the ends. Next note, that each time
>
> $$(N-1)!P! \tag{3}$$
>
> of the combinations thus obtained give the same symbol for the distribution (and give the same energy-grade to each resonator), viz. all those combinations which are formed from each other by the permutation of the P elements $\boldsymbol{\epsilon}$ or the $(N-1)$ elements $\bigcirc$. The number of the *different* symbols for the distribution and that of the distributions themselves required is thus obtained by dividing (2) by (3). q.e.d.

The essential premise of the derivation is therefore the permutability of the signs $\bigcirc$ among each other, and similarly for the signs $\boldsymbol{\epsilon}$. With regard to the former permutability, there is no problem: the small circles obviously are purely formal devices introduced in order to demarcate the individual oscillators from their neighbors; switching two of these signs does not correspond to any physical change. The permutability of the energy elements $\boldsymbol{\epsilon}$ may seem more problematic. One might be tempted to think that these signs refer to energy quanta conceived as individual physical systems, especially in light of Einstein's light quantum hypothesis (Einstein 1905). If that interpretation of the $\boldsymbol{\epsilon}$ signs were adopted, questions would arise about the *identity* of the represented quanta: it is not self-evident from the outset that there can be no differences whatsoever between situations in which individual light particles have been switched.

Ehrenfest and Kamerling-Onnes (1914) devote an Appendix to this issue, entitled *The contrast between Panck's hypothesis of the energy-grades and Einstein's hypothesis of energy-quanta* (this appendix is actually longer than the main text of their paper). They emphasize and warn:

> The permutation of the elements $\boldsymbol{\epsilon}$ is a purely formal device, just as the permutation of the elements $\bigcirc$ is. More than once the analogous, equally formal device used by Planck, viz. distribution of P energy-elements over N resonators, has by a misunderstanding been given a physical interpretation...
>
> Planck does not deal with really mutually free quanta $\boldsymbol{\epsilon}$; the resolution of the multiples of $\boldsymbol{\epsilon}$ in separate elements $\boldsymbol{\epsilon}$, which is essential in his method, and the introduction of these separate elements have to be taken *cum grano salis*; it is simply a formal device entirely analogous to our permutation of the elements $\boldsymbol{\epsilon}$ or $\bigcirc$. The real *object which is counted* remains the number of all the different distributions of N resonators over the energy-grades $0, \boldsymbol{\epsilon}, 2\boldsymbol{\epsilon}, ...$ with a given total energy $P\boldsymbol{\epsilon}$.

Ehrenfest and Kamerling-Onnes (1914) back up their claim that the energy elements should not be thought of as physical entities but have a merely formal significance by pointing out that Einstein (1905) had assumed his light quanta to be independent of each other, in the statistical sense, which leads to Wien's radiation law (indeed, Einstein had introduced the notion that light could be considered—in certain respects—as consisting of individual corpuscles with energy $h\nu$ in the context of an investigation of the consequences of Wien's law). Planck's law (which is the empirically correct radiation law), however, requires a probability distribution of energy elements in which the number of equiprobable cases is given by Eq. (1)—what we now call the Bose-Einstein distribution. The difference between the two distributions is illustrated by Ehrenfest and Kamerling-Onnes (1914) with the help of an example in which three energy elements have to be distributed over two oscillators: following Einstein's 1905 line of reasoning this can be done in $2^3 = 8$ ways (each of the three individual light quanta has an independent choice between two oscillators), whereas formula (1) tells us that there are only four ways of doing this. The difference is due to the fact that according to Eq. (1) only situations in which the total oscillator energies are different count as distinct, whereas according to Einstein's original line of reasoning it would also make a difference *which* energy element is in *which* oscillator. Ehrenfest and Kamerling-Onnes (1914) therefore conclude: "Planck's *formal device* (distribution of P energy-elements $P\boldsymbol{\epsilon}$ over N resonators) *cannot be interpreted in the sense of Einstein's light-quanta*."

2.2 Particles without identity

The situation changed drastically when Bose (1924) applied the statistics of Eq. (1) to a "gas" consisting of light quanta (interpreted as physical objects instead of energy elements) and was thus able to derive Planck's law; and when Einstein (1924) generalized this idea by using the same statistics to calculate the entropy of a mono-atomic ideal quantum gas. This made the distribution (1) into a distribution of *particles*, despite Ehrenfest's and Kamerling-Onnes' qualms about the lack of statistical independence between such particles. An important consequence of using the new particle distribution is, as we have seen, that permutations between particles, even when they are in different states (as in the permutation from "particle 1 in state A and particle 2 in state B" to "particle 2 in state A and particle 1 in state B"), do not give rise to new configurations.

This change of perspective in which Bose-Einstein statistics became the statistics of quantum particles was consolidated by the development of modern quantum mechanics, with its symmetrization rules for particles of the same kind ("identical particles"). Suppose, in analogy with the example discussed by Ehrenfest and Kamerlingh-Onnes, that we have three identical quantum particles that each can be in one of two pure quantum states, $|A\rangle$ or $|B\rangle$. In this case at least two particles must occupy the same state, so the particles have to be bosons and the state must be symmetric according to the symmetrization rules. Quantum mechanics assigns a state to this three-particle system in the tensor product Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$, where the factor spaces $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ are one-particle Hilbert spaces. The standard interpretation is that $\mathcal{H}_i$ is the state space of particle i, so that the labels of the three factor spaces also label the particles.

There are now four possible states:

$$|\Psi\rangle_1 = |A\rangle_1 |A\rangle_2 |A\rangle_3 \tag{4}$$

$$|\Psi\rangle_2 = |B\rangle_1 |B\rangle_2 |B\rangle_3 \tag{5}$$

$$|\Psi\rangle_3 = \frac{1}{\sqrt{3}}\{|B\rangle_1 |A\rangle_2 |A\rangle_3 + |A\rangle_1 |B\rangle_2 |A\rangle_3 + |A\rangle_1 |A\rangle_2 |B\rangle_3\} \tag{6}$$

$$|\Psi\rangle_4 = \frac{1}{\sqrt{3}}\{|A\rangle_1 |B\rangle_2 |B\rangle_3 + |B\rangle_1 |A\rangle_2 |B\rangle_3 + |B\rangle_1 |B\rangle_2 |A\rangle_3\} \tag{7}$$

That there are only these four possibilities is analogous to Ehrenfest's and Kamerling-Onnes' example in which an undifferentiated amount $3\boldsymbol{\epsilon}$ of quantized energy was distributed over two resonators. In that case all energy could be possessed by resonator A, or all could be possessed by B; or A could have the energy $2\boldsymbol{\epsilon}$ and B the energy $\boldsymbol{\epsilon}$, and *vice versa*. In the statistical calculations needed to determine thermodynamic quantities these four possible cases were assigned equal weights.

In the statistical mechanics of many-particle quantum systems the states (4)–(7) are similarly assigned equal probabilities. However, now we are dealing with descriptions of particles instead of energy quanta, and the form of the states (4)–(7), together with the usual understanding of the labels as particle markers, indicates that there are only four possible particle configurations: all three particles can be in state $|A\rangle$; all three can be in state $|B\rangle$; two particles, either the pair $\{2,3\}$, the pair $\{1,3\}$ or the pair $\{1,2\}$, can be in state $|A\rangle$ and the remaining one in state $|B\rangle$, and the similar possibility with $|A\rangle$ and $|B\rangle$ interchanged (states (6) and (7)).

The symmetry with respect to particle labels, plus the fact that states (6) and (7) receive the same statistical weights as (4) and (5), suggests that particle permutations do not correspond to physical differences. This suggestion motivates what French and Krause have termed the *Received View*. As French and Krause (2006) write (p. 143):

> from the point of view of the statistics, the particle labels are otiose. The implication, then, is that the particles can no longer be considered to be individuals, that they are, in some sense, 'non-individuals'. This conclusion expresses what we have called the 'Received View': classical particles are individuals but quantum particles are not. ... As we shall see in the rest of the book, one can in fact go beyond mere metaphor and underpin the Received View with an appropriate logico-mathematical framework.

The formal framework here referred to is that of "quasi-set theory" (French and Krause 2006, Ch. 7). In this variation on standard (ZFU) set theory two sorts of "atoms" ("*Urelemente*") are admitted, via the introduction of predicates $m(x)$ and $M(x)$ signifying that x is an m-atom or an M-atom, respectively. The intended interpretation is that m-atoms will refer to quantum particles, whereas M-atoms refer to classical objects. Because of the motivating idea that quantum particles do not possess individuality, the domain of application of the concept of identity ($=$) in quasi-set theory is restricted, so that it excludes m-atoms. There *is* a (defined) notion of extensional identity in quasi-set theory, but its applicability is limited to (quasi-)sets and

M-objects, so that $x = y$ is not a well-formed formula if at least one of the elements x and y is an m-atom. However, a weaker relation of "indistinguishability" ("$\equiv$") is introduced, which is general and so *can* be applied to m-atoms. A quasi-set all of whose members are indistinguishable m-atoms will have a "quasi-cardinal" indicating the number of elements of the quasi-set, but not an ordinal, since the atoms cannot be labeled (labeling would provide individual names and thus bestow individuality on the m-atoms). Without going into further details (see for such details (French and Krause 2006; Arenhart and Krause 2014; Arenhart 2017)), we may say that in this quasi-set theory m-atoms are handled as elements of collections of objects, that they are denoted by variables and that it is possible to quantify over them, and that they can be different in the sense of not being elements of the same set—everything just as in ordinary set theory. Nevertheless, certain arguments that are valid in ordinary set theory are blocked for m-atoms because the notion of identity does not apply to them.

For example, two m-atoms may be indistinguishable, $x \equiv y$, sharing all properties, without being the same. In ordinary set theory this is impossible: one of the shared properties would correspond to "being identical to x", since x is identical to itself, $x = x$. But in quasi-set theory the notion of self-identity is not applicable to m-atoms, so that Leibniz's principle does not hold. By the same token, the singleton set a cannot be formed if a is an m-atom: this set would consist of all atoms that are identical to a, but the notion of identity is not defined for m-atoms.

This axiomatic quasi-set theory is meant to capture the nature of quantum particles as entities without identity and to provide a formal background to the Received View. For instance, permutations of m-atoms lead to indistinguishable situations according to quasi-set theory, which mimics "one of the most basic facts regarding indistinguishable quanta" (Domenech at al. 2010). The theorem in quasi-set theory which formalizes this indistinguishability of permuted configurations reads (Domenech at al. 2010, p. 3086):

> Let x be a finite quasi-set such that x does not contain all elements indistinguishable from z, where z is an m-atom such that $z \in x$. If $w \equiv z$ and $w \notin x$, then there exists w' such that $(x - z') \cup w' \equiv x$.

Here z' and w' stand for quasi-sets with quasi-cardinal 1 whose only elements are indistinguishable from z and w, respectively. The idea is that if an element of a quasi-set is replaced by an element that is indistinguishable from it, but originally was not in the same quasi-set, the final situation cannot be distinguished from the original one.

2.3 Quasi-sets of quantum particles

If the labels that are standardly employed in many-particle quantum mechanics turn out to be otiose in the case of particles of the same sort, one expects that it should be possible to construct a formalism that does without particle labels from the very start. The task to reconstruct the quantum mechanics of many-particle systems accordingly was undertaken by Domenech, Holik and Krause (2008). Their *ab initio* label-less quantum theory of particles without identity takes the following form.

Consider a set of eigenvalues of a quantum mechanical quantity (an "observable"); for the sake of concreteness take the eigenvalues of the Hamiltonian H of the system, so that $H|\varphi_i\rangle = \epsilon_i|\varphi_i\rangle$, with $|\varphi_i\rangle$ the energy eigenstates. Now introduce the notion of a "quasi-function"; this is a mapping associating a finite quasi-set with each value ϵ_i, so that disjoint quasi-sets are associated with different ϵ-values. It is assumed that the sum of the quasi-cardinals of the quasi-sets occurring in this mapping is finite. If the quasi-set x is associated with ϵ_i, the interpretation is that the energy level ϵ_i has the quasi-cardinal $qc(x)$ as its occupation number. An alternative way of representing the situation is with symbols like $f_{\boldsymbol{\epsilon_1\epsilon_1\epsilon_1\epsilon_1\epsilon_2\epsilon_2\epsilon_4}}$, meaning that the level ϵ_1 has occupation number 4 while the levels ϵ_2 and ϵ_4 have the occupation numbers 2 and 1, respectively. The levels that do not appear are understood to have occupation number zero. (Note the similarity to the "symbols" of Ehrenfest and Kamerling-Onnes discussed in section (2.1)).

The use of pure quasi-sets (*i.e.*, quasi-sets solely consisting of m-atoms) makes the use of particle labels not only superfluous, but indeed impossible— as noted before, quasi-sets cannot be ordered. As Domenech, Holik and Krause (2008) remark, "the only reference is to the occupation numbers, because permutations make no sense here, as it should be."

As a tentative critical aside, we note that in our view this remark hits the nail on its head: if labels cannot be defined, permutations as ordinarily defined *make no sense*. It would seem natural to take this as a signal that there are no physical grounds to assume the existence of more than one physical objects that populate the various states; however, this would eliminate the very purpose of the introduction of quasi-sets of quantum objects occupying a state. We will say more about this worry in the next section.

As Domenech, Holik and Krause (2008) show, state descriptions of the form $f_{\epsilon_1\epsilon_1\epsilon_1\epsilon_1\epsilon_2\epsilon_2\epsilon_4}$, in which quasi-cardinalities of quasi-sets denote the occupation numbers of eigenstates like $|\varphi_i\rangle$, can be taken as building blocks (by converting them into vectors constituting a basis) for the construction of a many-particle Hilbert space. Not surprisingly, the resulting formalism is identical to the Fock space formalism of quantum field theory, which makes it clear that the constructed Hilbert space is isomorphic to either the symmetric (bosons) or antisymmetric (fermions) sector of the usual tensor product many-particle Hilbert space[1].

The Received View thus comes in two flavors. First, there is the labeled tensor product Hilbert space formalism. In this formalism labels are assumed to refer to particles, but label differences are declared to not represent any physical differences because of permutation symmetry. Second, there is the quasi-set formalism in which labeling is ruled out from the outset. Still, the existence is accepted of particles that are different from each other and can be permuted (in the quasi-set theoretical sense explained in section 2.2). These particles lack identity and therefore cannot possess any individuating physical properties—they must all share the same characteristics, just as in the labeled formalism. In short, the quasi-set version of the Received View is equivalent to the labeled version with the added refinement that labeling is impossible.

3 Criticism of the Received View

The history of the Received View, briefly sketched in the previous sections, shows how the notion of particles without identity originates in the application to particles of statistics devised for cases in which there is an undifferentiated whole. This amalgam of different conceptual frameworks suffers from internal tensions: an undifferentiated whole does not consist of non-arbitrary parts, whereas particles as traditionally conceived are non-arbitrary individuals.

In classical physics it is obvious that particles are such individuals: classical particles are impenetrable localized entities that travel along well-defined

[1] An alternative method to handle identical particles without any labels was proposed and applied in quantum information theory by Lo Franco and co-workers (Lo Franco and Compagno 2016; Sciara, Lo Franco and Compagno 2017; Dieks 2020). This approach also boils down to the Fock space formalism of quantum field theory.

spatial paths. They can consequently be distinguished from each other at any instant of time and can be followed over time (they possess "genidentity"). This makes it possible to label classical particles in a physically meaningful way: their labels can be associated with identifying physical properties (at least position, possibly also other properties like mass, charge etc.).

Philosophical discussions concerning identical quantum particles may suggest that in quantum mechanics the use of the concept "particle" is completely different. It then perhaps comes as a surprise that paradigm cases of the use of "particle" in experimental quantum physics are very much like the examples from classical physics. Individual elementary particles can be identified by their paths in devices like a bubble chamber; single electrons can be trapped in a potential well and kept there for a long time; an electron gun can be set to fire one single electron, and a bit later this same electron hits a detector; and so on. In such laboratory situations the attribution of identity to particles is considered a matter of course, even in the case of identical quantum particles.

This is not surprising. The experimental practice of physics for a large part consists in handling things, objects. The use of the particle concept comes naturally in this context: the very purpose of using the notion of a particle in experimental practice is the possibility of distinguishing and pinpointing individual entities. But also on the level of physical theory there is the need to conceptualize the world in terms of entities, things that can be different from each other. This is recognized even in quasi-set theory: although the notion of identity has there been removed from the game, quasi-set theory is still about "atoms" of which there are definite numbers in their quasi-sets and that can belong to different quasi-sets, so differ from each other. However, as soon as we are committed to things that exist in definite numbers and can be distinct, the notion of identity is implicitly there. As Berto (2017) analyzes (see also (Jantzen 2011; Dorato and Morganti 2013; Bueno 2014)):

> That a sentence of the form $a = b$ is true, ..., means that we need to count one thing: the thing named a, which happens to be the thing named b... That we, instead, count two things, means that that sentence is false. But then its negation, $\neg(a = b)$, is true. So a and b are different. And if the concept of difference meaningfully applies to a and b, the one of identity does as well. $a = b$ is meaningful together with its negation: adding or removing a

> negation in front of such a meaningful sentence cannot turn it into a meaningless one. The concept of identity cannot but apply to whatever the concept of difference applies to: if—to use Ryle's jargon—we have no category mistake in the latter case, we have no such mistake in the former. When the number of things (in a system) is given by positive integer n, these things cannot lack self-identity.

This argument hinges on a conceptual analysis of what it *means* to be a countable entity, an object, a thing: it is part and parcel of this notion of an entity that it can be different from other entities and that it must be identical to itself. As a semantic point this is true regardless of the epistemic question of whether we are actually always able to verify differences between any two entities. But the analysis has a methodological consequence. In empirical sciences like physics we need some justification, in the final analysis based on empirical data, for introducing distinctions and concepts. In our case, we need some empirical motivation for the introduction of the notion of an entity, a "particle", in the theoretical treatment of physical situations. If it were accepted that no demarcation lines between proposed hypothetical units can exist, that they lack identity so that they cannot be delineated from each other even in thought—let alone in experiments—there would be no ground for conceptualizing wholes as being built up from such proposed constituents.

Again, the analogy of deposit money in a bank account illustrates the predicament well: there is nothing in the sum of money (the account's buying power)—as it is in the account, as opposed to what may happen in transfers and withdrawals—that suggests the existence of smaller constituent entities. To discuss bank accounts in terms of quasi-sets of monetary units lacking identity, with quasi-cardinalities, does not illuminate the nature of bank accounts—it rather confuses the issue. Indeed, the introduction of quasi-sets and the possibility of quantifying over the atoms in them, not as an eliminable *façon de parler* but as an explanatory device, cannot but entail the actual existence of entities working together to compose the value of the account, and this is something we should reject.

It is exactly this problematic introduction of identity-less entities for the purpose of discussing what can unproblematically be considered as undifferentiated wholes that lies at the heart of the Received View. As we have seen in section 2.3, the Received View[2] associates different quantum states (which can be compared with different bank accounts) not only with occupation numbers (comparable to the values of the individual bank accounts), but also with quasi-sets whose quasi-cardinals equal the occupation numbers. That is, instead of saying that state $|\varphi_i\rangle$ has occupation number n_i, so that there is an energy $n_i.\epsilon_i$ in the "mode" represented by $|\varphi_i\rangle$, the Received View holds that there are n_i separate, though identity-less particles, in the state $|\varphi_i\rangle$.

These quantum particles without identity cannot be ordered and labeled by natural numbers and therefore cannot be counted in the ordinary sense of the word.[3] Thinking in this way about particles is far removed from physical practice. As already pointed out, the laboratory practice of quantum physics is replete with talk about particles that are countable in the ordinary sense, possess identity, and behave more and more like classical particles when the classical limit of quantum mechanics is approached. The same particle concept is standard in theoretical considerations. Think, for example, of the well-known arguments concerning how the uncertainty relations make quantum particles different from their classical counterparts. The uncertainties in position and momentum of a quantum particle obey the uncertainty relation $\Delta Q.\Delta P \geq \hbar$, so that a very small value of ΔQ implies a large value of ΔP. Since $P = mv$, with m the mass and v the speed of the particle, we can nevertheless have a very small uncertainty in v if the particle is macroscopic (with a very large mass). This is always taken to mean that a single individual quantum particle to a very high degree of approximation will have a well-defined trajectory and will become indistinguishable from a classical particle

[2] In the form elaborated by Domenech, Holik and Krause (2008) and Domenech at al. (2010).

[3] Adherents of the Received View retort that counting identical quantum particles need not involve a mapping to the natural numbers. For example, Krause and Arenhart (2019) state that there exist alternative counting procedures, like the *weighing* of a total amount, that are able to determine numbers of identity-less entities. Quasi-cardinalities can similarly be determined by measuring the total amount of energy in mode $|\varphi_i\rangle$. This is analogous to arguing that the euros in a bank account *can* be counted, namely by looking at the value of the account. But clearly, the existence of a total account value cannot be regarded as support for the actual existence of quasi-sets of euros in an account; nor can the existence of an occupation number be seen as support for the existence of several quasi-particles in state $|\varphi_i\rangle$. Quite the opposite: in such cases the non-existence of a counting procedure in the ordinary sense, *i.e.* a mapping to the natural numbers, disconfirms the existence of well-defined building blocks that constitute the whole.

in the macroscopic limit. It follows that at least in these cases particles in the quantum regime are conceived of as identifiable and distinct from their fellow particles of the same sort—and therefore can be assigned an identity. There thus proves to be a mismatch between the quantum particles allowed by the Received View, which never possess identity, and the objects called particles in the actual practice of quantum physics.

4 Identical Particles as Distinguishable Objects

The discrepancy between the particle notion used in physical practice and that of the Received View can be illustrated again by the following warning issued by Domenech, Holik and Krause (2008, p. 974) in their explanation of the Received View:

> before to continue we would like to make some few remarks on a common misunderstanding... People generally think that spatio-temporal location is a sufficient condition for individuality. Thus, two electrons in different locations *are* discernible, hence *distinct individuals*... we recall that even in quantum physics, fermions obey the Pauli Exclusion Principle, which says that two fermions (yes, they 'count' as more than one) cannot have all their quantum numbers (or 'properties') in common. Two electrons (which are fermions), one in the South Pole and another one in the North Pole, *are not individuals in the standard sense* (and we can do that without discussing the concepts of space and time). Here, by an individual we understand an object that obeys the classical theory of identity of classical (first or higher order) logic (extensional set theory included). In fact, we can say that the electron in the South Pole is described by the wave function $\psi_S(x)$, while the another one is described by $\psi_N(x)$ (words like 'another' in the preceding phrase are just ways of speech, done in the informal metalanguage). But the wave function of the joint system is given by $\psi_{SN}(x_1, x_2) = \psi_S(x_1)\psi_N(x_2) - \psi_N(x_1)\psi_S(x_2)$ (the function must be anti-symmetric in the case of fermions, that is, $\psi_{NS}(x_1, x_2) = -\psi_{NS}(x_2, x_1)$), a superposition of the product wave functions $\psi_S(x_1)\psi_N(x_2)$ and $\psi_S(x_2)\psi_N(x_1)$. Such a super-

position cannot be factorized. Furthermore, in the quantum formalism, the important thing is the square of the wave function, which gives the joint probability density; in the present case, we have $||\psi_{SN}(x_1, x_2)||^2 = ||\psi_S(x_1)\psi_N(x_2)||^2 + ||\psi_S(x_2)\psi_N(x_1)||^2 - 2\mathrm{Re}(\psi_S(x_1)\psi_N(x_2)\psi_S(x_2)^*\psi_N(x_1)^*)$. This last 'interference term' (though vanishing at large distances), cannot be dispensed with, and says that nothing, not even *in mente Dei*, can tell us which is the particular electron in the South Pole (and the same happens for the North Pole). As far as quantum physics is concerned, they really and truly have no identity in the standard sense (and hence they have not *identity* at all).

What is attacked in this quotation, and branded "a common misunderstanding", is the view that a two-electron wave function of the form[4]

$$\psi_{SN}(x_1, x_2) = \frac{1}{\sqrt{2}}\{\psi_S(x_1)\psi_N(x_2) - \psi_N(x_1)\psi_S(x_2)\} \tag{8}$$

represents one electron located at the South Pole and one at the North Pole. This latter interpretation is certainly common, and it is also true that it conflicts with the Received View; but it should not be dismissed as a misunderstanding. Rather, it can be shown to be part of a coherent and simple interpretation of (anti-)symmetric many-particle states that has the great advantage of not requiring the dubious introduction of entities lacking identity, and that is in accordance with physical practice.

That it is consistent to interpret (anti-)symmetrized product states like the one in Eq. 8 as describing individual particles with their own identifying properties was first pointed out by Ghirardi, Marinatto and Weber (2002). Details of an encompassing interpretation, rival to the Received View, in which this result is incorporated were discussed in (Dieks and Lubberdink 2011; Dieks 2020; Dieks and Lubberdink 2020), to which we refer. In the following we will summarize a number of key points.

The objection put forward in the above quotation hinges on the fact that states of identical particles must be symmetric or anti-symmetric under permutations of the labels that occur in the state. As a consequence of this permutation symmetry the labels do not correspond uniquely to pure one-particle states: in the example of Eq. (8) neither the label "1" nor the

[4]We have inserted a factor $\frac{1}{\sqrt{2}}$ for the purpose of normalization.

label "2" belongs uniquely to the state ψ_S or the state ψ_N—each of the two labels attaches equally to ψ_S and ψ_N. This motivates the statement, in the quotation, "that nothing, not even *in mente Dei*, can tell us which is the particular electron in the South Pole (and the same happens for the North Pole)." The thought is that it is impossible to say whether particle 1 or particle 2 finds itself at any particular Pole.

The silent premise of this argument is that the labels 1 and 2 are *particle labels*: they refer to *particle 1* and *particle 2*, respectively, and are therefore not just mathematical quantities labeling the factor Hilbert spaces in the total tensor product Hilbert space. The fact that neither of these labeled particles can be said to be in a state uniquely located at either the North or the South Pole is interpreted in terms of some kind of indeterminacy: it is objectively undetermined which particular electron is where.

More generally, the (anti-)symmetry of states of particles of the same kind has the consequence that the particle labels are "evenly distributed" over all one-particle states occurring in the total state. The Received View considers this as proof that not even God could tell which particle is in which one-particle state.

The basic interpretative maneuver that dismantles this line of thought, and which we will defend here, is to associate particles *not* with *labels of factor spaces*, but instead with the *one-particle states* that occur in the total N-particle state.[5] In the example of Eq. (8) this means that we will consider the indices 1 and 2 as having only a mathematical significance, as labels of the two factor Hilbert spaces. By contrast, we will define and identify the *particles* by the two orthogonal states ψ_S and ψ_N that build up the total state. This interpretative step will immediately eliminate the question of "which particle is in which state". The particle defined by ψ_S is at the South Pole, the particle defined by ψ_N at the North Pole, and both statements are true by definition.

In the case of fermions of the same kind (for example electrons) we have to work with anti-symmetric states. The one-particle states that occur in anti-symmetrized product states are all mutually orthogonal and appear only once in the total state—this is an expression of Pauli's exclusion principle. The particles that are defined by these orthogonal states, according to our

[5]The doctrine that the indices of the factor Hilbert spaces are also particle labels was dubbed "factorism" by Caulton (2014). The position that we will defend here is therefore "anti-factorist".

interpretation, are completely distinguishable at any instant of time (they are "absolutely discernible", *i.e.* distinguishable by means of monadic physical properties (Dieks and Versteegh 2008)), since orthogonal quantum states can always be distinguished perfectly. This, then, bestows identity on fermions: distinguishability implies identity. There *may* also be identity over time, *genidentity*. For example, this is the case if there is no interaction between N fermions, so that each one-particle state evolves independently and unitarily. Orthogonal states remain orthogonal under unitary evolution, so that in this special case an N-fermion system corresponds to N orthogonal one-particle states that trace out distinguishable paths in the total Hilbert space.

Another situation in which there is at least approximate genidentity is that of the classical limit. In this limiting case quantum particles as defined here, by one-particle states, will gradually transform into their classical counterparts.

However, in the most general quantum regime the existence of unrestricted genidentity cannot be guaranteed. Wave packets of particles defined at one instant of time will generally soon overlap with each other and be transformed by interactions, after which it may become undetermined which of the original particles is the same one as any given post-collision particle (see sections 5 and 6).

It may also happen that the total state is not an (anti-)symmetrized product, but rather a coherent superposition of product states. In such cases a simple interpretation in terms of the usual quantum particles (like electrons, protons, etc.) may prove to be not possible at all. This signals a basic feature of quantum mechanics in the interpretation that we are discussing: the concept of a particle is not basic and has no claim to general applicability in the quantum realm, but rather is *emergent* (Dieks and Lubberdink 2011; Dieks 2020; Dieks and Lubberdink 2020). Only if certain conditions are fulfilled (*e.g.* relating to the classical limit) do particles emerge.

That the ordinary concept of a quantum particle is not always applicable in the quantum regime can easily be illustrated for the case of bosons. The one-particle states occurring in symmetric product states need not be singly occupied. We encountered this situation in the context of Planck's energy packets: any given oscillator could contain more than one energy unit. Instead of saying that there are n indistinguishable and identity-less bosons in a state with occupation number n, in this case our interpretation conceives of the number n as a mass noun. This parallels the example of a total quantity of n liters of a liquid, in which there is no subdivision into separate liter-entities. The statement "there are n bosons in exactly the same state" should accordingly be replaced by the statement that there is *one* object, represented by an n-fold excited "field mode", without the connotation of n well-defined parts. This corresponds to the standard interpretation of the Fock space formalism of quantum field theory.

Summing up, in the alternative to the Received View sketched here, physical entities are individuated by distinct physical properties, corresponding to mutually orthogonal quantum states; the factor Hilbert space indices that occur in the labeled tensor product formalism of the quantum mechanics of identical particles are *not* interpreted as particle labels. Quantum objects defined this way, via orthogonal states, always possess identity, since they are distinguishable. The notion of a quantum particle introduced this way does not have unrestricted validity, though: quantum particles are *emergent*. The latter point signals another difference with the Received View: according to the Received View there always are particles, albeit of a very mysterious kind, always lacking identity. Our alternative says that particles emerge in certain physical circumstances, and in these cases always possess identity.

5 The Physics and Philosophy of Identical Particles With Identity

In 1956, in a Letter to the Editor of the *Physical Review*, the later Nobel prize winner Hans Dehmelt announced an experimental physics research program focusing on individual atoms and ions. Dehmelt (1956) predicted that for individual charged particles "the intriguing possibility even exists to trap them by suitable fields". As Dehmelt recalled in a 1990 review of his own work (Dehmelt 1990), it took another 17 years before he finally succeeded in confining a single electron quasipermanently in an electromagnetic trap (Wineland, Ekstrom and Dehmelt 1973). A decade later Van Dyck, Schwinberg and Dehmelt (1986) achieved a similar feat with a single positron, which was kept in a trap and observed continuously during three months—Dehmelt baptized this specific elementary particle "Priscilla". In 1989 Dehmelt, together with Wolfgang Paul, was awarded the Nobel prize in physics "for the development of the ion trap technique."

In 2012 the Nobel Prize for physics went to Serge Haroche and David Wineland "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"—methods that elaborated on the work of Dehmelt. As the 2012 Nobel citation states (Royal Swedish Academy of Sciences 2012): "The Nobel Laureates have opened the door to a new era of experimentation with quantum physics by demonstrating the direct observation of individual quantum particles without destroying them. ... David Wineland traps electrically charged atoms, or ions, controlling and measuring them with light, or photons. Serge Haroche takes the opposite approach: he controls and measures trapped photons, or particles of light, by sending atoms through a trap." Indeed, Haroche and his research group had been able to study the behavior of a single photon that had been trapped in a cavity (Gleyzes et al. 2007).

In their report the Nobel committee emphasized that these achievements should not be seen as engineering feats with little relevance for fundamental or philosophical questions. As they wrote (Royal Swedish Academy of Sciences 2012, "Scientifc Background on the Nobel Prize in Physics"):

> These techniques have led to pioneering studies that test the basis of quantum mechanics and the transition between the microscopic and macroscopic worlds, not only in thought experiments but in reality. ... Wineland and coworkers ... created [Schrödinger] "cat states" consisting of single trapped ions entangled with coherent states of motion and observed their decoherence. Recently, Haroche and coworkers created cat states, measured them and made a movie of how they evolve from a superposition of states to a classical mixture. ... Today, the most advanced quantum computer technology is based on trapped ions, and has been demonstrated with up to 14 qubits and a series of gates and protocols.

The remarks in the report about the transition from the quantum to the classical world are particularly pertinent to our present theme. The trapped particles are defined by their individual states (well-localized in space, in the mentioned experiments) and can be counted in the ordinary sense. In the quantum computing case typical configurations consist of series of "qubits", positioned next to each other. In the classical limit distinguishable quantum particles of this sort mimic the behavior of classical particles and thus realize the transition to the world of classical mechanics. This is all in accordance with the way in which our alternative to the Received View defines particles. By contrast, the particles of the Received View itself do not possess any individuating characteristics, do not have their own states, play no role in the classical limit and cannot be the subject of investigations as described above.

There are countless other examples in the physics literature in which identical quantum particles are treated as individual entities. In fact, that identical particles can be dealt with as approximately localized objects that follow trajectories when the spatial sizes of their wave packets are small compared to their mutual distances, has been part and parcel of the practice of quantum mechanics from its beginning.[6] Proponents of the view that quantum particles of the same kind can never possess identity, as a point of principle, certainly have a reason to attack the fundamental relevance of this practice. In this spirit Toraldo di Francia wrote (Toraldo di Francia 1985, p. 209), (Dalla Chiara and Toraldo di Francia 1993, p. 266):

> ...an engineer, discussing a drawing, can *temporarily* make an exception to the anonymity principle and say for instance: 'Electron a issued from point S will hit the screen at P while electron b issued from T hits it at Q'. But this mock individuality of the particles has very brief duration.

Toraldo di Francia (1985) did not explain the use of the somewhat disparaging term *mock* individuality, but only made the brief remark that this "individuality" breaks down as soon as the electron encounters other electrons, for example by entering an atom. However, even if we set aside for the moment questions about what exactly happens in such processes, it remains obscure why an individuality that does not last forever, perhaps even lasts only very briefly, should be dismissed as a "mock individuality". After all, everyday macroscopic objects also usually have finite life spans but it would seem weird to deny, on that ground, their identity when they are still intact.

Dalla Chiara and Toraldo di Francia (1993) attempted to fill this lacuna in the argumentation and provided a more extensive explanation for the use of the term "mock identity". They start by admitting that particles could be defined, conventionally as they say, through their different states and thus even be given proper names (curiously, there is no reference to Dehmelt's Priscilla). They also note that the thus defined objects could last for a very long time. Still, they maintain, their *possibly* limited life span raises a serious worry concerning the identity of such objects and the value of the names that are given. They write (Dalla Chiara and Toraldo di Francia 1993, p. 267):

[6] Note that also here silent use is made of a characterization of particles via one-particle states, instead of by "particle labels".

> *Prima facie* one may be tempted to think that the case of particle a is not different from that of Aristotle. After all, there was no Aristotle before 384 b.C. or after 322 b.C. But, in general, the particle does not die! An electron may very well survive a close encounter with another electron. Suppose that we follow with continuity an electron—say Peter—going from point P to point Q in a vacuum. We would like to be able to say that in a possible world Peter might encounter other electrons on its path and finally be scattered to Q. But then no one could tell that that electron is still Peter. There is no *trans-world* identity. In this situation the meaning of 'rigid designator' becomes very fuzzy. Anyway, the term seems useless.

In order to judge this argument we need not enter into a discussion of possible worlds containing collisions that do not actually occur, the concept of trans-world identity over such other worlds, and the Kripkean notion of a "rigid designator". The physically relevant core of the above reasoning is that in a process in which electron Peter collides with another individual electron (Paul, say), after which there still are two individual electrons, it may be impossible to tell which of these post-collision electrons is Peter and which is Paul. This is deemed unacceptable, since Peter has not died.

It is true that a loss of distinguishability of the described kind might occur—we will discuss this in the next section. But the argument as stated is unconvincing nonetheless. If the identity of Peter and Paul were completely lost in their encounter[7], this would surely mean that both Peter and Paul have died as individual particles due to the collision, contrary to the premise in the above argument that particles in general do not die. It is as if Aristotle and his completely identical twin brother meet and coalesce, in a science fiction scenario, after which two perfectly equal look-alikes reappear and start following their own courses. It is certainly possible to devise scenarios of this kind in which it makes no sense to ask who of the later two persons is the original Aristotle. But it does not follow at all from this that it is futile to use the name "Aristotle" for the philosopher when he is still Aristotle, and that we should deny him his identity. The usefulness of giving names, and assigning identity, in analogous physics situations was exactly what was argued for in our above discussion of work leading to the 1989 and 2012 physics Nobel prizes.

[7]It is not absolutely necessary that such a loss occurs in an encounter, see section 6.

Décio Krause, one of the co-authors of the term "Received View" (French and Krause 2006) and one of the staunchest defenders of the position, has undertaken to investigate and criticize the claims of Dehmelt, Haroche and Wineland (Krause 2011, 2019). Krause concludes that contrary to those claims, identical quantum particles always lack identity—even when they are trapped, are distinguishable and bear unambiguous names (like Priscilla). His argument relies on the fundamental quantum postulate that identical particles must be in permutation invariant states. Krause takes this postulate to imply that it does not make any difference to a physical situation when a particle, even a particle in a trap, is replaced by an arbitrary other particle of the same kind. This means, the argument continues, that in Dehmelt's experiment there is no fact of the matter concerning the question *which* positron in the universe is Priscilla. No positron has an identity, and Priscilla is only a "mock name".

Krause devotes a similar analysis to a situation with two trapped electrons, one in the infinite potential well (trap) 1 and the other at some distance in a similar potential well 2. The two particle wave function for this case is anti-symmetric and has the form (Krause 2019)

$$\psi_{12}(a,b) = \frac{1}{\sqrt{2}}\Big(\psi_1(a)\psi_2(b) - \psi_1(b)\psi_2(a)\Big), \tag{9}$$

where ψ_1 and ψ_2 are wave functions that vanish everywhere except in well 1 and well 2, respectively; a and b are the labels of the two factor Hilbert spaces in the tensor product Hilbert space of the two-particle system. Krause comments:

> note that we are dealing with different Hilbert spaces, the space of a and the space of b. But, if the particles are indistinguishable, how can we know which particle is in well 1? In other terms, which particle has its states represented by vectors of the first space? There is no way to do it, for anyone of them could be in well 1.

The conclusion must therefore be the same as in the case of Priscilla:

> Although trapped in the infinite wells, they [*i.e., the quantum particles*] have only what Toraldo di Francia has termed mock individuality, an individuality (and, we could say, a 'mock identity')

> that is lost as soon as the wells are open or when another similar particle is added to the well (if this was possible). And this of course cannot be associated with the idea of identity. Truly, there is no identity card for quantum particles. They are not individuals, yet can be isolated by trapping them for some time.

Krause's argument consists in a consistent application of the Received View to the case of trapped particles, but it is not really a *defense* of the Received View against rival interpretations. The argument accepts from the outset a key point of the Received View, namely that the labels occurring in the total state refer to *particles*, and does not address possible alternatives. But this interpretation of the labels is precisely the main point on which the alternative view that we have been discussing, and which is implicit in much of the physics cited above, deviates from the Received View. Since the Received View interprets the labels as referring to particles, it sees the invariance under label permutations as proof that the particles do not possess individual characteristics. But this conclusion does not follow in our "anti-factorist" scheme: particles are here characterized by one-particle states occurring in the total state, instead of by labels. This different characterization immediately shows that in the alternative interpretation particles will be distinguishable individuals.

What Krause's argument shows is that application of the Received View philosophy to the findings of Dehmelt, Haroche and Wineland leads to the conclusion that all particles in the experiments performed by these Nobel physicists lacked identity. But since it is the very premise of the Received View that all identical quantum particles always and under all circumstances are indistinguishable and without identity, this conclusion teaches us nothing new. In fact, instead of being a defense of the Received View, Krause's reasoning neatly illustrates some of its awkward and implausible features. His argument hinges on the thought that we can never know *which* positron in the universe is Priscilla (and similarly *which* electron is in trap 1 or 2). This statement suggests ignorance about what is the case—but that cannot be intended, since it would imply that it makes sense to say that, unknown to us, this or that positron is in fact Priscilla. That would presuppose the individuality of the positron in question—whereas individuality is something no positron possesses, according to the Received View. Rather, the formal elaboration in terms of quasi-sets suggests that we should conceive of the lack of "which particle" knowledge as a kind of ontological indeterminacy.

But in that case it makes no sense to wonder "*which* positron" could be in Priscilla's trap: there is no definite "this or that" positron. The conceptual picture is therefore obscure and verges on the brink of inconsistency. This should be contrasted with the simple and natural picture offered by the rival interpretation: Priscilla is by definition the positron in Dehmelt's trap, and it is the positron so baptized by Dehmelt; electron 1 is the electron defined by ψ_1, therefore by definition in trap 1, and *vice versa* for electron 2.[8]

6 Loss of Distinguishability and Identity

In 1987 Hong, Ou and Mandel performed a famous experiment that started a tradition of research in which individual photons, or atomic particles of the same kind, traveling along distinguishable paths, are brought together so that their wave functions overlap—this leads to a loss of distinguishability manifested by interference phenomena (Hong, Ou and Mandel 1987; Ou and Mandel 1988; Lopes et al. 2015). A recent version of the scheme, applied to two electrons, goes as follows (Tichy et al. 2013).

Suppose that we have two electron guns, one to the Left and one to the Right, and suppose that each of these devices fires exactly one electron—one with spin up in the z-direction, the other with spin down. (We follow the way in which experiments of this kind are commonly described in the physics literature, in which particles are defined by one-particle states, in accordance with our rival to the Received View.) Since electrons are identical fermions, we have to anti-symmetrize the total wave function, so that it has the following anti-symmetrized product form:

$$\frac{1}{\sqrt{2}}\{|L\rangle_1|R\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2 \ - |R\rangle_1|L\rangle_2|\downarrow\rangle_1|\uparrow\rangle_2\}. \tag{10}$$

The two individual electron wave packets evolve independently, by free evolution; for the sake of simplicity we do not represent this (trivial) part of the evolution in the formulas. Now, after some time, each wave packet encounters a beam splitter and is split; this happens in such a way that, of

[8] It is actually incorrect that these electron identities are automatically and necessarily lost as soon as the potential wells are opened so that the electrons can escape. For example, in free evolution ψ_1 and ψ_2 remain orthogonal states, so that they still define distinct identities—see section 6.

both original packets, one half is directed to the location L' and the other half to the location R'.

The two beam splitters and the evolution after the transit through the splitters can be represented by unitary transformations and change the incoming states in the following way:

$$|L\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|L'\rangle + |R'\rangle\right), \tag{11}$$

$$|R\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|L'\rangle - |R'\rangle\right), \tag{12}$$

where the states (ket vectors) $|L'\rangle$ and $|R'\rangle$ correspond to wave packets localized at L' and R', respectively.

After the evolution, the total state is still an anti-symmetrized product. The two original orthogonal spatial states $|L\rangle$ and $|R\rangle$ have evolved, by the unitary evolution, into two new orthogonal states (the right-hand sides of (11) and (12))—let us call these ϕ and ψ, respectively. The final total state can so be written as

$$\frac{1}{\sqrt{2}}\{|\phi\rangle_1|\psi\rangle_2|\uparrow\rangle_1|\downarrow\rangle_2 \ - |\psi\rangle_1|\phi\rangle_2|\downarrow\rangle_1|\uparrow\rangle_2\}. \tag{13}$$

According to our interpretation this state still represents two individual and distinguishable particles, now defined by the orthogonal states $|\phi\rangle|\uparrow\rangle$ and $|\psi\rangle|\downarrow\rangle$, respectively. This individual particle interpretation will be confirmed if we perform measurements of observables like $|\phi\rangle\langle\phi|$, $|\psi\rangle\langle\psi|$, $|\uparrow\rangle\langle\uparrow|$ or $|\downarrow\rangle\langle\downarrow|$; such measurements are able to distinguish perfectly between the two states defining our particles.

However, if we perform *local* (in the spatial sense) spin measurements, by using electron spin detectors positioned at L' and R', an interpretation of the results in terms of independent individual particles may appear problematic. This is so because *both* of the two initial particles contribute to all detection results at L' and R', which leads to interference. A calculation shows (Tichy et al. 2013; Dieks 2020) that correlations between spin values obtained at L' and R' will be found that suggest the presence of an entangled state (*i.e.*, not a symmetrized product state) of the Einstein-Podolsky-Rosen type; this is a “holistic” state that does not represent two independently existing particles each possessing its own set of distinctive physical properties (Dieks and Lubberdink 2011; Dieks 2020; Dieks and Lubberdink 2020).

This example illustrates how overlap of wave functions may *veil* the individuality of particles, without destroying it. In the discussed case there were always two individual particles, also during their encounter. Each kept its own identity, and this could have been verified by performing appropriate measurements. But if only the results of local measurements in the overlap area are available this will lead astray and suggest a loss of individuality.

These comments address part of the worries expressed by Toraldo di Francia (1985); Dalla Chiara and Toraldo di Francia (1993); Krause (2011, 2019). They show that identity grounded in individual and orthogonal one-particle states is not *automatically* and immediately lost outside of traps and in "close encounters", contrary to what was assumed by these authors.

It is true, however, that in general interactions (anti-)symmetric product states may be converted into superpositions of such states, which are not products themselves. In such cases the interaction will lead to entanglement; and as a result there will be a real loss of identity, since it will no longer be possible to decompose the total state into one-particle states.

In the case of bosons there is the additional possibility that two originally orthogonal states, both with occupation number 1, will evolve into a new state with occupation number 2, and similarly for more bosons. This would also imply a loss of identity: two individual particles would merge into one object, one whole, which is no longer an elementary particle.

In all such situations the applicability of the concept of an individual elementary particle will be suspended. The original particle will "die", and its identity will die with it. However, as argued in section 5, a finite life span does not degrade identity into mock identity.

7 Conclusion

According to the Received View the world described by quantum mechanics consists of elementary building blocks that lack identity, share all their physical properties, and are impossible to identify. This picture is hugely different from what is suggested by the actual practice of quantum physics and by the limiting cases in which classical physics becomes applicable. We have sketched an alternative to the Received View, drawing on earlier work, and compared and contrasted it with its rival. According to this alternative, systems of identical quantum particles can in many cases be described as consisting of individual particles, possessing their own identity. This view makes sense of the way such systems are usually discussed in the practice of physics, and it also provides an understandable and simple story of the way in which the classical world emerges from the quantum realm.

Differently from the Received View, the alternative does not maintain that the world on its deepest quantum level is always particle-like. Particles *emerge*, under certain conditions. The typical quantum phenomenon of a Bose-Einstein condensate provides an example. According to the Received View such a condensate should be conceptualized as consisting of many bosons, different from each other but still lacking identity. This is analogous to thinking of a bank account in terms of different euros, existing independently of each other but without identity. The alternative view adopted here is that a bank account has a certain value, a purchasing power, that is not composed of independent constituents. Similarly, a Bose-Einstein condensate is one object, having a total mass and charge without independent components. But when this object is subjected to certain interactions or measurements, individual bosons (possessing identity!) may emerge. In the case of fermions, one-particle states can only be singly occupied, and this makes it easier for fermions than for bosons to manifest themselves as individual particles.

We believe that this alternative to the Received View not only fits physical practice better, but also provides a consistent and understandable perspective that improves on the obscure notion of identity-less physical objects.

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